On Performance and Limitations of Active Noise Control in Mobile Telephony

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Abstract

Mobile phone communication today is carried out in all kinds of environments that exhibit background noise, affecting the speech intelligibility for the near end user. An interesting approach to solve this issue would be to implement an active noise control (ANC) system on the mobile phone. The aim of this thesis is to discuss the feasibility of implementing such a system on a handheld mobile phone. A simulation environment was created based on measurements using a model of a phone and an artificial head. The simulation aim to model implementations of different ANC-methods and adaptive algorithms. The stability of the system was analysed, displaying sensitivity to alterations in position of the phone. Feedforward and feedback ANC was analyzed, with and without secondary path compensation, filtered-x. LMS, nLMS, leaky nLMS and RLS was used as adaptive algorithms. The filtered-x feedforward method was superior to the other ANC methods, and a deeper analysis was conducted for this method. Leaky nLMS turned out to be the most efficient algorithm for both stationary and nonstationary signals regarding attenuation of noise and stability. The RLS algorithm might however be preferable if there are high demands on having lower order filters in the system. In conclusion it is feasible to implement ANC on a mobile phone, as long as some demands are fullfilled by the user, holding the phone in a steady position, pressed towards the ear. Which algorithm is to be used depends on what decision algorithms and what computational power is available. If the filter order of the adaptive filter is allowed to be of higher order the leaky nLMS is preferable, with the highest attenuation of noise.

Preface

This masters project was carried out in collaboration with Lund Institute of Technology(LTH), Blekinge Institute of Technology(BTH) and ST Ericsson in Lund. The project was initiated in January 2012 after an inquiry from ST Ericsson to investigate the feasibility of implementing ANC on a mobile phone. The project had two supervisors: Dr. Jörgen Nordberg, ST Ericsson; and Professor Lars Håkansson, School of Engineering at BTH. Measurements were performed in the acoustical lab at ST Ericsson, Lund. Analysis and evaluation was done at the Department of Electrical and Information Technology(EIT) at LTH. The thesis was examinated by Professor Leif Sörnmo, EIT. The analytical study, design of simulation environment and finally formulating all the information gathered into this thesis was an effort from both authors, combining and relying on the expertise of each other.

Acknowledgements

The authors would first like to thank Jörgen Nordberg, who helped us lay a firm foundation to build the thesis on, and gave valuable guidance through each step of the project. We would also like to thank Jonas Lunbäck for his expertise in understanding and conducting measurements in the Acoustic Laboratory at ST Ericsson, Leif Sörnmo for aiding with administration during the course of the project and Lars Håkansson for his firm grip on the subject and valuable knowledge in the field, guiding us to understand and interpret the results. Finally we would like to thank ST Ericsson for the opportunity to start this project and for providing the equipment necessary to see it through.

Nomenclature

A/D Analog to Digital

AA Adaptive Algorithm

ANC Active Noise Control

CSS Composite Synthesis Signal

D/A Digital to Analog

DSP Digital Signal Processing

EIT Electrical and Information Technology

ERM Error Reference Microphone

ES External Sound Source

FIR Finite Impulse Response

GUIDE Graphical User Interface Design Environment

HATS Heads and Torso Simulator

IIR Infinite Impulse Response

IS Internal Sound Source

LMS Least Mean-Squares

nLMS Normalized Least Mean-Squares

NRM Noise Reference Microphone

PSD Power Spectral Densities

PSM Primary Speech Microphone

RLS Recursive Least Squares

SNR Signal to Noise Ratio

SPL Sound Pressure Level

SVD Singular Value Decomposition

WGN White Gaussian Noice

WSS Wide Sense Stationary

Contents

Chapter 1

Introduction

1.1 Background

Communication have evolved through the years to the point where it today is possible to talk to virtually anyone from anywhere. The mobile phone usage is widespread and takes place in a vast variety of environments, some which are quiet, some that are noisy. Low frequency noise has a masking effect on speech, which degrades speech intelligibility for the *the near end user*, referring to the person in the noisy environment. In recent years noise reduction has been considered for mobile phones because of the increasing interest from mobile phone manufactures to incorporate it in their phones. One common approach is to try to 'cancel' noise by means of superposition, i.e. destructive interference of sound fields. Experiments on superpositioning of sound fields have been conducted since the end of the 19th century and this principal is what Active Noise Control (ANC) is based upon[12]. ANC can be performed using both analog and digital control, where as the latter is more common. The ANC using digital control used to be hard to implement, primarily because of lack in processing power, but as technology evolves, more and more fields are applicable to "digital" ANC solutions. Currently ANC is widely integrated into headsets of different types, taking advantage of the insulation that earplugs or headphone muffs accommodates. A handheld mobile phone lacks this luxury, which is the main reason why it is tough to implement ANC solutions, in such applications. Lack of insulation creates more variables for the ANC to handle, suggesting a need for systems that can handle large computation complexity, which has been a limiting factor in mobile phones.

Naturally this raises the question: Is it feasible to implement an ANC system on a handheld mobile phone using the technology available today? In order to investigate this a simulation environment is to be designed. To run simulations ST Ericsson constructed a mobile phone model, which will be used for estimating the channels that constitute the simulation environment. The ANC systems performance and stability properties will be examinated using different ANC setups and adaptive algorithms.

1.2 Objectives

The purpose of the project is to investigate the possibilites of implementing an ANC system on a handheld mobile phone. To decide whether or not it is helpful to implement and run ANC, this project aims to examine the algorithms and conditions required for adequate noise control using measured data. How much reduction can the system attain, and under what circumstances?

1.3 Methodology

The thesis is divided into three different parts, spanning seven chapters. The first part of the thesis lays the foundation for understanding the concepts of ANC, describing different approaches for implementing an ANC system and the underlying mathematical tools needed. The second part focuses on the measurements necessary to construct an ANC-system. The third and final part deals with the construction of the simulation environment followed by a performance analysis of the different ANC-algorithms leading up to the conclusion.

1.4 Thesis Outline

- **Chapter 2: Active Noise Control**
  This chapter aims to give an understanding of ANC on a theoretical level. Feedforward and Feedback ANC-systems will be discussed, accompanied by a few different methods to improve the performance.

- **Chapter 3: Filter Design, Adaptive Algorithms and Signal**

Analysis
Lays the foundation of signal processing and moves on to define the different adaptive algorithms used in this thesis.

- **Chapter 4: Signal Analysis**
  Presents tools for analyzing the signals to be measured.

- **Chapter 5: Measurements and Modeling of a Mobile Phone**
  A description of the different acoustic and electroacoustic paths dealt with in the model, followed by the procedure of initial calibration and measurements. An outline of the noise signals used and finally the preparation and syntax of the resulting audio files.

- **Chapter 6 Design and Evaluation of the ANC-system**
  A description of the process of estimating channels from the noise measurements to design a virtual model of the ANC-system followed by a performance analysis of the different algorithms available in the simulation environment.

- **Chapter 7: Conclusion**
  This chapter aims to summarize the work through the thesis and what conclusions were drawn.

Chapter 2

Active Noise Control (ANC)

In *Handbook Of Acoustical Measurements And Noise Control*[9] acoustic noise is defined as *unwanted sound*. In any acoustic environment such noise may be interfering, in some cases to a point where it is desired to dampen or completely remove the noise from the environment. The methods for doing so are many but can all be summed up in one phrase: Noise Control. By applying noise control, one aims to obtain an acceptable noise environment. Acceptable in this context means that it satisfies the requirements set for the system it is applied to. There are two main methods of controlling noise — passive and active noise control[11]. Passive noise control aims to modify the environment that the noise source operates in, and does not need a source of power in order to operate[11]. A common method in passive noise control is the use of insulators, absorbers and reflectors in order to reach an acceptable noise environment. The other method is Active Noise Control (ANC), active in the sense that it uses a power source in order to achieve an acceptable noise environment[13, 1]. There are numerous different methods of performing Active Noise Control, and in this chapter some of these methods will be discussed. In Chapter 1 the problem with noisy environments for a handheld mobile phone was introduced. To perform Passive Noise Control on such a size-constrained system is not feasible, and thus this concept will not be discussed in detail here. For more information about Passive Noise Control see *Active Noise Control Primer*[11], and to find out more about what materials are commonly used, how and why, see *Modern Recording Techniques*[15]. ANC is a general concept that can be applied to various systems with different complexity. Because of the characteristics of the mobile phone, only single-input/single-output ANC systems will be discussed in the thesis. The concepts in these sections can however also be applied to multiple-input/multiple-output systems, see *Active Noise Control: A Tutorial Review*[2] or *Active Control Of Sound*[12] for more information regarding this.

2.1 Superposition of waves

Sound is longitudinal waves travelling through a medium, oscillating the molecules along the line of propogation[16]. In *Physics For Scientist and Engineers*[16] waves in general and superpositioning of waves is discussed. Superpositioning of waves means essentially that two waves traveling in the same or different directions will additively interact[16, 12]. In figure 2.1 this concept is illustrated, with the following explanation: Suppose there is two sine waves travelling the same direction. One of the sine waves $x_1(t)$ has a constant amplitude, while the other, an inverse of $x_1(t)$, $x_2(t)$ has a varying amplitude. Suppose both the signals can be observed independently, as well as the resulting signal $x_3(t)$ at an arbitrary position. When the inverted signal has the same amplitude as the non-inverted signal the resulting wave will be zero, the two waves completely cancel each other out. If however the amplitudes of the signals differ, the resutling wave will be a dampened version of the signal with the highest amplitude, which can be seen in the bottom of figure 2.1. This is called the principle of destructive interference. ANC is a method derived from this principle, using destructive interference of sound fields. By generating and transmitting an inverted signal of observed noise, ANC aims to ideally cancel the observed noise. It is important to consider what happens if a signal is subject to interference with an inverse of the signal, but with higher magnitude or a different phase, since this may result in an even more noisy soundfield. In theory complete cancelation of a whole soundfield is possible, but in reality the wave properties of sound will cause the complete cancelation to only apply to a single point in space.

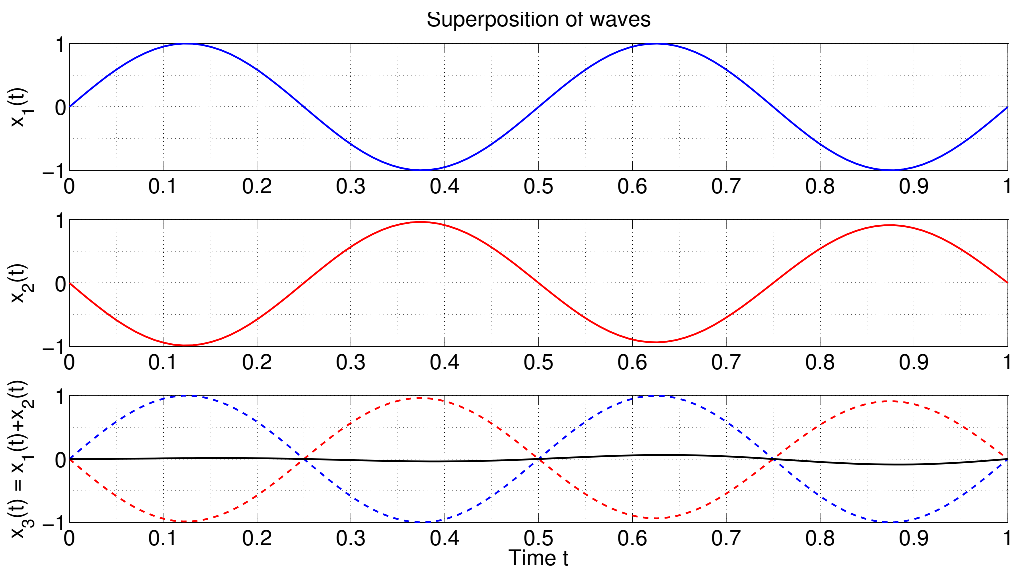

Figure 2.1: Superposition of waves. *Top:* x_1, sine wave with constant amplitude, *Middle:* x_2, inverted sine wave with varying amplitude, *Bottom:* x_3, resulting wave after superpositioning of x_1 and x_2

2.2 General ANC Concepts

There exists a lot of litterature discussing ANC, and there are a number of fields it can be applied to[1]. In this thesis ANC on acoustic systems will be discussed. Active Noise Control aims to actively dampen the noise in a noisy environment. Since the ANC using digital control will be the main focus of this thesis, the following discussion will be from a digital point of view. Consider the system described in figure 2.2. The input signal $x(n)$ can be considered acoustic noise, that is passed through an arbitrary acoustic path, resulting in the acoustic noise signal $d(n)$. The aim of the single channel ANC system is to dampen $d(n)$, using a reference sensor that picks up $x(n)$ alternatively estimates it. The unknown channel in the figure, is in this thesis referred to as the *primary path*, thus denoted $P(z)$ in the z-domain. Time-frequency transformations and z-transforms will be used frequently in order to describe the different systems and signals[6, 4].

The ANC system aims to sense the noise before the primary path to estimate

Figure 2.2: Unknown channel with input $x(n)$ and output $d(n)$

the desired signal, $d(n)$, in order to output an inverted version of $d(n)$, ideally resulting in complete cancellation. The signal that is used to cancel out or dampen $d(n)$ will be referred to as *anti-noise signal*. This basic model of the ANC system is fairly simple but when more aspects of the system are considered the model quickly increases in complexity. Initially consider an ANC system, with a constant primary path. To be able to generate the anti-noise signal, a finite impulse response (FIR)[17] filter $W(n)$ will be utilized. The reference signal $x(n)$ is then passed through this filter to generate the estimate of $d(n)$, $y'(n)$[1, 2] via the secondary path, see figure 2.3. The Secondary channel, $S(z)$, is an electroacoustic system that brings the signal $y(n)$ into the acoustic domain, resulting in $y'(n)$[12]. The difference between this signal and the acoustic noise $d(n)$ is referred to as the error signal, denoted $e(n)$. There are two types of noice that are commonly discussed in relation to ANC: Narrowband noise and Broadband noise. Narrow band noise is noise consisting primarily of one frequency or a narrow frecuency band when observed in the frequency domain. The other type is broadband noise, characterized by having a broad frequency band. Theses types of noise will be differently damped, depending on the ANC system used. There are two types of general ANC methods: Feedforward ANC and Feedback ANC, and these two approaches will be discussed in the following sections.

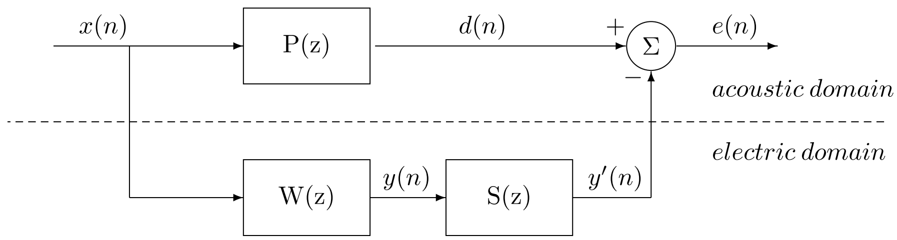

Figure 2.3: Basic ANC setup with input $x(n)$ and output $d(n)$

2.3 Feedforward ANC

Acoustic feedforward ANC utilizes a noise reference microphone, a loudspeaker and an error reference microphone to suppress noise passed though the primary path as depicted in figure 2.4. In its simplest form the reference signal $x(n)$ is passed through $W(z)$, a model of the primary path, $P(z)$, in order to generate an anti-noise signal $y(n)$[1, 2]. This signal is then subtracted from the desired signal $d(n)$ resulting in the error signal $e(n)$. The filter is based on a common system identifier used to model the channel $P(z)$[2]. The identifier uses an adaptive algorithm that adjust the filter taps of $W(z)$, $\mathbf{w}(n) = [w_0(n), \ldots, w_M(n)]^T$ using the error signal with the reference signal[1]. There are numerous adaptive algorithms that can be applied to the ANC system in order to adjust the coefficients $\mathbf{w}(n)$, these algorithms will be discussed later in chapter 3.2. For now an arbitrary adaptive algorithm (AA) will be used to adjust the coefficients $\mathbf{w}(n)$. Ideally this system completely cancels the noise. When an adaptive filter algorithm is used to adjust the coefficents $\mathbf{w}(n)$, the error sensed by the error reference microphone becomes essential for adaptive performance. The error signal recieved in the electric domain is affected by the transfer from the output of the filter, $W(z)$, to the input of the ANC control system. This path, commonly referred to as the secondary path may have some serious effects on the performance of the ANC system[1].

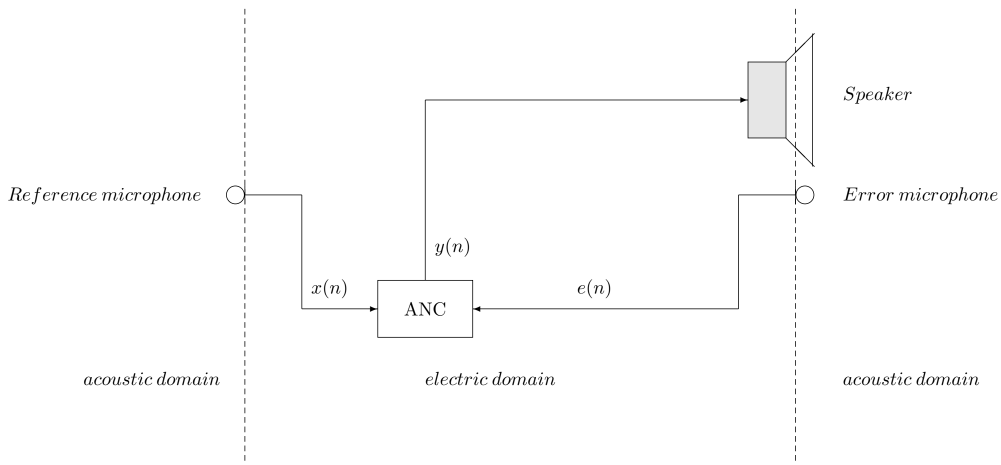

Figure 2.4: A Feedforward ANC system for a handheld mobile phone

2.3.1 Secondary Path

The secondary path is not only the analog signal path from the loudspeaker to the error microphone[2]. It also includes all analog and digital components the signal passes through such as analog to digital (A/D) and digital to analog (D/A) converters, filters and amplifiers[25]. Here the loudspeaker and microphone affects the signal and may introduce delays, alter the frequency content as well as introduce nonlinear distortion[9]. This may cause the anti-noise signal to be less correlated to the reference signal, affecting the performance of the ANC system and may even cause instability. Using adaptive feedforward control without compensating for the secondary path, the controller filter $W(z)$ will try to estimate

$$W(z) = \frac{P(z)}{S(z)}.$$

This may require $W(z)$ to be a very long FIR filter or an infinite impulse response (IIR) filter[1]. None of these two options are very practical, since this requires the secondary path to be time invariant, which in most cases it is not. It is therefore common to try to compensate for the secondary path effects using other methods. There are two general ways of addressing the effects of the secondary path. The first is to connect an inverse of the secondary path in series with it, and the second alternative is to compensate for the effects the secondary path have on the error signal, by filtering the reference signal with a estimate of the secondary path before it is passed to the adaptive algorithm[2, 1]. These methods will be explained in more detail in the following sections.

Inverse modeling

A direct method for dealing with the problems introduced by the secondary path is to model the inverse[1] and connecting it in series with the channel itself. Theoretically this should efficiently cancel out the effects of the secondary path. There are however a few problems involved with using this modeling technique. Suppose that the transfer function of the secondary path has a number of zeroes in it. These zeroes will become poles in a inverse of the transferfunction. To represent poles one need to either use an IIR filter, or approximate it with a high order FIR filter. The second is not very practical to use, since it will be heavy computationally and increase the delay in the ANC system. Implementing an IIR filter is generally not a good idea either, since IIR filters usually have stability issues when implemented in a fixed point environment[3]. Another major issue with this method is that the the secondary path is not guaranteed to be invertible at all, since the secondary path often is time varrying and not minimum phase[2].

Feedforward filtered-x ANC

The filtered-x method[1] is a common way of compensating for the effects of the secondary path. In figure 2.5 the filtered-x ANC block diagram is shown. The reference signal is filtered through a estimate of the secondary path, $\hat{S}(z)$, ideally affecting it in the same way as the real secondary path $S(z)$. As seen in equation (2.2) the filtered reference signal vector, $\mathbf{x}'(n)$ is the result of convolving the impulse response of the secondary paths estimate $\hat{\mathbf{s}}$ with the reference signal vector $\mathbf{x}(\mathbf{n})$. This will increase the correlation between the reference signal and the error signal[1], which are the two signals used to update the adaptive filter coefficients $\mathbf{w}(\mathbf{n})$, and this will therefore generally improve the ANC performance. It is important that the secondary channel does not introduce more delay than the system can handle. The delay of the primary path must be longer or equal to the delay the adaptive filter and secondary channel introduce together[1]. The filtered-x ANC system is tolerant to errors in the estimate of the secondary channel, and can handle phase errors of up to nearly 90 degrees between the real channel $S(z)$ and the estimated channel $\widehat{S}(z)$, under slow adaption[2, 23]. For faster convergence a smaller phase error of less than 50 degrees is recommended[23]. The ANC system is also sensitive to magnitude changes in the frequency response of the secondary path. A larger error in magnitude between the estimated secondary path and the real secondary path will make the tolerance of phase error decrease[23]. The equations for this filtered-x feedforward system are provided below.

$$e(n) = d(n) - \mathbf{s}(n) * [\mathbf{w}^T(n) \cdot \mathbf{x}(n)] \tag{2.1}$$

$$\mathbf{x}'(n) = \widehat{\mathbf{s}}(n) * \mathbf{x}(n) \tag{2.2}$$

$$\mathbf{w}(n+1) = AA(\mathbf{w}(n), \mathbf{x}'(n), e(n)) \tag{2.3}$$

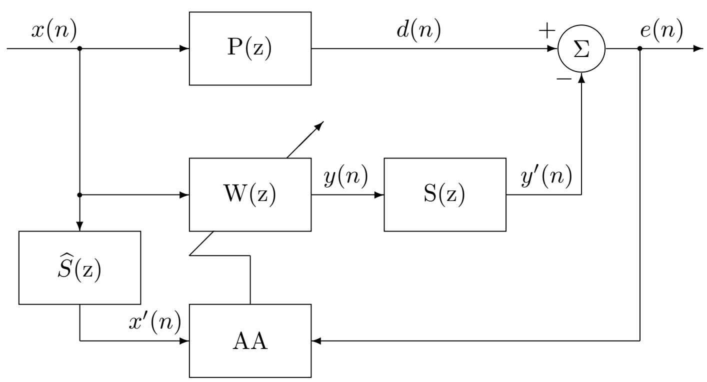

Figure 2.5: The Filtered-x Feedforward ANC, with arbitrary adaptive algorithm

2.3.2 Acoustic Feedback

When generating an anti-noise signal to reduce the primary noise, some of the anti-noise signal might leak back to the reference microphone. This acoustic feedback can greatly affect the performance of the system and should

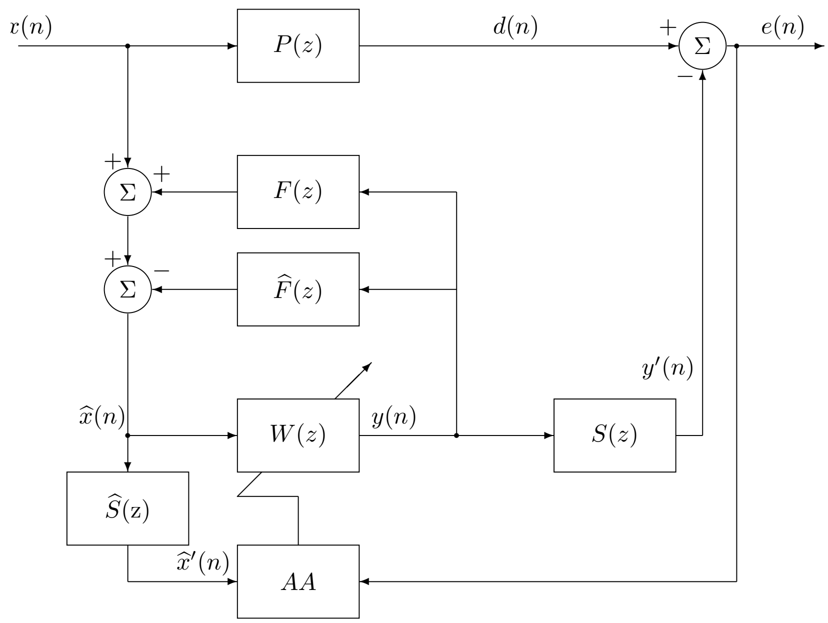

Figure 2.6: Feedback neutralization implemented in a Filtered-x Feedforward ANC system

be considered depending on the requirements of the system. There are numerous ways of dealing with the acoustic feedback. Some of the methods introduce new hardware such as loudspeakers and microphones to deal with the feedback[1]. However, the most common way to deal with the effects of the acoustic path $F(z)$ is to apply feedback neutralization or sometimes called feedback cancellation[12].

