Two-layer Topology Design of Wireless Optical Networks

Masoud Khazaee masoud.khazaee.933@student.lu.se

Department of Electrical and Information Technology Lund University

Advisor: Prof. Michal Pióro

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Abstract

Free Space Optical (FSO) communications is an evolving wireless technology, capable of providing point to point broadband links using direct light-beam connections. Currently, FSO links have the range of up to several kilometers, and realize transmission rates of the order of 2.5 Gb/s. At the same time, FSO links do not interfere with each other and thus all FSO links within the network can work simultaneously, contrary to the radio-based wireless networks. Therefore, the optical WMN (OWMN) is a promising networking solution for high capacity transport layer in metropolitan area networks. High link bandwidth and lack of interference are important advantages over traditional radio WMN solutions based on the IEEE 802.11 family standards.

However, FSO requires direct visibility between the end points of the installed optical links, and is sensitive to weather conditions (e.g., fog). To solve the former issue, mirrors can be used to reflect optical signals to connect pairs of nodes which are not in the line of sight. With this motivation, we elaborate a mixed-integer programming-based model incorporating mirrors to assure a connected graph with a minimum cost of deployment. This optimization model of OWMN reflects a two-layer network topology design that includes the mirror-layer and the optical node layer.

This thesis consists of three main parts. Firstly, we give a survey of the FSO technology, including components, working mechanisms, modulation and coding schemes, and applications, summarizing advantages and drawbacks of the FSO technology. Then we present our mixed-integer programming model for the two-layer design of wireless optical networks. The upper layer consists of the FSO nodes while the lower layer consists of the mirror nodes. The end nodes of a link in the upper layer which are not in the line of sight can be connected by a path along links in the lower layer. Finally, we discuss the topologies optimized with the proposed model for a set of selected network examples.

Keywords: FSO, mesh networks, topology design, mixed integer programming model

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Introduction

Chapter

1.1 Motivation

The increasing number of mobile users results in increasing traffic demand and different types of services which must have the ability to support high data rate. Currently, the fiber optic technology has successfully met all expectations such as high bandwidth over long distance, data security and immunity to electromagnetic interference. However, in some applications using fiber is not a viable solution. For example considering the situation that we need only a few MB/s and installation of optical fiber can not be affordable. There are two solutions. The first one is to use traditional radio technology. The second solution is the free space optical (FSO) technology, also called optical wireless communication (OWC). When there is line of sight, FSO supports GB/s over a distance of few kilometers on direct links. Because having a line of sight is a necessity for FSO links, RF links in this respect are more efficient because magnetic waves have ability to propagate through walls and around corners; however RF solution requires frequency licenses and the major drawback is that the price per bit in contrast to FSO technology. As a conclusion, FSO has better performance in terms of distance and interference. In this thesis we propose a model for the optical wireless mesh network (OWMN), for using a cross layer topology design method. A mirror can be used when some obstacles such as high-buildings block the line-of-sight for transmission. For an illustration consider the Figure 1.1 wherein buildings A, B, C, D and E are connected through OWC links; building O is a barrier and prevents having a line of sight between buildings A and B. A mirror is used at building M in order to obtain a broadband connection between building A and B [2]. The advantage of using the mirrors is decreasing the cost of connectivity in contrast with other methods. In this research we set up a mathematical model by considering the cost and the number of feasible hops and distance.



Figure 1.1: An example of OWC network [2].

1.2 Thesis outline

The thesis is organized as follows.

In chapter 2 we discuss the optical wireless communication (OWC) by giving a brief introduction of free space optic to build optical backbones with light beam to deliver high speed wireless communications through the air which is both reliable and secure.

In chapter 3 we give an introduction to the theory of minimum spanning trees and steiner trees and then turn our attention to modeling of network entities and functions and introduce single and two layer network modeling by bringing the introduced mathematical framework. We discuss network architecture and explain why and how various technologies are combined in backbone networks. Architecture in our case is composed of two layers of resources. Also we introduce a modeling framework, network design problems studied in the thesis are presented in a form of multi-commodity flow optimization models.

In chapter 4 we start with a simple example to illustrate our model and then provide a numerical example to investigate several cases by changing the cost of some factors to illustrate the efficiency of the model. Discussion is added for each implementation.

In chapter 5 we summarize the contributions of the thesis, provide the general conclusion of the work and identify directions for future work.

Chapter 2

Free Space Optical Networks

2.1 Background

In the last decade, we have seen a growing level of research and development activities in the emerging field of free space optical (FSO) communication systems. FSO is the kind of technology that transmits data by point to point communication links from rooftop to rooftop of tall buildings. Due to limitation on scalability and bandwidth of radio communication, FSO networks can be considered as a supplementary option. FSO technology consists of a modulated laser beam and a sensitive photo detector respectively for the transmitter and the receiver. When the line of sight is available, a two way full duplex FSO is established between transmitter and receiver with a leaser beam [3].

FSO technology has been utilized more than 30 years for providing wireless communications. New developments in FSO technology have opened up mainstream communications applications, from short-term solutions for short distance network bridges to an attractive and viable alternative for service providers to deliver the promise of all-optical networks. As an optical technology, FSO is a natural extension of the metro optical network core, bringing cost-effective, reliable and fast optical capacity to the network's edge [5]. Although the optical fiber technology has obtained acceptance in the communications industry, FSO communications is still relatively novel. FSO technology enables bandwidth transmission capabilities as fiber optics, using the same optical transmitters and receivers. FSO can even enable wavelength division multiplexing (WDM) technology capable of operation through free space [9]. Integrated FSO, fiber communication systems and (WDM) systems are currently in experimental stages and not deployed in the market [5]. Terrestrial FSO has now proven to be a viable complementary technology in addressing the contemporary communication challenges; most especially the bandwidth/high data rate requirement of end users at affordable cost [5]. There are several ways to transmit data. Fiber optic is the first choice and without any doubt most reliable to provide optical communication. However, problems such as digging, delays and related costs to lay a fiber make it economically expensive. Furthermore when fiber is deployed it is extremely difficult and prohibitive to redeploy if a customer switch to another service provider. Using radio frequency (RF) is a second option. However, it supports shorter distance and RF-based networks need substantial fund investment to obtain a spectrum license compared to FSO. Current bandwidth of RF is 622 Mb while FSO bandwidth is about 2.5 Gb. Therefore, for service providers it does not make sense to use RF technology to extend optical networks [5]. Wire and copper based technology is the third option. Availability of copper infrastructure almost everywhere is a reason to use this technology for connectivity but the bandwidth restriction of 2 Mb to 3 Mb is a drawback of this technology.

