Small antennas: Theory and applications

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IEEE AP-S Distinguished Lecturer Program

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Outline

IEEE AP-S Distinguished Lecturer Program

- Institute of Electrical and Electronics Engineers (IEEE).
- IEEE Antenna and Propagation Society (AP-S).

IEEE AP-S Distinguished Lecturer Program.

- provides AP-S chapters around the world with talks on topics of interest and importance to the AP community.
- possess a broad range of expertise within the area of AP.
- http://www.ieeeaps.org/distlecturers.html

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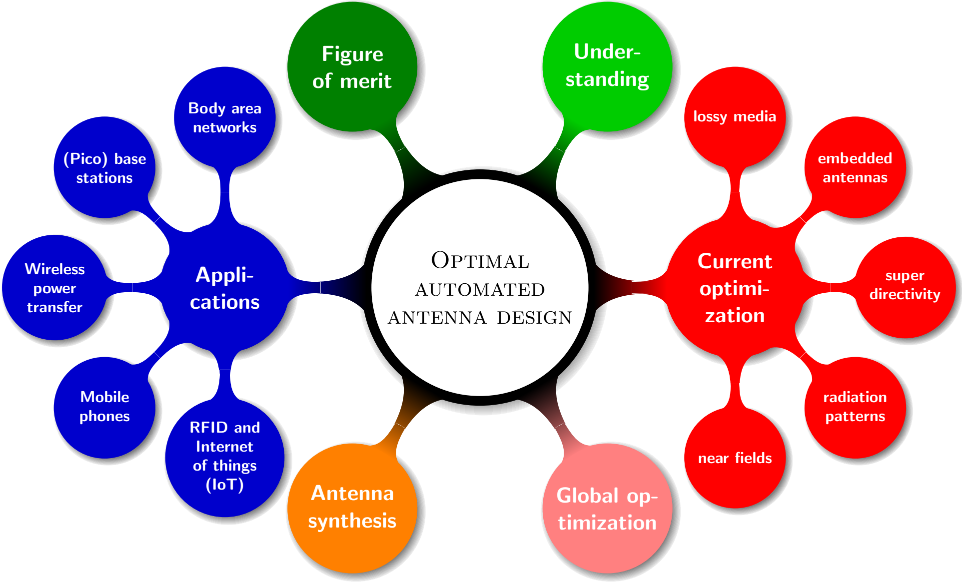
VETENSKAPSRÅDET
THE SWEDISH RESEARCH COUNCIL

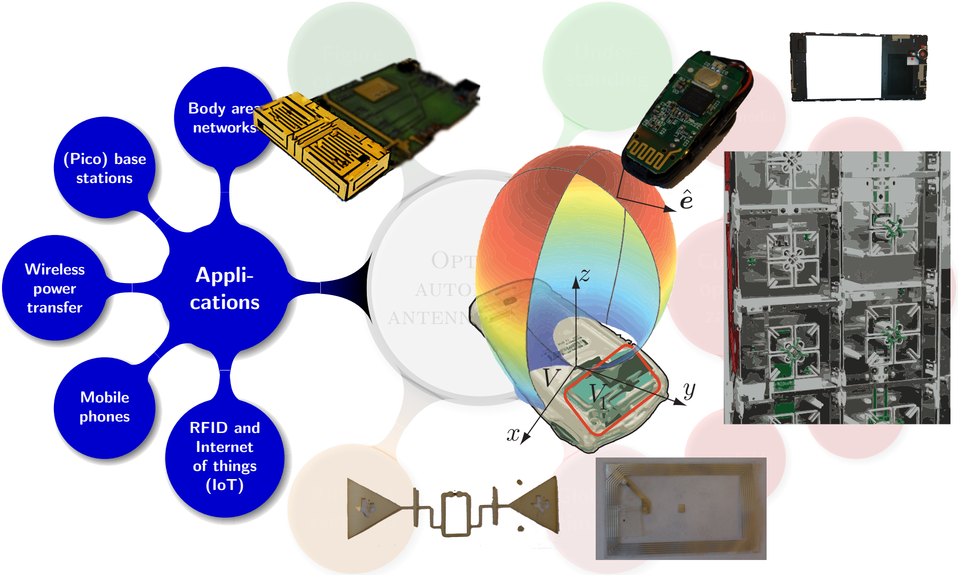
SWEDISH FOUNDATION *for* STRATEGIC RESEARCH

Lund University

- Lund university was founded in 1666.
- Sweden's largest university.
- Approximately 40 000 students.
- Department of Electrical and Information Technology: Broadband Communications, Circuits and Systems, Communication, Electromagnetic theory, Networking and Security, Signal Processing.

Outline


Optimal automated antenna design
Figure of merit
Under-standing
Current optimi-zation
lossy media
embedded antennas
super directivity
radiation patterns
near fields
Global op-timization
Antenna synthesis
Appli-cations
Body area networks
(Pico) base stations
Wireless power transfer
Mobile phones
RFID and Internet of things (IoT)


Appli-
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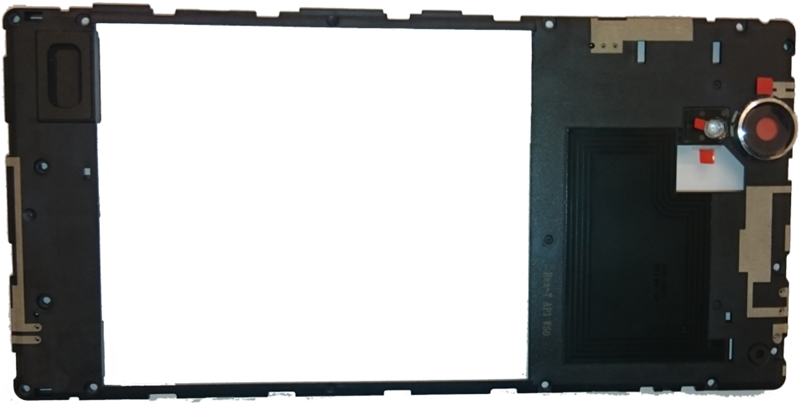
Mobile phones
RFID and Internet of things (IoT)
$\hat{e}$
z
y
x
V
V_{I}

Frame integrated antennas (Sony Xperia)



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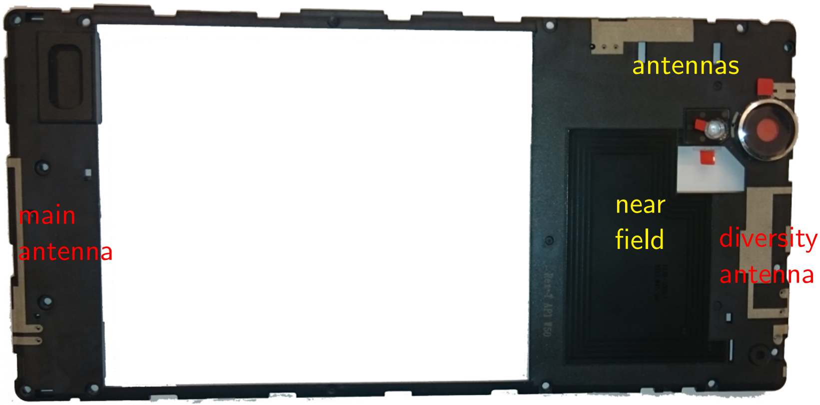
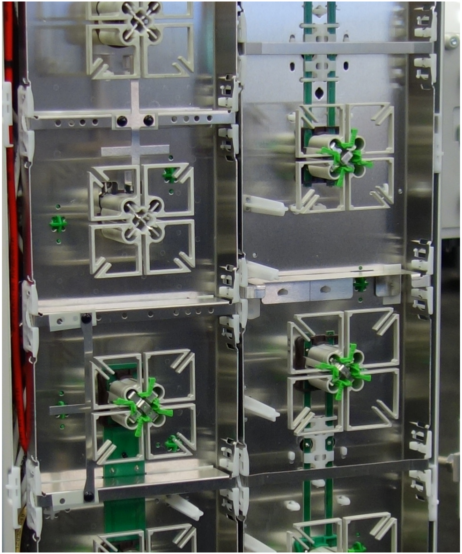
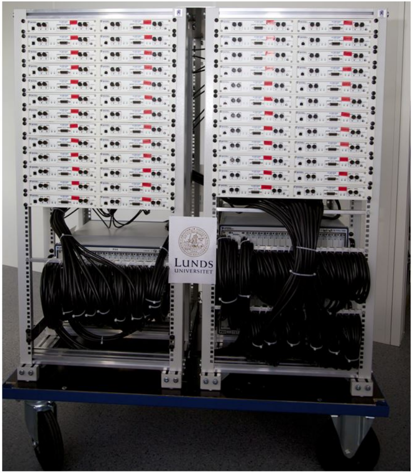


