On the extraordinary transmission through sub-wavelength apertures in perfectly conducting sheets

Mats Gustafsson, Daniel Sjöberg, Iman Vakili

Department of Electrical and Information Technology
Lund University, Sweden

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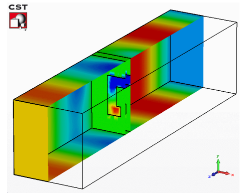
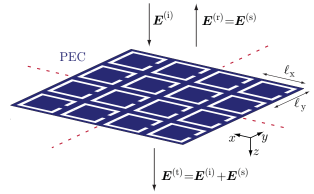
Extraordinary transmission

- Extraordinary optical transmission (EOT) is the phenomenon of greatly enhanced transmission of light through subwavelength apertures in an opaque metallic film.
- Ebbesen *et al.*, Extraordinary optical transmission through sub-wavelength hole arrays, Nature 1998.
- Localization of light, subwavelength optics, sensing, optoelectronics.
- Often in silver films with thickness $\approx 0.2\,\mu\mathrm{m}$.
- Bandpass frequency selective surface.
- Here, we analyze apertures in PEC (perfectly conducting) sheets (zero thickness).





Array of apertures.

Illustration: localization of EM fields

unit cell: $\ell = 1.5\,\mu\mathrm{m}$, SRR $\ell_1 = 0.8\,\mu\mathrm{m}$, $f = \{60, 75, 100\}\,\mathrm{THz}$, $\lambda = \{5, 4, 3\}\,\mu\mathrm{m}$ or $\lambda/\ell = \{10/3, 8/3, 4, 2\}$

E-field, E_y

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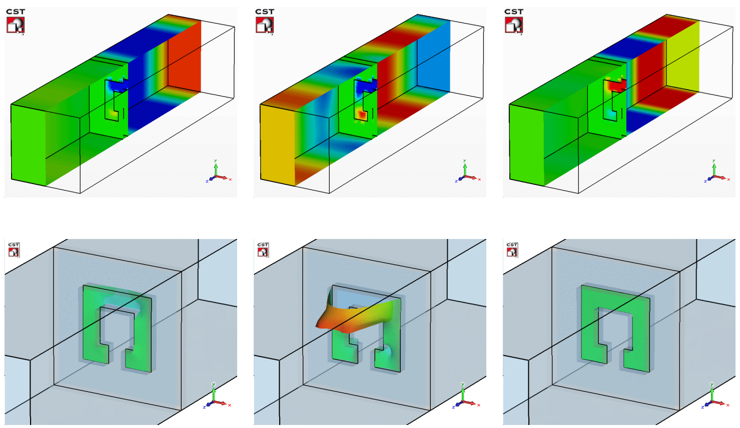
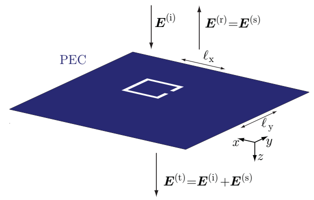
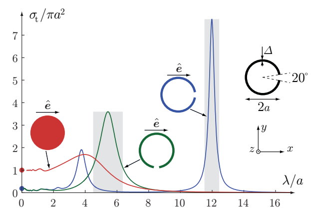


Transmission cross section

- Incident wave $\boldsymbol{E}^{(\mathrm{i})}$ with power flux $\mathcal{P}_\mathrm{i} = |\boldsymbol{E}^{(\mathrm{i})}|^2/(2\eta_0)$.
- Transmitted power through the aperture P_t.
- Transmission cross section $\sigma_\mathrm{t} = P_\mathrm{t}/\mathcal{P}_\mathrm{i}$, unit m^2.
- Extraordinary transmission $\sigma_\mathrm{t} \gg \mathcal{A}$, aperture area $\mathcal{A}$.
- The transmitted power of a wave, with wavelength λ, through a single circular aperture with radius a is proportional to $(a/\lambda)^4$ as $a/\lambda \to 0$. The transmission cross section, σ_t, is hence negligible if $\lambda \gg a$.



