Electromagnetic energy and antennas

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Acknowledgments

- The Swedish Research Council.
- Swedish Foundation for Strategic Research (SSF).

Collaboration with:

- Marius Cismasu, Lund University
- Doruk Tayli, Lund University
- Sven Nordebo, Linnæus University
- Lars Jonsson, KTH
Optimal (automated) antenna design

Applications
- Body area networks
- (Pico) base stations
- Wireless power transfer
- Mobile phones
- RFID and Internet of things (IoT)

Figure of merit

Understanding
- lossy media
- embedded antennas
- super directivity
- radiation patterns
- near fields

Current optimization

Global optimization

Physical bounds

Optimal current distribution

Optimal (automated) antenna design

Applications

Performance of an antenna design in relation to the optimal performance

▶ % from the optimal design
▶ a useful number to compare designs
▶ is it worth to improve a design?

...
Optimal (automated) antenna design

Current optimization, lossy media, embedded antennas, super directivity, radiation patterns, near fields.

Applications: Body area networks, (Pico) base stations, Wireless power transfer, Mobile phones, RFID and Internet of things (IoT).

Figure of merit, understanding performance of an antenna design in relation to the optimal performance, ▶ % from the optimal design, ▶ a useful number to compare designs, ▶ is it worth to improve a design?

Physical bounds, current distributions, polarizability, optimal current distribution, physical bounds, convex optimization...

Antenna design using genetic algorithms (GA), particle swarm, ant colony, ...

Chu bound, \( D / Q / (k \alpha)^3 \) 

\[ \frac{d}{\lambda} \approx 0.11 \]
\[ \frac{d}{\lambda} \approx 0.18 \]
\[ \frac{d}{\lambda} \approx 0.24 \]
\[ \frac{d}{\lambda} \approx 0.27 \]
Optimal (automated) antenna design

Performance of an antenna design in relation to the optimal performance

- % from the optimal design
- a useful number to compare designs
- is it worth to improve a design?
- …
Optimal (automated) antenna design

Physical bounds
Current distributions
Polarizability

Figure of merit

Understanding

Body area networks
Wireless power transfer
Mobile phones
RFID and Internet of things (IoT)
(Pico) base stations
Body area networks
Current distributions
Antenna synthesis
Optimal currents
Optimal antenna design

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Optimal (automated) antenna design

- Optimal current distribution
- Physical bounds
- Convex optimization
- ...
Optimal (automated) antenna design

Antenna design using genetic algorithms (GA), particle swarm, ant colony, ...

$\frac{l}{\lambda} \approx 0.11, \frac{l}{\lambda} \approx 0.18, \frac{l}{\lambda} \approx 0.24, \frac{l}{\lambda} \approx 0.27$

Global optimization

Applications

Optimalization

- Body area networks
- (Pico) base stations
- Mobile phones
- Wireless power transfer
- RFID and Internet of things (IoT)

Performance of an antenna design in relation to the optimal performance

- ▶ % from the optimal design
- ▶ a useful number to compare designs
- ▶ is it worth to improve a design?

Physical bounds

Current distributions

Polarizability

Optimal current distribution

Physical bounds

Convex optimization

...
Can we use optimal currents to synthesize antennas?
Optimal (automated) antenna design

**Applications**
- Body area networks
- (Pico) base stations
- Wireless power transfer
- Mobile phones
- RFID and Internet of things (IoT)

**Optimal automated antenna design**

**Figure of merit**

**Understanding**
- Lossy media
- Embedded antennas
- Super directivity
- Radiation patterns
- Near fields

**Current optimization**

**Global optimization**

**Antenna synthesis**

**Performance of an antenna design in relation to the optimal performance**

▶ % from the optimal design
▶ A useful number to compare designs
▶ Is it worth to improve a design?

▶ Physical bounds
▶ Current distributions
▶ Polarizability

**Convex optimization**

**Optimal current distribution**

**Physical bounds**

**Chu bound**, $D / Q / (k \alpha)^3$

$l / \alpha = 0.11$
$l / \alpha = 0.18$
$l / \alpha = 0.24$
$l / \alpha = 0.27$

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Optimization of antenna currents: examples

Gain over Q

\[
\begin{align*}
\text{minimize} & \quad \text{Stored energy} \\
\text{subject to} & \quad \text{Radiation intensity} = P_0
\end{align*}
\]

Q for superdirectivity \( D \geq D_0 \).

\[
\begin{align*}
\text{minimize} & \quad \text{Stored energy} \\
\text{subject to} & \quad \text{Radiation intensity} = D_0 \frac{P_{\text{rad}}}{4\pi} \\
& \quad \text{Radiated power} \leq P_{\text{rad}}
\end{align*}
\]

Embedded structures

\[
\begin{align*}
\text{minimize} & \quad \text{Stored energy} \\
\text{subject to} & \quad \text{Radiation intensity} = P_0 \\
& \quad \text{Correct induced currents}
\end{align*}
\]

Need to:

1. Express the stored energy in the current density \( J \).
2. Solve the optimization problems.
What is (stored) EM energy?