Feedback Neutralization

The concept of feedback neutralization is to model the acoustic feedback path with a filter, $\widehat{F}(z)$, see figure 2.6 for a block diagram of the feedback neutralisation. The anti-noise signal is passed through the model of the feedback channel and subtracted from the reference signal[12, 1]. Ideally this will completely cancel the feedback. This filter can be implemented using either a fixed or adaptive filter, depending on the system requirements. Since the channel is subject to changes, it is common to use some sort of adaptive implementation. The model can usually be estimated offline, however, since the acoustic feedback path occupies the same space as the primary path, the acoutic feedback path might be subject to similar variations as the primary path. In these case the model might have to be estimated online. If the feedback neutralization filter $\widehat{F}(z)$ is modeled online, the filter will however not stop to adapt even when all the feedback is cancelled, and the filter will consequently try to remove the reference signal from the input. This could become a problem that would have to be considered if online estimation is to be used[1, 25].

2.4 Feedback ANC

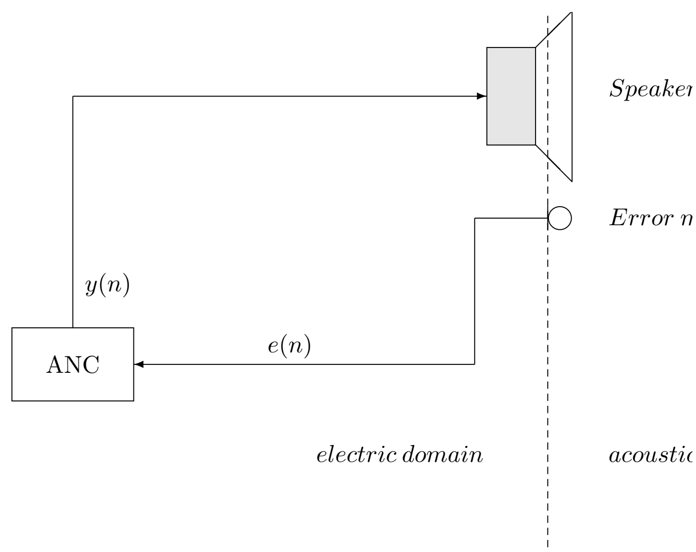

Figure 2.7: Conceptual Feedback ANC on a mobile phone.

Feedforward ANC works very well when a coherent reference signal is available[1]. In some cases, for practical reasons, a coherent reference signal is not available to the reference sensor. In feedback ANC the reference signal is instead

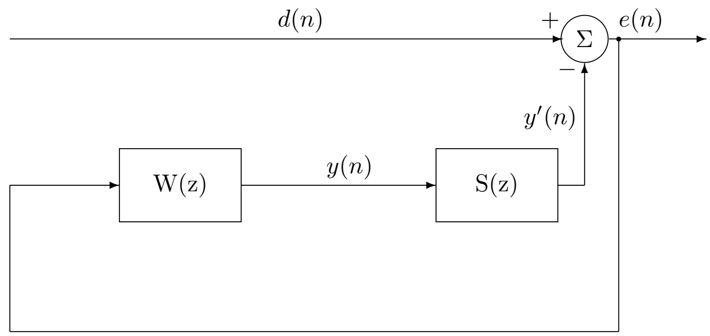

Figure 2.8: Block diagram of non-adaptive feedback ANC.

synthesized from estimations based on the anti-noise signal $y(n)$ and the error signal $e(n)$[1, 2]. A schematic picture of a feedback ANC system is shown in Figure 2.7. An important feature of the Feedback ANC is that an acoustic feedback path from the loudspeaker to reference sensor does not exist[1]. The most basic setup uses a non adaptive filter $W(z)$ as a controller to generate the anti-noise signal, as seen in Figure 2.8. The output of the controller, $y(n)$ passes through the secondary channel. This signal is then inverted and added to the primary noise $d(n)$. The error sensor measures the remaining residual noise $e(n)$, which is fed back to the controller. The feedback ANC aims to predict the primary noise and can thus be interpreted as a predictor. The z-transform of the error signal can be expressed as

$$E(z) = D(z) - S(z)W(z)E(z) \Rightarrow E(z) = \frac{D(z)}{1 + S(z)W(z)} \tag{2.4}$$

This concept can then be extended to deal with adaptive algorithms for updating the filter coefficients of $W(z)$. As previously stated in section 2.3.1 the secondary path is an important aspect to take under consideration. The feedforward filtered-x ANC system was discussed as a good way to deal with the effects of the secondary path. This method of filtering the reference signal can also be applied to the feedback ANC[2].

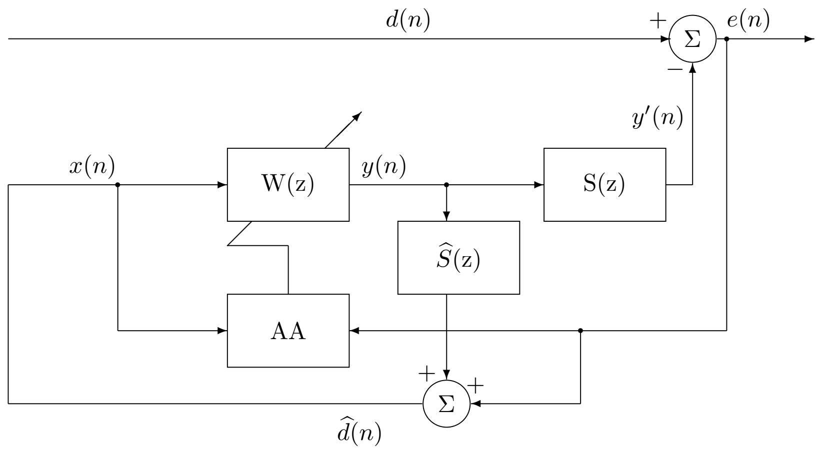

Figure 2.9: Block diagram adaptive feedback ANC.

2.4.1 Feedback filtered-x ANC

By filtering the output of the controller and then adding it with the error signal, an estimate of the filtered primary noise is synthesized[1]. This signal is then used as the reference signal, and is filtered through the model of the secondary path before it is fed to the adaptive block, AA, as can be seen in Figure 2.10. The filtered reference signal can then be expressed as[2]

$$X'(z) = X(z)\widehat{S}(z) \tag{2.5}$$

$$X(z) \equiv \widehat{D}(z) = E(z) + \widehat{S}(z)Y(z) \tag{2.6}$$

2.4.2 Feedback ANC Performance

Feedback ANC has a few flaws as well as a few advantages in comparison to feedforward ANC. Feedback ANC does not need a reference sensor, implying that there is no need for the coherence between the signals sensed with different sensors[1]. As previously mentioned in section 2.4, since there is no reference sensor, there is no risk of acoustic feedback. On the other hand, since the feedback ANC can be interpreted as a predictor it works best for

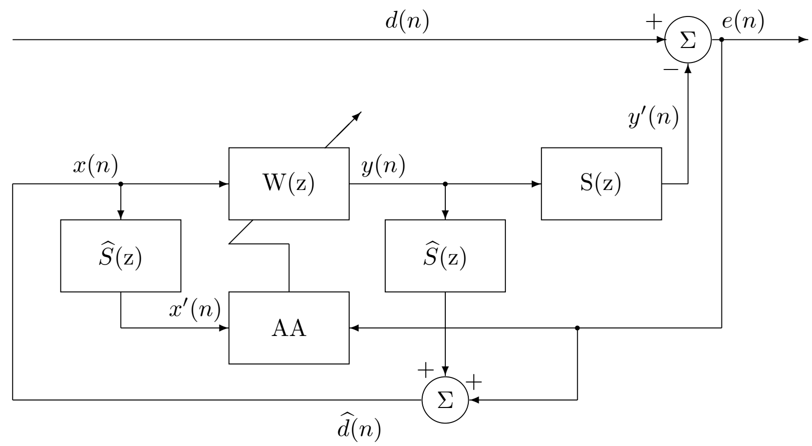

Figure 2.10: ANC filtered-x feedback set-up with arbitrary adaptive algorithm for finding the filter coefficients $W(z)$. Anti-noise signal $y(n)$ and estimated reference signal $x(n)$ are passed through an estimation of the secondary channel $\widehat{S}(z)$.

predictable signals, i.e. narrow band noise, and cannot attenuate wide band noise very well[1]. Another aspect one need to take under consideration is that the delay in the system heavily influences the performance and sets a constraint for how high frequencies it can handle[1].

2.5 Hybrid ANC

For some applications neither the feedback nor the feedforward approach satisfies the requirements for the system. In these cases it may be of interest to use a combination of both, that utilizes the positive characteristics of both methods. In *Active Noise Control Systems — Algorithms and DSP Implementations*[1], Kuo and Morgan suggest such a system called Hybrid ANC. This ANC system is however quite complex and has a high computational complexity. However it might be of interest to further investigate for future applications, as technology keep evolving. Moore's law predicted in the 1960's that the numbers of transistors that can be integrated on a single die would grow exponentially with time, which has proven to hold, and the computational power has increased with it. With this said, it might soon be feasible to implement a hybrid system on a mobile phone, however it is out of scope for this thesis which will be limited to examine the previously mentioned forms of ANC - feedforward and feedback.

2.6 ANC comparison

This is a short summary of the pros and cons of feedforward and feedback ANC systems. The table is an extract taken from *Active Noise Control Systems–Algorithms and DSP Implementations* p.212 Table 6.2[1]. A block-diagram illustrating plant noise can be seen in figure 2.11. Plant noise is additive noise $v(n)$ uncorrelated with the reference signal, introduced along the primary path.

Table 2.1: Adaptive ANC comparison

	Feedforward ANC	Feedback ANC
Filter order	Moderate	High
Spectral Capability	Broadband & narrowband	Narrowband only
Plant Noise	Not canceled	Good cancelation
Noise field coherence	Coherent only	Coherent and incoherent

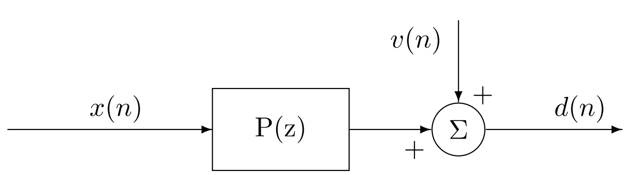

Figure 2.11: Block diagram showing how plant noise $v(n)$ is added to the desired signal $d(n)$ along the primary path $P(z)$

2.7 On the Market

Currently their are no handsets on the market to be found utilizing ANC to reduce background noise as previously described. It is important to differ between noise canceling on the sender and receiver side of the communication. There are many products available for the speech microphone on the sender side of the transmission, but none that operates on the loudspeaker on the receiver end of the communication link. Such technology primarily exists in hands-free headsets[26] where it is in combination with passive ANC. These typically use a feedback algorithm to cancel the noise that leak through the passive ANC[26]. After extensive search no handsets were found implanting any kind of ANC on the reciever side of the communication.

Chapter 3

Filter Design and Adaptive Algorithms

In order to design an efficient ANC system it is important to choose a well suited algorithm to perform the adaptation. This chapter will examine optimal filter designs and some important adaptive algorithms.

3.1 Optimal Solutions Signal Processing

It is very common that an desired signal is not directly observable[3], in which case it has to be estimated based on some other observable signal. In order to produce the best estimate of the desired signal, a filter has to be designed–an optimal filter. In *"Statistical Digital Signal Processing and Modelling"*[3], different methods for finding the optimal filter, in particular the FIR and IIR Wiener solutions are discussed. In the following section the basic concepts of finding these optimal solutions are described.

3.1.1 FIR Wiener filter

The wiener filter was designed to produce the optimal estimate of a signal measured or observed in a noisy environment[3]. $x(n)$ is the observed signal, and the wiener solution aims to find the optimal channel estimate that will give a signal as close to the desired signal, $d(n)$ as possible when $x(n)$ is passed through an unknown channel, see figure 3.1

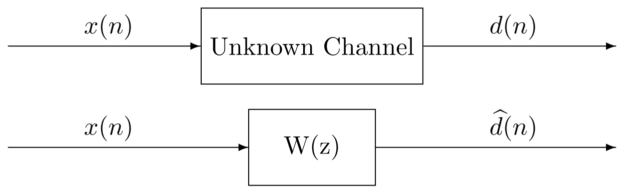

Figure 3.1: W(z) is the wiener solution, the optimal solution for identifying the unknown channel

The wiener solution assumes that both the observed signal $x(n)$ and the desired signal $d(n)$ are Wide Sense Stationary (WSS)[7]. Further the cross-correlation $r_{dx}(k)$, and the autocorrelations $r_d(k)$ and $r_x(k)$ have to exist and be known. The system function of the Wiener filter $W(z)$ with order $p-1$ is defined in equation (3.1)[3]

$$W(z) = \sum_{k=0}^{p-1} w(k) z^{-k}. \tag{3.1}$$

The estimate of the desired signal is the result of convolving the wiener filter coefficients with the observed or measured signal.

$$\hat{d}(n) = \sum_{k=0}^{p-1} w(k) x(n-k). \tag{3.2}$$

The difference between the desired signal and the estimate of the desired signal is known as the error $e(n)$. By minimizing the mean-square error $\xi = E(|e(n)|^2)$ an optimal solution for the wiener filter can be found[3]. The optimal solution is given by

$$\sum_{l=0}^{p-1} w(l) r_x(k-l) = r_{dx}(k), \quad k = 0, 1, ..., p-1, \tag{3.3}$$

where $r_x(k)$ and $r_{dx}(k)$ are the auto and cross-correlations, defined in chapter 4. This is known as the Wiener-Hopf equations, and can be written more concisely as[3]:

$$\mathbf{R}_x \mathbf{w} = \mathbf{r}_{dx}, \tag{3.4}$$

where $\mathbf{R}_x$ is a $p \times p$ positive-semidefinite Hermitian Toeplitz matrix of auto-correlations, $\mathbf{w}$ is the filter coefficients and $\mathbf{r}_{dx}$ is the cross-correlation between the observed and desired signal. The minimum mean-square error is given by

$$\xi_{min} = r_d(0) - \mathbf{r}_{dx}^H \mathbf{w}. \tag{3.5}$$

The correlations are not always known and sometimes they have to be estimated from the signal. Using the definition of auto- and cross-correlation as well as the property of WSS signals, these correlations can be computed as seen in equations (3.6) and (3.7)[3]

$$r_x(k) = E[x(n)x^*(n-k)] \tag{3.6}$$

$$r_{dx}(k) = E[d(n)x^*(n-k)], \tag{3.7}$$

where $E[\cdot]$ is the expectation value operator[7]. This solution yields the causal FIR wiener filter. In some cases the correlation matrix $\mathbf{R}_x$ may be singular or close to singular, i.e it has eigenvalues that are equal to, or close to zero. In these cases it is not feasible to take the inverse of $\mathbf{R}_x$. Instead one can use a pseudo-inverse of $\mathbf{R}_x$: $\mathbf{R}_x^{\dagger}$[6]. This is accomplished using singular value decomposition.

Singular Value Decomposition

Singular Value Decomposition (SVD) and how it is used to find a pseudo inverse is a useful tool in signal processing. In *"Mathematical Methods and Algorithms for Signal Processing"*[6], Moon describes SVD thoroughly. The following section is a summary of the concepts described by Moon.

A matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$ can be factored using SVD

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{3.8}$$

where $\mathbf{U} \in \mathbb{C}^{m \times m}, \mathbf{V} \in \mathbb{C}^{n \times n}$ are unitary matrices. $\mathbf{\Sigma} \in \mathbb{R}^{m \times n}$ is a diagonal matrix containing the singular values σ_i of $\mathbf{A}$. Usually the singular values are ordered by size such that

$$\mathbf{\Sigma} = diag(\sigma_1, \sigma_2, ..., \sigma_p) \tag{3.9}$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_p \geq 0 \tag{3.10}$$

with $p = min(m, n)$.

Given a system of the form $\mathbf{Ax} = \mathbf{b}$, where one aims to solve for $\mathbf{x}$, the system can then be re-written using SVD as $\mathbf{U\Sigma V}^H\mathbf{x} = \mathbf{b}$ and

$$\mathbf{U\Sigma V}^H\mathbf{x} = \mathbf{b} \Leftrightarrow \mathbf{U}^H\mathbf{U\Sigma V}^H\mathbf{x} = \mathbf{U}^H\mathbf{b} = \mathbf{\Sigma V}^H\mathbf{x}.$$

To find the solution for x the inverse of $\mathbf{\Sigma}$ needs to be computed as

$$\mathbf{\Sigma}^{-1} = diag(1/\sigma_1, 1/\sigma_2, ..., 1/\sigma_p). \tag{3.11}$$

However if the singular values are close or equal to zero the equations are poorly conditioned and computing the inverse is not feasible. Instead a pseudo-inverse of $\mathbf{\Sigma}$ can be used to approximate the solution. By defining a bound σ_{bound} for how small singular values are allowed and setting all the non-allowed singular values to zero the pseudo inverse can be computed as follows:

$$\sigma_r \geq \sigma_{bound} \geq \sigma_{r+1} \Rightarrow \hat{\mathbf{\Sigma}} = diag(\sigma_1, \sigma_2, ...\sigma_r, 0, ..., 0) \tag{3.12}$$

$$\hat{\mathbf{\Sigma}}^\dagger = diag(1/\sigma_1, 1/\sigma_2, ...1/\sigma_r, 0, ..., 0). \tag{3.13}$$

With the equation system defined previously this yields the approximate solution for $\mathbf{x}$

$$\mathbf{Ax} = \mathbf{b} = \mathbf{U\Sigma V}^H\mathbf{x} \Leftrightarrow \tag{3.14}$$

$$\Leftrightarrow \mathbf{\Sigma V}^H\mathbf{x} = \mathbf{U}^H\mathbf{b} \Rightarrow \tag{3.15}$$

$$\Rightarrow \hat{\mathbf{x}} = \mathbf{V}\hat{\mathbf{\Sigma}}^\dagger\mathbf{U}^H\mathbf{b}. \tag{3.16}$$

This will generate a robust approximation of $\mathbf{x}$ with minimum norm error. By applying this method to ill-conditioned equation systems, an approximation of the optimal wiener solution can be computed even when the correlation matrix is not invertible. The selection of the bound is problem-dependent[6].

3.1.2 IIR Wiener filter

Another form of solution is the noncausal IIR Wiener solution[3] given by

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}. \tag{3.17}$$

The Wiener-Hopf equations for the noncausal IIR filter are[3]

$$\sum_{l=-\infty}^{\infty} h(l)r_x(k-l) = r_{dx}(k), \quad -\infty < k < \infty \tag{3.18}$$

which consequently can be expressed as

$$r_{dx}(k) = h(k) * r_x(k). \tag{3.19}$$

In the frequency domain this convolution becomes a multiplication, which yields an expression for the IIR Wiener filters frequency response as shown below[3]

$$\mathcal{F}(h(k) * r_x(k)) = H(e^{j\omega})P_x(e^{j\omega}) = \mathcal{F}(r_{dx}(k)) = P_{dx}(e^{j\omega}) \Rightarrow \tag{3.20}$$

$$\Rightarrow H(e^{j\omega}) = \frac{P_{dx}(e^{j\omega})}{P_x(e^{j\omega})},$$

where $P_x(e^{jw})$ is the power spectral density of $x(n)$ and $P_{dx}(e^{jw})$ is the cross power spectral density of $x(n)$ and $d(n)$, defined in chapter 4.

3.2 Adaptive Algorithms

In order to adjust the filter coefficients of an impulse response, an adaptive algorithm may be utilized. In this section some of the most common adaptive algorithms are covered. There are two main methods for adaptive filtering, a stochastic gradient method and a deterministic method[5]. The stochastic gradient method uses least mean-square error estimations to estimate the gradient and update the filter coefficients[5]. The least mean-squares algorithms performance are dependent on some statistical properties of the signals as it follows directly from the wiener solution, which assumes that the signals are WSS[3]. In a lot of cases the gradient is estimated using instantaneous estimates of the ensembles averages[3] see equation (3.21).

$$\hat{E}[e(n)x^*(n-k)] = e(n)x^*(n-k) \tag{3.21}$$

Sometimes this method leads to slow convergence or a too large excessive error. In these cases a deterministic approach might be more suitable. The Least Squares method does not depend on any assumptions regarding statistical properties and it aims to minimize the sum of the squares of the error[5], more on this in section 3.2.5.

3.2.1 Steepest Decent

The method of Steepest Descent is a recursive method, using known statistics, $\mathbf{R}_x$ and $\mathbf{r}_{dx}$, to find the Wiener filter coefficients. The method saves computation time by using the Wiener-Hopf equations recursively so that no inversion of $\mathbf{R}_x$ is needed[5]. This is done by defining a cost function $J(\mathbf{w}(n)$ and a constraint on the cost function that is $J(\mathbf{w}_o) \leq J(\mathbf{w}(n))\ \forall n$[5]. $\mathbf{w}_o$ is the optimal wiener solution and $\mathbf{w}(n)$, is the filter coefficients after n iterations. For simplicity reasons $J(\mathbf{w}(n))$ will be denoted $J(n)$. The definition of the cost function can be seen in equation (3.23). The gradient of the cost function $\nabla_w J(n)$ is used to find the direction in which the cost function decreases most rapidly. The recursive update equation for the filter as well as the gradient of the cost function is shown in equations (3.24) and (3.25) respectively[5]. As seen in equation (3.25) the filter coefficients are updated by taking a step towards the opposite direction of $\nabla_w J(n)$ with step size μ.

$$e(n) = d(n) - \mathbf{w}^H(n)\mathbf{x}(n) \tag{3.22}$$

$$J(n) = E[e(n)e(n)^*] = E[e(n)(d(n) - \mathbf{x}(n)\hat{\mathbf{w}}^H(n))] \tag{3.23}$$

$$\nabla_w J(n) = 2E[-e(n)\mathbf{x}^*(n)] = -2\mathbf{r}_{dx} + 2\mathbf{R}_{\mathbf{x}}\mathbf{w}(n) \tag{3.24}$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{1}{2}\mu\nabla_w J(n), \tag{3.25}$$

By repeating this the least mean-square error is reduced successively and the optimum filter coefficient is approached. This method works well with known statistics, but cannot operate on signals with unknown statistics[5].

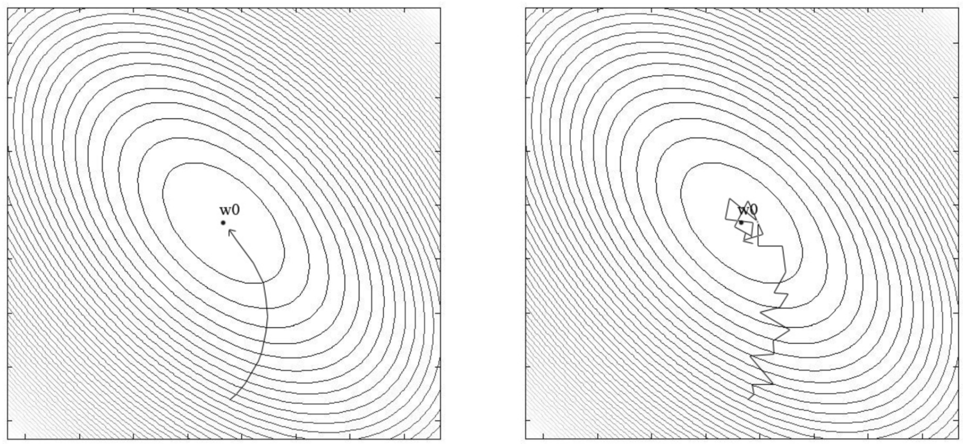

Figure 3.2: *Left*:Steepest descent *Right*:LMS. Example of how gradient noise perturbs the convergence path for the LMS algorithm. $w0$ is the optimal wiener solution.