Base station antenna (Ericsson)



Massive MIMO



(a)

(b)

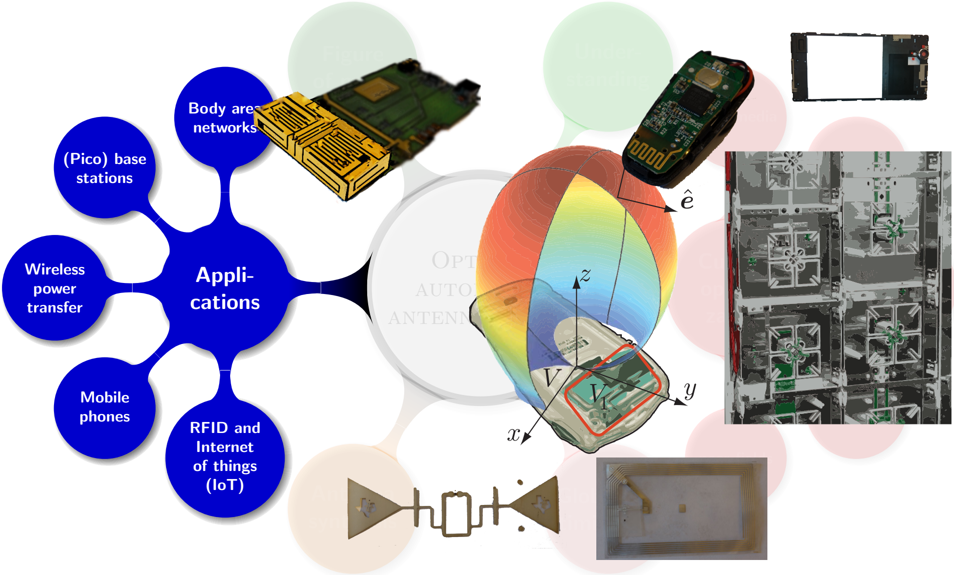

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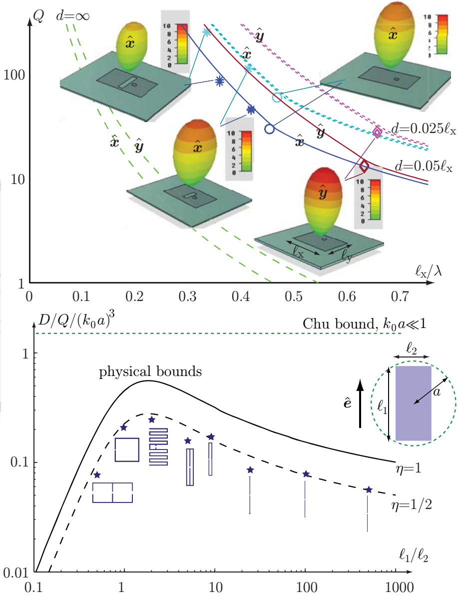
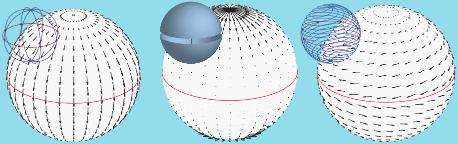
Figure of merit

Performance of an antenna design in relation to the optimal performance

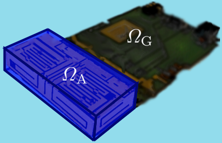
- % from the optimal design
- a useful number to compare designs
- is it worth to improve a design?
- ...



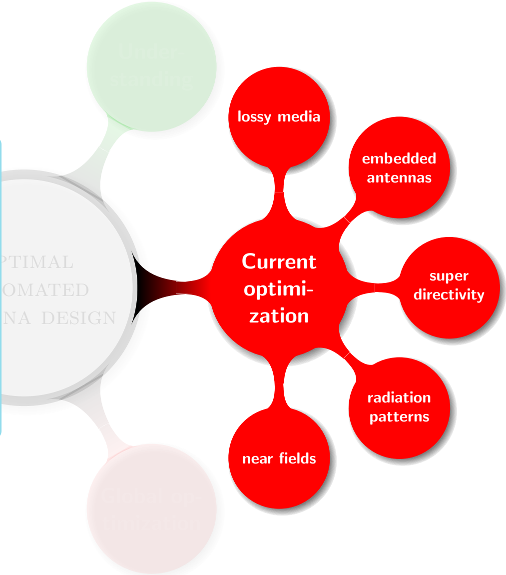
Under-standing

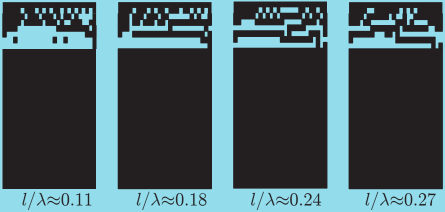


- Physical bounds
- Current distributions
- Polarizability



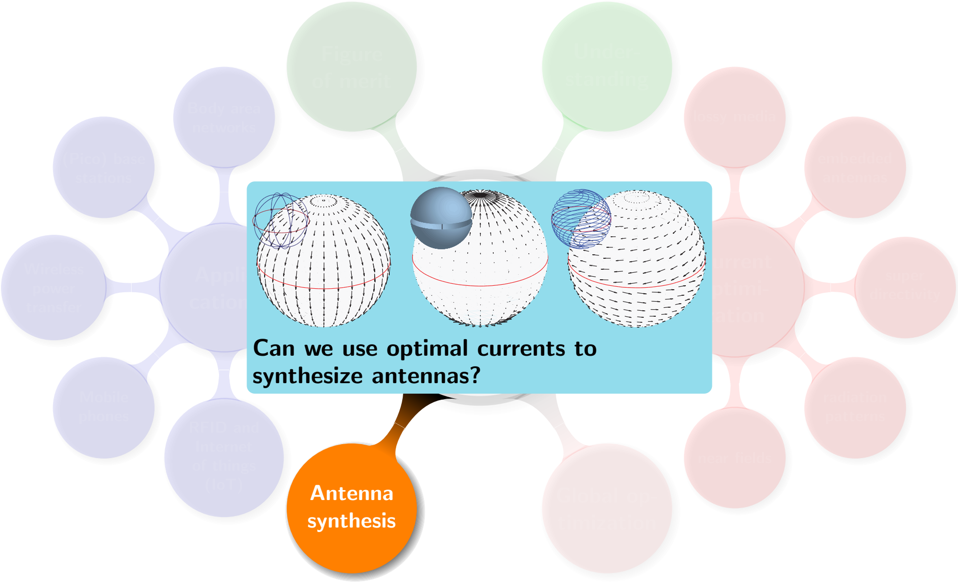
- **Optimal current distribution**
- **Physical bounds**
- **Convex optimization**
- **...**