Single aperture.

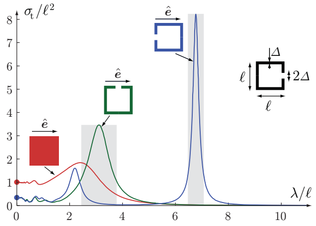


Transmission cross section for a SRR.

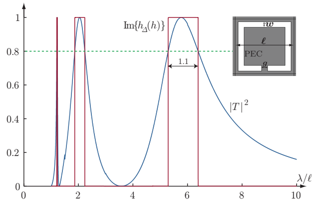
Sum rules on passive systems

1. Identify a linear and passive (and causal) system (construct a Herglotz or positive real function). Here, *e.g.*, the forward scattering.
2. Determine the low- and high-frequency asymptotic expansions. Expressed in the polarizability of the complementary structure.
3. Use integral identities (for Herglotz functions) to relate the dynamic properties to the asymptotic expansion, *e.g.*,

$$\frac{2}{\pi}\int_0^\infty \frac{\operatorname{Im} h(k)}{k^2}\,\mathrm{d}k = a_1 - b_1 \le a_1$$



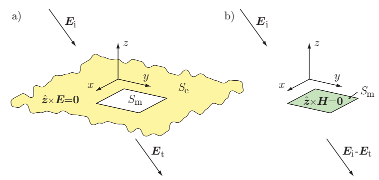
Transmission cross sections of a SRR aperture.



Transmission through a SRR aperture array.

Sum rules and constraints on passive systems J. Phys. A: Math. Theor. 44 145205, 2011

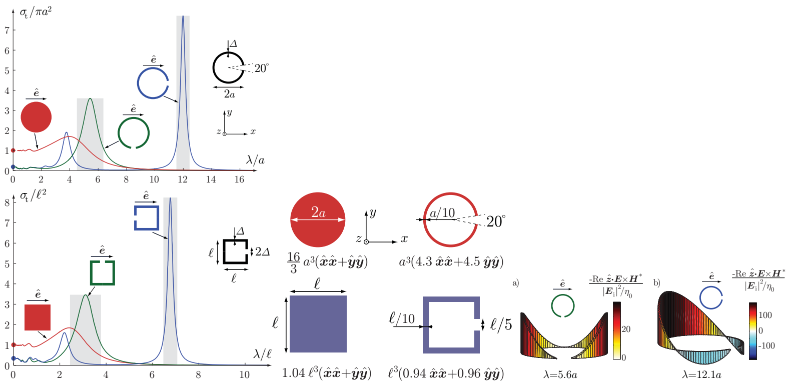
Babinet's principle



- The Babinet's principle replaces the PEC sheet with a finite PMC object.
- Forward scattering, $\sigma_{\text{ext}} = \operatorname{Im} h$, low-frequency expansion, $h(k) \sim \gamma k$, where $\gamma = \big(\hat{\boldsymbol{e}}\cdot\boldsymbol{\gamma}_{\text{e}}\cdot\hat{\boldsymbol{e}} + (\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\cdot\boldsymbol{\gamma}_{\text{m}}\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\big)$.
- Forward scattering sum rule.
- Sum rule for the transmission cross section, $\sigma_{\text{t}} = \sigma_{\text{ext}}/2$

$$\frac{2}{\pi^2}\int_0^\infty \sigma_{\text{t}}(\lambda;\hat{\boldsymbol{z}},\hat{\boldsymbol{e}})\,\mathrm{d}\lambda = (\hat{\boldsymbol{z}}\times\hat{\boldsymbol{e}})\cdot\boldsymbol{\gamma}_{\text{m}}\cdot(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{e}}).$$

Split ring resonators



- σ_{t}=transmission cross section = transmitted power/incident power flux.
- Known area

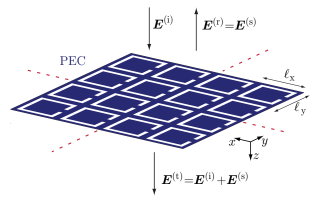
$$\frac{2}{\pi^2}\int_0^\infty \sigma_{\text{t}}(\lambda;\hat{\boldsymbol{z}},\hat{\boldsymbol{e}})\,\mathrm{d}\lambda = (\hat{\boldsymbol{z}}\times\hat{\boldsymbol{e}})\cdot\boldsymbol{\gamma}_{\text{m}}\cdot(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{e}}).$$

- Concentrates the EM power to a sub-wavelength scale.