3.2.2 Least Mean-Squares

The Least Mean-Squares (LMS) algorithm is based on the same concept as the Method of Steepest Descent, however it uses estimated statistics to obtain the optimal filter and is hence used when the statistics are not known[5]. Since the algorithm is dependent on updated estimates of the signal, the algorithm will adapt to changes in the signal. Since $\mathbf{R}_x$ and $\mathbf{r}_{dx}$ are not known, these have to be estimated. The LMS algorithm uses minimum mean-squared error estimations to approximate the gradient. The cost function $J(n)$ can then be summarized to equation (3.29)[5]. The update equation (3.28) show how the estimated gradient is used to update the filter coefficients. This step is sometimes called stochastic gradient descent[6], since the gradients are based on estimates. The convergence rate is governed by the stepsize applied to the algorithm, and it will converge in mean with a stepsize within the bounds set in equation (3.26)[5], where λ_{max} is the maximum eigenvalue of $\mathbf{R}_x$.

$$0 < \mu < \frac{2}{\lambda_{max}} \tag{3.26}$$

$$e(n) = d(n) - \mathbf{w}^H(n)\mathbf{x}(n) \tag{3.27}$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu}{2}\nabla_w J(n) \tag{3.28}$$

The LMS algorithm suffers from gradient noise. In every iteration the LMS algorithms takes a step towards the optimal solution, the optimal solution that the algorithm see however changes depending on the estimates of the statistics. The gradient noise will cause the convergence path to become noisy, see figure 3.2 and also prevent the LMS algorithm from complete convergence[5]. When the number of iterations, n, tends towards infinity, the LMS-algorithms cost functions will reach a constant final value $J(\infty)$. The distance between this and the minimum error J_{min} is the excess mean-square error $J_{ex}(\infty)$[5]. This relation is decribed by equation (3.31).

$$J(n) = |e(n)|^2 \tag{3.29}$$

$$\nabla_w J(n) = -2e(n)\mathbf{x}^*(n) \tag{3.30}$$

$$J(\infty) = J_{min} + J_{ex}(\infty) \tag{3.31}$$

3.2.3 nLMS

The LMS algorithm suffers from whats called a *gradient noise amplification problem*[5]. This means that when the energy in the input signal of the system changes, it will greatly effect the performance of the algorithm. A constant step size will thereby cause slow convergence rate when μ is small in comparison to the upper bound in equation (3.26) and may cause instability when μ is close to or above this upper bound [8]. Since real signals commonly are non stationary, it would be beneficial to use a time varying step size that adapts to the input signals energy. Normalized LMS achieves this by normalizing the step size using the energy of the input signal. The derivation of this algorithm follows from the *the principle of minimal disturbance*. In *Adaptive Filter Theory*[5] p.321 this is described as follows "From one iteration to the next, the weight vector of an adaptive filter should be changed in a minimal manner, subject to a constraint imposed on the updated filter's output". Equation (3.32) show the weight norm that is formulated by this principle, and equation (3.33) the constraint. For a detailed derivation see *Adaptive Filter Theory*[5]. The weight vector and the constraint result in the cost function seen in equation (3.34). Minimizing this to find the optimum value of the update multiplier λ is done by taking the derivative of the cost function with repect to the new weight vector $\mathbf{w(n+1)}$, and then setting the result to zero, see equation (3.35). Solving for λ results in the optimum update multiplier seen in equation (3.36)[5]. Where $||\cdot||_F$, is the euclidean norm operator[6]

$$\delta\mathbf{w}(n+1) = \mathbf{w}(n+1) - \mathbf{w}(n) \tag{3.32}$$

$$\mathbf{w}^H(n+1)\mathbf{x}(n) = d(n) \tag{3.33}$$

$$J(n) = (\mathbf{w}(n+1)-\mathbf{w}(n))^H(\mathbf{w}(n+1)-\mathbf{w}(n))+Re[\lambda^*(d(n)-\mathbf{w^H(n+1)x(n)})] \tag{3.34}$$

$$\frac{\delta J(n)}{\delta\mathbf{w}(n+1)} = 2(\mathbf{w}(n+1)-\mathbf{w}(n)) - \lambda^*\mathbf{x}(n) = 0 \tag{3.35}$$

$$\lambda = \frac{2e(n)}{||\mathbf{x}(n)||_F^2} \tag{3.36}$$

By combining the weight vector equation (3.35) with the optimal update multiplier equation (3.36), the equation for updating the adaptive filter is formulated, see equation (3.37). $\tilde{\mu}$ is the normalized step size, which is introduced in order to control the size of each step without disturbing the direction[5]. In order for the nLMS algorithm to be stable in the mean-square error sense, the nLMS step size is bounded by (3.38)[5, 3].

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\tilde{\mu}}{||\mathbf{x}(n)||_F^2}\mathbf{x}^*(n)e(n) \tag{3.37}$$

$$0 < \tilde{\mu} < 2 \tag{3.38}$$

This however introduces a new problem. If the energy in the input tends to zero, the step size tends towards infinity. One way to deal with this is to introduce a small constant ϵ and add to the norm of the input signal in the update equations which yileds a more stable version of nLMS, see equation (3.39)[3]. The computation cost for each iteration when using nLMS opposed to LMS is not very high[3], which is why nLMS is preferrable, especially for input signals with unknown energy.

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\tilde{\mu}}{||\mathbf{x}(n)||_F^2 + \epsilon}\mathbf{x}^*(n)e(n) \tag{3.39}$$

3.2.4 Leaky LMS

In some cases the the autocorrelation matrix of the input signal $\mathbf{x}$ have zeros in the eigenvalues. In [3], Hayes describes this problem and the implications it may have on the adaptive algorithms performance. The convergence rate of the LMS algorithm depends on initial value of filter coefficients and eigenvalue spread of the autocorrelation matrix, $\chi(\mathbf{R}_x)$ defined in equation 3.40[5].

$$\chi(\mathbf{R}_x) = \frac{\lambda_{max}}{\lambda_{min}} \tag{3.40}$$

The algorithm converges much slower in the directions of the small eigenvalues, compared to the directions of the larger ones. When an eigenvalue of the autocorrelation matrix is zero, there will be no convergence in this direction. This may cause the filter to drift in the directions of the zero eigenvalue, and can cause instability[5]. For a more detailed explanation see [5] or [24].
Leaky LMS compensates for this drift by introducing a leakage factor that leaks energy from the impulse response of the adaptive filter[5]. This leakage can easily be seen in the cost function, equation (3.41), that consists of two contributing terms, the mean-square error and the filter coefficients[5]. If the solution starts to drift, the filter coefficients' contribution will be very large, and the cost will be very high. This eventually leads to divergence. By introducing a leakage factor, α to penalize the filter coefficients' contribution to the cost function, the cost function can be stabilized and thereby the filter[5]. Minimizing the cost shown in equation (3.41) eventually leads to to the update equation (3.42). For simpler control of the leakage, the leakage factor $(1 - \mu\alpha)$ is represented with γ, and for better convergence properties, the step size is normalized as in section 3.2.3, yielding equation (3.43)[3]. The leakage factor should be relatively close to 1, for good adaptive properties.

$$J(n) = e(n)^2 + \alpha||\mathbf{w}(n)||_F \tag{3.41}$$

$$\mathbf{w}(n+1) = (1 - \mu\alpha)\mathbf{w}(n) + \mu\mathbf{x}^*(n)e(n) \tag{3.42}$$

$$\mathbf{w}(n+1) = \gamma\mathbf{w}(n) + \frac{\tilde{\mu}}{||\mathbf{x}(n)||_F^2}\mathbf{x}^*(n)e(n) \tag{3.43}$$

3.2.5 Recursive Least Squares

Recursive Least Squares (RLS) uses the the method of least squares, an approach where the difference between a desired and estimated signal is squared and summed in order to find a best fit. This concept can be extended to adaptive filtering. In *Adaptive Filter Theory* [5], Haykin discusses Least Squares and Recursive Least Squares adaptive filtering. RLS uses a deterministic approach to adaptively find a best fit filter for the system[18]. Since RLS is deterministic, it does not depend on assumptions ragarding the statistics of the signals. The cost function, $C(n)$, that aims to be minimized can be seen in equation (3.44). $\beta(n,i)$ is a weighting factor introduced to penalize old data[5]. With no weighting factor the RLS algorithm would have infinite memory and thus not adapt very well to changes in the system. A common forgetting factor to use is λ, known as an exponential forgetting factor defined in equation (3.45)[5].

$$C(n) = \sum_{i=1}^{n} \beta(n,i)|e(i)|^2 \tag{3.44}$$

$$\lambda^{n-i} = \beta(n,i), \qquad i = 1, 2, .., n \tag{3.45}$$

The RLS algorithm is initialized by setting the initial filter taps to zero as well as defining $\mathbf{P}(0)$, where $\mathbf{P}$ is the recursively computed inverse of the correlation matrix, see equation (3.46) and (3.47). δ is a perturbation factor, called the regularization parameter, that should be set to a small positive constant for high signal to noise ratio (SNR) and a large positive constant for low SNR[5]. $\mathbf{k}(n)$ is referred to as the gain vector, similiar to step size in the LMS adaptive algorithms. The RLS algorithm uses *a priori error* to update the filter equation, see equation (3.51) which is the error that would be produced if the filter coefficients were not updated[3]. This is opposed to the *a posteriori error*: the error used in the LMS approach.

Inititialization

$$\hat{\mathbf{w}}(0) = \mathbf{0} \tag{3.46}$$

$$\mathbf{P}(0) = \delta^{-1}\mathbf{I} \tag{3.47}$$

Iterations

$$n = 1, 2, ... \tag{3.48}$$

$$\mathbf{g}(n) = \mathbf{P}(n-1)\mathbf{x}(n) \tag{3.49}$$

$$\mathbf{k}(n) = \frac{\mathbf{g}(n)}{\lambda + \mathbf{x}^H(n)\mathbf{g}(n)} \tag{3.50}$$

$$\xi(n) = d(n) - \mathbf{w}(n-1)^H\mathbf{x}(n) \tag{3.51}$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{k}(n)\xi^*(n) \tag{3.52}$$

$$\mathbf{P}(n) = \lambda^{-1}\mathbf{P}(n-1) - \lambda^{-1}\mathbf{k}(n)\mathbf{x}^H(n)\mathbf{P}(n-1) \tag{3.53}$$

The convergence rate of the RLS algorithm are typically faster than those of the LMS algorithms. This is achieved by recursively using the inverse of the correlation matrix of the input signal in order to update the filter[5], basically by calculating the LS solution for the filter coefficient vector in each iteration.

3.2.6 Convergence and Stability Comparison of Adaptive Algorithms

When deciding what algorithm to use, it is important to understand what sets them apart. The RLS and LMS algorithms use two different methods to solve the same problem and these two methods have some pros and cons. The RLS method typically have an order of magnitude faster convergence rate than the LMS algorithm, and does not theoretically depend on the eigenvalue spread of the input signal[5]. When the RLS have a stationary input signal the excess error will tend towards 0 resulting in good convergence. The regularization parameter, δ can be compared to the stepsize μ in the LMS algorithms, since δ heavily affects the dependence on the input signal. When δ^{-1} is large the algorithm will be less robust, and similar if μ is large the LMS algorithm will be less robust[5]. The normalized LMS algorithm uses the input signal's energy to update the stepsize continously, which will improve the robustness and convergence, especially for dynamic signals. When the system is subject to non stationary input signals, a leakage factor can be of importance. The γ paramter in the leaky normalized LMS algorithm and λ in the RLS algorithm aims to serve this purpose, and will increase the robustness for the two algorithms.

Chapter 4

Signal Analysis

In order for the simulations to run properly it is important that the measurement data is being processed accurately. The signals should be observed and analyzed in the frequency domain, which implies that a time/frequency transform will be used. The signals power spectral densities (PSD) and cross power spectral densities need to be observed in order to identify disturbances and nonlinearities. Suppose a system is described by equation (4.1), where $d(n)$ is the output, $x(n)$ is the input signal and $h(n)$ is the system impulse response. The transfer function can then be described by equation (4.2). The corresponding auto and cross correlations are shown in equation (4.3), assuming weak stationarity[3].

$$d(n) = x(n) * h(n) \Leftrightarrow D(z) = H(z)X(z) \tag{4.1}$$

$$H(z) = \frac{D(z)}{X(z)} \tag{4.2}$$

$$\begin{gathered} r_{xx}(k,l) = E[x(k)x^*(l)] \\ r_{dx}(k,l) = E[x^*(k)d(l)] = E[x^*(k)x(l) * h(l)] = r_{xx}(k,l) * h(l) \end{gathered} \tag{4.3}$$

In *"Modern Spectral Estimation–Theory & Application"*[10], the definition of the discrete power spectral density, and the cross power spectral density are stated, these definitions can be seen in equation (4.4) and (4.5) respectively, where f denotes the discrete frequency.

$$P_{xx}(f) = \mathcal{F}(r_{xx}) = \sum_{k=-\infty}^{\infty} r_{xx}(k)e^{-j2\pi fk} \tag{4.4}$$

$$P_{dx}(f) = \mathcal{F}(r_{dx}) = \sum_{k=-\infty}^{\infty} r_{dx}(k)e^{-j2\pi fk} \tag{4.5}$$

If this is applied to the system described above in equation (4.1), then the cross power spectral density can be rewritten as displayed in equation (4.6), which means that the transferfunction can be written as the cross power spectral density divided by the power spectral density of the input signal.

$$P_{dx}(f) = P_{xx}(f)H(f) \Leftrightarrow H(f) = \frac{P_{dx}(f)}{P_{xx}(f)} \tag{4.6}$$

Another interesting property to look at is the coherence of the system. Suppose the system is perturbed by additive noise $v(n)$ uncorrelated with $x(n)$, see equation (4.7). In this case $y(n)$ and $x(n)$ are assumed to be WSS. The coherence between the two signals is defined in equation (4.8)[1] and will give a measure of the how much of the signal $x(n)$ can be directly related to $y(n)$. It is desirable to have a constant coherence for different input amplitudes, and that the coherence is 1 or close to 1 over the designated frequency interval, i.e that the additive noise have as little impact on the system as possible[1].

$$d(n) = x(n) * h(n) + v(n) \Leftrightarrow D(z) = H(z)X(z) + V(z) \tag{4.7}$$

$$C_{xd}(f) = \frac{|P_{xd}(f)|^2}{P_{xx}(f)P_{dd}(f)} = \frac{|P_{xx}(f)H(f)|^2}{P_{xx}(f)(P_{xx}(f)H(f)^2 + P_v(f))} \tag{4.8}$$

A rough estimate of the the best possible performance of the ANC system can be conducted by looking at the coherence[1]. The error functions power spectral density is given by equation (4.9)[1]. The optimum filter minimizing the error power spectral density is given by equation (4.10) $W(f)$, which gives the minimal error shown in equation (4.11)[1].

$$P_{ee}(f) = (1 - C_{dx}(f))P_{dd} + \left| W(f) - \frac{P_{dx}(f)}{P_{xx}(f)} \right|^2 P_{xx}(f) \tag{4.9}$$

$$W_o(f) = \frac{P_{dx}(f)}{P_{xx}(f)} \tag{4.10}$$

$$P_{ee}(f) = (1 - C_{dx}(f))P_{dd}(f) \tag{4.11}$$

An estimate of the best possible reduction by the ANC system at frequency f will therefor be given by equation (4.12)[1]

$$PSD_{ANCperformance}(f) = -10log_{10}(1 - C_{dx}(f)) \tag{4.12}$$

4.1 Spectral Estimation

Periodogram spectral estimator, $\hat{P}_{PER}(f)$, is a common tool for estimating the PSD[3]. The periodogram spectral estimation is defined in equation (4.13). The variance of the PSD using the periodogram spectral estimator can sometimes become an issue. There are however numerous methods of how to estimate the PSD using the concept of periodogram but with different windows and averaging techniques to remove uncertainties from the estimation. Welch metod of averaging periodograms[3, 10] averages windowed periodograms with an desired overlap to reduce the variance of the estimated PSD. The PSD estimate using Welch's method $\hat{P}_W(f)$ can be expressed as seen in equation (4.14), where N is the total number of samples, L is the block length, K is the number of sequences and D is the overlap in samples. W is the applied window, and W_E denotes the average energy in the window, see equation (4.15).

$$\hat{P}_{PER}(f) = \frac{1}{N}\left|\sum_{k=0}^{N-1} x(k)e^{-jk2\pi f}\right|^2 \tag{4.13}$$

$$\hat{P}_W(f) = \frac{1}{NLW_E}\sum_{i=0}^{K-1}\left|\sum_{k=0}^{L-1} w(n)x(k+iD)e^{-jk2\pi f}\right|^2 \tag{4.14}$$

$$W_E = \frac{1}{L}\sum_{k=0}^{L-1}|w(k)|^2 \tag{4.15}$$

The overlap used in Welch's method is typically 50%[3]. Choice of windows depend on the type of signal that is to be measured, it is usually desirable to have high attenuation in the sidelobes for a more accurate broadband noise PSD. More on windowing techniques can be found in [20, 3]. The resolution of the PSD is given by $\frac{F_S}{L}$. Since the blocklength directly affects the resolution, the length should be decided such that the desired resolution is attained. There is however a tradeoff between resolution and variance of the PSD, which have to be considered. Chosing a too short window size will also result in biasing error of the PSD estimate. See figure 4.1 for an example of PSD with different block sizes.

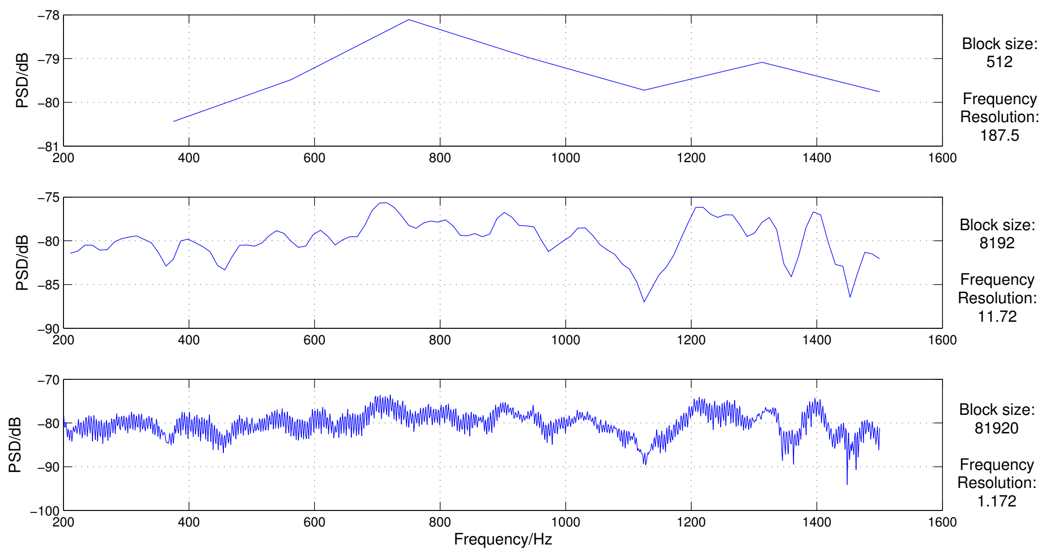

Figure 4.1: PSD of noise recorded at 96kHz. Blocksizes: 512, 8192, 81920 samples

Chapter 5

Measurements and Modelling of a Mobile Phone

When performing measurements on acoustical vibrations i.e. sound, a few questions have to be considered. Inspiration for setting up the measurements was found in "Handbook of Acoustical Measurements and Noise Control"[9]

- What data is required, what quantities are to be measured and to what accuracy?
- Are there influential ambient noise and other interfering sources?
- What is the dimensions and characteristics of the main noise source and measuring microphones? How are these affecting the emission/recording?
- What are the directional characteristics of the source and microphones?
- What instruments are to be used, what are their operating ranges?

The following sections will propose answers to these questions.

5.1 Measurement Setup

The purpose of these measurements is to gather data needed to run an ANC simulation of a handheld mobile phone, following the concept displayed in figure 5.1. In order to measure the data two plastic dummy phones were construced at ST Ericsson. The constructions can be seen in figure 5.2 and

5.3. These will be used to measure the acoustic and electroacoustics path in the ANC system using Head Acoustics *Head and Torso Simulator*(HATS) with artificial ear version 3.3. The HATS head is 16 cm from ear to ear, 20 cm deep and 20 cm tall. The height including the torso is 40 cm.

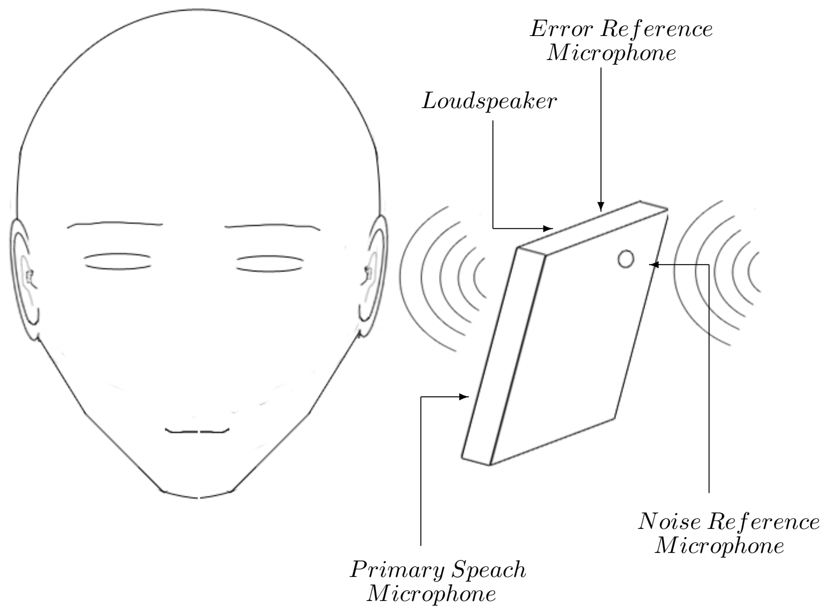

Figure 5.1: Head Acoustics HATS and dummy phone for recording noise, displaying the loudspeaker and microphones placements.

The channels that needs to be estimated are:

- **Primary Path** (P)
  The acoustic path between Noise reference microphone and the Error reference microphone needs to be estimated since this path will set important physical limitations to our ANC system. The channel may vary depending on position that the mobile phone is held in, and should be estimated and evaluated for different position in order to acquire the necessary information concerning the dynamic properties.

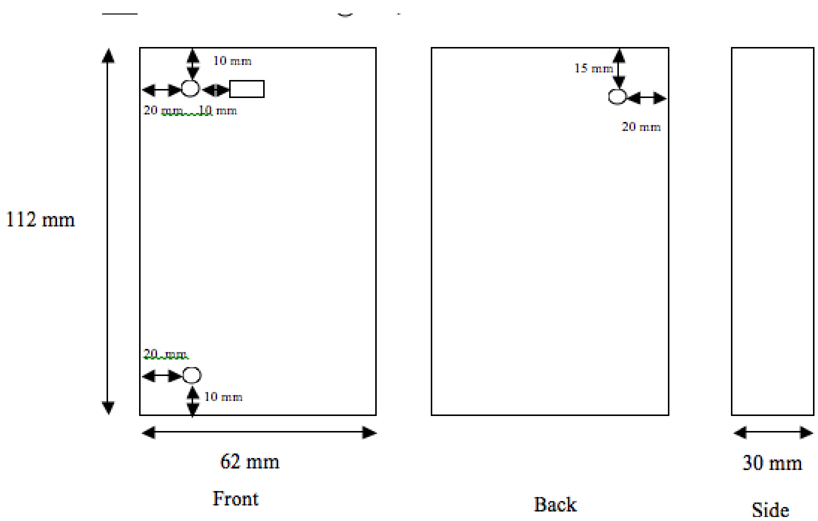

Figure 5.2: Model 1. *Front*-Top microphone denoted: Error Reference Microphone, lower microphone dentoted: Primary Speech Microphone, loudspeaker denoted: Loudspeaker. *Back*-Microphone dentoed: Noise Reference Microphone

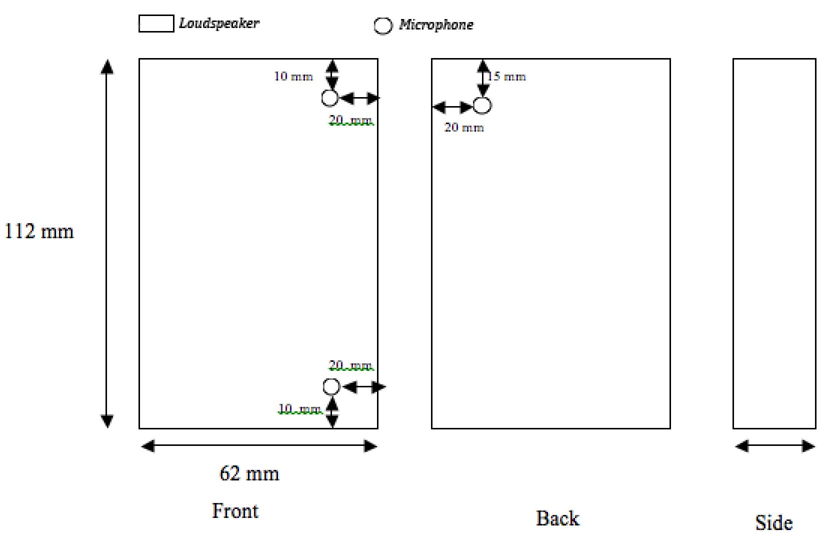

Figure 5.3: Model 2. *Front*-Top microphone denoted: Error Reference Microphone, Lower microphone dentoted: Primary Speech Microphone. *Back*-Microphone denoted: Noise Reference Microphone

- **Secondary Path** (S)
  The channel between the loudspeaker and the error microphone (including the effects of the microphone and the loudspeaker themselves with filters, amplifiers and converters) is required to be estimated both for the feedforward and the feedback ANC performance evaluation. This path should be estimated for a number of different holding positions of the dummy phone in order to acquire information concerning the dynamic properties of the secondary path for these different positions.

- **Acoustic Echo Path** (E)
  The acoustic echo path is the channel between the loudspeaker and the primary speech microphone. Acquiring data regarding this channel is of special importance for echo canceling considerations. The path should be estimated for a number of different holding positions of the dummy phone in order to acquire information concerning the dynamic properties of the acoustic echo path for these different positions.