- Time average energy density: 
  \[ \varepsilon_0 \frac{|\mathbf{E}|^2}{4} \] and 
  \[ \mu_0 \frac{|\mathbf{H}|^2}{4}. \]
- What is stored and radiated?
- How can we express the (stored) energy in the current density?
- Here, currents in free space.

Lumped elements

- Capacitors: 
  \[ W^{(E)} = \frac{C|V|^2}{4} = \frac{|I|^2}{4\omega^2C} \]
- Inductors: 
  \[ W^{(M)} = \frac{L|I|^2}{4} \]
Stored energy and $Q$

- Far fields and $X'_{in}$
- Brune
- Input impedance $Z_{in}$
- Current density $Q$
- $QZ'$
- RCL $QZ'$

Mathematical expressions:

\[
Z_{in} = V_{in} I_{H} H' I_{T} Z' I_{in} = |I_{T}| X'_{in} I_{in} - \frac{1}{2} \eta_0 \text{Im} \int_{\Omega} F' \cdot F^* d\Omega
\]

\[
\epsilon_0 \frac{1}{4} \int_{R} |E|^2 - |F|^2 r^2 dV
\]

\[
|I_{0}|^2 Z'_{in} - \frac{1}{2} \eta_0 \text{Im} \int_{\Omega} F' \cdot F^* d\Omega
\]

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Stored energy and $Q$

Far fields and $X'_{\text{in}}$

Current density

$Q$

Brune

Input impedance $Z_{\text{in}}$

$QZ'$

$RCL$

$QZ'$

\[
\frac{\varepsilon_0}{4} \int_{\mathbb{R}^3} |E|^2 ds \frac{|F|^2}{r^2} \, dV
\]
\[
\frac{\epsilon_0}{4} \int_{\mathbb{R}_+^3} |E|^2 ds \frac{|F|^2}{r^2} dV \\
\frac{|I_0|^2}{4} \frac{X'_{\text{in}}}{2\eta_0} \text{Im} \int_{\Omega} F' \cdot F^* d\Omega
\]
Stored energy and $Q$

Current density

$Q_{Z'}$

$Q$

$\frac{\epsilon_0}{4} \int_{R^3} |E|^2 ds \frac{|F|^2}{r^2} dV$

$\frac{|I_0|^2}{4} X'_{\text{in}} = \frac{1}{2\eta_0} \text{Im} \int_{\Omega} F' \cdot F^* d\Omega$

Far fields

Kirchhoff

$ZI'_{\text{Brune}} \equiv V$

$\frac{1}{4} I^H X'I$

Input

$Z_{\text{in}} I_{\text{in}} = V_{\text{in}}$

$|Z'_{\text{in}}| = \frac{|I^T X'I|}{|I_{\text{in}}|^2}$

$Z_{\text{in}} I_{\text{in}} = V_{\text{in}}$
Stored energy and $Q$

\[ \frac{\varepsilon_0}{4} \int_{\mathbb{R}^3} |E|^2 \frac{|F|^2}{r^2} \, dV \]

\[ \frac{|I_0|^2}{4} X'_\text{in} = \frac{1}{2\eta_0} \text{Im} \int_{\Omega} F' \cdot F^* \, d\Omega \]

Kirchhoff

\[ ZI = V \]

\[ \frac{1}{4} I^H X'I \]

MoM

\[ ZI = V \]

\[ \frac{1}{4} I^H X'I \]

Input

\[ Z_{\text{in}} I_{\text{in}} = V_{\text{in}} \]

\[ \frac{1}{4} |I^T Z'I| \]

\[ |Z'_\text{in}| = \frac{|I^T X'I|}{|I_{\text{in}}|^2} \]
\[
\frac{\epsilon_0}{4} \int_{\mathbb{R}^3} \left| \mathbf{E} \right|^2 \frac{|\mathbf{F}|^2}{r^2} \, dV
\]

\[
\frac{|I_0|^2}{4} \quad \text{Far fields and } \quad X'_\text{in} = \frac{1}{2\eta_0} \Im \int_{\Omega} \mathbf{F}' \cdot \mathbf{F}^* \, d\Omega
\]

\[
\frac{1}{4} \mathbf{I}^H \mathbf{X}' \mathbf{I}
\]

\[
\text{MoM} \quad \frac{Z\mathbf{I}}{Z} = \mathbf{V}
\]

\[
\frac{1}{4} |\mathbf{I}^T \mathbf{Z}' \mathbf{I}|
\]

\[
\frac{1}{4} |\mathbf{I}^T \mathbf{Z}' \mathbf{I}|
\]

\[
\frac{1}{Z'_\text{in}} = \frac{|\mathbf{I}^T \mathbf{X}' \mathbf{I}|}{|\mathbf{I}_{\text{in}}|^2}
\]

\[
\text{Kirchhoff} \quad \frac{1}{4} \mathbf{I}^H \mathbf{X}' \mathbf{I}
\]

\[
\text{Input} \quad Z_{\text{in}} I_{\text{in}} = V_{\text{in}}
\]

\[
\frac{1}{Z_{\text{in}}}
\]

\[
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\]
MoM for $Q$ and $Q_{Z'}$ (I)