Finally a viable alternative solution is FSO. The capacity of FSO is comparable with that of an optical fiber-based system but at relatively lower cost and is more environmentally friendly as it requires no digging of trenches or cutting roads. Free space optic technology utilizes atmosphere [9]. Most important thing is that basic FSO technology needs lines of sight without any physical obstruction [5].

2.1.1 Features of FSO

The basic features of the FSO technology are given below [5]:

1. Huge modulation bandwidth: In any communication system, the amount of data transported is directly

related to the bandwidth of the modulated carrier. The allowable data bandwidth can be up to 20 percent of the carrier frequency. Using optical carrier whose frequency ranges from $10^{12}to10^{16}$ Hz could hence permits up to 2000 THz data bandwidth. Optical communication therefore, guarantee an increased information capacity.

- 2. **Narrow beam size:** The optical radiation prides itself with an extremely narrow beam. This implies that the transmitted power is concentrated within a very narrow area.
- 3. Unlicensed spectrum: Due to congestion the RF spectrum, interference from adjacent carriers is a major problem. However, the optical frequencies are free from this.
- 4. **Cheap:** The cost of deploying FSO is lower than that of an RF with a comparable data rate. FSO can deliver the same bandwidth as optical fiber without the extra cost.
- 5. **Quick to deploy and redeploy:** The time it takes for an FSO link to become fully operational starting from installation down to link alignment could be as low as few hours. It can as well be taken down and redeployed to another location quite easily.
- 6. Weather dependent: The performance of FSO is tied to the atmospheric conditions. The properties of the FSO channel undoubtedly pose the greatest challenge. Although this is not peculiar to FSO, as RF and satellite communication links also experience link outages during heavy rainfall and in stormy weather.
- 7. Line of sight: Requires line of sight and strict alignment as a result of its beam narrowness.

2.1.2 Areas of application

Features of FSO make this technology very attractive as a data bridge between the backbone and the end users. Following areas have been found suitable for using FSO technology [5].

- 1. Last mile access: FSO can be used to bridge the bandwidth gap that exists between the end-users and the fiber optic backbone. Links ranging from 50 m up to a few kilometers are readily available in the market with data rates covering 1 Mbps to 2.5Gbps.
- 2. Optical fiber back up link: Used to provide back-up against loss of data.
- 3. Cellular communication back-haul: Can be used to back-haul traffics between base stations and switching centers in the 3rd and 4rd generation networks.
- 4. **Disaster recovery/temporary links:** The technology finds application where a temporary link is needed be it for a conference or ad-hoc connectivity in the event of a collapse of an existing communication network.
- 5. Multi-campus communication network: FSO can be used to interconnect campus networks.
- 6. **Difficult terrains**: Such as across a river, a very busy street, rail tracks or where fiber is not available or too expensive to pursue, FSO is an attractive data bridge in such instances.

2.2 Overview of FSO

2.2.1 How Free Space Optics works

The basis of FSO is the transmission of signals/data/information from one point to another point using optical radiation as the carrier signal through an unguided channel [5]. For this reason FSO systems should have telescopes with sensitive photon detector to collect photon streams of digital data such as internet messages, video images, radio signals and so on [6].

Both line of sight between source and destination and sufficient transmitter power are necessary in order to use FSO technology. There are two variants for FSO communication. The conventional FSO shown in Figure

2.1 is for point-to-point communication with two similar transceivers; one at each end of the link, what permits for a full-duplex communication. As shown in Figure 2.2 a modulated retro-reflector (MRR) is the second variant. Laser communication links with MRRs are composed of different terminals and hence are asymmetric links: on one end of the link there is the MRR while the other hosts the interrogater [5].



Figure 2.1: Conventional FSO system block diagram.



Figure 2.2: Modulated reflector FSO system block diagram.

2.2.2 FSO operation wavelength

Operation frequencies for FSO links are from Mb/s to Gb/s and links range are from a few hundred meters to a few kilometers. Standard and popular protocol such as SONET/SDH,T1 1.544Mb/s, or E12.048Mb/s, E3/DS3, and 10/100/1000 are used for transmission signal. Currently available FSO hardware can be classified into two categories depending on the operating wavelength-systems that operate near 800 nm and those that operate near 1550 nm. There are compelling reasons for selecting 1550 nm FSO systems due to eye safety, reduced solar background radiation, and compatibility with existing technology infrastructure.