- **Acoustic feedback path** (F)
  Estimate of the acoustic feedback between loudspeaker and Noise reference microphone. Knowledge of dynamic properties of this path may affect the selection of controller structure. This path should be estimated for a number of different holding positions of the smart phone model in order to acquire information concerning the dynamic properties of the acoustic feedback path for the different positions.

In order to estimate these channels, a few different measurement setups will be needed. The basic layout of the ANC system is shown in figure 5.5. In this block diagram no feedback or echo is presented. There will be two main sources available in order to conduct the measurements; an external source simulating background noise, and the loudspeaker in the dummy phone Model 1. In figure 5.4, the setup using external speakers is shown. The measurements are carried out at ST Ericssons acoustic lab in Lund. The primary channel P will be estimated using the external source, while all the other channels will be estimated using the dummy phones loudspeaker, see figure 5.6 for measurement setups. For more advanced performance analysis, measurements have to be made in real-time when the ANC system is online, since the channels may vary a lot when a person is holding it. These measurements are however out of scope for this project, but should naturally be conducted before implementation.

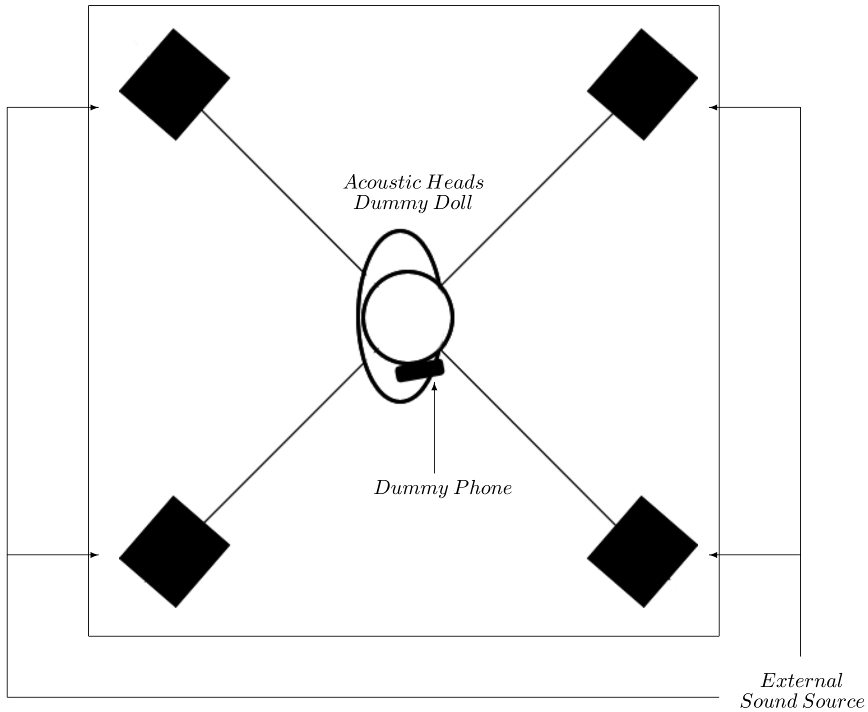

Figure 5.4: Conceptual picture of ST Ericssons acoustic laboratory displaying placement of external speakers and dummy doll for measurements

5.2 Initial Calibration and Measurements

The microphones used in the dummy phone are Panasonic *Omnidirectional Back Electret Condenser Microphones* from the WM-61A series. The microphones have a fairly smooth frequency response and should probably not introduce any issues. The loudspeaker used in the dummy phones is a Philips 16mm MALT SPEAKER with spring contacts. The frequency response for this loudspeaker was observed in the data sheet. It is not designed for the low frequency range and has large attenuation for low frequencies, but since it is part of the secondary path, it should not introduce any issues, unless there are non linearities in the response. The signals used for estimating the primary path both experience the same effect from the microphones frequency response, thus their influence in the estimate of the primary path can be neglected. In order for the signals to be as accurate as possible it

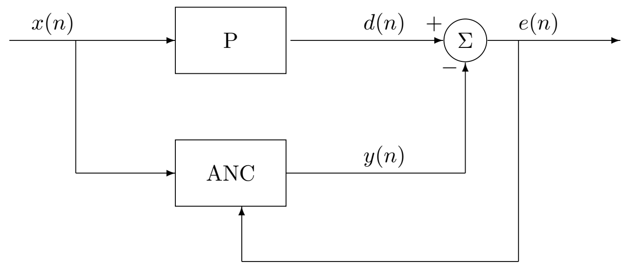

Figure 5.5: Block diagram of feedforward setup. $x(n)$ is the reference signal, recorded with the noise reference microphone. P is the primary path. $d(n)$ is the desired signal, recorded with the error reference microphone. $y(n)$ is the anti-noise signal, generated using the loudspeaker. ANC-block for generating the anti noise signal, and $e(n)$ is the error signal, the resulting signal after interference between the desired signal and the anti-noise signal.

is recommended to set the gain so that the signals digital representations are as resolute as possible. As the measurements are performed in a sound insulated environment, ambient noise sources should not influence the results significantly. It is also important for accurate estimations that there are no other interfering noise sources. For analysis purposes it may however be interesting to look at the ambient noise level during the measurements, which could be recorded as an initial measurement.

5.3 Noise signals for ANC evaluation

In order to identify the five paths, two different kinds of measurements will be needed: The first is when the output is recorded using the dummy phones loudspeaker and the second measurements is when the signal is recorded using a background noise generator. All measurement will be done using 96kHz sampling rate and 16 bit word length, which is the highest sampling rate the sound card can handle. In order to collect all the necessary data some measurements are to be performed. Every measurement should be performed at three different sound pressure levels. This is to observe what effects the sound pressure level (SPL) have on the linearity of the transfer functions at three different realistic phone application sound levels. For instance a relative

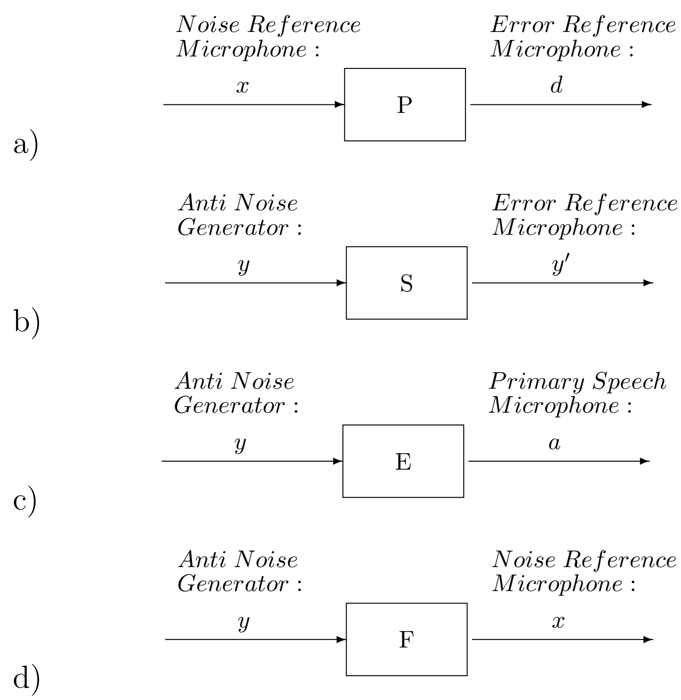

Figure 5.6: Measurement setups: a) Block diagram for estimating the primary path P, from x to d. External source recorded with Noise Reference Microphone and Error Reference Microphone. b) Block diagram for estimating secondary path S, from y to y'. Anti noise signal through loudspeaker recorded with Error Reference. c) Block diagram for estimating acoustic echo path E, from y to a. Anti noise signal through loudspeaker recorded with Primary Speech Microphone. d) Block diagram for estimating acoustic feedback path F, from y to x. Anti noise signal through loudspeaker recorded with Noise Reference Microphone.

volume of -10dB, 0 dB, +10dB will be used for the internal loudspeaker. The SPL was measured 1 cm from the internal loudspeaker, setting the reference level 0dB as 63dB SPL. When the noise is generated and passed through the external loudspeaker, the sound pressures 58.3 dB SPL, 68.3 dB SPL and 78.3 dB SPL was used. That is a relative gain, again of -10dB, 0 dB, +10dB. The dummy phone can be set to be pressed towards the ear with different pressure, how this perturbs the system is also important to evaluate. The pressure towards the ear is usually tested for around 2N and 8N at ST Ericsson, representing relaxed respectively normal pressure of the phone towards the ear. The third pressure of 0N towards the ear, was tested to see how barely touching the ear would affect the acoustic and electroacoustic paths. Different positions of the phone in relation to the head will also be recorded and evaluated. The angle describes the angular difference from a straight line from the ear to the mouth with the ear as axis. A positive angle is down towards the throat and a negative angle is up towards the forehead. The microphones that will be recorded are:

- Noise Reference Microphone: NRM
- Error Reference Microphone: ERM
- Primary Speech Microphone: PSM

The two different sound sources will be called

- External Sound Source: ES
- Internal Sound Source: IS

The channels will be estimated using WGN but to evaluate the performance of the ANC model some other signals will be used. The following list describes the signals of interest. A detailed measurement list is available in Appendix B.1.

Signals for channel estimations:

- White Gaussian Noise (WGN)

Signals for ANC performance evaluation
Stationary:

- WGN

Non-Stationary:

- Café noise
- Chirp signal: Sweeping sinusoid between 100 and 1100 Hz

5.4 Preparation and Syntax

The sound interface used for these measurements is Edirol FA-101 Sound card - 192 kHz - 24-bit. Input 1 and 2 have microphone preamps, and will not be used, instead Input 3, 4 and 5 will be used. A schematic picture of how the measurement equipment is connected can be seen in figure 5.7. The noise for the measurements using the internal speaker was generated in MATLAB and passed through the external Sound Card to the Secondary Source. When performing measurements using the built in loudspeaker, delay from the Sound Card and computer was introduced to the system, currently measured to 73 samples. This system delay will be removed from the measurements used in the analysis. When generating sound fields using the external speakers, Head Acoustics *Head Auto EQ* was used. The noise signals were measured for 1 minute, and after recording, 2 seconds were cut off from begining and end of the data, resulting in 56 seconds of data with a sampling rate of 96kHz. The simulation tools have been designed to handle *.wav* files to set up the channel estimations. The estimations can then be saved in a *.mat* MATLAB file from the Estimation GUI, see appendix A, so that the desired simulation setups can be saved and re-loaded.

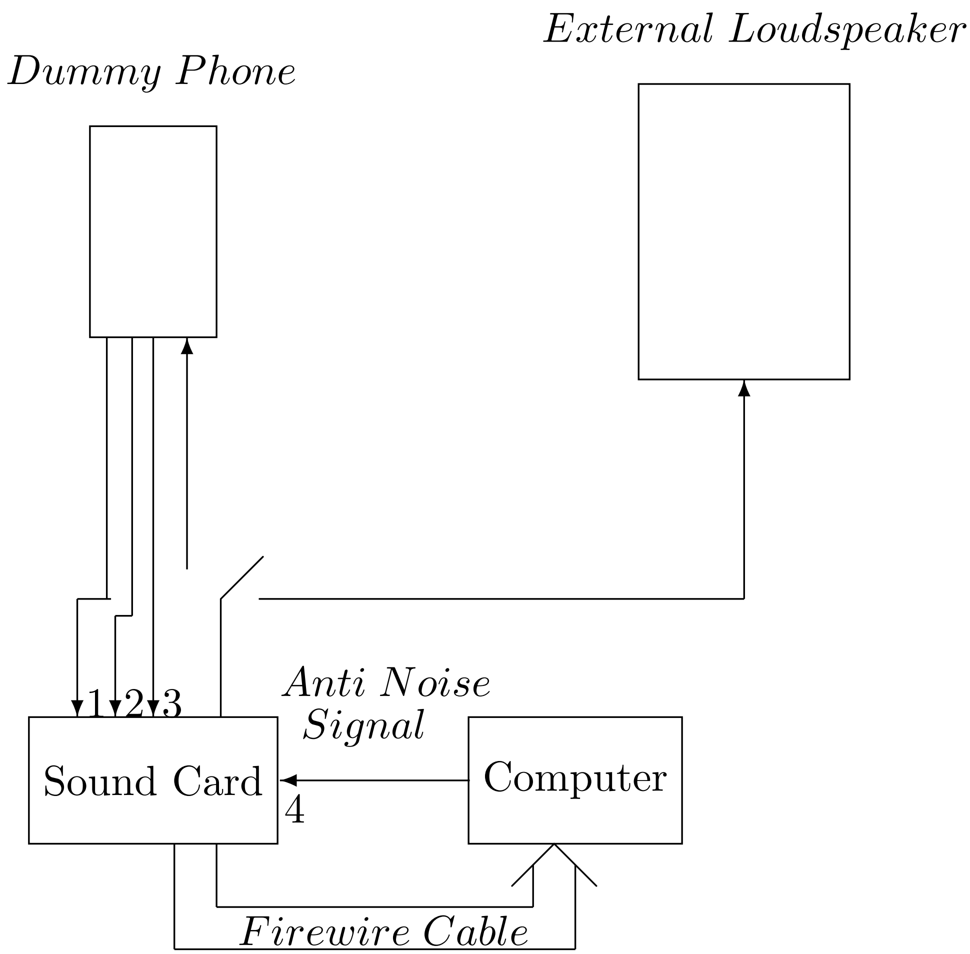

Figure 5.7: Layout of measurement setup and signal paths. 1: NRM to input 3 on sound card; 2: ERM to input 4 on soundcard; 3: PSM to input 5 on sound card; 4: Noise signal from computer to input 6 on sound card. The noise was outputted from the headphone jack on the soundcard to the loudspeaker.

Chapter 6

Design and Evaluation of the ANC-system

The object of the measurements described in chapter 5 is to provide sufficient information about the mobile phone's acoustic and electroacoustic behavior, so that simulations of an ANC system can be performed. In order to run these simulations and further analyze the results, a simulation tool was created using the graphical user interface design environment (GUIDE) in MATLAB. Description and manual for the simulation tool can be found in appendix A. The simulation tool uses models of the channels to pass sound signals with various characteristics through the ANC system. Different methods and algorithms can be applied to evaluate how much noise attenuation is achievable. In *Active Noise Control Systems – Algorithms and DSP Implementations*[1] a general hierarchy of levels of performance analysis of an ANC system are suggested and these are listed below.

- Fundamental limitations
- Practical constraints that limit performance
- Performance balanced with complexity
- How to design a practical architecture

The scope of this thesis deals primarily with the first two evaluation levels. The ANC performance will also be evaluated but only little attention will be given complexity. The architectural design will not be discussed. This chapter aims to describe the methods used for collecting the necessary data needed to set up appropriate simulations and to discuss and evaluate different aspects of the ANC system.

Frequency domain

For evaluation purposes, signals as well as impulse responses will often be displayed in the frequency domain. The frequency domain representations are computed using Welch's method of averaged periodograms, with a 50% overlap and a hanning window. The window size was derived by looking at PSD graphs for different signals, observing the tradeoff between resolution and variance of the PSD. The data signals were 56 seconds long, which gives a sequence length of 5376000 samples. A block size of 32000 samples was chosen, resulting in 334 averaged periodograms. To validate this choice of block size, a comparison of different block sizes were performed for a signal with five times longer sequence length. The same observations could be made in this case, and 32000 seemed to be the most fitting block size. With this confirmed, this set of parameters will be used in all frequency transforms, unless otherwise stated, to guarantee that the results are consistent.

Standard Case

The measurements described in chapter 5 are all based around one standard setup: The phone is pressed towards the ear with pressure 8N, 0 degrees deviation from the line between the ear and the mouth, using 0dB relative gain. This setup represents a natural holding position of the phone and will be used as the *standard case*.

6.1 Fundamental Limitations

In chapter 5 the channels involved with the dummy phone were identified and described. The measured signals described in the measurement list in appendix B.1 will give important information about the system. Initially the recorded signals PSD and time domain representation are observed to ascertain that the signals look accurate. In figure 6.1 an example of a signal in the time domain is shown, and figure 6.2 shows the estimated PSD for the same scenario.

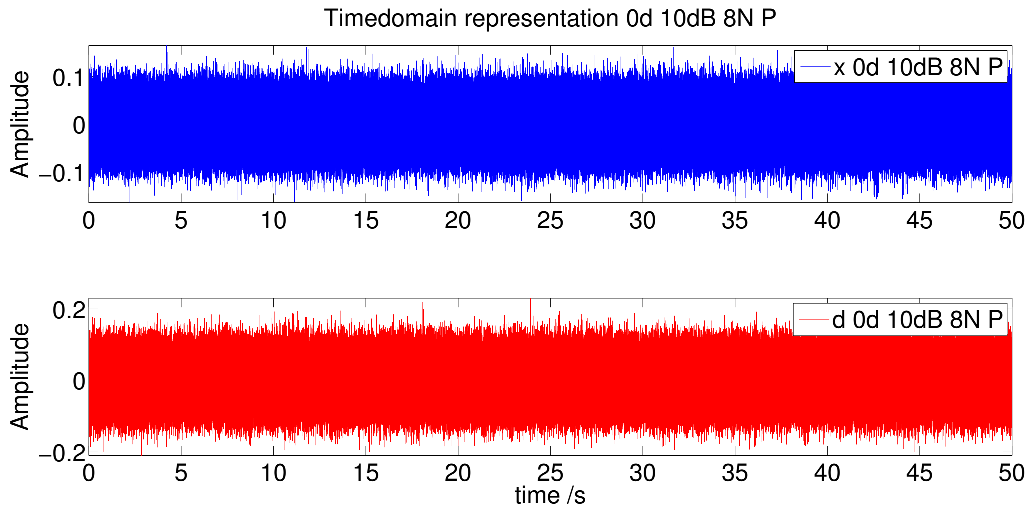

Figure 6.1: Time domain representation of the noise reference signal $x(n)$ and the desired signal $d(n)$ measured with the error reference microphone to estimate the primary path, for the measurement setup; 0 degrees, 0dB, 8N.

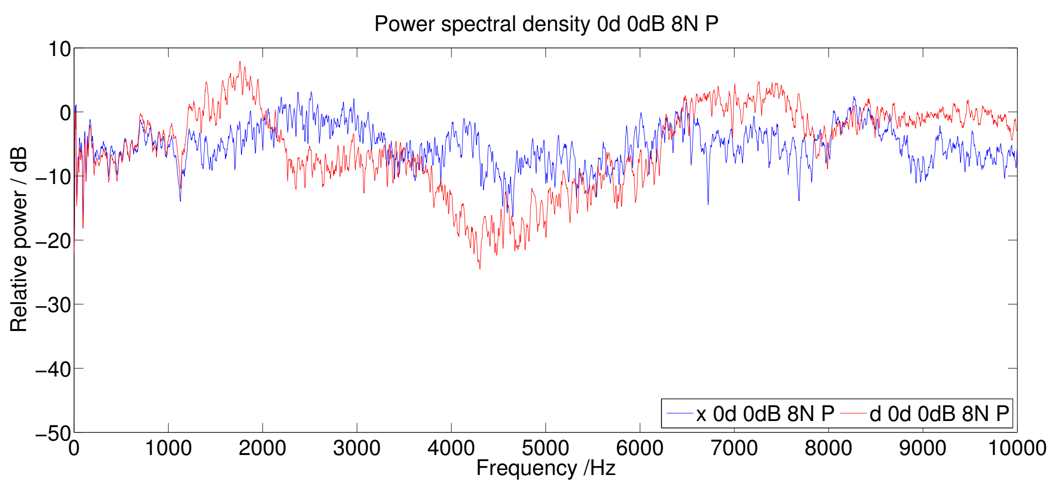

Figure 6.2: Frequency domain representation of the noise reference signal $x(n)$ and the desired signal $d(n)$ measured with the error reference microphone. These signals are used to estimate the primary path for the measurent setup; 0 degrees, 0dB, 8N.

6.1.1 Coherence Analysis

To determine what fundamental limitations in terms of frequency range the system imposes, the coherence of the system can be used as described in chapter 4. The channels that will primarily limit the performance are the primary and secondary path. In order for the ANC system to work properly the coherence in these channels should be close to one in the desired frequency range. By looking at what frequencies the coherence is lost, the system can be frequency bounded to only operate in the designated range.

Frequency Limits

The coherence dependent on relative loudness for the secondary path and the primary path is displayed in figure 6.3 for frequencies 0 Hz to 10000 Hz. The frequency interval where there is good coherence on the primary path is limited to a maximum frequency of around 1100Hz, where there is a big dip in the spectrum. The secondary path have bad coherence for low frequencies, setting a lower limit for the system at around 100-200Hz. For practical purposes the graphs in the frequency domain will therefore be limited to display the range 100Hz to 1500Hz. The coherence of the different channels were thoroughly examined by holding some of the parameters defining the measurement setup fixed, while comparing the effects of changing the other parameters. The ANC system was set to operate in the frequency range $100Hz < F < 1100Hz$. At 1100 Hz the wavelenght is around the same length as the broadness of the head and freaquencies above this will be subject to diffraction[16].

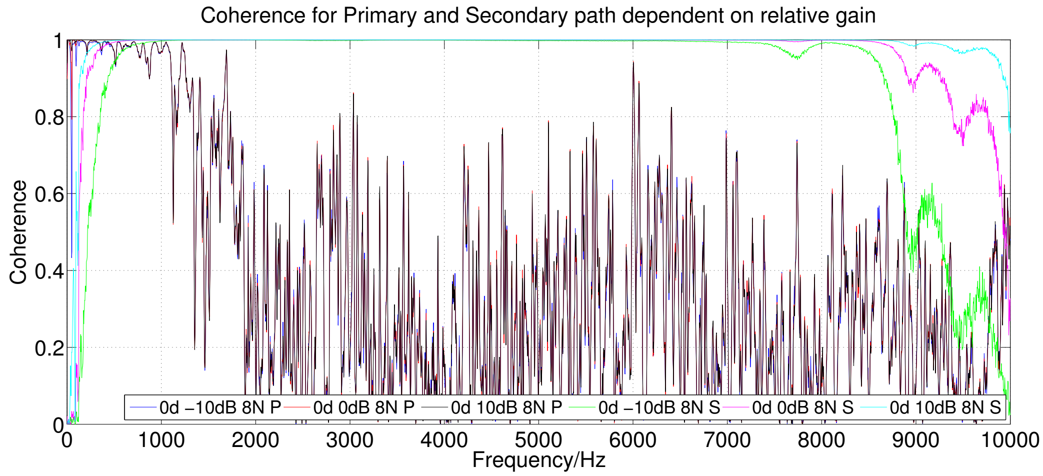

Figure 6.3: Coherence of the primary path and the secondary path dependent on relative gain; 0 degrees deviation and 8N pressure towards the ear.

Attenuation Limits

As mentioned in chapter 4, one can use the coherence to get a rough estimate of the maximum attenuation that can be achieved in the ANC system. In figure 6.4 this estimate is shown for the secondary path and the primary path over all the setups described in the measurement list in appendix B.1 for 0dB relative gain. The system has a frequency interval between 400 Hz and 1000 Hz where around 10dB attenuation is likely achieved. The coherence for the higher frequencies are lost in the primary path, and the secondary path set a limit for the lower frequencies. Overall attenuation between 5dB and 15dB in the interval 100Hz to 1100Hz is likely attainable.

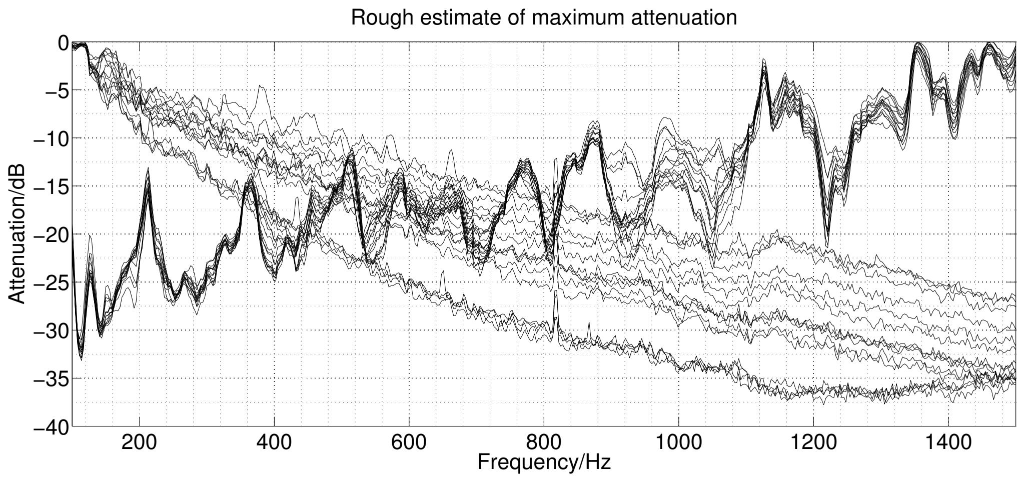

Figure 6.4: Rough esitmation of achievable attenuation.

