Är världens mest kända ekvation $E = mc^2$ korrekt?

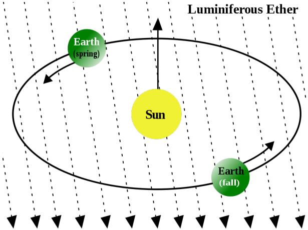
Galilean transformation 1632

Any two (inertial) observers moving at constant speed and direction with respect to one another will obtain the same results for all mechanical experiments.

Imagine you are inside a ship which is sailing on a perfectly smooth lake at constant speed.

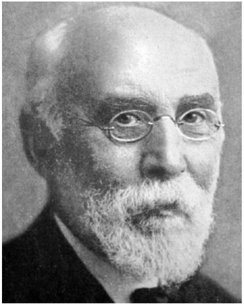
Can you determine that the ship is moving?

Luminiferous ether

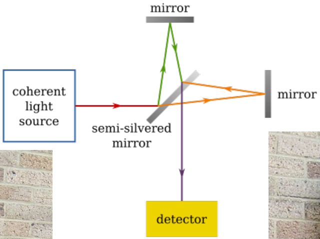



(a) James Clerk Maxwell


(b) Henri Poincaré


(c) Hendrik Lorentz

Michelson and Morley



Albert A Michelson

Edward W Morley

Special relativity, Einstein 1905

1. All the laws of physics are the same for inertial observers.
2. The speed of light is independent of the motion of the observers.

Maxwell's equations (1861) $\bar{D} = \epsilon\epsilon_0 \bar{E}, \quad \bar{H} = \frac{1}{\mu}\frac{1}{\mu_0}\bar{B}$

Electromagnetic wave velocity in vacuum

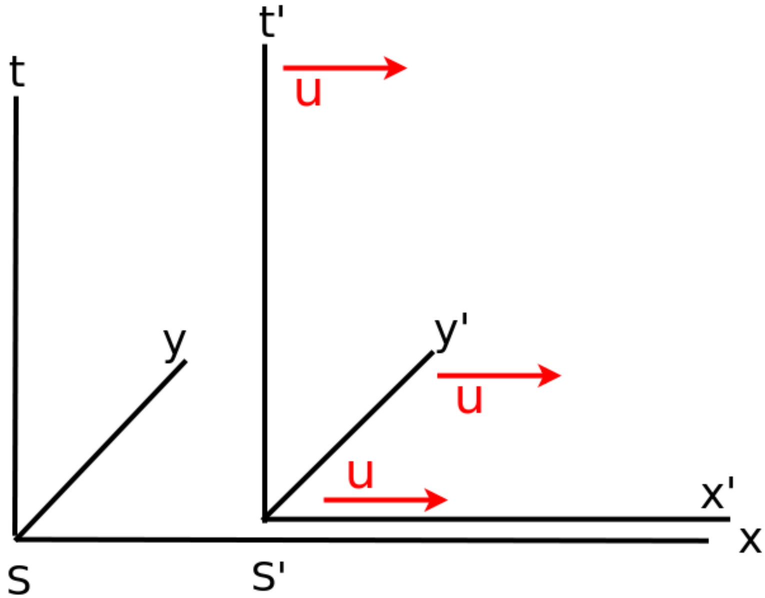
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 299\ 792\ 458 \text{ m/s}$$ is a Constant !

1 meter is defined as the distance traveled by light during

1/299792458 sec

Einstein in Bern

Frames of reference



Lorenz transformation

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut), \quad t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2)$$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

If we consider two events labelled 1 and 2, the coordinate differences obey

$$\Delta x' = \gamma(\Delta x - u\Delta t), \qquad \Delta t' = \gamma(\Delta t - u\Delta x/c^2)$$

where $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$ and differences are not necessarily small.

Time dillation

Your clock is moving with the frame. Thus, in

$$t = \gamma(t' + ux'/c^2) \text{ put } x' = 0, \text{ getting}$$

$$t = \gamma t'$$

Your clock is slow compared to mine. All moving clocks are slow.

Examples:

$$u = 0.866c = 2.596 \times 10^8 \text{m/s} \rightarrow \gamma = 2$$

One hour of my time corresponds to 30 minutes of your time.

$u = 120$ km/h Your clock will be slow by 0.195 μsec per year.