Use a method of moments (MoM) formulation of the electric field integral equation (EFIE). Impedance matrix $Z = R + jX$

$$Z_{mn} = j \int_S \int_S \left( k^2 \psi_{m1} \cdot \psi_{n2} - \nabla_1 \cdot \psi_{m1} \nabla_2 \cdot \psi_{n2} \right) e^{-jk r_{12}} \frac{1}{4\pi k r_{12}} dS_1 dS_2$$

where $\psi_{n1} = \psi_n(r_1)$, $\psi_{n2} = \psi_n(r_2)$, $n = 1, \ldots, N$, and $r_{12} = |r_1 - r_2|$. The current density is $J(r) = \sum_{n=1}^N I_n \psi_n(r)$ with the expansion coefficients determined from

$$ZI = V \quad \text{or} \quad I = Z^{-1}V = YV$$

where $V$ is a column matrix with the excitation coefficients. The input admittance is

$$Y_{in} = 1/Z_{in} = V^TYV/V_{in}^2$$

where $Z_{in} = R_{in} + jX_{in}$ is the input impedance.
Differentiate the MoM impedance matrix

$$\frac{k}{\eta} \frac{\partial Z_{mn}}{\partial k} = \int_V \int_V j \left( k^2 \boldsymbol{\psi}_m \cdot \boldsymbol{\psi}_n + \nabla_1 \cdot \boldsymbol{\psi}_m \nabla_2 \cdot \boldsymbol{\psi}_n \right) \frac{e^{-jk r_{12}}}{4 \pi k r_{12}} + \left( k^2 \boldsymbol{\psi}_m \cdot \boldsymbol{\psi}_n - \nabla_1 \cdot \boldsymbol{\psi}_m \nabla_2 \cdot \boldsymbol{\psi}_n \right) \frac{e^{-jk r_{12}}}{4 \pi} dS_1 dS_2$$

Differentiated input admittance (frequency independent V (MoM))

$$V_{in}^2 Y_{in}' = \left( V^T Y V \right)' = V^T Y' V = -I^T Z' I.$$ 

The stored energy determined from $X' = \text{Im} Z'$

$$W^{(E)}_X + W^{(M)}_X = \frac{1}{4} I^H X' I$$

is identical to the stored energy expression (free space) introduced by Vandenbosch (IEEE-TAP 2010), see ?? and already considered by Harrington (1968, 1971, 1975).
\( Q \) and \( Q_{Z'} \) for free-space self-resonant antennas

Assume for simplicity a **self-resonant** antenna (circuit)

\[
Q_{Z'} = \frac{\omega |Z_{in}'|}{2R_{in}} = \frac{\omega |I^TZ'I|}{2I^HR_I}
\]

and using MoM with the stored energy by Vandenbosch

\[
Q = \frac{2\omega \max\{W_{X'}^{(E)}, W_{X'}^{(M)}\}}{P_d} = \frac{\omega I^H X'I}{2I^H R_I}
\]

Transpose for \( Q_{Z'} \) and Hermitian transpose for \( Q \)

- \( I^H X'I \geq 0 \) for positive semidefinite matrices \( X' \).
- \( |I^TZ'I| = 0 \) for some \( I \) (rank > 1).

See also Capek+etal. IEEE-TAP 2014 for \( Q_{Z'} \) using \( I^H \) and \( I' \) and for stored energy and Q-factors in lumped circuits.
Convex optimization

minimize \( f_0(x) \)
subject to \( f_i(x) \leq 0, \ i = 1, \ldots, N_1 \)
\( Ax = b \)

where \( f_i(x) \) are convex, i.e., \( f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y) \) for \( \alpha, \beta \in \mathbb{R}, \alpha + \beta = 1, \alpha, \beta \geq 0 \).

Solved with efficient standard algorithms. No risk of getting trapped in a local minimum. A problem is 'solved' if formulated as a convex optimization problem.