6.2 Practical Constraints

Practical constraints primarily describes the limitations imposed on the ANC system by the mobile phone. This includes transfer functions and characteristics of electrical components involved in the ANC system.

6.2.1 Loudspeaker

The loudspeaker sets some constraint on the model of the secondary path. As stated in section 5.2, the loudspeaker is not designed for low frequencies. This notion is strengthened by the transfer function for the secondary path, see figure 6.6, where there is almost 40dB attenuation at 100 Hz and between 20dB and 5dB attenuation at 1100Hz. The order of the secondary channel model have to be high enough to account for the coloring of the noise.

6.2.2 Delay

The delay in the primary path sets a practical constraint on the ANC system. There is a peak in the correlation function for the primary path estimated to be delayed by 6 samples. This delay turned out to be important for ANC performance, implying that the electroacoustic path should have less delay than this. 6 samples delay corresponds to around 62.5 μs at 96 kHz sampling rate. The dummy phone is 3 cm thick, which should correspond to 8-9 samples at 96 kHz sampling rate for sound travelling through air, when the angle of incidence is perpendicular to the surface of the back of the dummy phone. However in a soundfield this incidence angle is not guaranteed, and may be the reason for the correlation peak being delayed by only 6 samples. The inside of the dummy phone is more hollow than a real phone, which may also affect the delay since the microphones are omni-directional. The dummy phone is much thicker than the common phone on the market, which means that the delay in reality is probably even less. This will set an important limitation to the system when implemented on a real phone. It however does not affect the simulation results. The highest correlation peak should therefore be detected for the specific phone in which an ANC system shoul be implemented, and for good performance the delay in the ANC system have to be less than the delay to this correlation peak.

6.2.3 Stationarity of Channels

The channels in the system may be subject to non stationarities, that is if the physical environment changes, the channels will change as well. For good performance the changes that might occur in the primary path, secondary path and acoustic feedback path have to be considered. Coherence describes how much of a signal can be directly linearly related to another signal, which is why high coherence is desirable for good ANC performance. Low coherence does however not necessarily indicate non-linearities in the transfer function between the signals, but can also be due to plant noise, which is why the transfer functions are important to examine.

Transfer Functions

The transfer functions were estimated using MATLAB's *tfestimate* function, which uses the PSD of the signals as described in section 4. Transfer function in this thesis denotes the estimated frequency response function between two signals. By comparing the transfer functions under the different conditions from the measurements, the channels stationarities can be examined. Depending on how stationary a channel is, generalizations regarding the channel can be used to simplify the simulation environment.

Primary Path

In figure 6.5 all the transfer functions for the primary path at 0dB relative gain can be observed, in order to investigate how it is affected by the environment. It shows that in a stationary environment the channel is more or less stationary, independent of the different measurement setups. This will primarily aid in setting up a simplified model. In reality it will change depending on position in a room or when walking through a door, from one environment to another. Since the adaptive filter of the ANC systemsis heavily dependentn on the primary path it should be able to track changes in this path. Different adaptive algorithms have different tracking properties which will affect the tracking. How well the system adapts naturally depends on how big the change in the channel and noise to be reduced is. The primary path is further described by complementary coherence and transfer function graphs in appendix B.3.1.

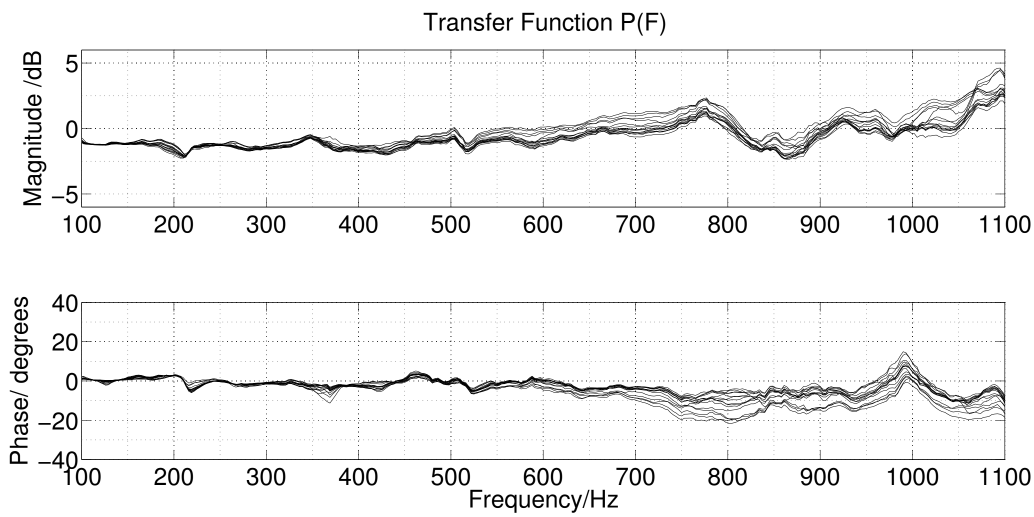

Figure 6.5: Frequency Transfer Function Primary Path, for all measurement positions and 0dB relative gain.

Secondary Path

For good performance it is desirable to have a secondary path estimate with phase and magnitude that does not change too much from the real secondary path while the ANC system is active. Table 6.1 show the phase differences with the standard scenario as reference. The table aims to describe how much change from the standard case the system can be subject to, whithout loosing its convergence properties, even though some of the changes may seem a little irrational.

Table 6.1: Maximum phase difference on Secondary Channel at different measurement setups between 100 and 1100Hz, given in degrees

Angle/Position	Sound Pressure	Pressure			
		0N	2N	8N	No HATS
-20 degrees	-10dB	-31.4365	-22.4296	-41.1718	-131.9693
	0dB	-28.1517	-19.5398	-21.2693	-87.8989
	10dB	-23.8259	-28.0584	-24.8032	-87.5547
-10 degrees	-10dB	-35.2193	-61.8703	41.8085	
	0dB	-25.6890	-35.0092	-28.2380	
	10dB	-34.2778	-28.8293	-25.9952	
0 degrees	-10dB	-29.8045	-42.9961	-44.1494	
	0dB	-31.1968	-41.2014	0	
	10dB	-27.7226	-26.2590	-25.7852	
10 degrees	-10dB	45.4673	-44.9902	-58.1687	
	0dB	-31.9929	-28.3456	-27.4931	
	10dB	-24.3471	-24.3434	-25.3996	
20 degrees	-10dB	-62.4760	-59.0574	-50.3101	
	0dB	-61.8601	-58.6121	-50.3443	
	10dB	-61.7920	-58.5452	-50.2660	

As can be seen there is quite a lot of phase variation between different scenarios. As described in section 2.3.1 a small phase error is acceptable for the secondary path estimate, but for good convergence it should be less than 50 degrees, with no convergence at 90 degrees phase error. The phase is dependent on angular placement, pressure and sound level with some angular differences well over 50 degrees. Even worse is the scenario with no HATS where the phase difference at -10dB is as high as 132 degrees. This is an unacceptable scenario for ANC convergence but very realistic, for instance if the user takes the phone away from the ear. For these cases, some sort of decision algorithm is needed to turn on and off the ANC system. Further, a dynamic solution for the secondary path estimate should help the performance. This can be realized using a filter bank in combination with a decision making algorithm or slowly converging adaptive algorithms. To be able to apply ANC on the mobile phone, some demands on the user is necessary. For good ANC performance the phone should preferably be pressed towards the ear, in a stationary position so that an isolated acoustic environment is created and maintained between phone and ear.

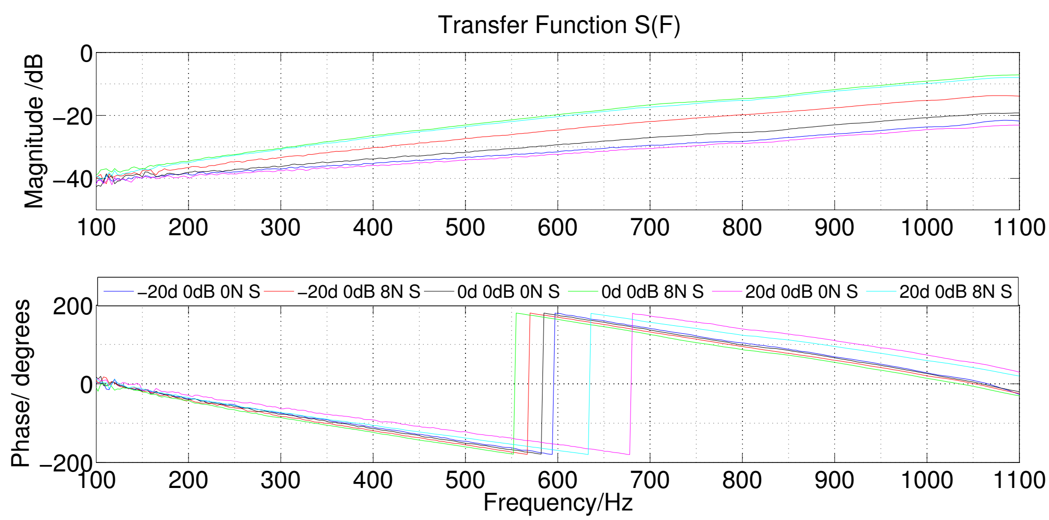

Figure 6.6: Frequency Transfer Function Secondary Channel for six different cases at 0dB relative gain. -20 degrees of angular position for 0N and 8 N, 0 degrees of angular position for 0N and 8N as well as 20 degrees of angular poistion for 0N and 8N.

As a complement to the table of phase differences, the transfer functions for six different cases are displayed in figure 6.6. The figure displays a difference in magnitude of around 15dB at 1100Hz between two of the extreme cases, indicating that the system would be even more sensitive to phase errors between model and reality going from one extreme case to the other. The secondary path is further described by complementary coherence and transfer function graphs in appendix B.3.2.

Acoustic Feedback Path

The acoustic feedback path can impose some problems for the ANC system. By analyzing this channel, some conclusions regarding the effect it has on the ANC performance can be drawn. The coherence for the acoustic feedback path can be observd in figure 6.7. For the lower SPL levels the coherence is very bad.
To ensure that this did not have to do with non-linearities, the noise floor's magnitude was measured. This magnitude was compared to the magnitude of the noise with -10dB relative gain passed through the internal loudspeaker and recorded with the NRM. This signal had around 1-2dB higher magnitude than the measured noisefloor, and for some frequencies the difference was even less. A general limitation to performance is always the uncorrelated background noise. The lower the measurement signal is compared to the noise floor, the more it will affect the coherence.

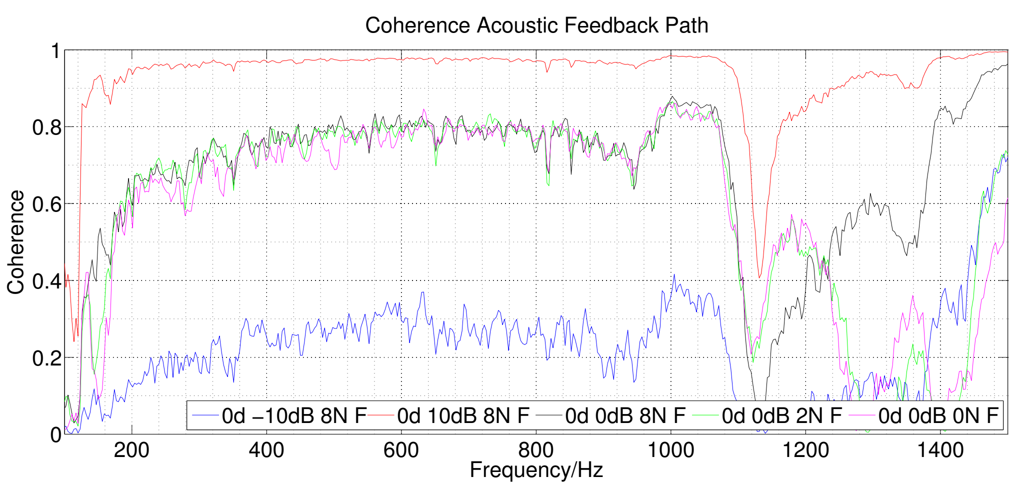

Figure 6.7: Coherence Acoustic Feedback Path dependent on relative gain and pressure towards the ear of the HATS.

The transfer function for the acoustic feedback path can be observed in 6.8. The figure shows a transfer function with around 40dB attenuation for more or less all frequencies in the interval of interest. Due to the high attenuation in this channel, the feedback path does not likely affect the performance of the ANC-system noticeably. This theory is strengthened by running simulations to examine how much the feedback would degrade the ANC performance, compared to being compensated with a perfect feedback neautralization filter. Perfect in this context means that the same estimate is used for both model and compensating filter. The result from this simulation is shown in figure 6.9. As can be seen the the effect of the feedback path is more or less negligible. Implementing a feedback compensator could actually degrade the performance more than it helps, because of errors in the model, round off errors and an increase in computation complexity. The acoustic feedback path is further described by complementary coherence and transfer function graphs in appendix B.3.3.

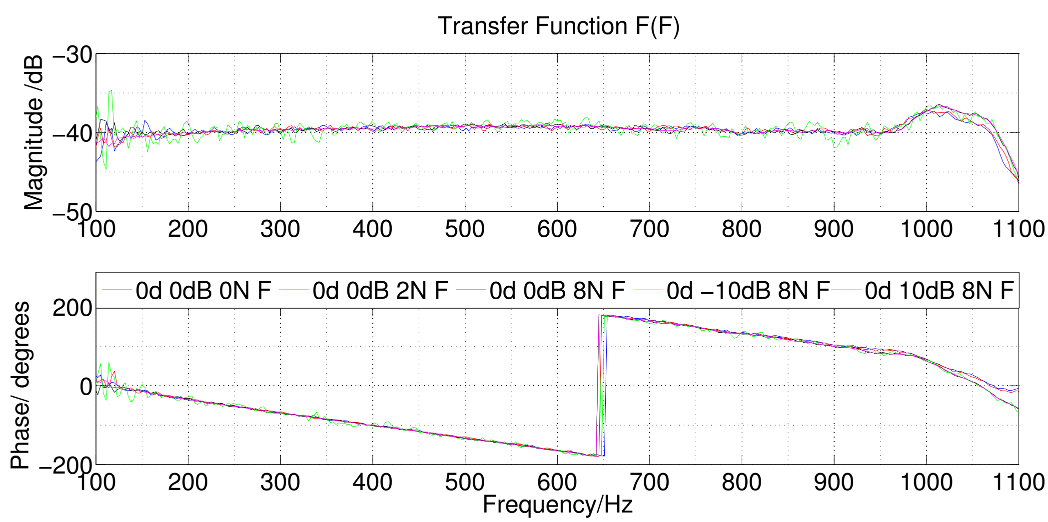

Figure 6.8: Frequency Transfer Function Acoustic Feedback Path for five different cases with 0 degrees angular position.

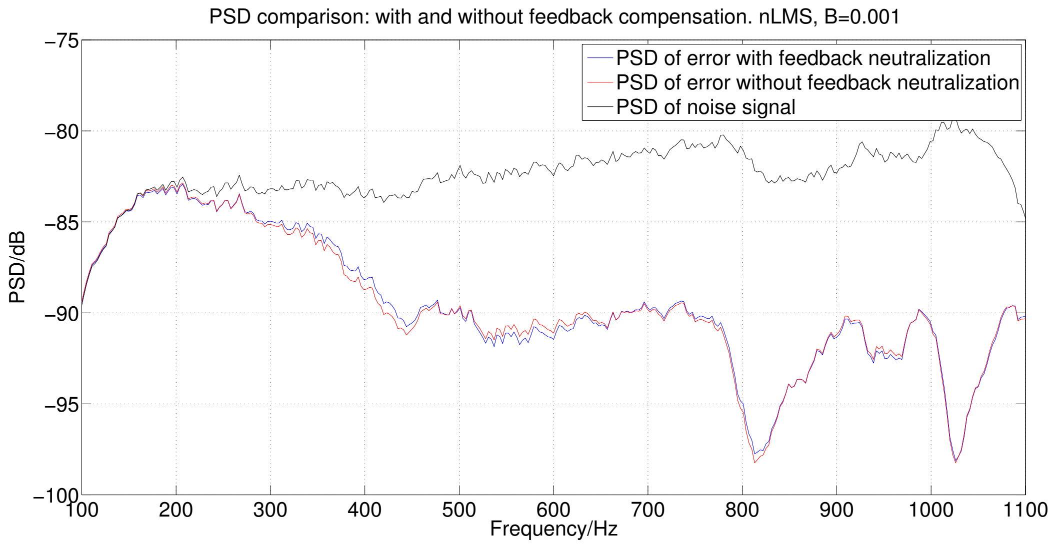

Figure 6.9: Comparison of attenuation with and without feedback compensation, using nLMS with stepsize 0.001, adaptive filter order 120, and estimated secondary channel order 120.

The Echo Path

The echo path should be treated separately since it will serve no purpose to model this channel for ANC performance. The effect of the echo path can become a source of interference for the two way communication, and it is important that the signal from the secondary source does not leak too much of the anti noise signal into the primary speech microphone, which will degrade the speech intelligibility. For this purpose the coherence and transfer function of this channel was analyzed. The transfer function dependence on pressure and relative gain can be observed in figure 6.10. The transfer function of the acoustic echo path looks similar to the transfer function of the acoustic feedback path with an attenuation of around 40dB for all frequencies. Today's mobile phones have pretty complex noise reduction systems using at least two microphones to deal with unwanted noise and echo that is introduced on the transmission line. These already implemented noise reduction tools should provide sufficient noise attenuation of the noise transferred via the acoustic echo path. This conclusion will thereby end the discussion of the acoustic echo path. Coherence plots and transfer function graphs describing the echo path can however be found in appendix B.3.4.

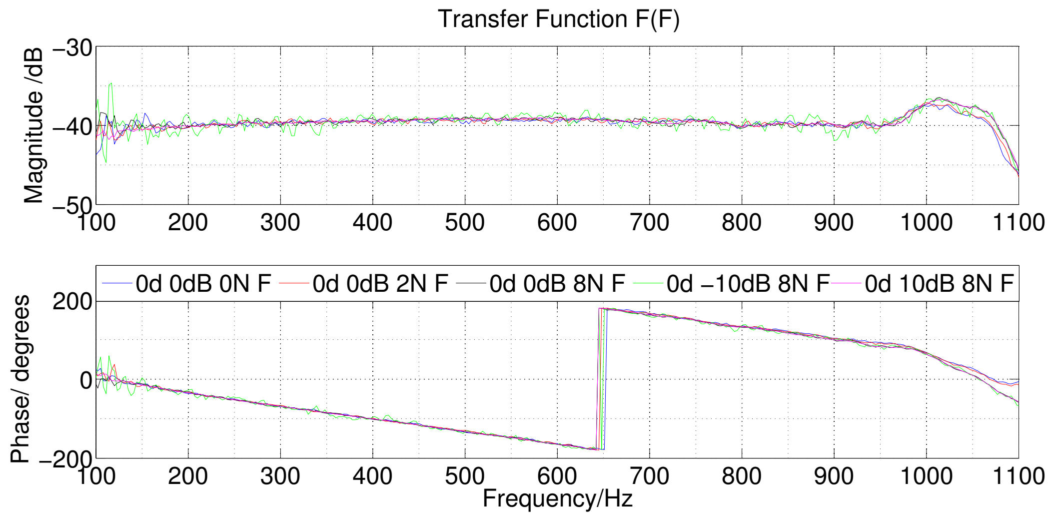

Figure 6.10: Frequency Transfer Function Acoustic Echo Path for five different cases with 0 degrees angular position.

6.3 Channel Estimation and Model Design

In order to analyze the feasibility of implementing ANC-algorithms in a hand-held mobile phone a simulation environment is needed to model all relevant aspects of the phone. Creating such an environment will allow performance analysis of the different ANC-algorithms without having to set up the actual measurement systems, making it possible to evaluate performance in a more controlled environment. This section will focus on how to design such an environment.

Designing a model is the first, and one of the most important steps to creating an actual simulation environment. The channels to be modeled are covered in chapter 5 and these will together constitute a complete simulation environment. The accuracy of the each separate channel model will affect all simulations using it, hence the models have to be as close to reality as possible. To ensure a good simulation the model has to be compared to 'reality', which in this thesis will be portrayed with a transfer function, estimated using Welch's method of averaged periodograms, as described in the beginning of this chapter.

With the 'reality' in check, the next step is to determine how to estimate the model of the channel. Since the input signals for estimating the channels are WGN, they are assumed to be WSS. Therefore optimal FIR wiener filters, covered in section 3.1.1, are used to represent the channels. In order to get a correct representation of reality the filter order have to be of an appropriate length. A too short filter will cause inaccuracy, while too long filters will be difficult to handle computationally. A good way to determine an appropriate filter length is to first look at the cross correlation of the signals defining the channel. In figure 6.11 this is shown for the primary path at standard conditions.

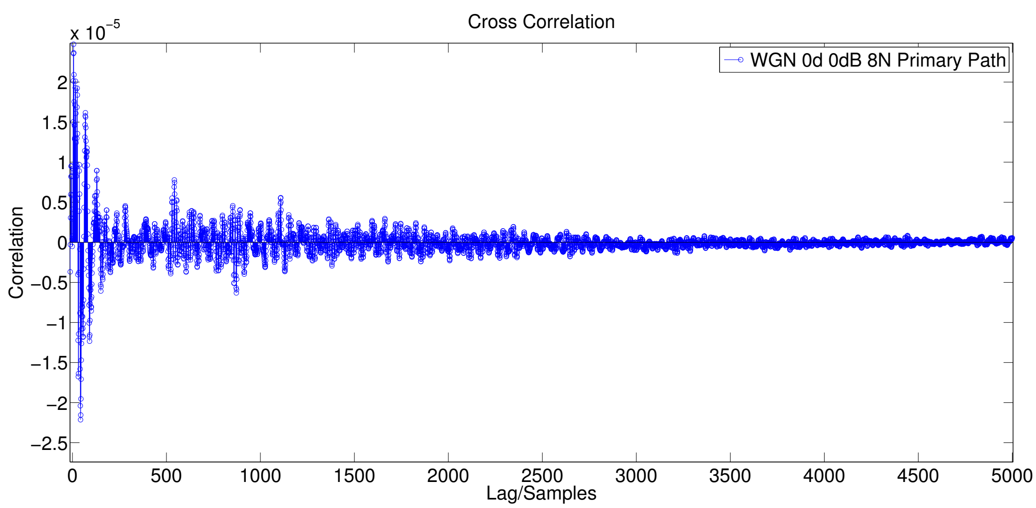

Figure 6.11: Cross correlation of the NRM and ERM for standard conditions.

As can be observed, the correlation starts out quite volitile and rings out after around 3000 samples which gives a good idea of the needed filter length. After this is determined, the filter is estimated using different filter lengths. The filters ability to model the real transfer function can be evaluated by looking at the energy of the error between the estimate and the real transfer in the desired frequency range. In figure 6.12 the energy of the error can be seen for different filter lengths. It is clearly visible that for orders of around

2000-3000 and higher, the error is quite stable, while relatively high for lower filter orders. By looking at the correlation function and the error between the model and the estimated real frequency response, the filter order of the estimated channel was set to 3000, which should give a good representation of reality.

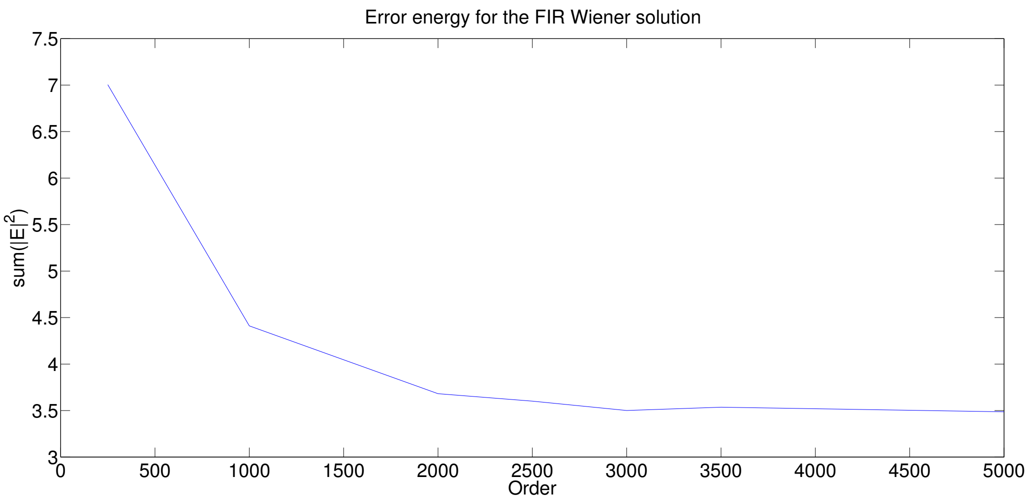

Figure 6.12: Energy of the error (deviation) from the transfer function for different orders of Wiener FIR estimates of the primary path at standard conditions.