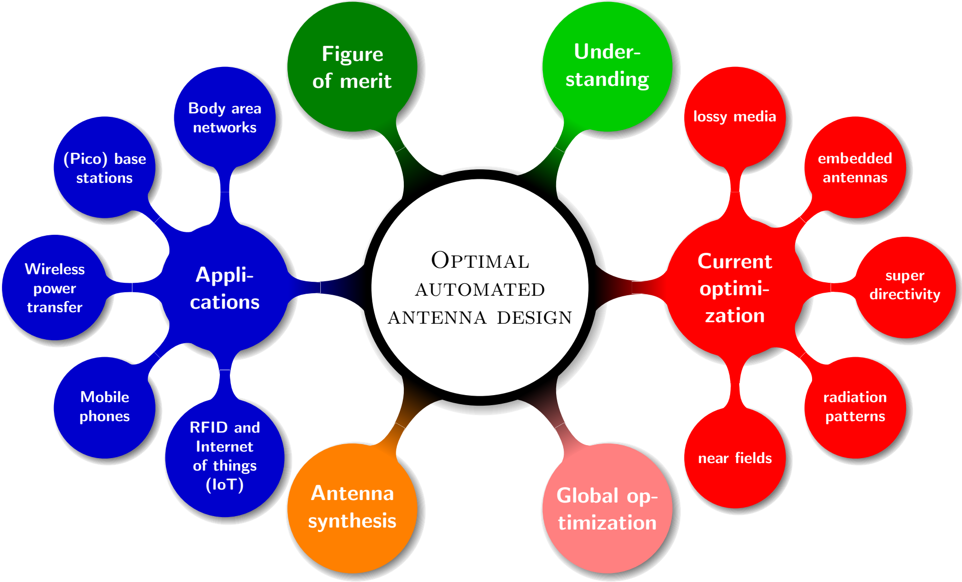




Antenna design using genetic algorithms (GA), particle swarm, ant colony, ...

Global optimization


Can we use optimal currents to synthesize antennas?
Antenna synthesis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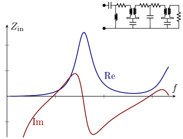
Outline

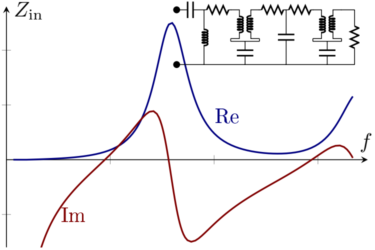
Stored electromagnetic energy

- How is the energy expressed?
  - Fields
  - Currents
  - System/Circuit
- Stored according to what?
  - From input impedance
  - In material
  - For scatterer
- Why are we interested?
  - Physics, EM-theory
  - Antenna bandwidth
  - Physical bounds



There are several proposals for the stored energy. They agree for many cases but differ for some. Differences often due to different interpretations, assumptions, and applications.

Stored energy expressed in fields, currents, and circuits



Stored energy expressed in fields, **currents**, and circuits

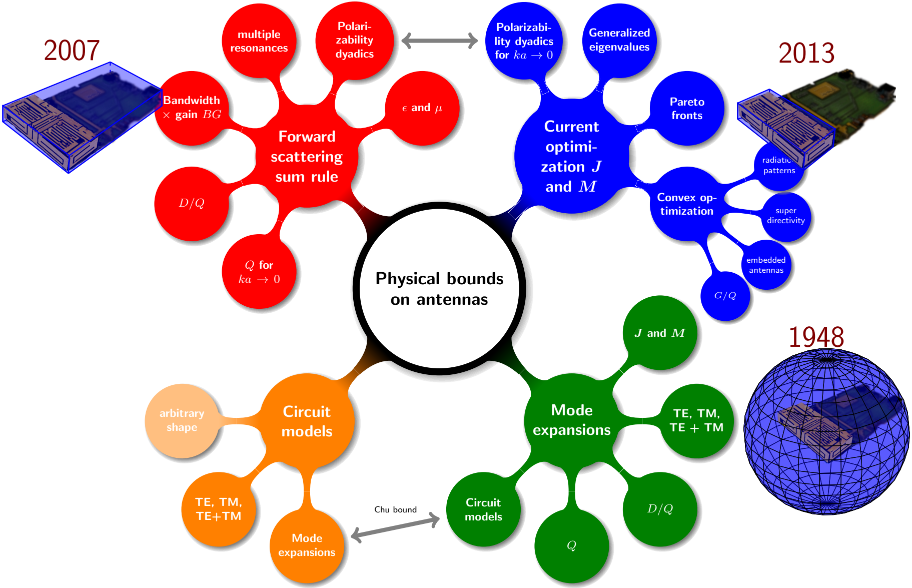
Stored electric energy by Vandenbosch 2010 (Geyi 2003b, $ka \to 0$).

$$W_{\mathrm{e}} = \frac{\eta_0}{4\omega} \int_{\Omega} \int_{\Omega} \nabla_1 \cdot \boldsymbol{J}(\boldsymbol{r}_1) \nabla_2 \cdot \boldsymbol{J}^*(\boldsymbol{r}_2) \frac{\cos(k|\boldsymbol{r}_1 - \boldsymbol{r}_2|)}{4\pi|\boldsymbol{r}_1 - \boldsymbol{r}_2|}$$
$$- \frac{k}{2} \big(k^2 \boldsymbol{J}(\boldsymbol{r}_1) \cdot \boldsymbol{J}^*(\boldsymbol{r}_2) - \nabla_1 \cdot \boldsymbol{J}(\boldsymbol{r}_1) \nabla_2 \cdot \boldsymbol{J}^*(\boldsymbol{r}_2)\big) \frac{\sin(k|\boldsymbol{r}_1 - \boldsymbol{r}_2|)}{4\pi} \, \mathrm{dV}_1 \, \mathrm{dV}_2$$

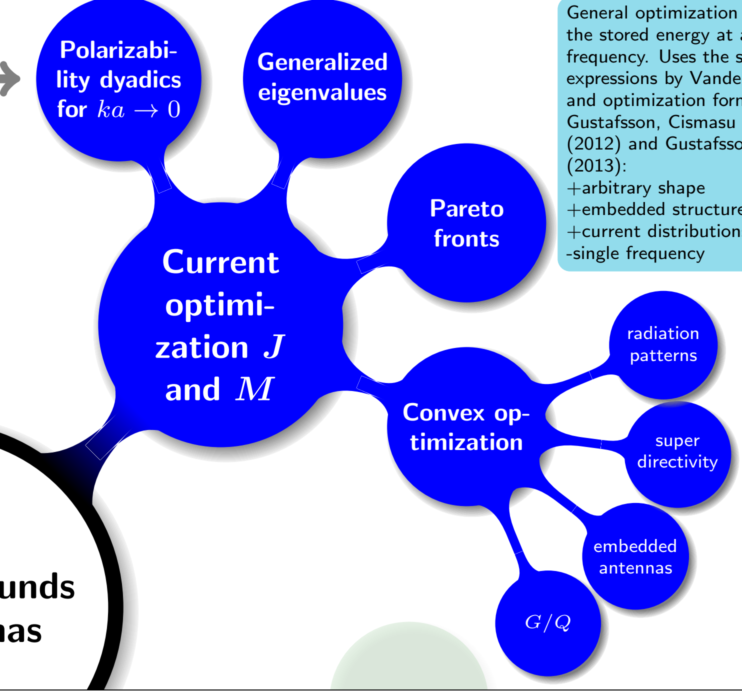
- Can be derived from the subtracted far-field energy (Gustafsson and Jonsson 2015b).
- Negative values (Gustafsson, Cismasu, and Jonsson 2012).
- Need only the current density.
- Can be used in convex optimization.
- Extensions to temporal dispersion.