2.2.3 Modulation techniques

There exist different types of modulation schemes that are suitable for optical wireless communication systems. The type of modulation to be used depends on speed, distance and weather conditions. The simplest is digital coherent detection with On-Off keying modulation (OOK). All commercially available systems use OOK with a fixed threshold. This results in sub-optimal performance in turbulence regimes. This is primarily due to its simplicity and resilience to laser non-linearity. In OOK, a digital data bit d(t) = 0 is transmitted as an absence of the light pulse and d(t) = 1 as a pulse of finite duration. OOK is well studied and is known for its simplicity but requires an adaptive threshold to perform optimally in a turbulent atmosphere [5]. Another modulation technique improves the power efficiency of OOK but at the expense of an increased bandwidth requirement and a greater complexity. In PPM, each block of (log_2M) data bits is mapped to one of M possible symbols. Subcarrier Intensity Modulation (SIM) is the third modulation technique. In optical SIM, an RF subcarrier signal premodulated with the source data is used to modulate the intensity of the optical carrier. SIM is employed to

Туре	Radius (μm)	Size Parameter x_0	Scattering Process
Air Molecules	0.0001	0.00074	Rayleigh
Haze particle	0.01 - 1	0.074 - 7.4	Rayleigh – Mie
Fog droplet	1 - 20	7.4-147.8	Mie-Geometrical
Rain	100 - 10000	740-74000	Geometrical
Snow	1000 - 5000	7400-37000	Geometrical
Hail	5000-50000	37000-370000	Geometrical

Table 2.1: Typical atmospheric scattering particles with their radii and scattering process at λ = 850 nm [5]

avoid the need for an adaptive threshold required by optimum OOK. Moreover, SIM has the capability to increase the system capacity by modulating multiple digital and/or analogue information sources into different electrical subcarriers, which are then used to modulate the intensity of a continuous-wave laser that serves as the optical carrier. This, however, comes at the price of increased signal-to-noise ratio (SNR) at a given level of bit error rate (BER) performance. Hence, multiple SIM can only be used when increased capacity is of a paramount importance over the power requirement [5].

2.2.4 The atmospheric channel

An optical communication channel is different from the conventional Gaussian-noise channel. Quality and performance of FSO links are generally affected by the link distance and weather conditions like environmental temperature and light, sun, fog, snow, smoke, haze and rain. The effects of different weather conditions for FSO link are as follows.

- 1. **Rain**: The effect of rain is photon absorption which is not significant. In this condition we can increase transmit optical power [3].
- 2. **Fog**: Fog is the major challenge to FSO communications simply because fog is composed of water droplets which have few hundred microns in diameter and can modify or completely prevent the passage of light through a combination of absorption, scattering and reflection. An effective way to solve this problem is to shorten FSO link length [3].
- 3. Aerosols Gases and Smoke: Mie scattering, Photon absorption, Rayleigh scattering are the effects of aerosols gases and smoke [3].

2.2.4.1 Atmospheric channel attenuation

The atmospheric attenuation includes absorption and scattering. The absorption is an interaction between the propagating photons and molecules along its path which is wavelength dependent. In order to reduce the effect of absorption, suggested by Ghasemlooy et.al., [5], the wavelength used is located within atmospheric transmission windows such as 785 nm and 1550 nm.

Scattering results in angular redistribution of the optical field with and without wavelength modification [5]. It depends on the radius, r, of the particles encountered during propagation. One way of describing this is to consider the size parameter $x_0 = 2\pi r/\lambda$ [5]. If $x_0 \ll 1$ the scattering is classified as Rayleigh scattering, if $x_0 \approx 1$ it is Mie scattering and for $x_0 \gg 1$ the scattering process can then be explained using the diffraction theory (geometric optics). The detail can be found in [5] [9].

As shown in Table 2.1 and Figure 2.3, it is obvious that fog is a major photon scatterer and it contributes as the major power attenuation [5].



Figure 2.3: Atmospheric attenuation as function of visibility λ =785 nm [8]

As indicated in Figure 2.3 the visibility in dense fog for operating wavelength at 785 nm is very short. It is around 50 meters and the attenuation is around 300 dB/km [8] where in practical applications it can shut-down the FSO links.

Scattering caused by fog is categorized as Mie scattering. It is described based on an empirical formula expressed in terms of the visibility range V in km, as shown in Equation(2.1) [5].

$$\beta_a(\lambda) = \frac{3.91}{V} \left(\frac{\lambda}{550}\right)^{-\delta} \tag{2.1}$$

where λ is wavelength in *n*m, β_a is atmospheric attenuation and δ is size of scattering particles defined by using Kim and Kruse model see [5][8].

As shown in Figure 2.4 that system uses wavelength 1550 nm is slightly less attenuated than the system with a lower wavelength at 785 nm. The scatter model used in Figure 2.4 is the Kruse model [5][8].

Attenuation caused by other scattering objects such as rain and snow are low compared to fog. Heavy rain attenuation is around 7 dB/km while attenuation from snow is around 34 dB/km [6].

For most commercial FSO deployments, operation in heavy fog environments requires keeping the distance between FSO transceiver short in order to maintain the required level of availability. The link power margin of most vendors equipment allows for availability that exceeds 99.99% if distances are kept below 200 m [9].

In addition, the total attenuation/extinction coefficient, $\alpha(\lambda)$ in m^{-1} , is the sum of the absorption and the scattering coefficients from particles constituents of the atmosphere [5].

The robustness of the design of any optical communication system can be effectively verified by critically applying performance checks on the system. The evaluation criteria should provide a precise determination and separation of dominant system limitations, making them crucial for the suppression of propagation disturbances and performance improvement. The Bit Error Rate (BER) of an optical link is the most important measure of the faithfulness of the link in transporting the binary data from transmitter to receiver. FSO faces a major challenge from scintillation introduced by atmospheric turbulence but improvement in performance of FSO Link has been observed with the use of small beam divergence angles. The performance of varying divergence angle under controlled turbulence environment has been studied for FSO communication link that The simulation is performed for several divergence angles from the range 0.1 mrad to 3mrad. The simulation system performance improvement for the FSO link compared to 3mrad divergence. It is also concluded that the link can tolerate more attenuation by decreasing the divergence angle observed that decreasing the beam divergence from 3 mrad to 0.1 mrad has significant effect on the improvement of the performance of the FSO link [7].



Figure 2.4: Comparison atmospheric attenuation as function of visibility for λ =785 and 1550 nm [8].