This procedure is performed for each channel since they have different correlation properties. When the proper filter length for each channel model have been decided, the final estimations of the different paths can be done. After examinating the different channels, it was concluded that an filter order of 3000 was suitable for all channels. An estimation GUI, simEst was designed for this purpose and is described in detail in appendix A.2. Once all electroacoustic and acoustic channels are modelled, the filters for the ANC algorithm have to be estimated as well. These should have lower filter order than the simulation models. The length of these filters are part of the evaluation of the ANC performance, and will be discussed in the following section.

6.4 Performance of the ANC-system

Once the fundamental limitations to the ANC-system are set, and all the necessary channels have been defined and estimated, different adaptive algorithms can be compared and evaluated in terms of performance. The simulation tool, described in appendix A.1, is designed to handle different ANC setups combined with different adaptive algorithms and can simulate the following ANC-systems.

- Feedforward ANC (FF)
- Feedforward ANC utilizing filtered-x (FFFX)
- Feedback ANC (FB)
- Feedback ANC utilizing filtered-x (FBFX)

These can be evaluated using LMS, nLMS, nLeaky-LMS and RLS adaptive algorithms. The feedforward ANC is subject to acoustic feedback from the loudspeaker, initially this path is set to zero in order to simulate a simplified model. In section 6.2.3 the effect of the feedback path was discussed, and the conclusion drawn was that the effects it has on the ANC performance is negligible. With this notion the simplified model was used to evaluate the Feedforward ANC system. Simulations will be run using WGN to compare the results using different filter lengths, step sizes and algorithms. By applying WGN noise on the input, the ANC systems ability to attenuate broadband stationary noise will be tested. The results from these simulations will then be used to inspect how well the system performs with different noise characteristics on the input signal.

Attenuation Evaluation

The most interesting parameter to study for an ANC-system is the level of noise attenuation the system can achieve. To be able to evaluate performance, the frequency band 100Hz-1100Hz where the comparison takes place, was split into two sub bands: 100Hz-600Hz and 600Hz-1100Hz. Average attenuation at each frequency over these bands will be presented in tables to give an initial idea of the achievable performance. The tables present the attenuation in terms of the two frequency intervals: $L : (100 < F < 600)$, $H : (600 < F < 1100)$. When the setup completely fails to attenuate the noise, attenuation is presented as not available (N/A). The different simulations are noted as follows: ANC system is defined by feedback (FB) or feedforward (FF), followed by (FX) if the filtered-x method is applied. M defines the adaptive filter order for $W(z)$, K defines the order of the secondary path estimate $\widehat{S}(z)$, and the adaptive algorithm is referrred to by the previously defined abbreviations, followed by the parameters and their values. Leaky nLMS is denoted lnLMS. The sequences used to simulate the ANC performance are 10 seconds long, considering this being a resonable time to give the algorithms to converge. In order to examine the best possible ANC setup some initial analysis was performed.

6.4.1 Initial analysis

Secondary path compensation

The feedforward ANC system was compared to the feedforward filtered-x system to examplify how much the secondary path compensation can improve the performance. The best results attained can be seen in table 6.2. The filter order for both secondary path estimate and adaptive filter is 120. The attenuation properties for the filtered-x system outperforms the system without secondary path compensation, and is less likely to diverge. Especially for non stationary noise on the input, where the ANC without secondary path compensation fails to converge. The reason why was discussed previously in section 2.3.1. Because of the superiority in performance of the filtered-x system, only the systems using this method will be further investigated. The blockdiagrams of the feedforward and feedback filtered-x systems are displayed in figure 6.13 and 6.14 respectively.

Table 6.2: The table show the average attained attenuation over the two frequency intervals $L : (100 < F < 600$, $H : (600 < F < 1100)$

Noise	ANC simulation	M	K	Frequency interval	
				L	H
WGN 0dB	$FFnLMS, \mu = 0.015$	120	120	5.0354	1.3610
Café	$FFnLMS, \mu =$	120	120	N/A	N/A
WGN 0dB	$FFFXnLMS, \mu = 1.07$	120	120	12.8064	19.7869
Café	$FFFXnLMS, \mu = 0.625$	120	120	12.6927	11.5736

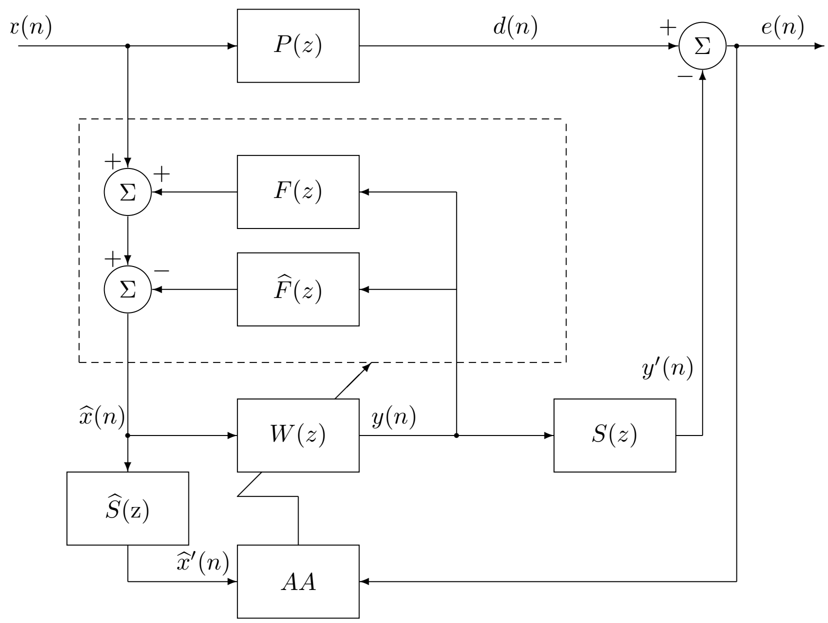

Figure 6.13: Block diagram of feedforward filtered-x ANC system for simulations, utilizing arbitrary adaptive algorithm for finding the filter coefficients $W(z)$.. The Feedback Path and Feedback neutralization are not modeled for all simulations

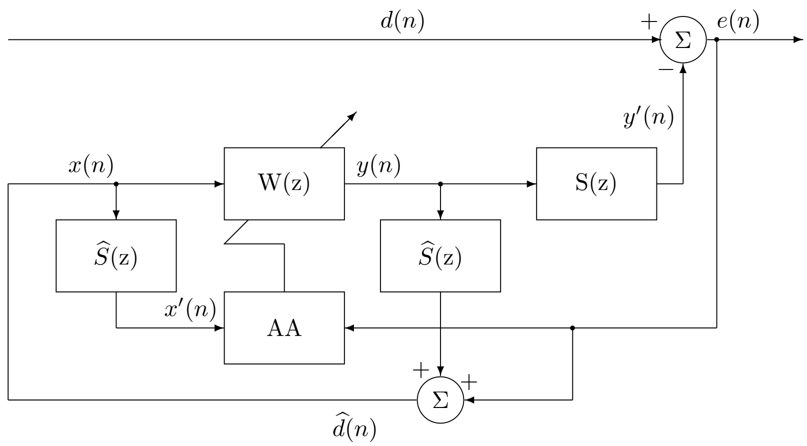

Figure 6.14: Block diagram for simulations of feedback filtered-x ANC system for simulations, utilizing arbitrary adaptive algorithm for finding the filter coefficients $W(z)$. Anti-noise signal $y(n)$ and estimated reference signal $x(n)$ are passed through an estimate of the secondary path $\widehat{S}(z)$.

Feedback filtered-x

In table 6.3, results from the feedback filtered-x system compared to the feedforward filter-x system simulations are presented. The feedback filtered-x system is not very good at handling the broadband noise on the input. Most of the achievable attenuation was located in the higher frequency range, $600Hz < F < 1100$ Hz, with a maximum average attenuation at around 9.3 dB for WGN on the input. When the input was non stationary noise, namely the Café noise, the gainable attenuation was much worse. The maximum attenuation gained with adaptive filter order 120 and secondary path estimate order 120, was 1.2556 dB over the high frequency interval. Tests with a sinusoid added to WGN showed that feedback have good attinuation for narrow banded signals while failing to adequately attenuate the rest of the frequencies.

Table 6.3: The table show the average attained attenuation over the two frequency intervals $L : (100 < F < 600$, $H : (600 < F < 1100)$

Noise	ANC simulation	M	K	Frequency interval	
				L	H
WGN 0dB	$FBFXnLMS, \mu = 0.0007$	120	120	N/A	9.3555
WGN 10dB	$FBFXnLMS, \mu = 0.0007$	120	120	N/A	9.3491
Café	$FBFXnLMS, \mu = 0.000051$	120	120	1.4321	2.4000
WGN 0dB	$FFFXnLMS, \mu = 1.07$	120	120	12.8064	19.7869
WGN 10dB	$FFFXnLMS, \mu = 1.07$	120	120	13.0819	20.0498
Café	$FFFXnLMS, \mu = 0.625$	120	120	12.6127	11.5736

These initial results have been produced to examplify the theory, and exclude some of the ANC setups from the more thourough investigation. The nLMS algorithm and RLS algorithms will be compared for stationary inputs and non stationary inputs on the feedforward filtered-x sytem, with and without leakage factors.

6.4.2 Feedforward filtered-x

This section aims to present and evaluate the performance of a feedforward filtered-x ANC system. The system will be examined using different input noises, starting with a thourough examination of stationary noise, i.e WGN. To be able to limit the number of simulations, the filter orders were generally tested for up to 120, with some higher order simulations. The simulations were run in such a way that best possible performance and stability bounds were assessed, using different combinations of filter orders and adaptive filter parameters.

Stationary Performance

In table 6.4 some important results from simulating the feedforward filtered-x ANC system are shown. Best performance is around 32 dB attenuation for the higher frequency subband and up to almost 23.5 dB for the lower frequency subband. The best attenuation performance for the high subband 600Hz-1100Hz was achieved with an order of 120 on the Secondary Channel estimate and using the leaky nLMS algorithm. There are some interesting properties that can be concluded from the results. The mean-squares algorithms have a distinguishable connection between performance and filter orders used to estimate the primary path. The best performance for the nLMS algorithm was found with both secondary path estimate order and adaptive filter order of 120, and $\mu = 1.1$. Introducing a leakage factor allowed the algorithms stepsize to be pushed further, giving results of up to 32 dB of attenuation for the higher frequency band. The leakage factor helps the nLMS distinvtively, however if the stepsize is pushed too hard, artifacts will be introduced to the resulting error signal. This will set the limit for the attenuation attainable for the leaky nLMS. Decreasing the order of the estimate of the secondary path does not seem to degrade the results noticeably. A very important result is the attenuation gained when using an adaptive filter order of 120 and a filter order of only 10 for the estimate of the secondary path. WIth this setup, the leaky nLMS was still able to attain very good noise reduction. The adpative filter order however have more impact on the system, but balancing the amount of leakage with the appropraite step size for the leaky nLMS algorithm, allows for bigger steps since a bigger portion of the enegy intoduced into the system is leaked out stopping the algorithm from diverging. This is true for larger filterorders, but when the filter orders become small, the stepsize will have to be small, and there is no longer much to gain from leaking out the energy in the filter. This is examplified by the performance of nLMS and leaky nLMS for adaptive filter order 10 and secondary path estimate of order 10. The RLS algorithm is not as dependent on filter size, and give more stable results for the different filter orders. In fact the RLS and the leaky nLMS give similar results for an adaptive filter order of 80 and secondary path estimate of order 10, and for filter orders lower than this, the RLS outperforms the nLMS and leaky nLMS.

To show the superiority of the nLMS algorithm over the LMS algorithm, the LMS best performance was compared to the nLMS, using the same filter orders. The result show, that even in a somewhat stationary environment, the LMS can not reach the same attenuation as the nLMS. The nLMS algorithm outperforms the LMS algorithm in every aspect, and the LMS algorithm will not be examined further for this reason.

Table 6.4: The table show the average attained attenuation for filtered-x feed-forward ANC over the two frequency intervals $L : (100 < F < 600$, $H : (600 < F < 1100)$

Noise	ANC simulation	M	K	Frequency interval L	H
WGN 0dB	$FFFXLMS, \mu = 1344$	120	120	9.8279	14.4508
WGN 10dB	$FFFXLMS, \mu = 108.65$	120	120	9.103	13.8646
WGN 0dB	$FFFXnLMS, \mu = 1.3$	200	200	14.1621	18.1631
WGN 0dB	$FFFXnLMS, \mu = 1.07$	120	120	12.8064	19.7869
WGN 0dB	$FFFXnLMS, \mu = 1.7$	120	120	-1.5909	6.5610
WGN 0dB	$FFFXnLMS, \mu = 0.01$	20	10	3.7126	6.1288
WGN 0dB	$FFFXnLMS, \mu = 0.01$	10	10	3.7976	6.2978
WGN 0dB	$FFFXlnLMS, \mu = 1.07, \gamma = 0.985$	120	120	19.3117	27.7191
WGN 0dB	$FFFXlnLMS, \mu = 1.75, \gamma = 0.986$	120	120	23.1146	32.0092
WGN 0dB	$FFFXlnLMS, \mu = 1.75, \gamma = 0.985$	120	120	23.2250	31.8153
WGN 0dB	$FFFXnLMS, \mu = 1.1$	120	50	9.8903	17.6289
WGN 0dB	$FFFXlnLMS, \mu = 1.7, \gamma = 0.985$	120	50	22.9546	31.6293
WGN 0dB	$FFFXlnLMS, \mu = 1.7, \gamma = 0.985$	50	120	13.3011	18.6982
WGN 0dB	$FFFXlnLMS, \mu = 1.4, \gamma = 0.985$	120	10	**22.0039**	**30.8459**
WGN 0dB	$FFFXlnLMS, \mu = 1.4, \gamma = 0.985$	100	10	21.8864	30.9450
WGN 0dB	$FFFXlnLMS, \mu = 0.65, \gamma = 0.96$	80	10	15.3575	18.172
WGN 0dB	$FFFXlnLMS, \mu = 0.3\gamma = 0.96$	60	10	10.7723	13.6728
WGN 0dB	$FFFXlnLMS, \mu = 0.3\gamma = 0.97$	50	10	8.2579	13.1908
WGN 0dB	$FFFXlnLMS, \mu = 0.025\gamma = 0.98$	20	10	1.9125	2.2016
WGN 0dB	$FFFXlnLMS, \mu = 0.0045\gamma = 0.96$	10	10	2.6670	2.8739
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 11, \lambda = 1$	120	120	13.6055	20.3759
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	120	120	14.1534	20.9769
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 11, \lambda = 1$	120	50	13.1727	20.1883
WGN 0dB	$FFFXRLS, \delta = 5\text{e} - 12, \lambda = 1$	120	10	13.4304	19.4325
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 11, \lambda = 1$	100	10	13.1185	19.9541
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 11, \lambda = 1$	80	10	12.9189	20.1618
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 11, \lambda = 1$	50	10	11.4467	18.7489
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	20	10	11.1503	16.9449
WGN 0dB	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	10	10	9.8611	15.5266
WGN 10dB	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	10	10	9.9712	15.6441
WGN 10dB	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	15	10	-5.2319	0.7973
WGN 10dB	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	20	10	11.4129	17.0722

For the RLS, higher attenuation is gained by lowering the regularization parameter, until the stability bound is reached and the system diverges. For the nLMS there is a distinct peak in performance where higher stepsize eventually leads to divergence, and lower stepsize yields better attenuation when using longer sequence lengths. The divergence limit for the nLMS can be pushed using a leakage factor, which is one of the reasons for it's high attenuation results. Evidently, there is a tradeoff between the adaptive filter order and the convergence properties of the filter. Lower order filters for the nLMS moves the stability bound of the stepsize towards 0, while lower order filters for the RLS moves the stablity bound for the regularization parameter towards 1. The bound of the RLS does not move as distinctively as the nLMS, however the best performance of the RLS is always dangerously close to the stability bound. This notion is illustrated in figure 6.15.

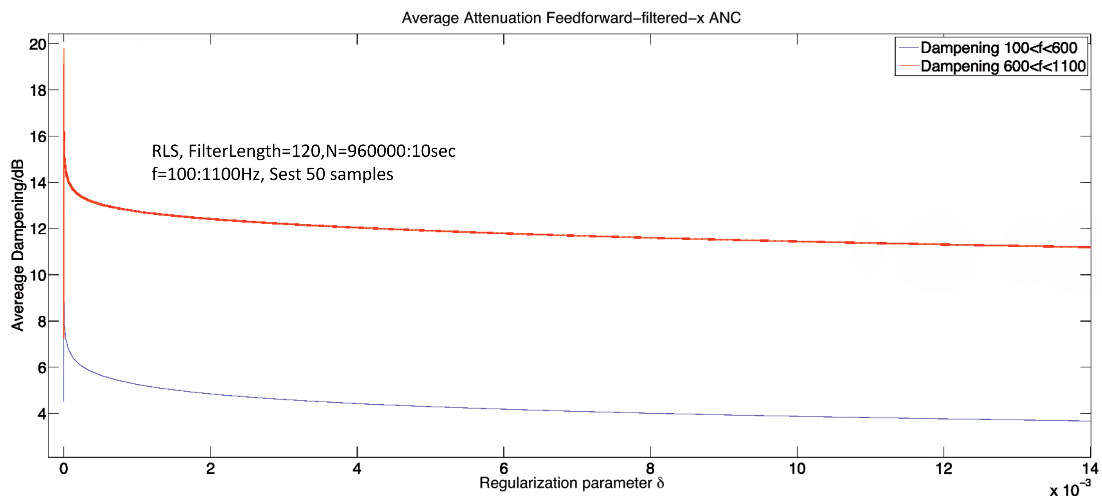

Figure 6.15: Attenuation dependent on regularization parameter δ for the RLS algorithm. Simulated with feedforward filtered-x ANC and WGN as input signal

There is also an interesting occurrence that happens for the RLS algorithm with secondary estimate of order 10. When the filter order of the adaptive filter of 10, 15 and 20 were compared, both order 10 and 20 outperforms a filter order of 15, for which the system does not converge. This is an interesting property cause it displays that the filter length of the secondary

path estimate carefully needs to be matched with the filterlength of the adaptive filter, in order to optimize performance.
The results in the table point towards better performance for the leaky nLMS algorithm, both in attenuation and stability. The parameters invovled in designing the appropriate algorithm are deeply entangled, meaning that parameters that diverges using one set of filter orders, does not necessarily diverge for another set. This is part of why evaluation of the algorithms is a complex matter. If there are any demands on filter sizes or computational power, these have to be matched with the right set of parameters to optimize the system performance.

Table 6.5: The table show the average attained attenuation over the two frequency intervals $L : (100 < F < 600,\ H : (600 < F < 1100)$

Noise	ANC simulation	M	K	Frequency interval L	Frequency interval H
Café	$FFFXlnLMS, \mu = 1.85, \gamma = 0.99$	120	120	25.711	29.4657
Café	$FFFXlnLMS, \mu = 1.85, \gamma = 0.99$	120	50	25.3715	28.7225
Café	$FFFXlnLMS, \mu = 1, \gamma = 0.99$	120	50	20.6036	24.0761
Café	$FFFXlnLMS, \mu = 1.6, \gamma = 0.96$	120	10	**23.0919**	**24.6541**
Café	$FFFXlnLMS, \mu = 0.7, \gamma = 0.96$	100	10	17.4722	17.5687
Café	$FFFXlnLMS, \mu = 0.25, \gamma = 0.96$	80	10	10.7517	11.3984
Café	$FFFXlnLMS, \mu = 0.14, \gamma = 0.96$	50	10	6.9024	7.3476
Café	$FFFXlnLMS, \mu = 0.045, \gamma = 0.96$	20	10	2.9819	2.3514
Café	$FFFXlnLMS, \mu = 0.025, \gamma = 0.96$	10	10	2.2286	1.6534
Café	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	120	120	16.2485	16.1366
Café	$FFFXRLS, \delta = 1\text{e} - 13, \lambda = 1$	120	50	15.7399	16.2352
Café	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 0.999999$	120	50	13.4493	13.1671
Café	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	120	10	15.3682	16.2213
Café	$FFFXRLS, \delta = 1\text{e} - 13, \lambda = 1$	100	10	14.5098	14.6355
Café	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	80	10	14.3958	14.1960
Café	$FFFXRLS, \delta = 1\text{e} - 12, \lambda = 1$	50	10	12.9895	11.7539
Café	$FFFXRLS, \delta = 6\text{e} - 14, \lambda = 1$	20	10	11.4900	11.2607
Café	$FFFXRLS, \delta = 5\text{e} - 14, \lambda = 1$	10	10	11.1278	10.5852
Chirp	$FFFXlLMS, \mu = 1.7, \gamma = 0.9$	120	50	23.1233	22.3092

Non Stationary Performance

As seen in table 6.5 the leaky nLMS algorithm has the best performance for both cafe noise and a chirp signal. Best results were achieved with a step size of 1.75 and a leakage factor of 0.985. This was however close to the divergence limit. The RLS does not perform poorly, but looking at maximum attenuation, it does not manage to compete with the leaky nLMS. When exposed to a chirp sound, the RLS algorithm does not converge at all. As expected, the leaky nLMS algorithm really shows its advantage here. Its ability to keep the algorithm stable while tracking changes in the environment produces very good attenuation results, over 20dB in both frequency intervals. It is interesting to note that RLS still seem to be less dependent on filter order, achieving better results with low filter orders than the leaky LMS. These results are highly relevant if implemented on a mobile phone. Some more simulation results can be found in table B.2 in appendix B. In figure 6.16 a graph displaying the PSD of the best performing cases for RLS and leaky nLMS is shown in order to conclude the presented results.

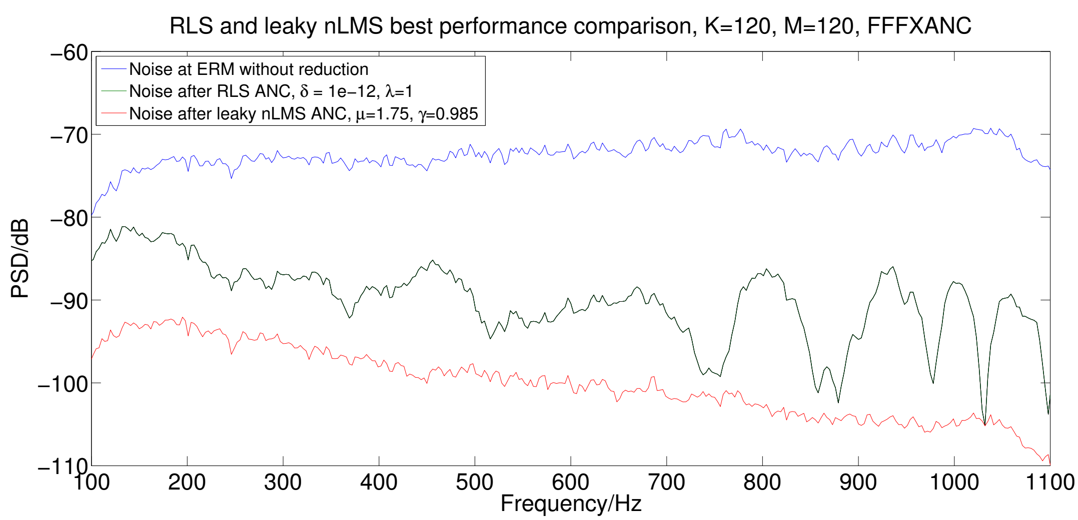

Figure 6.16: The PSD of the error signals without ANC and for the best performing ANC setups for leaky nLMS and RLS respectively.

6.5 General Discussion

Leaky nLMS seems to be the winning candidate when it comes to the adaptive algorithms. But is it really? It was shown in the previous section that the leaky nLMS needed higher orders of the filters in order to attain the same or higher attenuation as the RLS. The RLS algorithm on the other hand operates using matrices, while the LMS algorithms uses vectors. The RLS algorithm uses much larger amounts of memory compared to the leaky nLMS algorithms. The RLS might converge after fewer iterations, but each iteration takes longer time to preform. In ANC, speed is of the essence, especially when working with such short delays as on a handheld mobile phone, which could be a weakness for the RLS algorithm. One could argue that the gain from using a lower filter order is lost in the complexity of the algorithm. It really depends on the computational power of the mobile phone. The leaky nLMS is much lighter computationally but takes more iterations to converge and needs larger filter orders to gain the same results as the low order RLS implementations.

The simulation model constructed in the simulation environment is a good representation of the dummy phone. This has been confirmed by analyzing transfer functions, phase and coherence plots using a suitable block size to get a good balance between variance and bias. To avoid nonlinear areas, the signals were prefiltered using a bandpass filter between 100 and 1100 Hz. The prefiltering is applied in order to only observe the frequency interval where good performance is attainable. In a real application the input signal and error signal would have to be filtered in the same way, which will add to the delay in the system.