Outline

Physical bounds on antennas: methods



Gustafsson *etal*, Physical Bounds of Antennas, in Handbook of Antenna Technologies, Springer, 2015



General optimization formulation for the stored energy at a fixed frequency. Uses the stored energy expressions by Vandenbosch (2010) and optimization formulations, see Gustafsson, Cismasu & Jonsson (2012) and Gustafsson & Nordebo (2013):

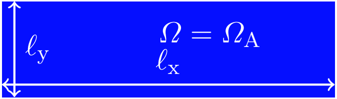
+arbitrary shape
+embedded structures
+current distributions
-single frequency

Antenna and antenna current optimization

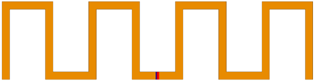
Device structure Ω with a maximal size for the antenna region Ω_{A}.

- **Antenna optimization:** determine the shape, material, and feed properties for optimal performance.
- **Antenna current optimization:** synthesize an optimal current distribution in the available geometry.

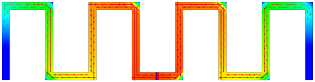
Maximal size of the antenna



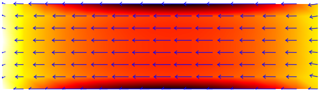
Antenna geometry with feed point



Current distribution on the antenna



Current distribution in the antenna antenna region



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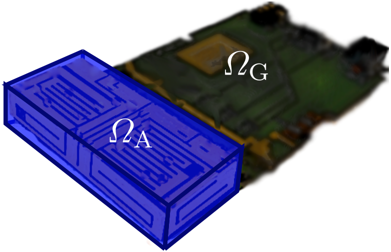
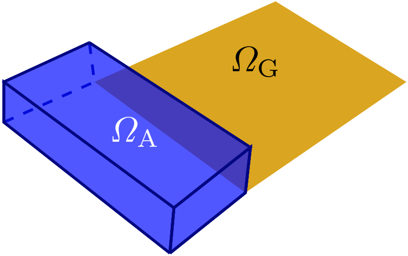




Optimization of antenna currents: examples

Q-factor

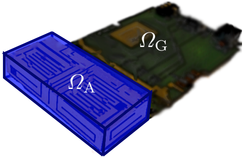
$$\begin{array}{ll} \text{minimize} & \text{Stored energy} \\ \text{subject to} & \text{Radiated power} = P_{\mathrm{rad}} \end{array}$$

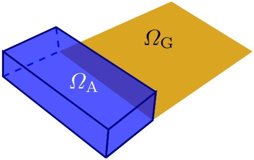
Q for superdirectivity $D \geq D_0$.

$$\begin{array}{ll} \text{minimize} & \text{Stored energy} \\ \text{subject to} & \text{Radiation intensity} = D_0 P_{\mathrm{rad}}/(4\pi) \\ & \text{Radiated power} \leq P_{\mathrm{rad}} \end{array}$$

Embedded structures

$$\begin{array}{ll} \text{minimize} & \text{Stored energy} \\ \text{subject to} & \text{Radiated power} = P_{\mathrm{rad}} \\ & \text{Correct induced currents} \end{array}$$





Need to:

1. Express the *stored energy* in the current density $\boldsymbol{J}$.
2. Solve the optimization problems.

Matrix expressions for the stored EM energies

Method of Moments approximation (expand $\boldsymbol{J}$ in basis functions)

$$W_{\mathrm{e}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \quad \text{stored E-energy, } \mathbf{X}_{\mathrm{e}} \text{ electric reactance}$$

$$W_{\mathrm{m}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \quad \text{stored M-energy, } \mathbf{X}_{\mathrm{m}} \text{ magnetic reactance}$$

$$P_{\mathrm{rad}} \approx \frac{1}{2}\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I} \quad \text{radiated power}$$

giving $\mathbf{Z} = \mathbf{R} + \mathrm{j}(\mathbf{X}_{\mathrm{m}} - \mathbf{X}_{\mathrm{e}})$. We also use

$$\hat{\boldsymbol{e}}^{*} \cdot \boldsymbol{F} \approx \mathbf{F}\mathbf{I} \quad \text{far field}$$

$$\boldsymbol{E} \approx \mathbf{N}\mathbf{I} \quad \text{near field}$$

$$\mathbf{I}_{\mathrm{G}} \approx \mathbf{C}\mathbf{I}_{\mathrm{A}} \quad \text{induced current on a PEC}$$

Matrix expressions for the stored EM energies

Method of Moments approximation (expand $\boldsymbol{J}$ in basis functions)

$$W_{\mathrm{e}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}$$ stored E-energy, $\mathbf{X}_{\mathrm{e}}$ electric reactance

$$W_{\mathrm{m}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}$$ stored M-energy, $\mathbf{X}_{\mathrm{m}}$ magnetic reactance

$$P_{\mathrm{rad}} \approx \frac{1}{2}\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}$$ radiated power

giving $\mathbf{Z} = \mathbf{R} + \mathrm{j}(\mathbf{X}_{\mathrm{m}} - \mathbf{X}_{\mathrm{e}})$. We also use

$$\hat{\boldsymbol{e}}^* \cdot \boldsymbol{F} \approx \mathbf{F}\mathbf{I}$$ far field

$$\boldsymbol{E} \approx \mathbf{N}\mathbf{I}$$ near field

$$\mathbf{I}_{\mathrm{G}} \approx \mathbf{C}\mathbf{I}_{\mathrm{A}}$$ induced current on a PEC

Pre-computed matrices used in the optimization.

Optimization of the current distribution

Characteristic modes

Modes with small Rayleigh quotients

$$\frac{\mathbf{I}^{\mathsf{H}}\mathbf{X}\mathbf{I}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}} = \frac{\mathbf{I}^{\mathsf{H}}(\mathbf{X}_{\mathrm{m}} - \mathbf{X}_{\mathrm{e}})\mathbf{I}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}}$$

Eigenvalue problem

$$(\mathbf{X}_{\mathrm{m}} - \mathbf{X}_{\mathrm{e}})\mathbf{I} = \nu\mathbf{R}\mathbf{I}$$

- Modes with low reactive power.
- Resonances $(\nu = 0)$
- Does not enforce low stored energy.

Stored energy

Minimize the energy Rayleigh quotient

$$\frac{\mathbf{I}^{\mathsf{H}}(\mathbf{X}_{\mathrm{m}} + \mathbf{X}_{\mathrm{e}})\mathbf{I}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}}$$

Eigenvalue problem

$$(\mathbf{X}_{\mathrm{m}} + \mathbf{X}_{\mathrm{e}})\mathbf{I} = \nu\mathbf{R}\mathbf{I}$$

- Modes with low stored energy.
- Does not enforce resonance.

Q-factor

Minimize the Q-factor quotient

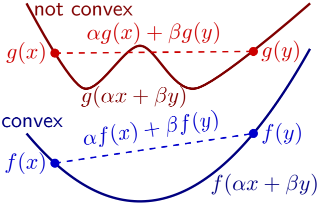
$$\frac{2\max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}\}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}}$$

- Currents with low Q-factors.
- Resonance by tuning.
- Need to solve these optimization problems $\Rightarrow$ convex optimization and eigenvalue problems.

Chen and Wang 2015; Garbacz and Turpin 1971; Harrington and Mautz 1971

Convex optimization

$$\begin{aligned} \text{minimize} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 0, \ i = 1, ..., N_1 \\ & \mathbf{A}\mathbf{x} = \mathbf{b} \end{aligned}$$



where $f_i(x)$ are convex, *i.e.*, $f_i(\alpha\mathbf{x} + \beta\mathbf{y}) \leq \alpha f_i(\mathbf{x}) + \beta f_i(\mathbf{y})$ for $\alpha, \beta \in \mathbb{R}$, $\alpha + \beta = 1$, $\alpha, \beta \geq 0$.

Solved with efficient standard algorithms. No risk of getting trapped in a local minimum. A problem is 'solved' if formulated as a convex optimization problem.