Length contraction

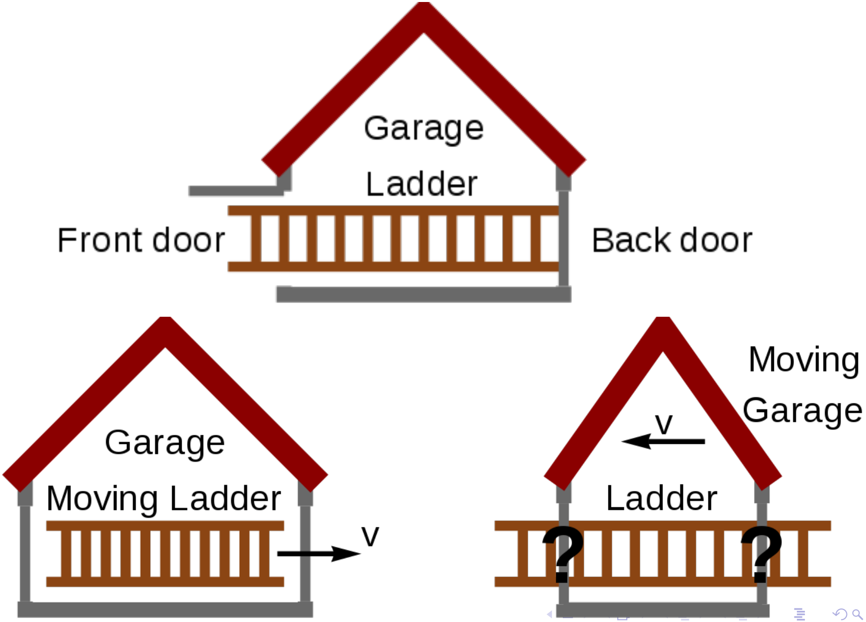
I have to determine the x-coordinates of the endpoints of your measuring rod at the same instant. Thus, in

$$x' = \gamma(x - ut) \text{ put } t = 0, \text{ getting}$$

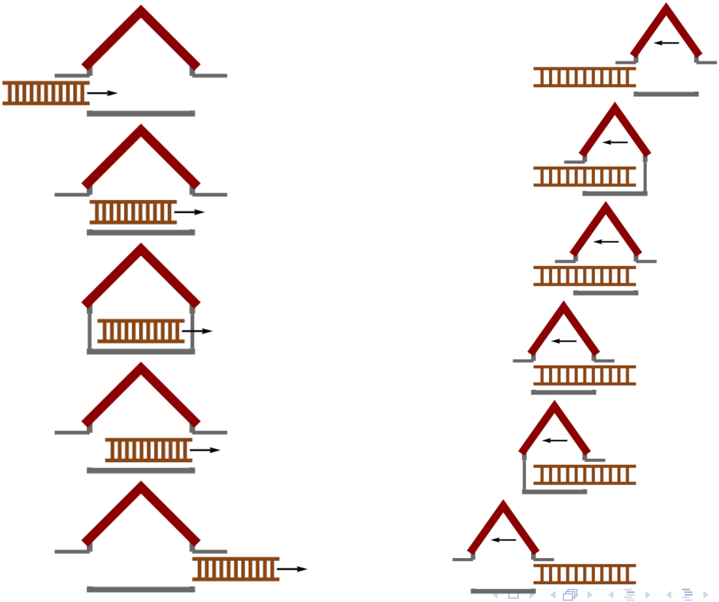
$$x' = \gamma x$$

Your one meter measuring rod is shorter than mine.

Ladder paradox



Scenarios in garage and ladder frame



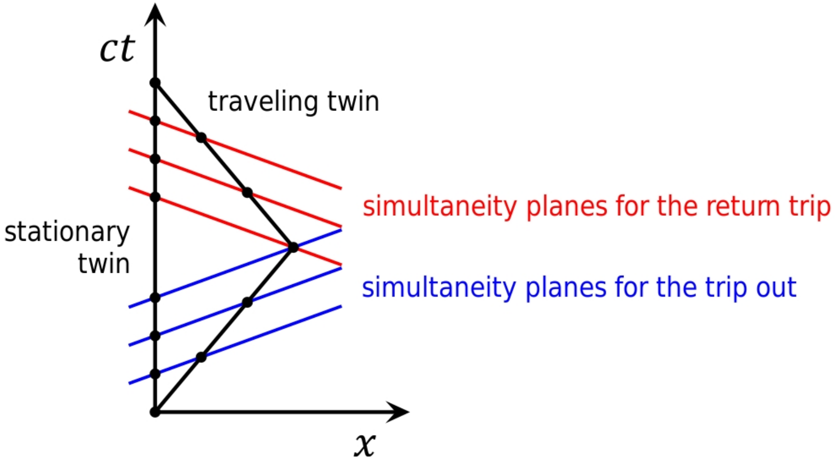
Relative simultaneity

The garage is 10 feet wide and the ladder is 12 feet long. The ladder is moving at velocity of $v = c\sqrt{1/2}$ therfore $\gamma = \sqrt{2}$
Use c = 1 ft/ns