Overall the simulations show the attenuation limits using different methods under optimal circumstances. These attenuations are most probably unrealistic, since the system is subject to continuous changes and stability is crucial. The results were derived pushing the system to it's maximum, and it is usually not recommended to be this close to the divergence limits when implemented on a real application. Further, a real application in a mobile phone would commonly use a fixed point implementation. This means that there will be round of errors on signals and filter coefficients, which will degrade the systems performance. The destructive interference can create a silent point in space. It is also possible that the sound the user hear does not have as good attenuation as the ANC system displays. This will need to be considered for real performance evaluation. With this in mind, will the attenuation gained in the frequency interval the ANC system operates in be enough to give noticible difference for speech intelligibility? In order for it to work on a real mobile phone, the decision algorithms governing the system have to be carefully considered. To ensure a properly working system, some demands should be set on the user. Preferably, one should not move around too much, or change the position of the phone. It should also be pressed pretty hard towards the ear to create a closed stable acoustic environment. This may seem like a lot to demand from the user, but given the situation that one is talking in a very noisy environment, it is not so farfetched that the user would press the phone towards the ear and not move around too much. With this the ANC system could give adequate noise reduction for speech intelligibility.

Chapter 7

Conclusions and Further Work

In chapter 1, the question was raised whether or not it is feasible to implement ANC on a handheld mobile phone. During the course of this project some limitations and demands for ANC performance has been established, and these conclusions are summed up in this chapter.

ANC can be implemented on a handheld phone, working on a frequency limited band. The frequency band should be from around 100/150 Hz up to 1100 Hz, where the lower bound is set by the secondary path and the upper bound by the primary path.

For good ANC performance the filtered-x method should be implemented. In a comparison of feedforward and feedback systems, the feedforward ANC system outperforms the feedback system by far, with broad band attenuations of more than 20 dB for some frequencies. Higher frequencies 600 Hz to 1100 Hz can attain more attenuation than the lower frequencies, 100 Hz to 600 Hz. It was also shown that the acoustic feedback path had negligible affect on the ANC system, and compensating for this would probably degrade the system more than help it.

There are two candidates for algorithms performing the adaption, and they both have some pros and cons. These are RLS and leaky nLMS. RLS have a way faster convergence rate, but does not attain as much attenuation as the leaky nLMS algorithm. RLS can manage to perform adequately with small filter orders on both secondary path estimate and adaptive filter, while the leaky nLMS performance is much more dependent on the adaptive filter order. The RLS algorithm can adapt to some changes in the system, but the leaky nLMS have much better tracking abilities, which can be crucial since the system itself is subject to non stationarities. The best results were attained using the leaky nLMS.

The reults from the simulations also show another important aspect of the system, there need to be some demands set on the user. For good noise reduction the user should not move around too much, keep the mobile phone in a steady position and press it towards the ear. If this set of directions are fullfilled, the system will be more or less stationary, giving the ANC system the best possible conditions for reducing the noise.

The analysis have given some basic ideas of how an ANC system would have to be designed in order to work on a handheld mobile phone. The performance analysis give some information on how to set the parameters for the algorithm, but this will be one of the major obstacles in implementing it on a real mobile phone. In order to do so, decision making algorithms have to be designed. These have to be able to turn on and off the ANC system depending on the environment–when the ANC system will help or not help the speech intelligibility. It also have to be able to adjust to some minor changes in the system, where either a online estimation for the secondary path have to be designed, or a decision algorithm choosing the appropriate estimate from a filter bank. Further analysis could also include examining hybrid solutions if there is enough computation power available. Overall the next step in realizing ANC on a handheld mobile phone would be to acquire specifications and limitations for the actual implementation. What order of filters are reasonable, what clock frequency will the algorithm work with, how much memory and computation power will be available? These answers have to be put in context with the systems and algorithms described in this thesis, and only then will the answer to the question arise; Is it feasible to implement an ANC system? The results from this projects says yes, it could be implemented, and yes it could help speech intelligibility, but there is still a lot of work to be done.

Appendix A

Simulation Tools

In order to analyze the the ANC system with different parameters and settings a simulation environment is needed that can handle all relevant properties of the ANC system. To achieve this two simulation tools were developed in MATLAB. This chapter will serve as a description of these tools, as well as a manual for operating them.

A.1 simGUI.m

The simulation environment itself is called simGUI.m and is the main program. The graphical user interface (GUI) is shown in figure A.1. On start up, the GUI is loaded with simple default filters.

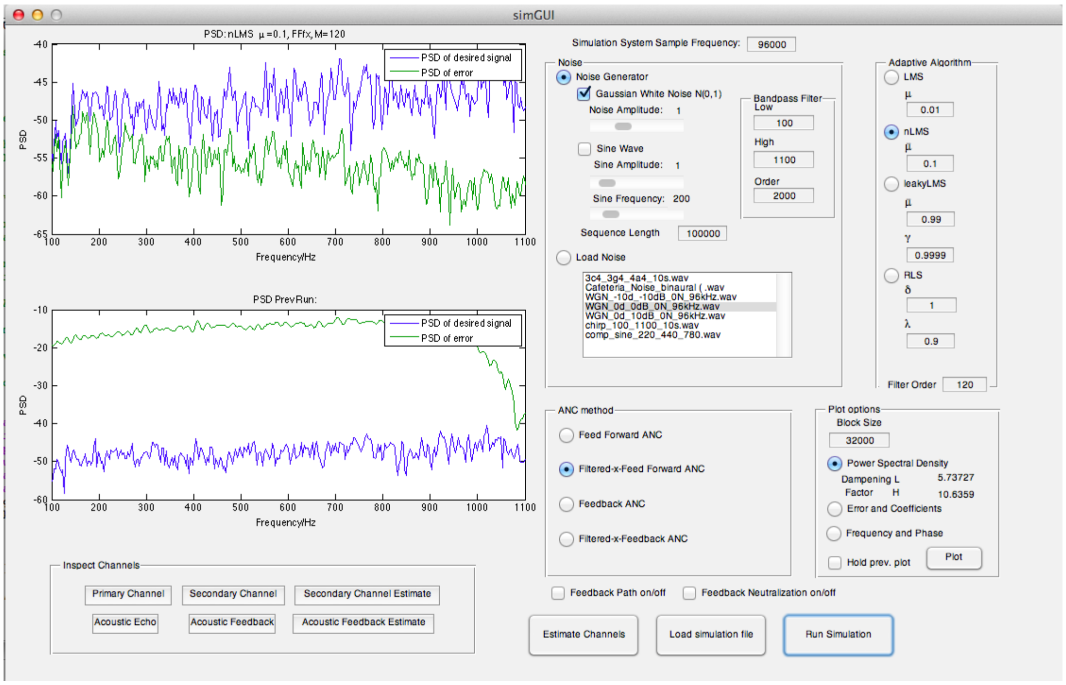

Figure A.1: Graphical user interface for simulation tool: simGUI.

A.1.1 Noise

Noise Generator

The noise section of the interface is for choosing what type of noise to feed the algorithm with. The noise generator can be set to either generate WGN, a sine wave with configurable frequency, or a mix controlled by the amplitude faders. The sequence length in number of samples can be set it its designated text box.

Load Noise

If some other noise is desirable a few other types of noise are available in the simulation environment. The following noise types are available by default:

- Chirp
- Cafe' Noise

- Street Noise
- Chirp

The noise file is chosen by simply checking load noise and marking the desired file.

A.1.2 Adaptive Algorithm

There are four different adaptive algorithms available for estimating the primary path. They are configured as follows:

- LMS
  - μ: Step size
- nLMS
  - $\tilde{\mu}$: Normalized step size
- leakyLMS
  - $\tilde{\mu}$: Normalized step size
  - γ: Leakage factor
- RLS
  - δ: Regularization factor
  - λ: Expential forgetting factor

The desired **filter size** is set in its designated text box.

A.1.3 ANC method

There are six different ANC methods available:

- Feed Forward ANC:
  A simple feed forward algorithm without any extras.
- Filtered-x Feed Forward ANC:
  The feed forward algorithm with added filtered-x filter.

- Inverse Modeling Feed Forward ANC:
  Feed Forward ANC with an inverse secondary channel filter instead of the filtered-x approach to compensate for the secondary channel.

- Feedback ANC:
  A simple feedback algorithm without any extras.

- Filtered-x Feed Feedback ANC:
  The feedback algorithm with added filtered-x filter and feedback compensation filter.

- Inverse Modeling Feedback ANC:
  Feedback ANC with an inverse secondary channel filter instead of the filtered-x approach to compensate for the secondary channel.

A.1.4 Plot Options

There are two different axes in the GUI operating a little differently depending on which plot option is chosen. To analyze the results of the ANC-algorithm there are three different plot options. The block size for the frequency transforms can be set in its designated text box.

Power Spectral Density

Calculates and plots the PSD of the input signal and the error signal using the previously set block size. The figure is plotted on the upper axes. On the lower axes the corresponding figure from the previous run is displayed.

Error and Coefficients

Displays the error signal at each iteration on the upper axes, used to see the convergence of the algorithm. On the lower axes the estimated coefficients from the final iteration is shown.

Frequency and Phase

On the upper axes the frequency response of the estimated filter is displayed. On the lower axes is the corresponding phase angle is shown.

The plot can be held to the next run by checking the "Hold prev. plot" check box.

A.1.5 Inspect Channels

Each channel in the simulation environment can be studied by pressing its corresponding button. The channels impulse response will then show up on the upper axes.

A.1.6 Main Buttons

Estimate Channels

This button is used to reach the estimation utility *simEst.m* which will be discussed in next section. Here the different channels can be estimated based on external sound files.

Load Simulation File

A simulation file saved in *simEst.m* can be loaded directly into the simulation environment using this button. All channels will be replaced with the channels stored in the simulation file.

Run Simulation

When all channels are loaded and parameters are set, the simulation is started by pressing this button. The simulation may take some time depending on the parameters chosen for the simulation.

A.2 simEst.m

In order to estimate models of the channels in the simulation environment *simEst.m* was created. By loading two correlated sound files, *simEst.m* estimates the channel between them. Included are also a few analysis tools to analyze the sound files prior to the estimation as well as for analyzing the resulting channel filter. The graphical user interface is shown in figure A.2

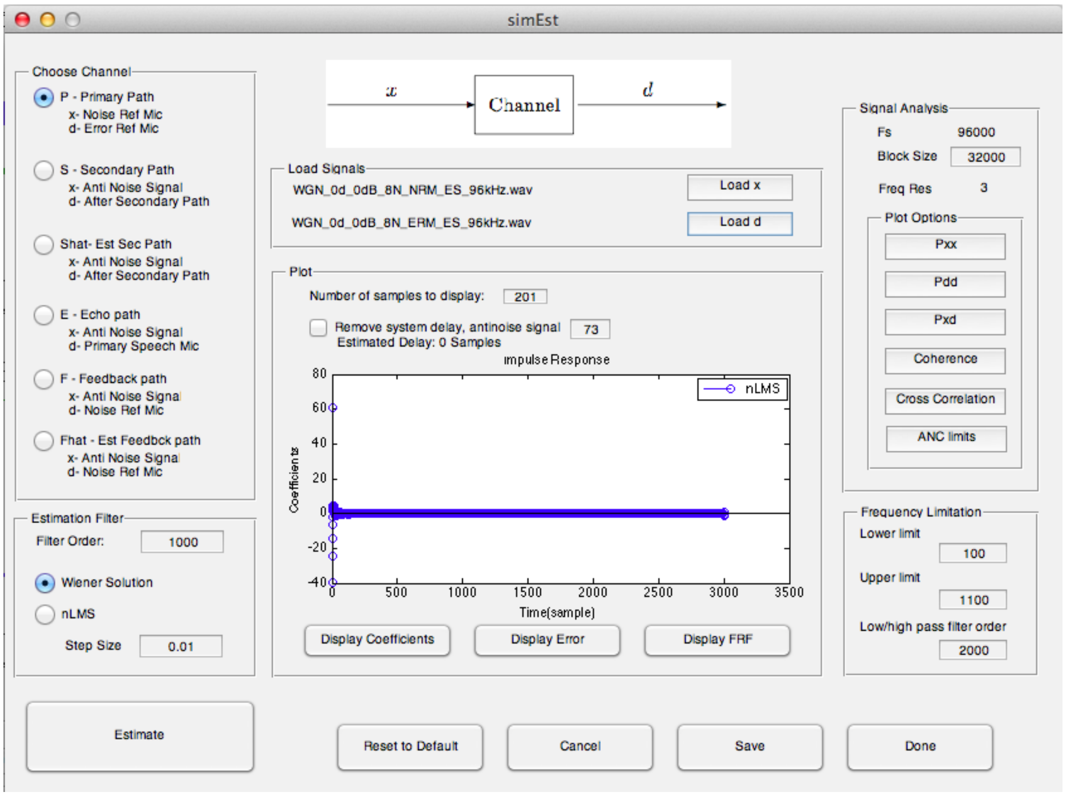

Figure A.2: Graphical user interface for estimating channels: simEst.

A.2.1 Choose Channel

There are seven different channels that can be estimated:

- P - Primary Channel
  The channel uses the NRM as input x and ERM as output d. Both signals recorded using the external noise source.
- S - Secondary Channel
  The channel uses the anti noise signal generated in MATLAB as input x and ERM recorded using the internal source as output d.
- Shat - Est Sec Channel
  Estimation filter to the ANC-algorithm to use with “filtered-x ANC” in

simGUI. The channel uses the anti noise signal generated in MATLAB as input, x and ERM recorded using the internal source as output d.

- E - Echo Channel
  The channel uses the anti noise signal generated in MATLAB as input x and PSM recorded using the internal source as output d.

- F - Feedback Channel
  The channel uses the anti noise signal generated in MATLAB as input x and NRM recorded using the internal source as output d.

- Fhat - Est Feedback Channel
  The estimated feedback compensation filter for the ANC-algorithm. The channel uses the anti noise signal generated in MATLAB as input x and NRM recorded using the internal source as output d.

A.2.2 Load Signals

The signals x and d, defined in the "Choose Channel" section, can be chosen here. The audio files imported have to be .wav files.

A.2.3 Signal Analysis

This area handles the pre-estimation analysis of the signal. Displays the current sampling rate, Fs, and enables the user to set desired block size. The current frequency resolution is shown in its designated text box.

Plot Options

Decides what to display on the axes. The following are available:

- Pxx
  Calculates and plots the PSD of the signal x using the user-defined block size.

- Pdd
  Calculates and plots the PSD of the signal d using the user-defined block size.

- Pxd
  Calculates and plots the cross-PSD of the signals x and d, using the user-defined block size.

- Coherence
  Calculates the coherence of the channel between signals x and d using the user-defined block size.

- Correlation
  Calculates the cross correlation between the signals x and d. Displays the number of lags that is set at the plot window. For removal of system delay, check the corresponding check box at the plot window.

- ANC limits
  Calculates the maximal theoretical dampening for the ANC-system by taking $-10\log_{10}(1-\text{coherence})$.

A.2.4 Estimation Filter

Settings for the filter to be estimated. The desired filter order is entered in the designated text box. The user have the choice between a Wiener FIR solution or a nLMS solution. If the latter is chosen, the desired step size should be entered in its designated text box. The possible system delay can be removed by checking the box at the plot window.

A.2.5 Plot

Displays desired figures on its axes. The number of samples to display can be set in its designated text box. The buttons at the bottom of the area chooses what property of the estimated filter to show.

A.2.6 Frequency Limitations

Sets the frequency limits to the signals and filters by band pass filtering the two signals x and d.

A.2.7 Main Buttons

- Estimate
  Starts the estimation of the selected channel with entered settings.

- Reset to Default
  Resets all channels to initial values received from the main program.

- Cancel
  Discards changes and returns to the main program.

- Save
  Opens a save dialog to save all channels and relevant settings to a .m-file. This can later be imported in the main program.

- Done
  Returns to the main program with the estimated channels and loads them.

A.3 Audio Files

The audio files recorded during the measuring sessions in the lab are available on request. The files are organized in a file tree as follows:

→ Angle
  → Pressure to ear
    → Sound level

How the audio files are named is covered in B.1.

Appendix B

Complementary Graphs and Data

B.1 Detailed Measurement List

The audio files was saved as:
'Noise Signal'_'Angle/Position'd_'SoundPressure'dB_'Pressure'N_
'Microphone'_'Sound Source'_96kHz.wav
example: WGN_0d_0dB_2N_ERM_ES_96kHz.

To perform all these measurements the plan described by table B.1, was created. The measurements should be performed in groups defined by the color dots in the table. For each dot the following need to be recorded/generated:

- Anti-noise Signal for output through Secondary Source
- NRM Using External Source - Estimating Primary Path
- ERM Using External Source - Estimating Primary Path
- NRM Using Internal Source - Estimating Acoustic Feedback
- ERM Using Internal Source - Estimating Secondary Path
- PSM Using Internal Source - Estimating Echo Channel

Table B.1: Desired Measurements Position

Angle/Position	Noise signal	Sound Pressure	Pressure			
			0N	2N	8N	no HATS
0 degrees	WGN	-10dB	●	●	●	●
		0dB	●	●	●	●
		10dB	●	●	●	●
	Chirp	-10dB	●	●	●	-
		0dB	●	●	●	-
		10dB	●	●	●	-
	Pink Noise	0dB	-	●	-	-
	Car Noise 100km/h	0dB	-	●	-	-
	Café Noise	0dB	-	●	-	-
	Street Noise	0dB	-	●	-	-
	Wind	0dB	-	●	-	-
-20 degrees	WGN	-10dB	●	●	●	-
		0dB	●	●	●	-
		10dB	●	●	●	-
-10 degrees	WGN	-10dB	●	●	●	-
		0dB	●	●	●	-
		10dB	●	●	●	-
10 degrees	WGN	-10dB	●	●	●	-
		0dB	●	●	●	-
		10dB	●	●	●	-
20 degrees	WGN	-10dB	●	●	●	-
		0dB	●	●	●	-
		10dB	●	●	●	-

Table B.2: The table show the average attained attenuation for filtered-x feed-forward ANC over the two frequency intervals $L : (100 < F < 600$, $H : (600 < F < 1100)$. The results in this table serves as complementary results to those presented in chapter 6.4.

Noise	ANC simulation	M	K	Frequency interval L	Frequency interval H
WGN 0dB	$FFFXnLMS, \mu = 1.1$	50	120	6.0555	19.6927
WGN 0dB	$FFFXnLMS, \mu = 0.1$	50	50	5.1248	14.2685
WGN 0dB	$FFFXnLMS, \mu = 0.2$	50	50	4.6514	14.9574
WGN 0dB	$FFFXnLMS, \mu = 1.1$	50	50	N/A	N/A
WGN 0dB	$FFFXnLMS, \mu = 1.1$	30	50	N/A	N/A
WGN 0dB	$FFFXnLMS, \mu = 0.01$	30	10	3.8433	7.7282
WGN 0dB	$FFFXnLMS, \mu = 0.02$	30	10	4.0552	7.69987
WGN 10dB	$FFFXnLMS, \mu = 0.01$	20	10	3.3557	6.0240
WGN 0dB	$FFFXlnLMS, \mu = 0.25\gamma = 0.985$	60	10	6.6381	16.9594
WGN 0dB	$FFFXlnLMS, \mu = 0.65, \gamma = 0.985$	80	10	8.6898	18.2437
Café	$FFFXlnLMS, \mu = 0.1, \gamma = 0.99$	30	30	4.2292	6.0851
Café	$FFFXlnLMS, \mu = 0.2, \gamma = 0.95$	30	30	8.5285	5.3061
Café	$FFFXlnLMS, \mu = 0.2, \gamma = 0.95$	50	30	9.8424	9.4786
Café	$FFFXlnLMS, \mu = 0.2, \gamma = 0.95$	50	50	9.5405	8.9598
Café	$FFFXlnLMS, \mu = 0.2, \gamma = 0.95$	120	50	10.0865	10.0756
Café	$FFFXlnLMS, \mu = 0.5, \gamma = 0.99$	120	50	15.5211	18.8103
Café	$FFFXlnLMS, \mu = 1.9, \gamma = 0.99$	120	50	N/A	N/A
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-9, \lambda = 1$	50	120	10.7941	18.5905
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-9, \lambda = 1$	50	50	10.7760	18.2023
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	50	50	12.2553	19.7693
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	30	30	11.9206	19.9047
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	50	30	11.8148	19.5936
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	30	20	11.8303	19.8663
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	30	10	11.2582	17.7280
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-11, \lambda = 1$	60	10	11.5931	18.9980
WGN 0dB	$FFFXRLS, \delta = 1\mathrm{e}-13, \lambda = 1$	120	120	N/A	N/A
Café	$FFFXRLS, \delta = 5\mathrm{e}-14, \lambda = 1$	120	50	N/A	N/A
Café	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	50	50	13.2597	13.2306
Café	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	50	30	13.0699	13.4576
Café	$FFFXRLS, \delta = 1\mathrm{e}-12, \lambda = 1$	30	30	13.4338	15.0906
Café	$FFFXRLS, \delta = 5\mathrm{e}-13, \lambda = 1$	30	30	13.7137	15.1181
Café	$FFFXRLS, \delta = 1\mathrm{e}-13, \lambda = 1$	30	30	14.1421	15.1641

B.2 Complementary Simulation Results

In order to find the best performance a number of simulations were run. This table displays some of the more interesting results as a complent to those presented in chapter 6.4.

B.3 Complementary Graphs

In this section some complementary graphs are displayed. The following sections will aim to further describe the acoustic and electro-acoustic paths invovled in the ANC system, as well as compare these paths to the models used in the simulations. The paths will be illustrated with coherence graphs and transfer functions. The transfer functions for the standard case described in chapter 6 will be compared to the models used in the simulation. To estimate the channels for the model a special Graphical User Interface was designed in MATLAB. To identify the channels necessary for setting up the simulation environment, optimal wiener solutions were used. The tool for estimating the channels is described in appendix A.2.

B.3.1 Primary Path

The primary path is one of the most crucial paths in the ANC system. With a somewhat stationary primary path the adaptive filter will be able to track diffrences in the environment. In this section graphs describing the primary path are presented as a compliment to those presented in the thesis.

Coherence

As illustrated by the following graphs, the system is quite stationary and does not heavily depend on either SPL, pressure to the ear or angular placement.

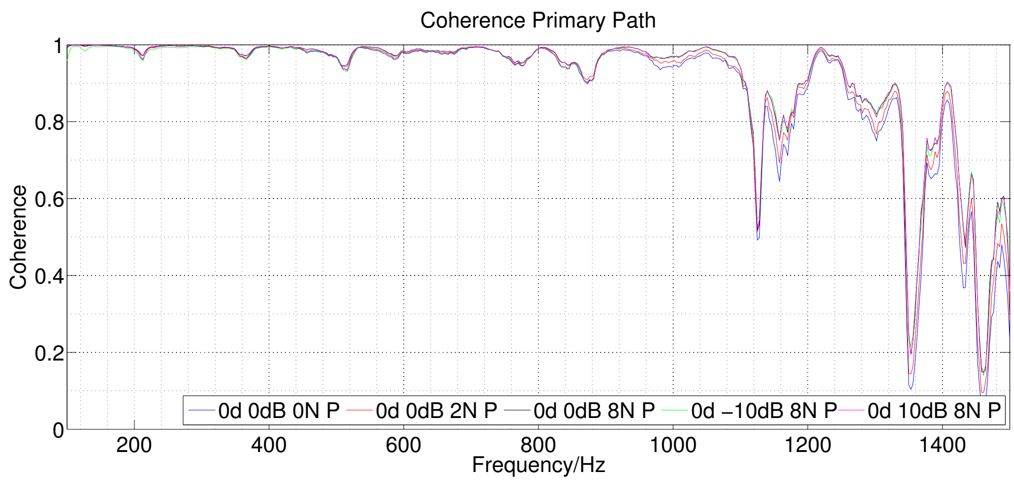

Figure B.1: Coherence primary path dependent on relative gain and pressure towards the ear of the HATS.

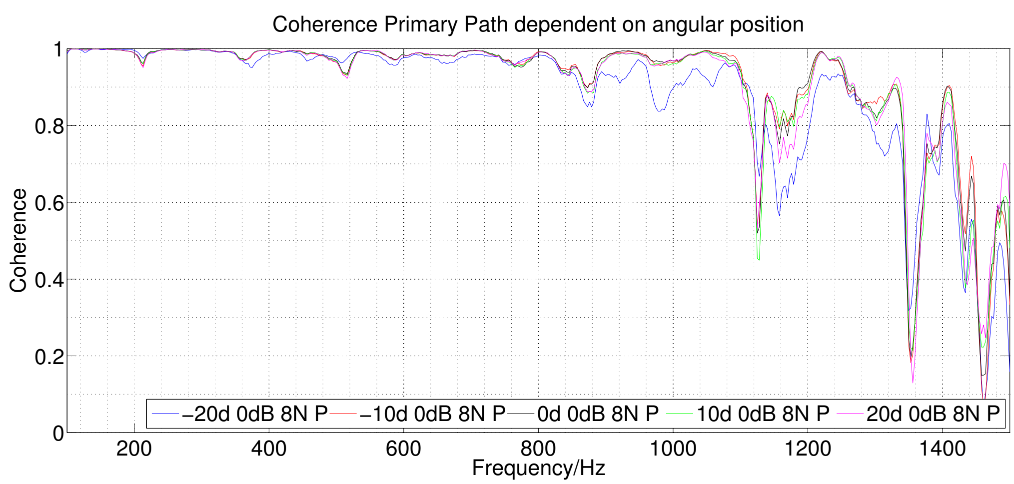

Figure B.2: Coherence Primary Path with different angular placements of the dummy phone.