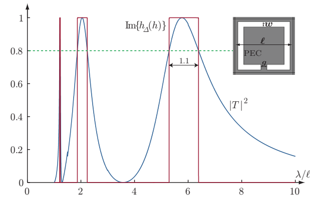
Sum rule for the transmission cross section of apertures in thin opaque screens, Optics letters, 34(13), 2009

Transmission coefficient

- Incident wave $\boldsymbol{E}^{(\text{i})}$ with power flux $\mathcal{P}_{\text{i}} = |\boldsymbol{E}^{(\text{i})}|^2/(2\eta_0)$.
- Transmitted co-polarized field $\boldsymbol{E}^{(\text{t})} = T\boldsymbol{E}^{(\text{i})}$.
- Transmitted power through the apertures per unit cell $P_{\text{t}}/A \ge |T|^2\mathcal{P}_{\text{i}}$.
- Extraordinary transmission $|T|^2 \gg \mathcal{A}/A$, aperture area $\mathcal{A}$.

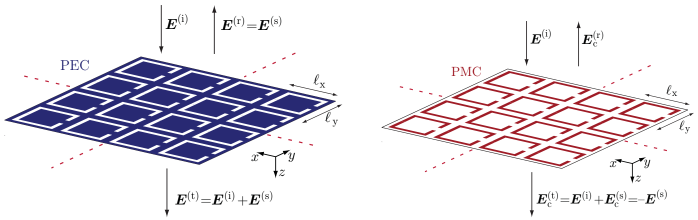


SRR aperture array.



Transmission $|T(\lambda)|^2$ for a SRR with $\mathcal{A}/A \approx 0.3$.

Babinet's principle



- Babinet's principle gives an array with disconnected PMC objects.
- Low-frequency expansion of T_c

$$T_c(k) \sim 1 + \frac{\mathrm{i}k}{2A}\big(\hat{\boldsymbol{e}}\cdot\boldsymbol{\gamma}_\mathrm{e}\cdot\hat{\boldsymbol{e}} + (\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\cdot\boldsymbol{\gamma}_\mathrm{m}\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\big) \quad \text{as } k\to 0,$$

where $\boldsymbol{\gamma}_\mathrm{e}$ and $\boldsymbol{\gamma}_\mathrm{m}$ are the electric and magnetic polarizability dyadics.
- Low-frequency expansion of T

$$T(k) \sim -\frac{\mathrm{i}k}{2A}\big(\hat{\boldsymbol{e}}\cdot\boldsymbol{\gamma}_\mathrm{e}\cdot\hat{\boldsymbol{e}} + (\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\cdot\boldsymbol{\gamma}_\mathrm{m}\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\big) \quad \text{as } k\to 0.$$

Transmission coefficient

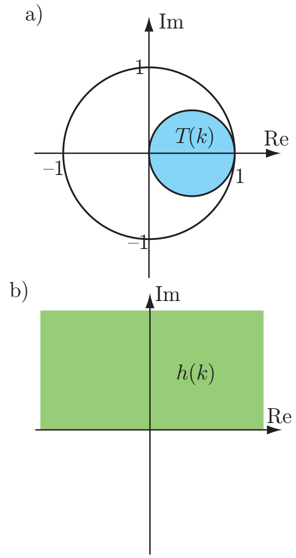
- Transmission and reflection coefficients $T = 1 + R$.
- Conservation of power $|T|^2 + |R|^2 \le 1$.
- Combine

$$-\operatorname{Re} T + |T|^2 = \left|T - \frac{1}{2}\right|^2 - \frac{1}{2^2} \le 0$$

- Construct a Herglotz function, *i.e.*, map to the upper half plane with

$$h(k) = \mathrm{i}\frac{1-T(k)}{T(k)}$$

- $\operatorname{Im} h = 0$ for $|T|^2 + |R|^2 = 1$ (lossless).
- $h = 0$ for $T = 1$ and $h = \infty$ for $T = 0$.