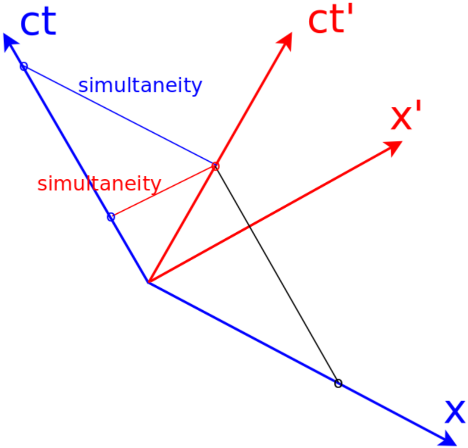
Open back door and close front door simultaneously (garage frame)

The front door will be closed 8.48 ns after the back door is opened (ladder frame)

Twin paradox



Loedel diagram



Stationary clock located at $x = 0$, moving clock at $x' = 0$

$$u/c = 0.866, \gamma = 2$$

Velocity addition

Follow a particle
as it moves an amount Δx in time Δt according to me

Letting the deltas be infintesimals going to zero, the velocities become

- $v = \frac{\Delta x}{\Delta t}$ according to me
- $w = \frac{\Delta x'}{\Delta t'}$ according to you
- $u =$ your velocity relative to me

$$w = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x - u\Delta t}{\Delta t - u\Delta x/c^2} = \frac{v - u}{1 - uv/c^2}$$

If we go from your description to mine we get

$$v = \frac{w + u}{1 + uw/c^2}$$, $w = c$ yields $v = c$ for all values of u

Space-time interval

The space-time interval $s^2 = {x_0}^2 - {x_1}^2$ between the origin and the point (x_0, x_1) is the same for all observers The four-dimensional dot product $X \cdot X)$ is similar to the invariant length of the (x, y, z) vector.

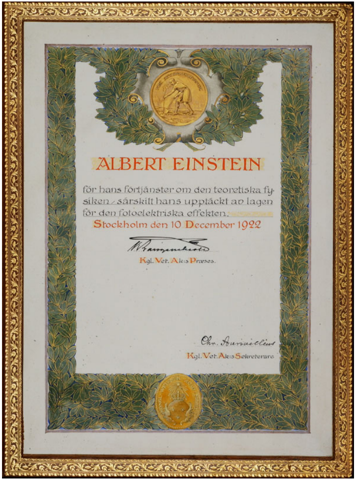
$$X \cdot X = {x_0}^2 - {x_1}^2 = {x'_0}^2 - {x'_1}^2$$

If there are two events separated in space by Δx_1 and time by Δx_0 then the square of the space-time interval between the events is

$$(\Delta s)^2 = (\Delta x_0)^2 - (\Delta x_1)^2$$

and is the same for all observers. Note that s^2 and $(\Delta s)^2$ are not positive definite.

Nobelpriset i fysik 1921 delades ut först 1922 tillsammans med 1922 års pris, som Niels Bohr fick för sin atommodell.



... /oberoende av det värde som / efter eventuell bekräftelse må tillerkännas relativitets- och gravitationsterorien / ...

Nobel Physics Committee

Allvar Gullstrand (1862-1930)

Ögonläkare.

He was the Chairman of the Nobel Physics Committee of the Swedish Academy of Sciences (1922-1929).

Nobelpris in Physiology or Medicine 1911 for his work on the dioptrics of the eye.

"Einstein must never receive a Nobel Prize, even if the whole world demands it"

He published work on general relativity and his name is attached to Painlevé-Gullstrand coordinates.

Among other things he criticised the absence of dynamic solutions (gravitational waves) in general relativity.