Transfer Function

The stationarity dependent on SPL of the channel is further described by analysing the transfer function.

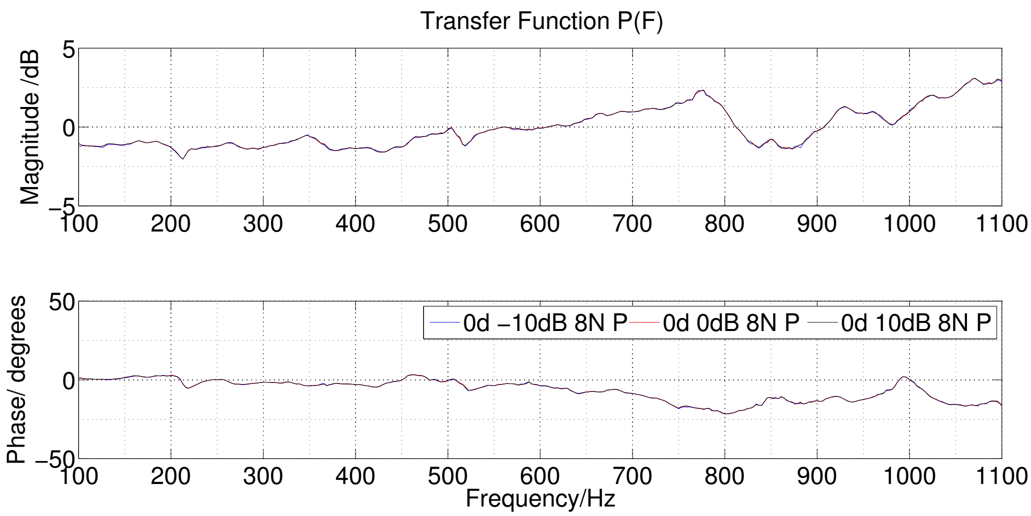

Figure B.3: Frequency Transfer Function Primary Channel, displaying dependency of relative gain

Model of Channel for Simulations

As can be seen, the model is a pretty rough estimate of the primary path, but t is able to identify most od the details in the path's frequency response.

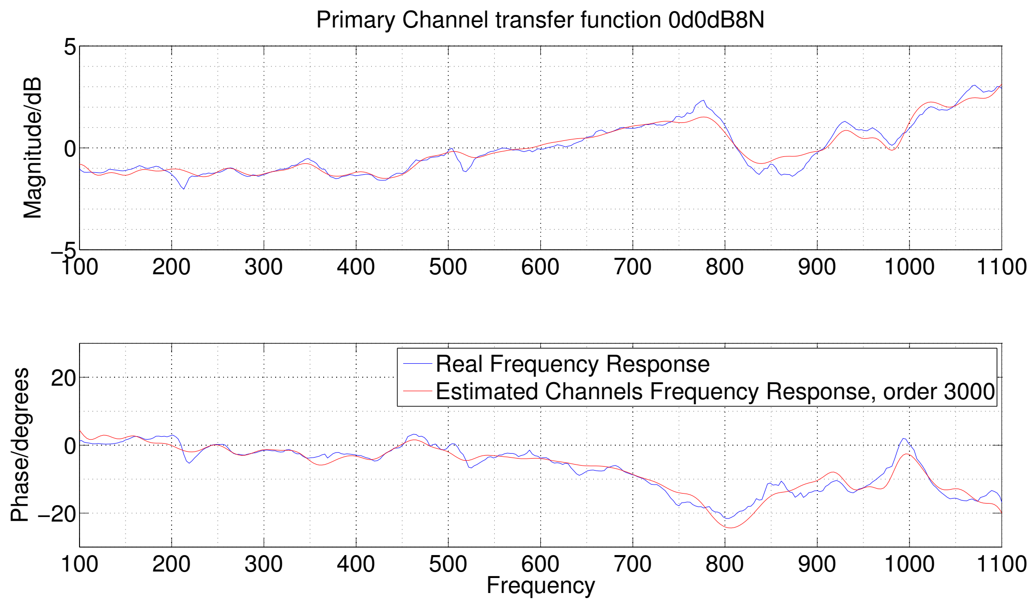

Figure B.4: Frequency Transfer Function of the Primary Path for model and estimated path

B.3.2 Secondary Path

The secondary path is one of the most crucial paths in the ANC system. With a somewhat stationary secandary path the adaptive filter will be able to track diffrences in the environment. In this section graphs describing the secondary path are presented as a compliment to those presented in the thesis.

Coherence

The stationarity of the secondary path is described by the following plots. The dependency on SPL was deducted to primarily an effect of interferrence from plant noise. It is also notable that the pressure towards the ear seem to affect the path distinctly with better coherence for higher pressure.

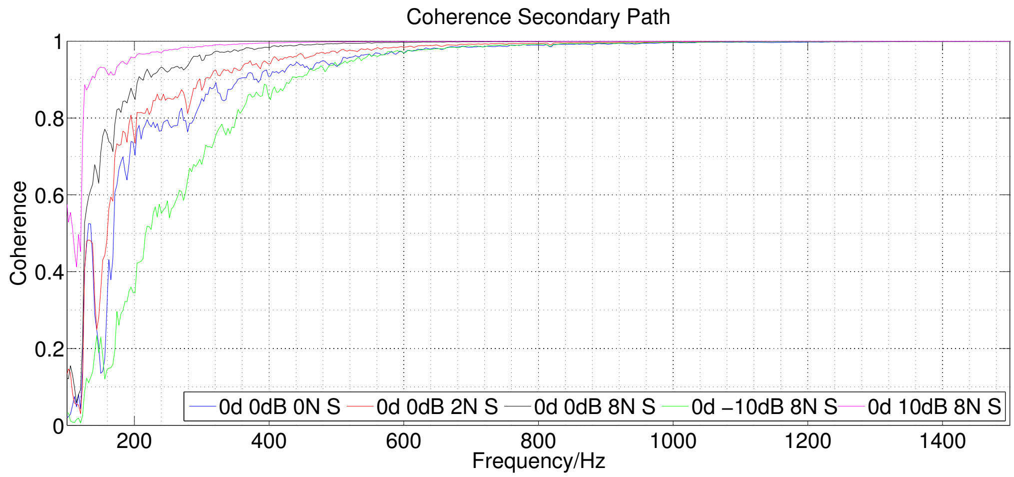

Figure B.5: Coherence Secondary Path dependent on relative gain and pressure towards the ear of the HATS.

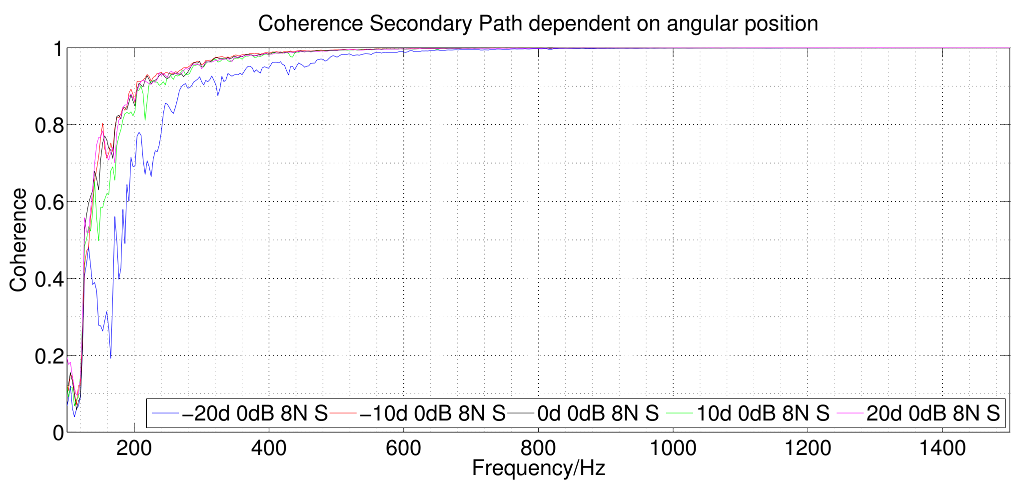

Figure B.6: Coherence Secondary Path with different angular placements of the dummy phone

Transfer Function

The nonstationarity of the secondary path is further described by the deviations in the frequency transfer function.

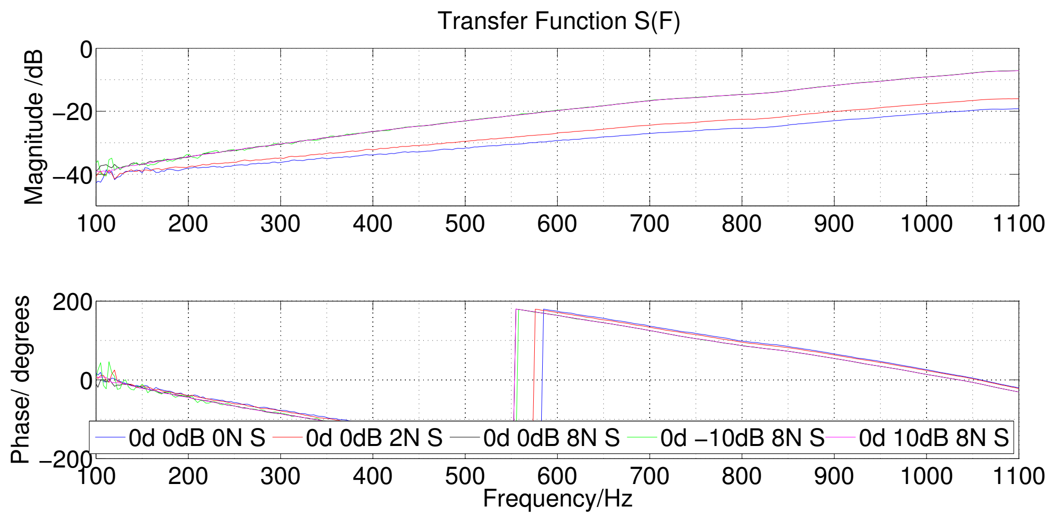

Figure B.7: Frequency Transfer Function Secondary Path

Model of Channel for Simulations

The model of the secondary path follows the real estimated frequency response very well. It is however noteable that the channel itseft suffers from nonstationarities, and the estimation presented is not general, but show the estimation for the standard case.

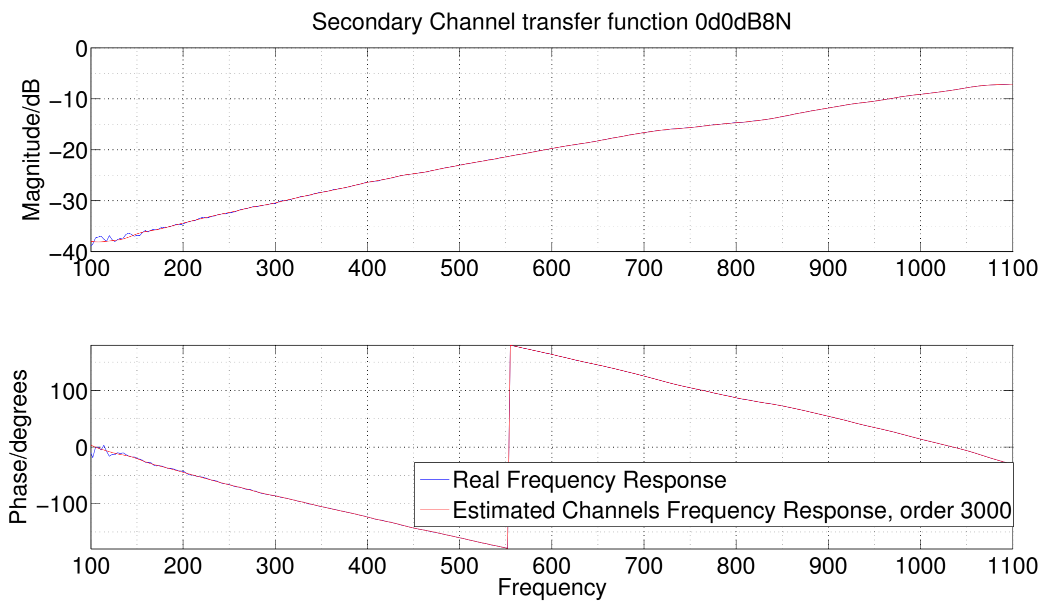

Figure B.8: Frequency Transfer Function of the Secondary Path for model and estimated path of the standard case

B.3.3 Acoustic Feedback Path

The acoustic feedback path describes how much of the signal sent out the secondary source is being fed back into the noise reference sensor. This path may be an obstacle in attaining good ANC performance and should therefore be considered. In this section graphs describing the acoustic feedback path are presented as a compliment to those presented in the thesis.

Coherence

The coherence plot describes the feedback path as rather stationary for different angular placements. The transfer of noise from the secondary source is so little that the coherence is heavily affected by the noise floor.

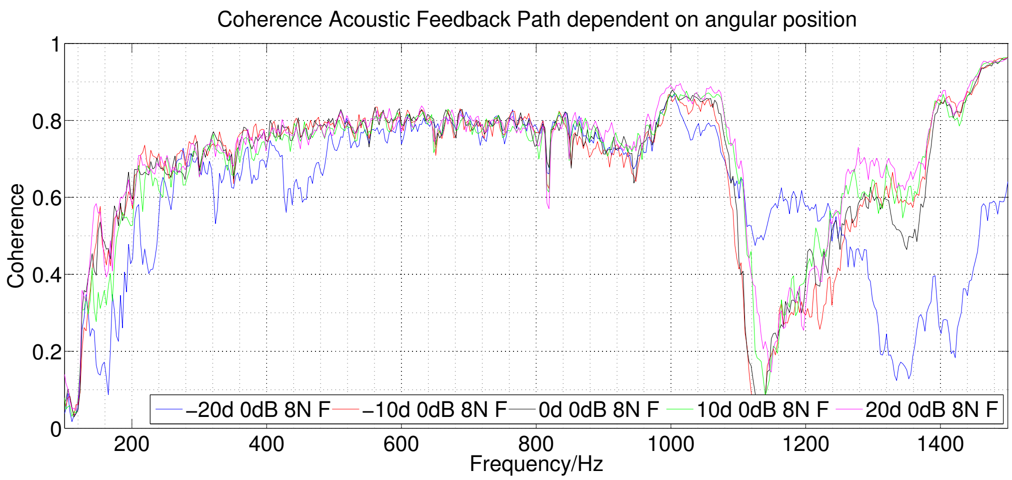

Figure B.9: Coherence Acoustic Feedback Path with different angular placements of the dummy phone

Transfer Function

The transfer function confirms the stationarity of the channel. For 20 degrees angular displacement the phase differse from the other scenarios with about 30 degrees. This should however not affect the adaption much.

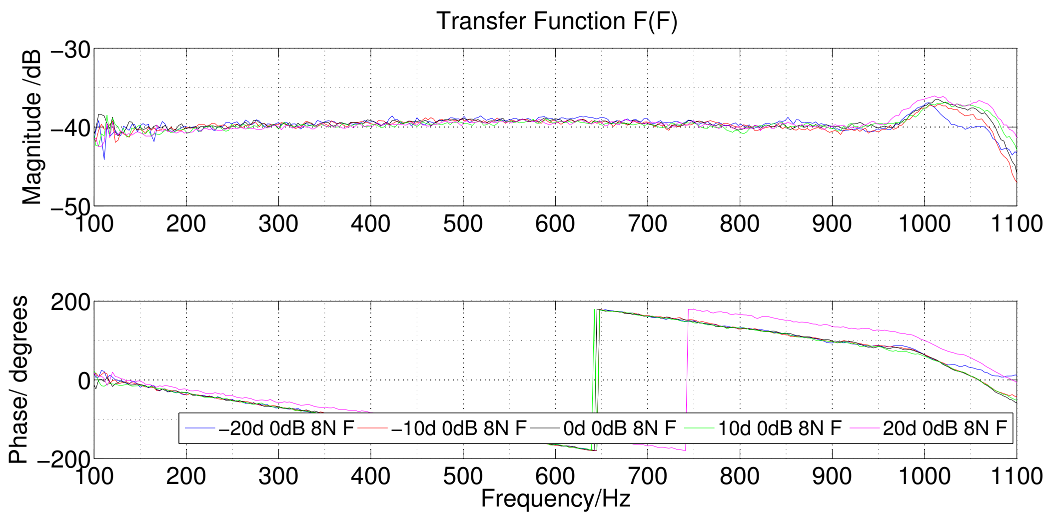

Figure B.10: Frequency Transfer Function Acoustic Feedback Path

Model of Channel for Simulations

The model of the feedback path is a pretty good estimation of the real path. The cariations in the real frequency responce is probably connected to the noise floor.

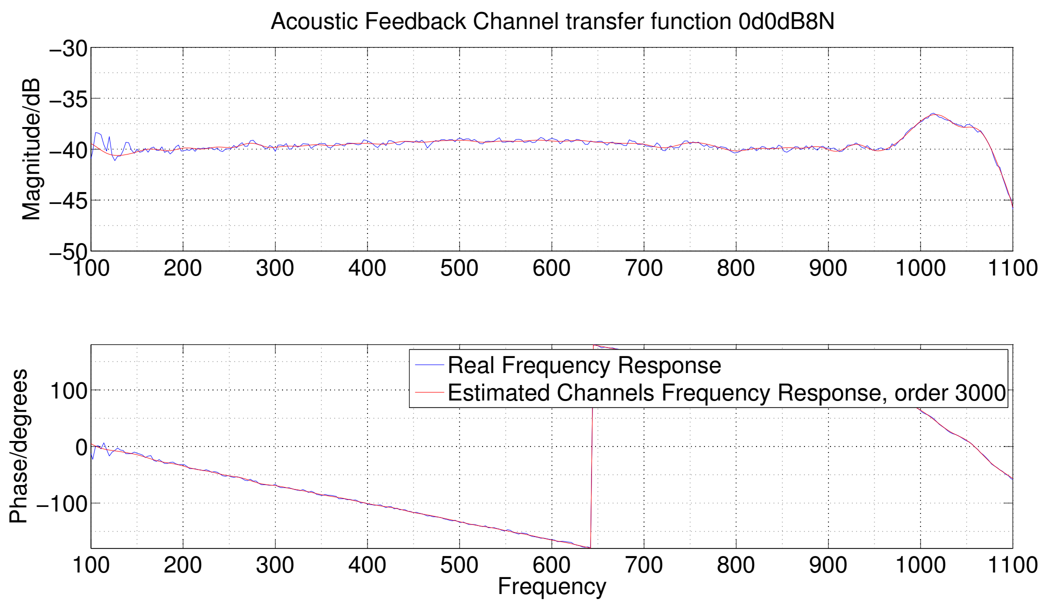

Figure B.11: Frequency Transfer Function of the Acustic Feedback Path for model and estimated path

B.3.4 Acoustic Echo Path

The acoustic echo path, describes how much of the anti-noise signal is being fed into the speech and onto the transmission line. If th.is path has a high transfer it might become an obstacle. In this section graphs describing the acoustic echo path are presented as a compliment to those presented in the thesis.

Coherence

The coherence show a rather stationary acoustic echo path. The only major variations is caused by SPL and is heavily affected by the noise floor.

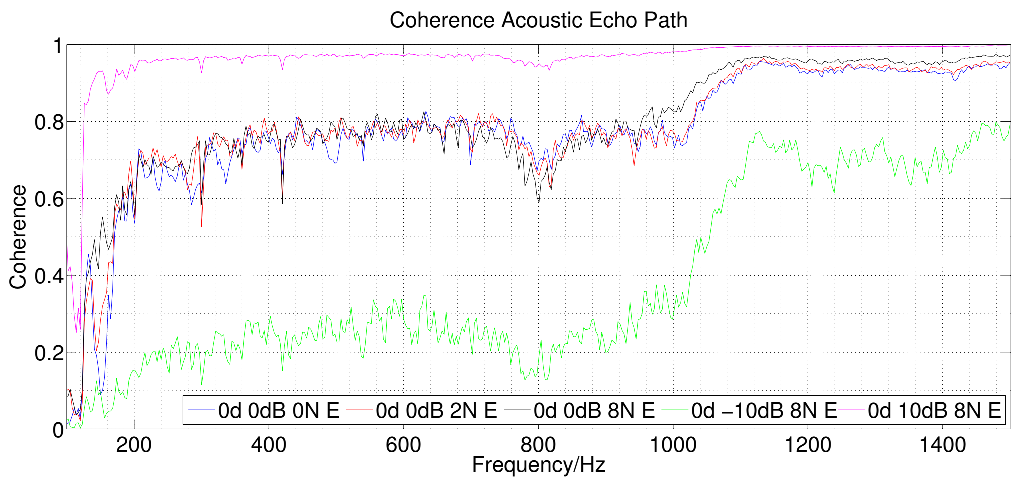

Figure B.12: Coherence Acoustic Feedback Path dependent on relative gain and pressure towards the ear of the HATS.

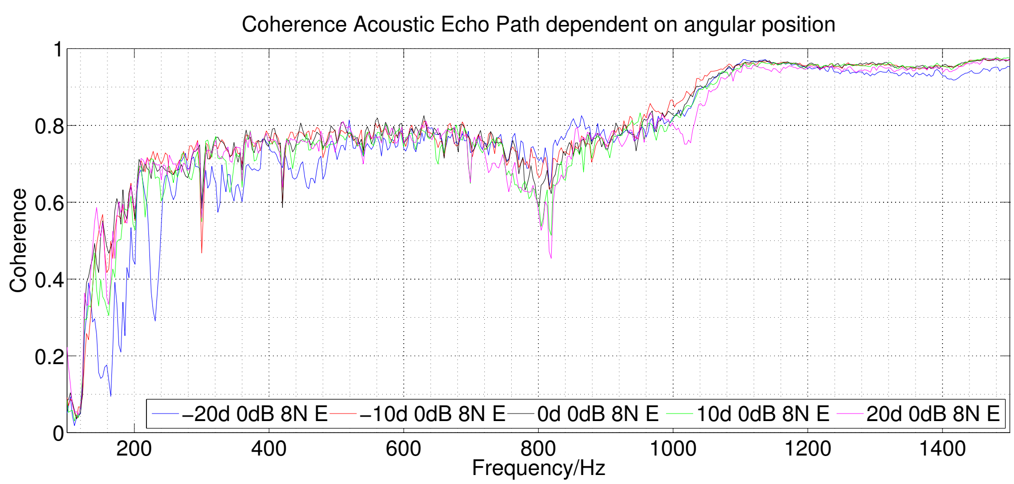

Figure B.13: Coherence Acoustic Echo Path. Comparison 2
Coherence Acoustic Echo Path with different angular placements of the dummy phone

Transfer Function

The transfer function of the acoustic echo path is pretty stationary, and has an attenuation around -40dB from 100Hz to about 1000Hz. For higher frequencies the channel attenuates less. This however will not be an issue if the frequency rages chosen to 100-1100Hz.

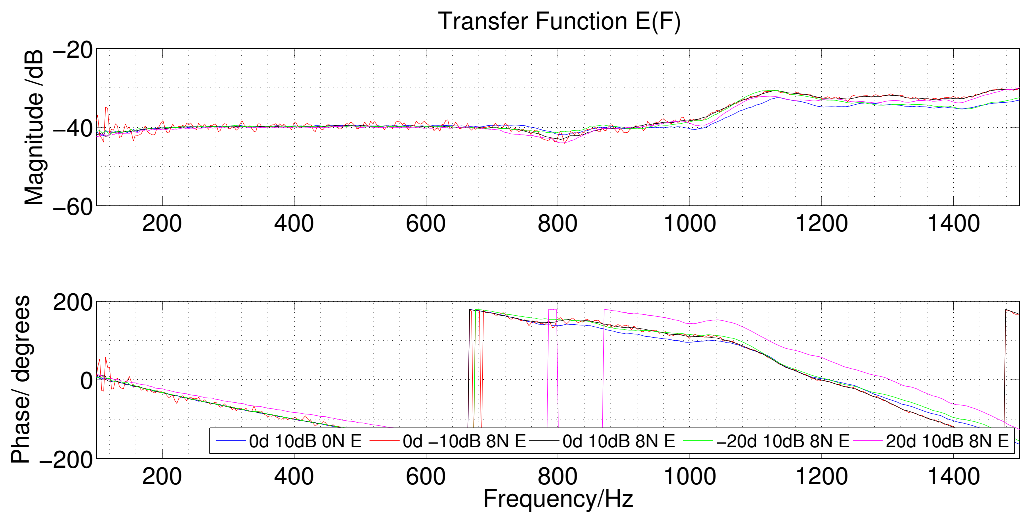

Figure B.14: Frequency Transfer Function Acoustic Echo Path

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