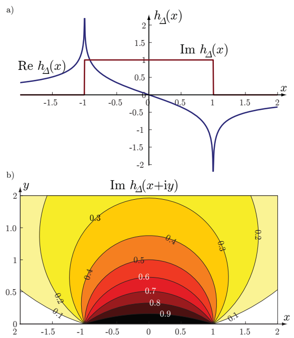
Sum rule I

- Interested in regions with $|T(k)|^2 \ge 1/(1+\varDelta^2)$ that corresponds to $|h(k)|^2 + 2\operatorname{Im} h \le \varDelta$ and hence $|h| \le \varDelta$.
- Compose with the pulse Herglotz function

$$h_\varDelta(z) = \frac{1}{\pi}\ln\frac{z-\varDelta}{z+\varDelta} \sim \begin{cases} \mathrm{i} & z\to 0 \\ -\frac{2}{\pi}\frac{\varDelta}{z} & z\to\infty \end{cases}$$

- Low-frequency asymptotic $T(k) \sim \mathrm{i}k\gamma/(2A)$ as $k\to 0$ imply

$$h(k) \sim \frac{-2A}{\gamma k} \quad \text{as } k\to 0$$



The Herglotz function $h_\varDelta(x+\mathrm{i}y)$ with $\varDelta = 1$.

Sum rule II

- The composed function, $h_\varDelta(h(k))$, has the low frequency expansion

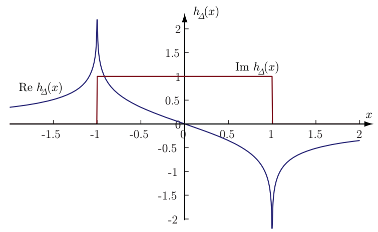
$$h_\varDelta(h(k)) \sim \frac{\gamma k\varDelta}{A\pi} \quad \text{as } k\to 0$$

- Integral identities for Herglotz functions give

$$\int_0^\infty \frac{\operatorname{Im}\{h_\varDelta(h(k))\}}{k^2}\,\mathrm{d}k = \frac{\gamma\varDelta}{2A}$$

- Rewrite in the wavelength $\lambda = 2\pi/k$ to get

$$\int_0^\infty \operatorname{Im}\{h_\varDelta(h(\lambda))\}\,\mathrm{d}\lambda = \frac{\gamma\varDelta\pi}{A}$$



Properties $z = x + \mathrm{i}y$

$$\begin{cases} \operatorname{Im} h_\varDelta(z) \le 1 & \text{for all } z \\ \operatorname{Im} h_\varDelta(z) \ge 1/2 & \text{for } |z| < \varDelta \\ \operatorname{Im} h_\varDelta(x) = 1 & \text{for } |x| < \varDelta \\ \operatorname{Im} h_\varDelta(x) = 0 & \text{for } |x| > \varDelta \end{cases}$$

Numerical example: array of SRR

unit cell: $\ell = 1.5\,\mu\mathrm{m}$, SRR $\ell_1 = 0.8\,\mu\mathrm{m}$, $f = \{60, 75, 100\}$ THz, $\lambda = \{5, 4, 3\}\,\mu\mathrm{m}$ or $\lambda/\ell = \{10/3, 8/3, 4, 2\}$