2.2.4.2 Atmospheric turbulence

The effects of atmospheric turbulence are beam steering, image dancing and beam spreading. Refractive index structure C_n^2 is another important parameter which displays the strength of the atmospheric turbulence [3]. This parameter is a function of the altitude and wind speed [5]. Ghasemlooy et.al., [5] discusses atmospheric turbulence factors such as weather phenomena and scintillation by pressure, humidity, and temperature in detail. The severe weather condition will significantly affect performance of a free space link. The channel models commonly used for atmospheric are log–normal, gamma–gamma [5][3]. Weak to moderate turbulence [5]. In this thesis, log–normal is considered The marginal distribution of light intensity fading induced by atmospheric turbulence can be statistically modeled as Equation(2.2) [3]:

$$f_I(I) = \frac{1}{2\sigma_x I \sqrt{2\pi}} exp\left\{-\frac{(ln(I) - ln(I_o))^2}{8\sigma_x^2}\right\}$$
(2.2)

where I_0 is the average received power intensity and σ_x^2 is the variance of the log amplitude fluctuation. The variance has the form Equation(2.3) [3]

The standard deviation can be approximated as

$$\sigma_x^2 = 0.30545 \left(\frac{2\pi}{\lambda}\right)^{7/6} C_n^2(\eta) z^{11/6}$$
(2.3)

where, λ , is wavelength in meter, $C_n^2(\eta)$, in is the index of refraction structure parameter in $m^{-2/3}$, with constant altitude η in meter, and z is the transmission distance in meter. For atmospheric channels near the ground, e.g., $\eta < 18.5$ m, C_n^2 ranges from $10^{-13}m^{-2/3}$ to $10^{-17}m^{-2/3}$ for strong to weak atmospheric turbulence, with a typical value $10^{-15}m^{-2/3}$. Under log-normal fading, the reliability of an FSO link can be computed as Equation(2.4) [3]:

$$\Gamma_{ij} = Pr\{I \ge I_{th}\} = \frac{1}{2} - \frac{1}{2}erf\left(\frac{(ln(I/I_{th}))}{2\sigma_x\sqrt{2}}\right)$$
(2.4)

where I_{th} is a threshold of received signal intensity. For fixed ratio of I_{th}/I_0 , Γ_{ij} is determined by the standard deviation σ_x , which is strongly influenced by the weather condition (i.e., a decreasing function of $C_n^2(\eta)$). With a suitable threshold, I_{th} , Γ_{th} , the potential link visibility of an FSO link can be obtained.

The severe weather condition will significantly affect performance of a free space link.



Figure 2.5: Link visibility (γ_{ij}) vs refraction coefficient (C_n^2) for different transmittance [3].

Figure 2.5 shows that the probability visibility of FSO link varies with atmospheric refraction turbulence at distance 4 km away from the transmitter using wavelength 1550 nm. It is clearly shown that the visibility link is varied and changed at different weather condition. A severe weather condition decreases the visibility of the links lower than 65% whereas at relatively clear weather $(C_n^2=10^{-16})$, the visibility link can reach up to around 5-nines (99.999%). The transmittance also defines the visibility. It makes sense that system with less attenuated intensity at receiver side (lower transmittance) has better visibility link compared to that system with higher attenuated intensity.

Furthermore, in Figure 2.6, at the same weather condition, visibility link of wavelength 1550 nm is greater than for a shorter wavelength. At $C_n^2 = 10^{-16}$, probability visibility of link for wavelength 1550 nm can reach above 90% whereas for wavelength 550 nm only about 80% for the same distance application. Equation(2.5) is used to define the potential of FSO link at certain level threshold.

$$\omega(n) = \begin{cases} \Gamma_{ij} & \text{if } \Gamma_{ij} \ge \Gamma_{th} \\ 0 & \text{otherwise} \end{cases}$$
(2.5)

In this thesis, probability visibility of FSO link, Γ_{th} is set fixed at 90%.

$$\tau(\lambda, R) = \frac{P_R}{P_T} = exp(-\alpha(\lambda)R)$$
(2.6)

where P_R is detected power intensity (in watt) at location R and P_T is initial transmitted power intensity (in watt). The ratio of them is also called transmittance [3]. Parameter $\alpha(\lambda)$, represents the total attenuation coefficient at certain wavelength. The received power is exponentially decreased with $\alpha(\lambda)$ and distance.



Figure 2.6: Link visibility (γ_{ij}) vs refraction coefficient (C_n^2) , for different wavelength [3].

2.2.5 Safety

Transmitting with high power can certainly alleviate atmospheric losses influences, however, laser sources beyond certain power threshold can be dangerous to human body including eyes [3]. There are laser safety standards like ANSI Z136.1 and IEC 60825-1 [9]. According to IEC 60825-1(Amendment 2) the allowable transmitting power for 850 nm wavelength is 0.78 mW while for 1550 nm it is up to 10 mW, this because for wavelength 1550 nm the aqueous fluid of the eye absorbs much more of the energy of the beam, preventing it from traveling to the retina and inflicting damage. However, the cost production of 1550 nm is higher than for a shorter wavelength [9].