Gullstrand, A.: “Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie” Ark. Mat. Astr. Fys. 16(8) 1–15 (1922)

Einstein 1923 lecture

ALBERT EINSTEIN Fundamentale Ideen und Probleme der Relativitätstheorie Lecture delivered to the Nordic Assembly of Naturalists at Gothenburg July 11, 1923

Four-vector momentum

For a space-vector we define the velocity of a particle located at (x, y, z) by

$$\bar{v} = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt})$$

and its momentum $\bar{p} = m\bar{v}$ by multiplying by the scalar m.
The equivalent to the scalar dt in four dimensions is the space-time interval:

$$ds = \sqrt{(dx_0)^2 - (dx_1)^2} = cdt\sqrt{1 - \left(\frac{dx}{cdt}\right)^2} = cdt\sqrt{1 - v^2/c^2}$$

where v is the velocity of the particle as seen by the observer who has assigned the coordinates X to that particle.

Proper time

Instead of ds we will use the quantity $d\tau$ with dimensions of time

$$d\tau = dt\sqrt{1 - v^2/c^2} = \sqrt{dt^2 - dx^2/c^2}$$

In the frame moving with the particle, the two events on its trajectory occur at the same point $dx = 0$. Thus, $d\tau$ is the time interval of the on-board clock (the proper time).

We define the (energy-)momentum four-vector as

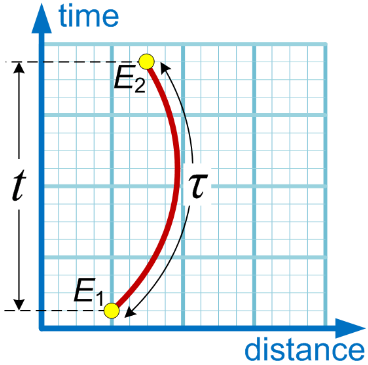
$$P = m\left(\frac{dx_0}{d\tau}, \frac{dx_1}{d\tau}, \frac{dx_2}{d\tau}, \frac{dx_3}{d\tau}\right)$$

We can trade τ derivatives of any function f for t derivatives:

$$\frac{df}{d\tau} = \frac{df}{dt} \cdot \frac{dt}{d\tau} = \frac{df}{dt} \cdot \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{and write}$$

$$P = (P_0, P_1) = \left(\frac{mc}{\sqrt{1 - v^2/c^2}}, \frac{mv}{\sqrt{1 - v^2/c^2}}\right)$$

Proper and coordinate time



Energy-momentum

We define the energy

$$E = cP_0 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = mc^2 + \frac{1}{2}mv^2 + \dots$$

The term mc^2 is called rest energy. The remainder is the kinetic energy with correction for faster particles.

The term $P_1 = \frac{mv}{\sqrt{1-v^2/c^2}} = v\frac{E}{c^2}$ is the momentum of the particle in relativistic theory. The last term can be used for a massless particle (photon)

We call E and P_1 Energy and Momentum because relativistic Lagrangians indicate that they are conserved in the cases where Newtonian Lagrangians indicate that the quantities $1/2mv^2$ and mv are conserved

Relativistic invariant and photon

$P_A \cdot P_B$ is invariant, where P_A and P_B are any two four-momenta.

For a single particlee ($A = B$) we obtain the invariant

$$P^2 \equiv P \cdot P = P_0^2 - P_1^2 = \frac{m^2 c^2 - m^2 v^2}{1 - v^2/c^2} = m^2 c^2$$

As the velocity of light c has the the same value for all observers the same is also true for the mass m

We denote the momentum of a photon by K. As the photon has no mass we get

$$K \cdot K = K_0^2 - K_1^2 = 0$$

Thus

$K_0 = K_1$ or $E = cp$ (the photon energy is c times its momentum)

Velocity dependent mass

Some people write

$$P_1 = \left(\frac{m}{\sqrt{1 - v^2/c^2}} \right) \cdot v \equiv m(v) \cdot v$$

where $m(v) = m/\sqrt{1 - v^2/c^2}$ is a velocity dependent mass

For us m is invariant, independent of the speed of the particle.

Note! The relation between mass and energy is NOT $E = mc^2$.
There is NO equivalence between total energy and mass.

For example, the photon has kinetic energy but no mass.

The correct relation is $E_0 = mc^2$ where E_0 is the rest energy.