Antenna performance expressed in the current density $\boldsymbol{J}$, *e.g.*,

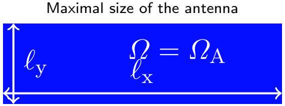
- Radiated field $\boldsymbol{F}(\hat{\boldsymbol{k}}) = -\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} \times \int_\Omega \boldsymbol{J}(\boldsymbol{r}) \mathrm{e}^{\mathrm{j}k\hat{\boldsymbol{k}}\cdot\boldsymbol{r}} \,\mathrm{dV}$ is affine.
- Radiated power, stored electric and magnetic energies, and Ohmic losses are positive semi-definite quadratic forms in $\boldsymbol{J}$.

Currents for maximal G/Q

Determine a current density $\boldsymbol{J}(\boldsymbol{r})$ in the volume Ω that maximizes the partial-gain Q-factor quotient $G(\hat{\boldsymbol{k}}, \hat{e})/Q$.

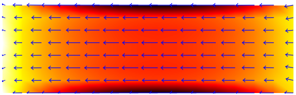
- Partial radiation intensity $P(\hat{\boldsymbol{k}}, \hat{\boldsymbol{e}})$

$$\frac{G(\hat{\boldsymbol{k}}, \hat{\boldsymbol{e}})}{Q} = \frac{2\pi P(\hat{\boldsymbol{k}}, \hat{\boldsymbol{e}})}{\mathrm{c}_0 k \max\{W_{\mathrm{e}}, W_{\mathrm{m}}\}}.$$



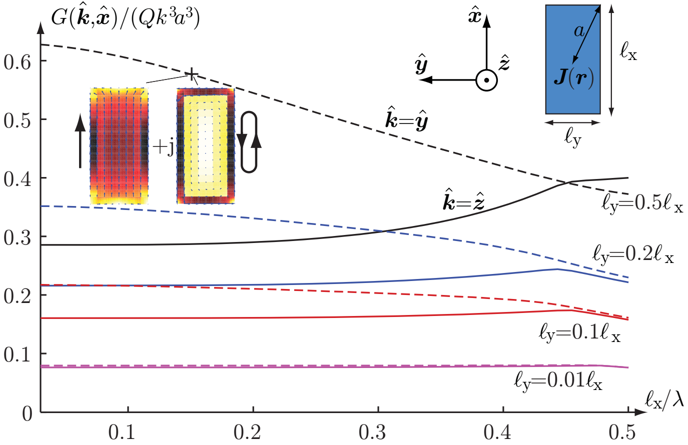
- Scale $\boldsymbol{J}$ and reformulate $\max. P$ as $\max. \operatorname{Re}\{\hat{\boldsymbol{e}}^* \cdot \boldsymbol{F}\}$.
- Convex optimization problem.

$$\begin{aligned} &\text{maximize} && \operatorname{Re}\{\mathbf{FI}\} \\ &\text{subject to} && \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \le 1 \\ & && \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \le 1 \end{aligned}$$



Determines a current density $\boldsymbol{J}(\boldsymbol{r})$ in the region Ω with maximal partial radiation intensity and unit stored EM energy.

Maximum $G(\hat{\boldsymbol{k}}, \hat{\boldsymbol{x}})/Q$ for planar rectangles



Solution for current densities confined to planar rectangles with side lengths ℓ_x and $\ell_y = \{0.01, 0.1, 0.2, 0.5\}\ell_x$.
Gustafsson and Nordebo 2013; Gustafsson et al. 2016

G/Q bounds

Typical (but not optimal) MATLAB code using CVX

```
cvx_begin
   variable I(n) complex;        % current density
   maximize(real(F*I))           % far-field
   subject to
      quad_form(I,Xe) <= 1;      % stored E energy
      quad_form(I,Xm) <= 1;      % stored M energy
cvx_end
```

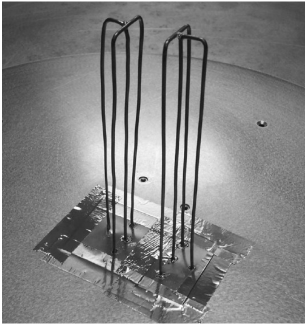
- Similar to the forward scattering bounds (2007) for TM.
- Can design 'optimal' electric dipole mode (TM) antennas.
- TE modes and TE+TM are not well understood.

We can reformulate the complex optimization problem to analyze superdirectivity, antennas with a prescribed radiation pattern, losses, and antennas embedded in a PEC structure.

M. Gustafsson, *etal*, Antenna current optimization using MATLAB and CVX, FERMAT, 2016

Superdirectivity

- A superdirective antenna has a directivity that is much higher than for a typical reference antenna.
- Often low efficiency (low gain) and narrow bandwidth.
- There is an interest in small superdirective antennas, *e.g.*, Best *etal.* 2008 and Arceo & Balanis 2011,



Best, *etal.*, An Impedance-Matched 2-Element Superdirective Array, IEEE-TAP, 2008

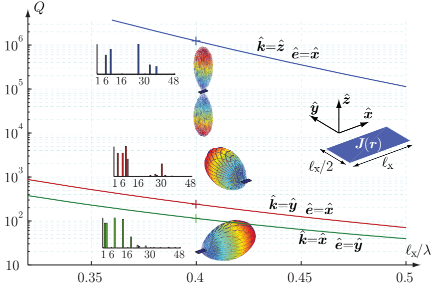
Here, we add the constraint $D \geq D_0$ to the convex optimization problem for G/Q to determine the minimum Q for superdirective lossless antennas. We can also add constraints on the losses.

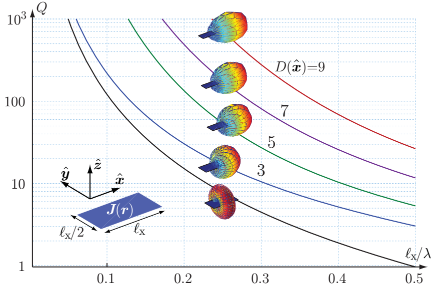
Superdirectivity: min. Q s.t. $D \geq D_0$

Add the constraint $P_{\mathrm{rad}} \leq 4\pi D_0^{-1}$ the get the convex optimization problem

$$\begin{aligned} \text{min.} \quad & \max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\} \\ \text{s.t.} \quad & \mathrm{Re}\{\mathbf{F}\mathbf{I}\} = 1 \\ & \mathbf{I}^{\mathsf{H}}\mathbf{P}\mathbf{I} \leq k^3 D_0^{-1} \end{aligned}$$

Example for current densities confined to planar rectangles with side lengths ℓ_{x} and $\ell_{\mathrm{y}} = 0.5\ell_{\mathrm{x}}$.





Currents for maximal G/Q for embedded antennas

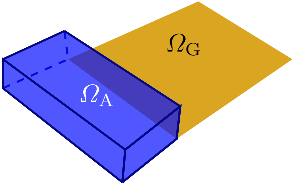
Determine an optimal current density $\boldsymbol{J}_{\mathrm{A}}(\boldsymbol{r})$ in the region Ω_{A}. Assume that the ground plane $\Omega_{\mathrm{G}} = \Omega \setminus \Omega_{\mathrm{A}}$ is PEC.