2.3 FSO network topologies

- 1. **Point to Point topology:** The simplest topology is the point-to-point topology, which requires two transceivers with in the line of sight (Los) in order to establish a link. This means that one transceiver, must be able to see the other transceiver, and these two transceivers are mounted on top of two buildings to establish a FSO link. In some cases, one or both transceivers can be mounted in the window [11].
- 2. **Ring topology:** The second topology is Ring topology. In terrestrial applications for which the link exceeds the permissible link length, it is necessary to add nodes to reduce the length of the FSO link. In such a case, FSO consists of more than one transceiver and they can form a ring topology. In fact, because the FSO links are full-duplex (bidirectional), the constructed ring is of the type two counter-rotating rings [11].
- 3. **Mesh topology:** The mesh topology is more complex than the ring and the PtP. This topology is capable to interconnect many nodes and although, depending on the application, the distance between nodes or inter-node link (INL) may be as short as few hundred meters or as long as few kilometers, the overall

network may extend over many kilometers square. On the negative side, the complexity of design and maintenance of each node in mesh topology increases as the number of (INLs) per node increases, and thus the network cost-efficiency, and thus network optimization, is required. On the positive side, mesh topology is applicable to multi-node networks in a scalable and expandable fashion, is capable of transporting ultra-high volume of standardized traffic at very high data rates (for FSO application up to 10Gb/s per link) and provides the best-network and service protection. Network scalability refers to the network ability and flexibility to add or delete a node , while service is provided. Generally the mesh topology is suitable to add or remove nodes [11]. Consider a situation in which there is a reason, for example a high building, that prevents possibility to have the line of sight between these two points. Consequently, it is not possible to transfer data directly; to do so one approach is using a mirror in the nearly nodes to transfer data. In Figure 2.8 there is no line of sight between node A and B.



Figure 2.7: Point-to-Point, ring and mesh topologies



Figure 2.8: The bidirectional links between B and C can be interrupted

Chapter 3

Optimization model

3.1 Problem description

The line of sight (LOS) is the propagation condition requiring transmitting and receiving stations are in view of each other without any sort of obstacles between them. In FSO networks, LOS is the necessary condition for a link to be set up. In practice, this condition may not be satisfied due to obstacles such as high buildings which destroy the LOS. However, these issues can generally be mitigated through pre-planning and use of additional technologies. In this thesis mirrors are used to solve this problem. There are two kind of nodes, optical nodes and mirror nodes. They are disjoint. Also there are two kinds of link, a direct optical-link has its end nodes in the LOS, and hence does not require an mirror-path to have connection. Another kind of link is indirect optical-link, if provided must be realized on an mirror-path. We set up a mathematical model to find a topology with minimized cost. Different factors are taken into account, including cost, distance, number of hops and degree of limitation. When all demand nodes use direct optical-link, the problem is to find a minimum spanning tree. However, when there is no LOS, we use indirect path consisting of mirrors to connect the nodes. In this case the problem is changed to finding a minimum Steiner tree.

3.1.1 Trees in graphs

A tree is an undirected graph in which any two vertices are connected by exactly one path. In other words, any connected graph without cycles is a tree. In this section, we give some facts about trees.

Spanning tree: A spanning tree connects all of the vertexes in a graph and has no cycles. A graph may have many Spanning trees. Figure 3.1b shows a tree for the Figure 3.1a. A, B, C, D are the vertexes of the graph. The numbers on each link is the link weights.

Minimum spanning tree (MST): A minimum spanning tree (MST) is a spanning tree whose edge wieghts sum up to minimum weight. In other words, a minimum spanning tree is a tree formed from a subset of edges in a given undirected graph, with two properties: (1) it spans the graph, i.e., it includes every vertex in the graph; (2) The total weight of all edges of the tree is minimum. In this thesis we will construct a minimum spanning tree according to the cost of links. For example Figure 3.1c represents a minimum spanning tree of Figure 3.1a



Figure 3.1: Example of trees

Steiner tree: The Steiner tree problem is similar to the spanning tree problem, given a set of points (vertices), interconnect them by a network (graph) of shortest length, where the length is the sum of the weights of all edges. The difference between the Steiner tree problem and the spanning tree problem is that, in the Steiner tree problem, extra intermediate vertices and edges may be added to the graph in order to reduce the total weights of the spanning tree. These new vertices introduced to decrease the total length of connection are known as Steiner points or Steiner vertices. There may be several Steiner trees for a given set of initial vertices [13].

Minimum Steiner tree: The minimum Steiner tree is to find a Seiner tree with the minimal total weight of links. Figure 3.2 gives a Steiner tree in which s1, s2 are Steiner points. We can see that the total weight of the minimum Steiner tree is smaller than the total weight of the minimum spanning tree in Figure 3.1c. In this thesis mirrors are Steiner points. Adding mirror nodes are a cost-effective way to have connection.



Figure 3.2: A minimum Steiner tree

3.1.2 A network example using mirrors

After these explanations we give a simple example which illustrates the idea of our model. As shown in Figure 3.3 there are 3 demand nodes and one mirror node. In this example nodes 1, 2, 3 are demanding nodes represented by red circles and node 4 is mirror node represented by a blue square. There are obstacles between nodes 1 and 3 and also nodes 2 and 3. In Figure 3.3 the dash line shows that there is not line of sight between demand nodes because of obstacles such as high buildings. Here we put triangles to represent obstacles between optical nodes. There is not direct optical-link between optical nodes 1, 2 because the distance between them is too far to set up a link. For this reason we use mirror node 4, to establish indirect optical-link with the help of mirror-path. We show path 1 and 2 in the Figure 3.3.





3.2 Mathematical modeling

3.2.1 Notations

The considered network, composed of a set of nodes \mathcal{V} and a set of links \mathcal{E} , is bi-directed. That is, if a link $(v, w), v, w \in \mathcal{V}$ exists in the network, then (w, v) also exists in the network. There are two disjoint types of nodes, nodes which could be installed with optical transceivers (o-nodes) and nodes which could be installed mirrors (m-nodes). Consequently, the links are classified to two disjoint groups, o-links (both ends are o-nodes) and m-links (at least one end is m-node).