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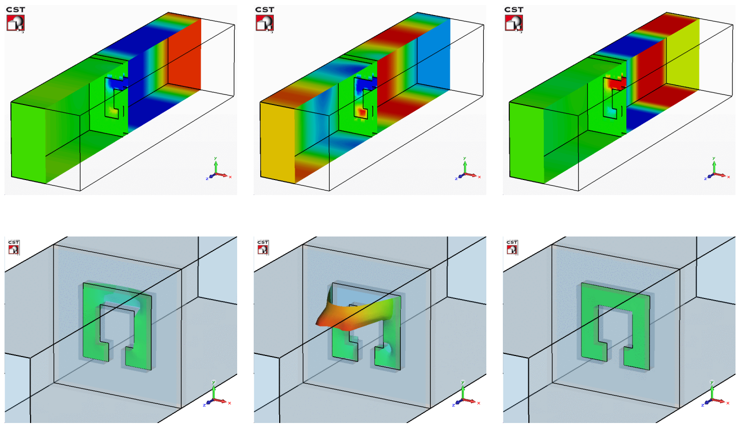
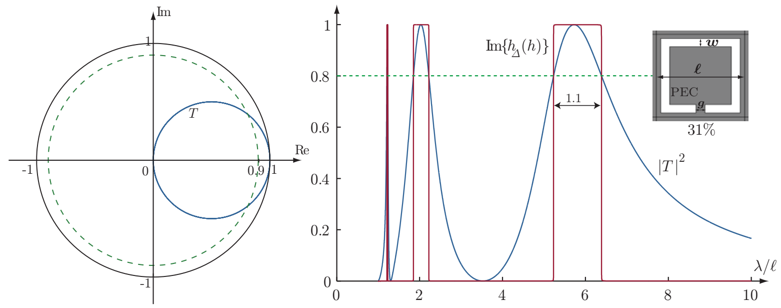
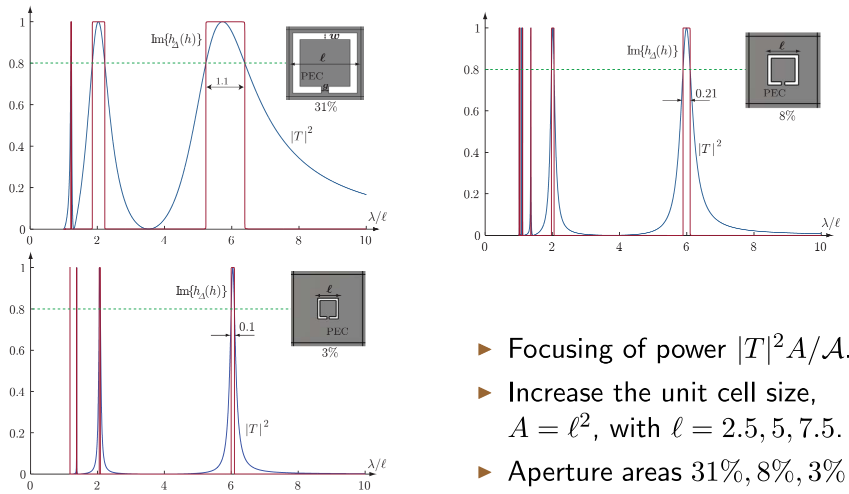


Numerical example: array of SRR



- Aperture array of split ring resonator.
- Threshold level of $|T|^2 \geq 0.8$ (at least 80% of the power is transmitted) or $\varDelta = 1/2$.
- The area under $\mathrm{Im}\{h_\varDelta(h(\lambda/\ell))\}$ is known: 1.56.
- Bandwidth with $|T|^2 \geq 0.8$ is approximately 1.1 and the corresponding bound is 1.56 giving a performance of 70%.

Numerical example: array of SRR



- Focusing of power $|T|^2 A/\mathcal{A}$.
- Increase the unit cell size, $A = \ell^2$, with $\ell = 2.5, 5, 7.5$.
- Aperture areas 31%, 8%, 3%.

Conclusions

- Apertures in PEC sheets.
- Sum rules for extraordinary transmission through single apertures and periodic aperture arrays.
- Polarizability of the complementary structure.
- Physical bounds on the bandwidth.
- Examples with SRR apertures.