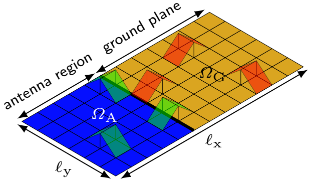
Can minimize the stored energy for given radiated field

$$\begin{aligned} &\text{minimize} && \max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\} \\ &\text{subject to} && \mathbf{F}\mathbf{I} = 1 \\ & && \mathbf{I}_{\mathrm{G}} = \mathbf{C}\mathbf{I}_{\mathrm{A}} \end{aligned}$$

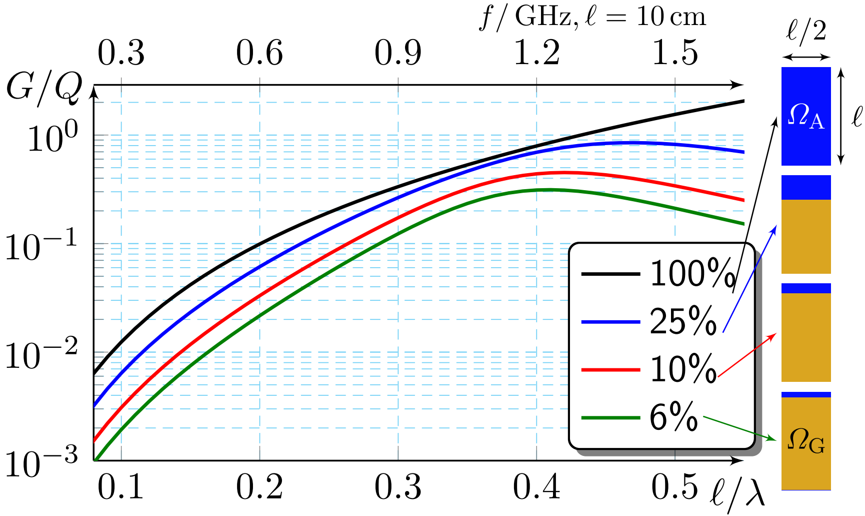
or maximize the radiated field for given stored energy

$$\begin{aligned} &\text{maximize} && \mathrm{Re}\{\mathbf{F}\mathbf{I}\} \\ &\text{subject to} && \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \leq 1 \\ & && \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \leq 1 \\ & && \mathbf{I}_{\mathrm{G}} = \mathbf{C}\mathbf{I}_{\mathrm{A}} \end{aligned}$$

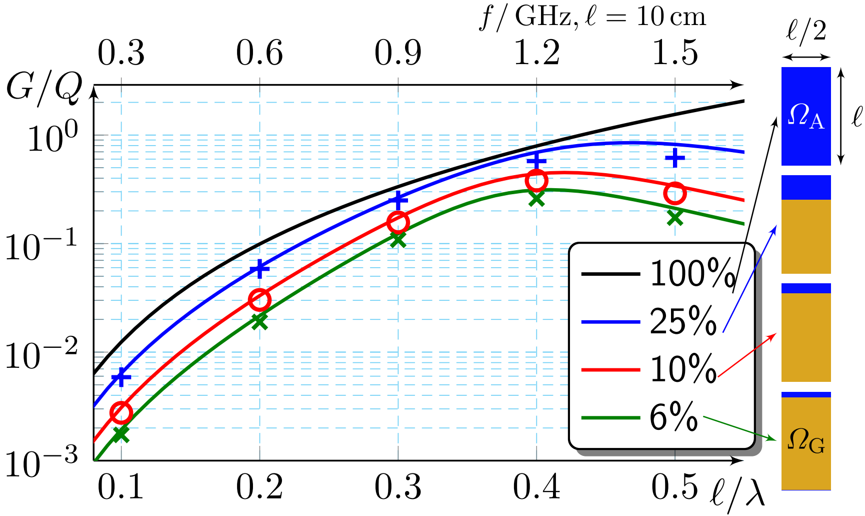




Finite ground plane with $\{6, 10, 25, 100\}\%$ antenna region

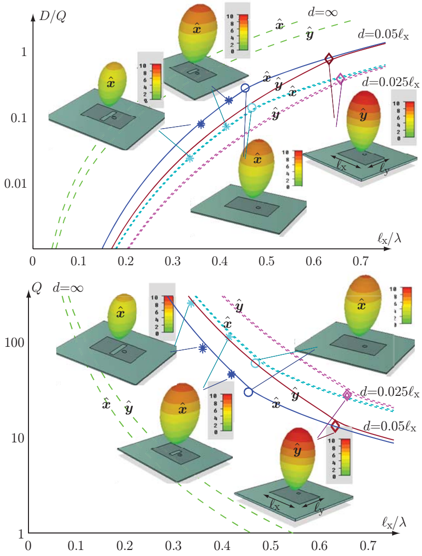


Finite ground plane with $\{6, 10, 25, 100\}\%$ antenna region



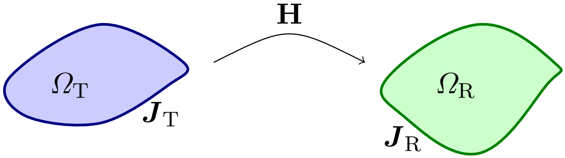
Antennas above ground planes

- Common with antennas above ground planes.
- Add mirror currents for the stored energy and radiated field.
- Results for rectangular structures at height d above the ground plane.
- Comparison with patch and slot loaded patches.



Tayli and Gustafsson 2016

Maximum capacity for MIMO antennas

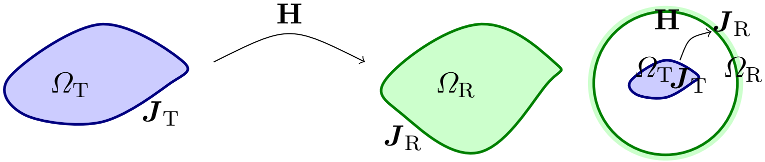


Can solve cases of the type (Ehrenborg and Gustafsson 2017)

$$\begin{aligned}
\text{maximize} \quad & \log_2 \det(\mathbf{1} + \gamma \mathbf{M}\mathbf{P}\mathbf{M}^{\mathsf{H}}) \\
\text{subject to} \quad & \mathrm{Tr}(\mathbf{X}_{\mathrm{e}}\mathbf{P}) \leq Q \\
& \mathrm{Tr}(\mathbf{X}_{\mathrm{m}}\mathbf{P}) \leq Q \\
& \mathrm{Tr}(\mathbf{R}_{\Omega}\mathbf{P}) = 1 - \eta \\
& \mathrm{Tr}(\mathbf{R}\mathbf{P}) = 1 \\
& \mathbf{P} \succeq \mathbf{0}
\end{aligned}$$

Total stored energy and total dissipated power. Note no simple relation between bandwidth and Q-factors for multiport antennas.

Maximum capacity for MIMO antennas

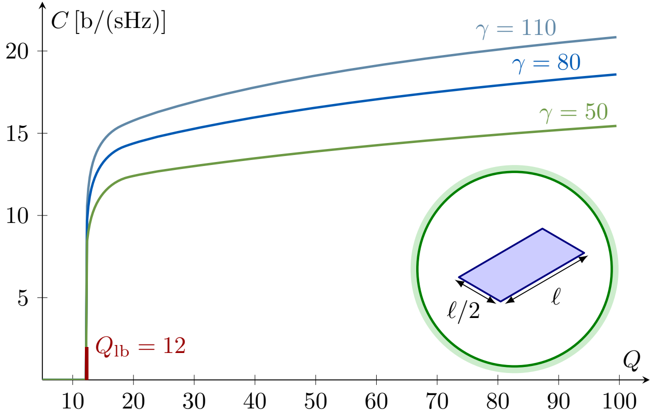


Can solve cases of the type (Ehrenborg and Gustafsson 2017)

$$
\begin{aligned}
\text{maximize} \quad & \log_2 \det(\mathbf{1} + \gamma \mathbf{M}\mathbf{P}\mathbf{M}^{\mathsf{H}}) \\
\text{subject to} \quad & \mathrm{Tr}(\mathbf{X}_{\mathrm{e}}\mathbf{P}) \leq Q \\
& \mathrm{Tr}(\mathbf{X}_{\mathrm{m}}\mathbf{P}) \leq Q \\
& \mathrm{Tr}(\mathbf{R}_{\Omega}\mathbf{P}) = 1 - \eta \\
& \mathrm{Tr}(\mathbf{R}\mathbf{P}) = 1 \\
& \mathbf{P} \succeq \mathbf{0}
\end{aligned}
$$

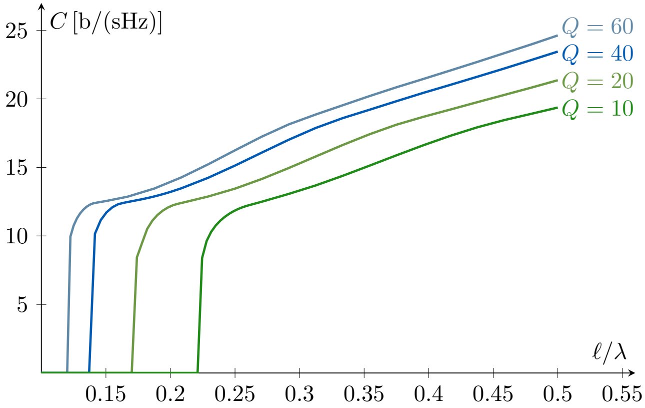
Total stored energy and total dissipated power. Note no simple relation between bandwidth and Q-factors for multiport antennas.