Table 3.1: Notations

sets	
\mathcal{V}	set of optical nodes (o-nodes)
\mathcal{W}	set of mirror nodes (m-nodes)
\mathcal{E}'	set of direct optical links (o-links, directed)
\mathcal{E}''	set of indirect optical links (o-links, directed)
${\mathcal E}$	set of all optical links (o-links, directed), $\mathcal{E} = \mathcal{E}' \cup \mathcal{E}''$
parameters	
s(e), t(e)	origin and destination node, respectively, of link $e \in \mathcal{E}$
${\cal F}$	set of mirror links (m-links) (m,m),(o,m),(m,o) (m-links, directed)
\mathcal{D}	set of (directed) demands
s(d), t(d)	origin and destination node, respectively, of demand $d \in \mathcal{D}$
$\delta^+(v)$	set of o-links originating in node $v \in \mathcal{V}$
$\delta^{-}(v)$	set of o-links terminating in node $v \in \mathcal{V}$
$\delta(v)$	set of o-links incident with node $v \in \mathcal{V}$, $\delta(v) = \delta^+(v) \cup \delta^-(v)$
$\Delta^+(v)$	set of m-links originating in node $v \in \mathcal{V} \cup \mathcal{W}$
$\Delta^{-}(v)$	set of m-links terminating in node $v \in \mathcal{V} \cup \mathcal{W}$
$\Delta(v)$	set of m-links incident with node $v \in \mathcal{V} \cup \mathcal{W}$, $\Delta(v) = \Delta^+(v) \cup \Delta^-(v)$
m	maximum number of hops of a m-path
$c_e, e \in \mathcal{E}$	unit cost of o-link $e \in \mathcal{E}$
$C_v, v \in \mathcal{V}$	cost of node $v \in \mathcal{V}$ equipped with optical transceivers
k	cost of one mirror
$K_w, w \in \mathcal{W}$	cost of node $w \in \mathcal{W}$ equipped only with mirrors
variables	
$x_{de}, d \in D, \ e \in E$	continuous variables, $x_{de} = 1$ when o-link <i>e</i> realizes demand <i>d</i>
$y_e, e \in \mathcal{E}$	binary variables, $y_e = 1$ when link e is realized
$z_{ef}, e \in E, f \in F$	binary variables, $z_{ef} = 1$ when m-link f realizes o-link e
$Y_v, v \in \mathcal{V}$	binary variables, $Y_v = 1$ when node v is equipped with optical transceivers
$Z_w, w \in \mathcal{W}$	binary variables, $Z_w = 1$ when node w is equipped with mirrors

3.2.2 Problem formulation

In this subsection we present the mixed integer programming model for the considered problem. Different factors, node cost, degree limitation, the number of hops are considered in the model.

Cost of deployment

One of the most important issues to design network is the cost to establish such network. We introduce different types of cost including the cost of optical link, the cost of node equipped with optical transceivers, cost of a mirror and cost of a node equipped only with mirrors. The cost of nodes equipped with optical transceivers is more expensive than the cost of nodes equipped only with mirror.

Degree limitation

In practice, the number of optical transceivers installed in a node is limited. Let b(v) ($v \in V$) denote the upper limit of the number of optical interface for node v.

Number of hops

In the model, mirrors are used to forward the optical signals which helps to transmitting signals. However, the number of consecutive mirrors used to realize an optical-link should be limited due to the path loss.

Full formulas

Taking into account these issues, we present a mixed integer programming model as

min
$$F = \sum_{e \in \mathcal{E}} c_e y_e + \sum_{e \in \mathcal{E}''} k(\sum_{f \in F} z_{ef} - y_e) + \sum_{v \in V} C_v Y_v + \sum_{w \in W} K_w Z_w$$
(3.1a)

$$\sum_{e \in \delta^+(v)} x_{de} - \sum_{e \in \delta^-(v)} x_{de} = \begin{cases} 1, & \text{if } v = s(d) \\ 0, & \text{if } v \notin \{s(d), t(d)\} \\ -1, & \text{if } v = t(d) \end{cases} \qquad d \in \mathcal{D}, \ v \in V$$
(3.1b)

$$x_{de} \le y_e, \quad d \in \mathcal{D}, \ e \in \mathcal{E}$$
 (3.1c)

$$\sum_{e \in \delta(v)} y_e \le b, \quad v \in \mathcal{V}$$
(3.1d)

$$\sum_{f \in \Delta^+(s(e))} z_{ef} = y_e, \quad e \in \mathcal{E}''$$
(3.1e)

$$\sum_{f \in \Delta^-(s(e))} z_{ef} = 0, \quad e \in \mathcal{E}''$$
(3.1f)

$$\sum_{f \in \Delta^{-}(t(e))} z_{ef} = y_e, \quad e \in \mathcal{E}''$$
(3.1g)

$$\sum_{f \in \Delta^+(t(e))} z_{ef} = 0, \quad e \in \mathcal{E}''$$
(3.1h)

$$\sum_{e \in \Delta^+(w)} z_{ef} - \sum_{f \in \Delta^-(w)} z_{ef} = 0, \quad e \in \mathcal{E}'', \ w \in W$$
(3.1i)

$$\sum_{f \in F} z_{ef} \le m+1, \quad e \in \mathcal{E}'', \ f \in F$$
(3.1j)

$$y_e \le Y_v, \quad v \in V, \ e \in \delta(v) \tag{3.1k}$$

$$z_{ef} \le Z_w, \quad e \in \mathcal{E}'', \ w \in \mathcal{W}, \ f \in \Delta(w)$$
(3.11)

The objective function and constraints are explained as follows.

1. (3.1a): In the objective function the first part is the summation of the cost of all optical links. The second part is the total cost of used mirrors. The term in brackets expresses the number of mirrors to connect a pair of nodes. The third part is the cost for leasing nodes for installing FSO transceivers. The last part is the cost for leasing nodes for installing mirrors.