Photosynthesis

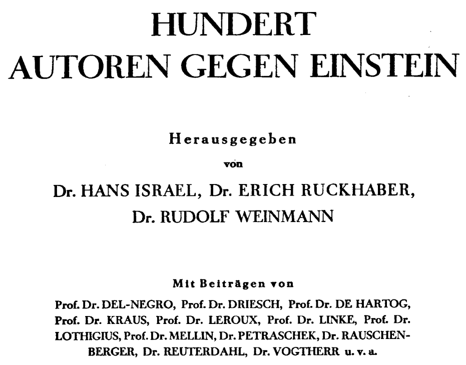
An everyday process where there is a change of mass according to $E_0 = mc^2$ is photosynthesis where the light of the sun is absorbed by vegitation

$$light + 6CO_2 + 6H_2O = 6O_2 + C_6H_{12}O_6$$

The kinetic energy of (the massless) photons is transformed into the rest energy (mass) of carbohydrates.

Against Einstein 1

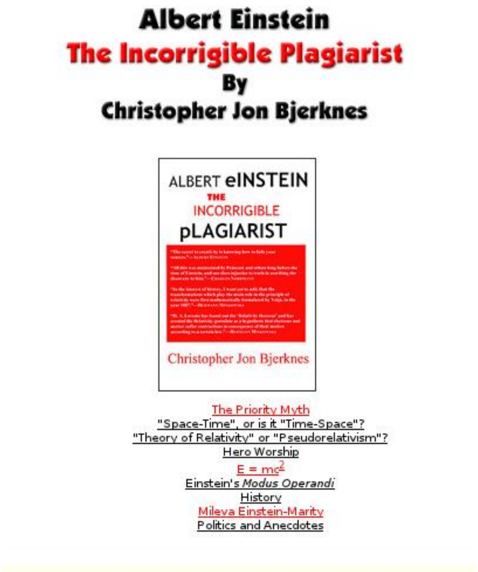
Hundert Autoren gegen Einstein

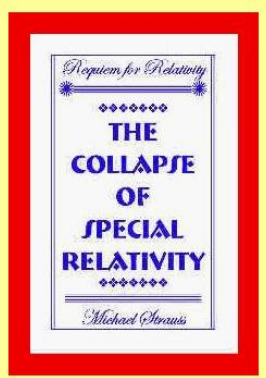

HUNDERT AUTOREN GEGEN EINSTEIN

Herausgegeben von Dr. HANS ISRAEL, Dr. ERICH RUCKHABER, Dr. RUDOLF WEINMANN

Mit Beiträgen von Prof. Dr. DEL-NEGRO, Prof. Dr. DRIESCH, Prof. Dr. DE HARTOG, Prof. Dr. KRAUS, Prof. Dr. LEROUX, Prof. Dr. LINKE, Prof. Dr. LOTHIGIUS, Prof. Dr. MELLIN, Dr. PETRASCHEK, Dr. RAUSCHENBERGER, Dr. REUTERDAHL, Dr. VOGTHERR u. v. a.

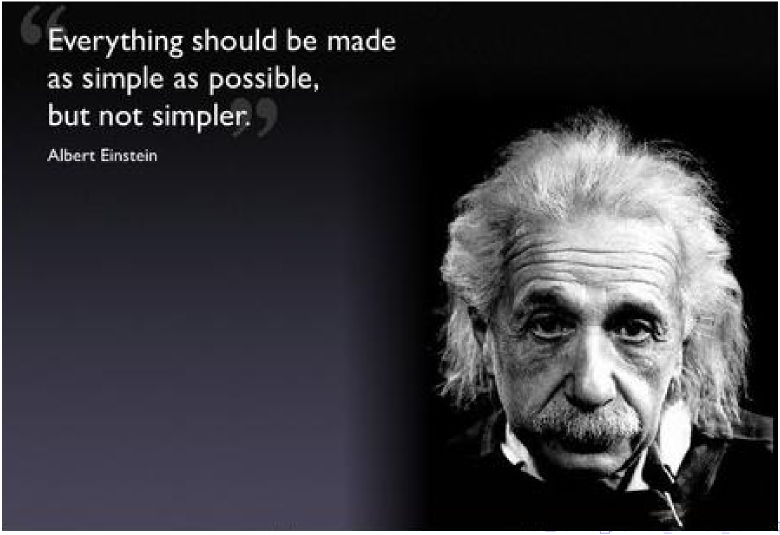
Against Einstein 2





Credo

$$E_0 = mc^2$$


"Everything should be made
as simple as possible,
but not simpler."
Albert Einstein