Maximum capacity for a planar rectangle



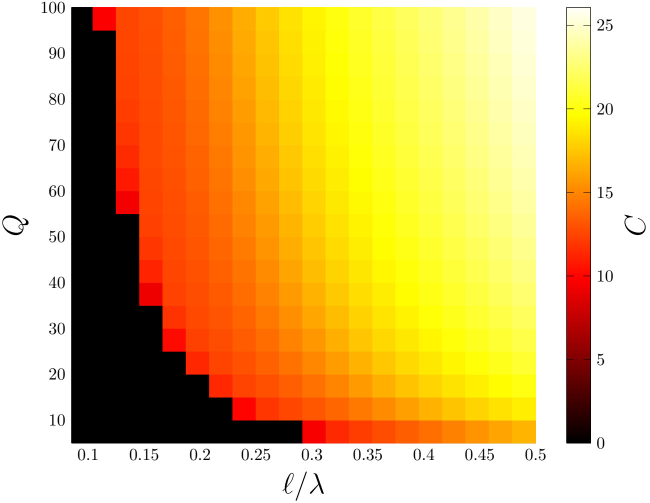
Capacity in $\mathrm{bits/s/Hz}$ for fixed noise level (Ehrenborg and Gustafsson 2017).

Maximum capacity for a planar rectangle



Capacity in bits/s/Hz for fixed noise level (Ehrenborg and Gustafsson 2017).

Maximum capacity for a planar rectangle

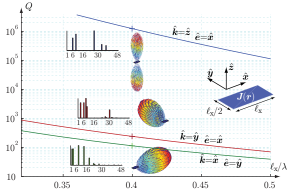


Capacity in $\mathrm{bits/s/Hz}$ for fixed noise level (Ehrenborg and Gustafsson 2017).

Simple optimization formulations

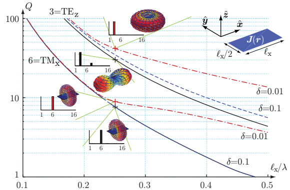
Superdirectivity:

$$\begin{aligned} \text{minimize} \quad & \max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\} \\ \text{subject to} \quad & \mathbf{F}\mathbf{I} = 1 \\ & \mathbf{I}^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I} \leq 4\pi/(\eta_0 D_0) \end{aligned}$$



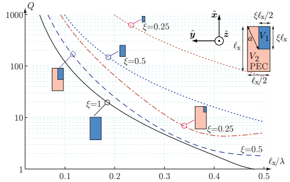
Prescribed far field:

$$\begin{aligned} \text{minimize} \quad & \max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\} \\ \text{subject to} \quad & \int_{\Omega} |\boldsymbol{F}(\hat{\boldsymbol{k}}) - \boldsymbol{F}_0(\hat{\boldsymbol{k}})|^2 \,\mathrm{d}\Omega_{\hat{\boldsymbol{k}}} < \delta \end{aligned}$$



Embedded antennas:

$$\begin{aligned} \text{minimize} \quad & \max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\} \\ \text{subject to} \quad & \mathbf{F}\mathbf{I} = 1 \\ & \mathbf{I}_{\mathrm{G}} = \mathbf{C}\mathbf{I}_{\mathrm{A}} \end{aligned}$$



Antenna current optimization

Can optimize the antenna current for many but not all antenna problems

- Mainly single frequency cases. Multi-frequency and UWB challenging.
- Well defined stored energy. Free space and sub-wavelength. Some recent results for lossy and dispersive media.

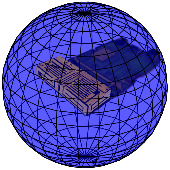
Formulations for or combinations of

- $\min. Q$, $\min. Q$ s.t. $D \geq D_0$, $\min. Q$ s.t. $\boldsymbol{F} \approx \boldsymbol{F}_0$, ...
- $\max. G/Q$,
- Efficiency
- Capacity
- ...

What is know about antenna performance?

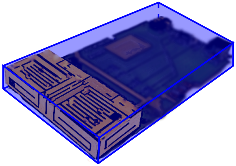
Outline

Antennas with close to optimal performance



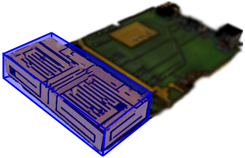
Spherical modes

- Best 2004, folded spherical helix
- Stuart *etal* 2007, 2008, 2009
- Kim *etal* 2010,2012



Arbitrary shapes

- Best 2009, meander lines, folded cylindrical helix
- Cismasu and Gustafsson 2014a GA
- Shahpari, Thiel, and Lewis 2014 Ant colony

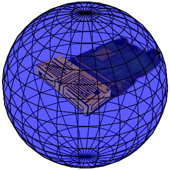


Complex

- Finite ground planes Cismasu and Gustafsson 2014a
- Patch antennas Tayli and Gustafsson 2016
- , ...

Need more antennas to compare with.

Antennas with close to optimal performance



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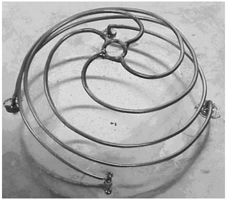
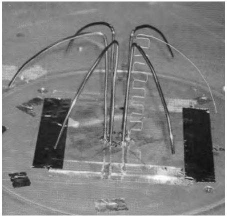
Best 2004

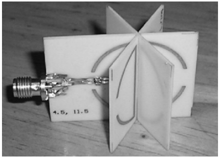


Best and Hanna 2010



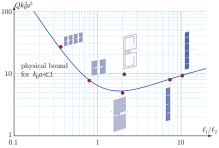
Kim 2010

Stuart and Tran 2007

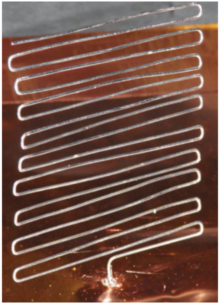


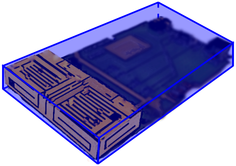
Antennas with close to optimal performance

Best 2009



Best 2015





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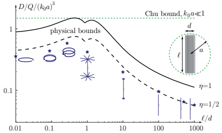
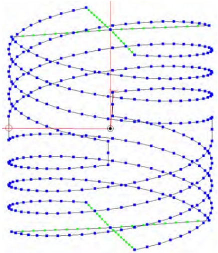
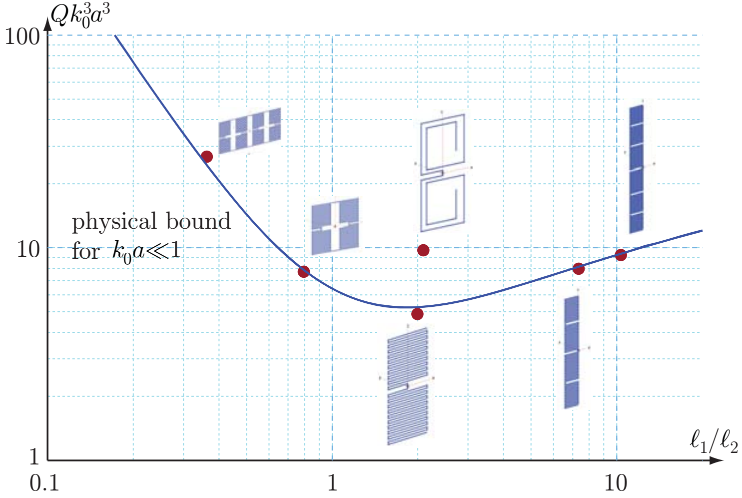