- 2. (3.1b): This inequality is the conservation law and makes sure that all optical-nodes will be connected. In this formula x_{de} is the amount of demand d allocated to optical link e.
- 3. (3.1c): This equation means if there is demand on optical link e, link e should be installed.
- 4. (3.1e), (3.1f), (3.1g), (3.1h), (3.1i): These inequalities express the flow conservation rule which use m-links to realize o-links. The first formula makes sure that the sum of all flows outing going from the source node of demand d should be equal to the capacity of optical-link e. The second formula means that the sum of all flows incoming to the source node of demand d be equal to zero. The same interpretation are true for third formula, with this explanation which t_e is terminating node. The last formula assures that the amount of flow coming into the intermediate node of a demand should equal to the outgoing from that node.
- 5. (3.1j): This inequality limits the number of mirrors used in a path. The number of consecutive mirrors used to realize an optical-link should be limited due to the path loss.
- 6. (3.1k): If an optical-link e is installed, the both ends of link e should be rented for installing transceivers.
- 7. (3.11): If a mirror-link m is installed, the both ends of link e should be rented for installing mirrors.

Note that optical nodes and mirror nodes are disjoint. A direct optical-link has its end nodes in the line of sight, and hence does not require an mirror-path. Each indirect optical-link, if provided, must be realized by a mirror-path.

Chapter 4

Numerical study and results

4.1 Generating network instances

Network instances are generated using a random data generator on the x-y plane. The codes for optimization models are developed in a Python environment. The optimization solver is a commercial package, GUROBI, a Mixed Integer Linear Programming (MILP) solver [12].

The following shows the steps to generate the instances which are used in the subsequent experiments.

1. Generating the first layer (optical layer)

First of all we generate n nodes randomly. We consider them as optical nodes in given square and we use these nodes as demand nodes in a given square. Demand nodes refer to those nodes which exchange data with each other. We select the position of optical nodes randomly on x-y plane. After this step it is necessary to calculate the distance between these nodes in order to find out if it is possible to establish direct optical links between them, by using an FSO channel modeled by the formulas given in chapter 2. According to the log-normal model, FSO link is available if the link reliability is larger than link reliability threshold.

2. Removing some links

In order to show that it is possible to use a mirror path when there is no line-of-sight, we remove some of direct optical links randomly. It means there are obstacles between optical nodes and we want to connect them again by using mirror paths.

3. Generating mirror paths (mirror layer)

By adding some random nodes and considering them as mirror nodes we want to use indirect mirror path in order to provide connection between optical nodes. If n is the number of optical nodes, and m is the number of mirror nodes, we select m = 2n. The reason is that because we select the position of optical nodes randomly. and the mirror nodes are Steiner nodes. We want to put more mirror nodes in order to select the cheapest mirror node to have connection by mirror path.

4. Calculating distance and testing connectivity

Owing to the fact that the mirror nodes should be in the acceptable range from the demand nodes, we should calculate the distance between mirror nodes and demand nodes in order to find out the possibility of connection between them according to FSO channel modeling. The purpose of adding the mirror nodes is to simulate when there is no line-of-sight, an idirect mirror path is used to provide a Steiner tree beetween all nodes.

4.2 Case studies and discussions

In this section, we test several network instances to illustrate the optimal solution of the model and show different optimal topologies by adjusting parameters including cost parameters, degree limitation and the number of hops. We uniformly generate nodes in a square area of $10 \ km$. Figure 4.1 is an example in which there are 15 nodes including optical nodes from 0 to 4 represented by red circles and mirror nodes from 5 to 14 represented by blue squares. The five optical nodes are also called demand nodes since traffic are only generated between every two nodes of them. By using log-normal for FSO channel given in chapter 2, we draw all possible direct optical-links. And then we remove some direct optical-links randomly due to obstacles which prevent the availability of line-of-sight. We show removed links by blue dash lines. For example if you pay attention to node 3 and 4 there is the blue dash line between this pair of optical nodes. Our purpose is to construct a minimum Seiner tree in which mirrors are Steiner points. The weights for some links are shown as the numbers in the figure.





Table 4.1 shows all demands between optical nodes in Figure 4.1. In this example we want to transfer data of all optical nodes from various sources to various destinations.

Demand (d)	Source $s(d)$	Destination $t(d)$
0	0	1
1	0	2
2	0	4
4	1	0
5	1	2
6	1	3
7	2	0
8	2	1
9	2	3
10	3	0
11	3	1
12	3	2
13	3	4
14	4	0
15	4	3

Table 4.1: Demands of optical nodes.

Table 4.2 lists the links which are removed due to obstacles. It means there is no line-of-sight between these pair of nodes.

Deleted link (e)	Origin $s(e)$	Destination $t(e)$
6	1	3
4	1	0
12	3	2
5	1	2
15	4	3
3	0	4
1	0	2
10	3	0

Table 4.2: Removed links because of obstacles

Figure 4.2 represents the optimal topology for Figure 4.1. All optical nodes connect to each other by mirror paths. By considering the weight of links all demand nodes receive data with mirror path in cost-effective path. Mirror nodes are Steiner points and using them for establishing mirror path is cheap. In the following, we will show some examples by adjusting the parameters. We set the cost of optical links $c_e = 50$, $e \in E$, the cost of optical nodes $C_v = 1000$, $v \in V$, the cost of a mirror k = 1, the cost of mirror nodes $K_w = 100, w \in W$, the degree limitation b = 5 and the hop of mirrors m = 5. Note that the cost of an optical node is significantly more expensive than the cost of a mirror node plus the cost of mirrors installed on them. These values are used for all the following part if they are not specified.

4.2.1 Case-1: changing the cost of mirror nodes

The cost of mirror nodes referring to the cost of leasing buildings for installing mirrors. In this case we select randomly between 10 and 100 for $K_w, w \in W$. Table 4.3 and Table 4.4 show two sets of the cost of mirror nodes.