Antenna performance expressed in the current density \( J \), e.g.,

- Radiated field \( F(\hat{k}) = -\hat{k} \times \hat{k} \times \int_V J(\mathbf{r})e^{jk\cdot\mathbf{r}} \ dV \) is affine.
- Radiated power, stored electric and magnetic energies, and Ohmic losses are positive semi-definite quadratic forms in \( J \).
Finite ground plane with \( \{6, 10, 25, 100\}\% \) antenna region

\[
\frac{G}{Q} \quad f / \text{GHz}, \ell = 10 \text{ cm}
\]

- Black: 100%
- Blue: 25%
- Red: 10%
- Green: 6%

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Finite ground plane with \{6, 10, 25, 100\}% antenna region

\[ f / \text{GHz, } \ell = 10 \text{ cm} \]

\[ G/Q \]

\[ 0.3 \quad 0.6 \quad 0.9 \quad 1.2 \quad 1.5 \]

\[ 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \]

\[ \ell/\lambda \]

\[ 10^0 \quad 10^{-1} \quad 10^{-2} \quad 10^{-3} \]

\[ 100\% \quad 25\% \quad 10\% \quad 6\% \]
Why convex optimization?

Solved if formulated as a convex optimization problem.

Consider the $G/Q$ problem

$$\begin{align*}
\text{minimize} & \quad \max \{ I^H X_e I, I^H X_m I \} \\
\text{subject to} & \quad F^H I = 1
\end{align*}$$

Many (optimization) algorithms can be used to solve this problem.

- Can e.g., use any of the solvers included in CVX.
  - Very simple to use.
  - Good for small problems but less efficient for larger problems.
- A dedicated solver for quadratic programs.
  - More efficient for larger problems.
- Random search, e.g., genetic algorithms (GA), particle swarms, ...
  - Very inefficient. Note you do not (should not) use (GA, ...) to solve e.g., $Ax = b$ (min.$\|Ax - b\|$).
- We also use a dual formulation
  - Computational efficient for large problems.
  - Illustrates dual problems and posteriori error estimates.
Why convex optimization: illustration

The upper bound on $G/Q|_{ub}$ is obtained by solving the dual (relaxed) problem, i.e., finding the minimum of the (blue) curve

$$
\frac{G}{Q}|_{ub} \leq \frac{G_\alpha}{\alpha Q_{e\alpha} + (1 - \alpha) Q_{m\alpha}}
$$

This is efficiently solved by golden section search and parabolic interpolation.

\[ G(\hat{z}, \hat{x})/Q \]

\[ G(\hat{y}, \hat{x})/Q \]

\[ \ell/\lambda \approx 0.1 \text{ or } ka \approx 0.35 \]
Why convex optimization: illustration

We also compute the actual $G/Q$ for the current $I_\alpha$ to get

$$\frac{G_\alpha}{\max\{Q_{e\alpha}, Q_{m\alpha}\}} \leq \left. \frac{G}{Q} \right|_{ub}$$

$\ell/\lambda \approx 0.1$ or $ka \approx 0.35$
Why convex optimization: illustration

The upper bound on $G/Q|_{ub}$ is obtained by solving the dual (relaxed) problem, i.e., finding the minimum of the (blue) curve

$$\frac{G}{Q}|_{ub} \leq \frac{G_\alpha}{\alpha Q_{e\alpha} + (1 - \alpha)Q_{m\alpha}}$$

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We also compute the actual $G/Q$ for the current $I_\alpha$ to get

$$\frac{G_\alpha}{\max\{Q_{e\alpha}, Q_{m\alpha}\}} \leq \frac{G}{Q}|_{ub}$$

$\ell/\lambda \approx 0.1$ or $k\alpha \approx 0.35$
Why: simple optimization formulations

Super directivity:

minimize \( \max\{I^H X_e I, I^H X_m I\} \)

subject to \( F^H I = 1 \)
\( I^H R_r I \leq 4\pi/(\eta_0 D_0) \)

Prescribed far field:

minimize \( \max\{I^H X_e I, I^H X_m I\} \)

subject to \( \int_\Omega |F(\hat{k}) - F_0(\hat{k})|^2 \, d\Omega \hat{k} < \delta \)

Embedded antennas:

minimize \( \max\{I^H X_e I, I^H X_m I\} \)

subject to \( F^H I = 1 \)
\( I_2 = CI_1 \)
Summary

▶ Stored energy expressed in the current density, fields, and input impedance.
▶ Can we define stored energy for a Herglotz function?
▶ Energy in complex media?
▶ Optimization of the antenna structure (global optimization) and the antenna currents (convex optimization).
▶ Convex optimization for bounds and optimal currents: $G/Q$, superdirective, embedded, ...
▶ Closed form solutions for small antennas.

Initial results for efficiency, more realistic geometries (phones), SAR, MIMO. Investigating dielectrics, volume currents, magnetic currents, ...

Slides at:
http://www.eit.lth.se/staff/mats.gustafsson
References

Current optimization and physical bounds


Stored energy expressed in the current density


Convex optimization


See also: http://www.eit.lth.se/staff/mats.gustafsson
References