Figure 4.2: Optimal topology

The optimal topologies corresponding to Table 4.3 and Table 4.4 are shown as Figure 4.3a with total cost 1424 and Figure 4.3b with total cost 1414 respectively. We can see that the optical nodes 0,1,2 switch from connecting to mirror node 9 in Figure 4.3a to connecting to mirror node 10 for the reason that the cost of node 10 is cheaper than the cost of node 9 in Figure 4.3b.

Node (w)	5	6	7	8	9	10	11	12	13	14
$\operatorname{Cost}\left(k_{w}\right)$	22	97	43	90	33	64	41	70	44	31

Table 4.3: the cost of some mirror nodes

Node (w)	5	6	7	8	9	10	11	12	13	14
$Cost(k_w)$	25	63	100	41	80	72	68	10	34	49

Table 4.4: the cost of some mirror nodes



Figure 4.3: Optimal topologies for adjusting cost of mirror nodes

4.2.2 Case-2: the degree limitation

In the model, the number of optical transceivers installed in an optical node is limited. It can also affect the optimal topologies. We test an network example to show the optimal topologies as Figure 4.4a with degree limitation 2 and Figure 4.4b with degree limitation 3. In this example, we only consider one demand from node 4 to node 1. Table 4.5 shows weights for some links. We can see that in the weight of link (4,2) is cheaper than the weight of link (4,6). But Figure 4.4a does not use this link due to the degree limitation. In Figure 4.4b, we increase the degree limitation to 3, and then link (4,2) is used in the optimal solution. The objective is also decreased by 50.



Figure 4.4: Optimal topologies for different degree limitations

Link (e)	(4,2)	(2,1)	(3,2)	(6,4)	(6,3)
Weight (c_e)	20	10	10	30	30

Table 4.5: the weight of some links

4.2.3 Case-3: the hop of mirrors

The hop of mirrors refers to the hops in the path consisting of mirrors which can limit the transmission distance of the optical signals. The reason is that the optical signals cannot be reflected many times from the source to the destination due to the attenuation. In this section we provide an example to show the effect of the hop of mirrors.

Figure 4.5 shows two optimal topologies with different value for the hop of mirrors. In the considered example, there are 5 demand nodes and an obstacle between node 0 and 1. Therefore, we need to find a path consisting mirrors to connect them. In this case the cost of mirror nodes are set as in Table 4.6. In Figure 4.5a we select the hops of mirrors m = 1. It means there is possibility to use just 1 hop in order to transfer data. We can see that node 8 is used in Figure 4.5a even if the cost of node 8 is expensive than the cost of node 5 plus node 10. In Figure 4.5b we select m = 3 and then node 5 and node 10 are used replacing node 8.

Node (w)	5	6	7	8	9	10	11	12	13	14
$Cost(k_w)$	10	100	100	50	100	10	100	100	100	100

Table 4.6: the cost of some mirror nodes



Figure 4.5: Network instances for number of hops

4.3 Discussions

In this chapter, we generate the network instances following realistic channel model of wireless optical communications and test the proposed model for several network instances. Some conclusions can be drawn from the experiments.

Firstly, mirrors are used when some optical nodes are not in the line-of-sight of each other. We can see that some mirrors are used in the optimal topology (Figure 4.2) for network instance of Figure 4.1 in which some optical nodes are not in line-of-sight. What is more, we could get the optimal number and the optimal locations of the mirrors. There are many potential places to install mirrors. As we can see in Figure 4.2, only 2 mirror nodes are used and the finding topology is the cheapest among all potential trees. When the cost of mirror nodes is changed, the optimal topology is changed as in Figure 4.3. The model always produce cheapest topology.

Then, we could also see that the degree limitation affects the cost of network and the optimal topology. In Figure 4.4, when we relax the degree limitation, changing degree limitation from 2 to 3, we can find an optimal topology Figure 4.4b which is cheaper than Figure 4.4a.

At last, we also show that the hops of mirrors can affect the optimal solution of the model. By relaxing the limitation on the hop of mirrors from 1 to 2, the obtained topology in Figure 4.5b is cheaper than the topology in Figure 4.5a.

In practice, the propose model can be examined by real networks like the true mobile backhand network and real parameters such as the commercial price for mirrors, optical transceivers, leasing cost of the top of buildings, etc. which will be great help for network operators. Of course, some other factors could also be considered in the model like the prorogation length of mirror path to make the model realistic.

Chapter 5

Concluding remarks

In conclusion, this thesis summarizes the development of free space communication (FSO) and studies a physical topology design problem in wireless optical networks. Technological features of FSO communications are discussed and analyzed, including hardware, operation wavelength, modulation techniques, applications, etc. Among all the factors affecting the performance of FSO links, the weather turbulence effects on the FSO links is studied in detail.

Although FSO has been considered for many years now, its usage for high-speed communications has not been fully exploited yet. The design of physical topology is an important and fundamental work for further research on routing protocols and applications in FSO networks. In this thesis, we propose a two-layer model, comprising the FSO layer and the mirror layer, to generate an optimal tree topology for FSO networks. Mirrors are adopted to connect the FSO nodes which are not in the line-of-sight. A mixed integer programming model is presented, taking into account different factors such as node degree limitation, hop constraint, etc. The developed model can deliver topologies with minimized deployment cost and find proper locations and the number of mirrors needed. Finally, we present a numerical study by showing several topology examples for different parameters that verify effectiveness of the proposed model.

In the future, the presented model could be extended to generate meshed networks to improve the resilience when single link or multiple links fail due to the adverse weather, and to improve the network performance like network throughput and delay. The described model could also be used to develop topology planning and optimization tools.

The main contributions of this thesis are as follows:

- 1. The features of FSO communications are investigated. The advantages and drawbacks are analyzed.
- 2. The considered pysical topology design problem is modeled as a two-layer model and the formulations are presented. Mirrors are incorporated in the mixed integer programming model to connect the nodes which are not in the line-of-sight.
- 3. A numerical study is conducted to test the model for a set of parameters and the results are analysed.

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