Best 2009



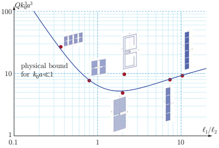
Meander line antennas in planar rectangles ($\ell_1 \times \ell_2$)



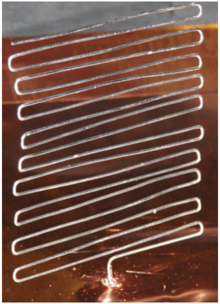
Best 2009; Best 2015 Note, here normalized with k^3a^3 and not Q_{Chu}.

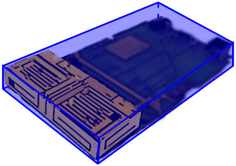
Antennas with close to optimal performance

Best 2009



Best 2015





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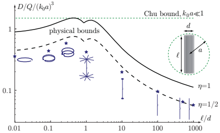
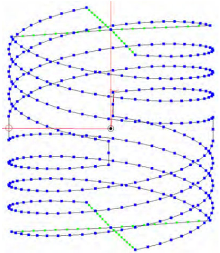
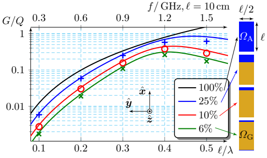


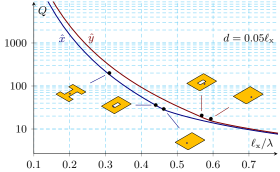
Best 2009



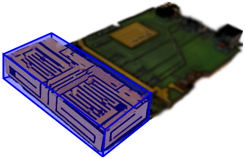
Antennas with close to optimal performance



Cismasu and Gustafsson 2014a



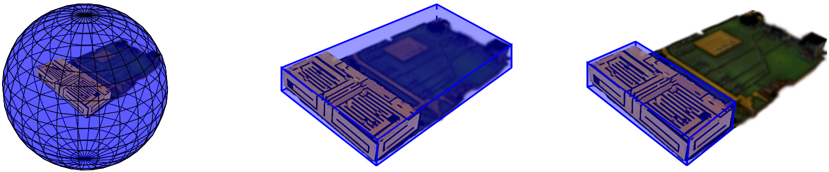
Tayli and Gustafsson 2016



Complex

- Finite ground planes Cismasu and Gustafsson 2014a
- Patch antennas Tayli and Gustafsson 2016
- , ...

Antennas with close to optimal performance



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Complex

- Finite ground planes Cismasu and Gustafsson 2014a
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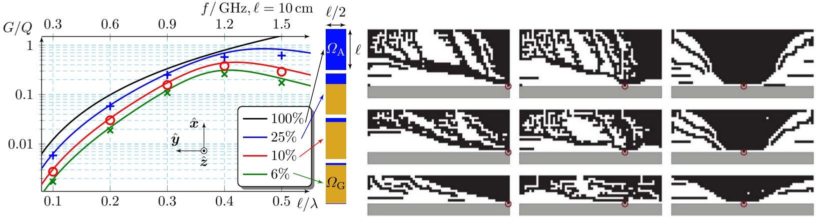
- , ...

Need more antennas to compare with.

Automatic design

- Metaheuristic approaches such as GA has been shown to produce antennas close to Q_{lb} or similar bounds for many geometries and TM radiation.
- Not much known for other cases.
- Machine learning

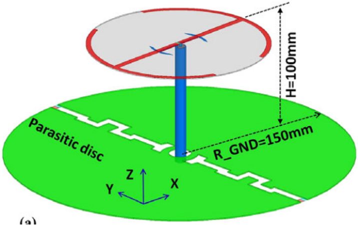


Cismasu and Gustafsson 2014a; Cismasu and Gustafsson 2014b

Outline

What can be done to overcome the limits?

- Limits are always based on assumptions.
- Often linear time invariant passive materials.
- Break some of the assumptions to beat the limit.
  - matching can increase the bandwidth. Bode-Fano limit $B \leq 27/(Q|\mathit{\Gamma}_{0,\mathrm{dB}}|)$.
  - non-Foster matching for further increase. Check SNR.
  - non-linearity, time varying, switches
  - ...



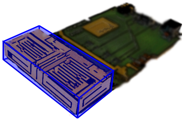
Ziolkowski, Tang, and Zhu 2013

Outline

Summary

- Stored energy in the current density.
- State-space approach for temporal dispersion.
- Convex optimization for bounds and optimal currents: Q, G/Q, superdirective, embedded, MIMO, losses, ...
- Physical bounds from spheres (Chu 1948) and arbitrary shapes (Gustafsson *etal* 2007) to embedded antennas...
- Non-Foster to overcome $B \sim 1/Q$ ...

M. Gustafsson *etal*, *Antenna current optimization using MATLAB and CVX*, FERMAT, 2016.



Slides: `http://www.eit.lth.se/staff/mats.gustafsson`

References I

Best, S. R. (2004). "The radiation properties of electrically small folded spherical helix antennas". *IEEE Trans. Antennas Propag.* 52.4, pp. 953–960.

— (2009). "A Low Q Electrically Small Magnetic (TE Mode) Dipole". *Antennas and Wireless Propagation Letters, IEEE* 8, pp. 572–575.

— (2015). "Electrically Small Resonant Planar Antennas: Optimizing the quality factor and bandwidth.". *IEEE Antennas Propag. Mag.* 57.3, pp. 38–47.

Best, S. R., E. E. Altshuler, A. D. Yaghjian, J. M. McGinthy, and T. H. O'Donnell (2008). "An impedance-matched 2-element superdirective array". *Antennas and Wireless Propagation Letters, IEEE* 7, pp. 302–305.

Best, S. R. and D. L. Hanna (2010). "A performance comparison of fundamental small-antenna designs". *Antennas and Propagation Magazine, IEEE* 52.1, pp. 47–70.

Boyd, S. P. and L. Vandenberghe (2004). *Convex Optimization*. Cambridge Univ. Pr.

Capek, M., P. Hazdra, and J. Eichler (2012). "A method for the evaluation of radiation Q based on modal approach". *IEEE Trans. Antennas Propag.* 60.10, pp. 4556–4567.

Capek, M., L. Jelinek, P. Hazdra, and J. Eichler (2014). "The Measurable Q Factor and Observable Energies of Radiating Structures". *IEEE Trans. Antennas Propag.* 62.1, pp. 311–318.

Capek, M., M. Gustafsson, and K. Schab (2016). "Minimization of Antenna Quality Factor". *arXiv preprint arXiv:1612.07676*.

Capek, M. and L. Jelinek (2016). "Optimal Composition of Modal Currents for Minimal Quality Factor Q". *IEEE Trans. Antennas Propag.* 64.12, pp. 5230–5242.

Carpenter, C. J. (1989). "Electromagnetic energy and power in terms of charges and potentials instead of fields". *IEE Proc. A* 136.2, pp. 55–65.

Chalas, J., K. Sertel, and J. L. Volakis (2011). "Computation of the Q limits for arbitrary-shaped antennas using characteristic modes". In: *Antennas and Propagation (APSURSI), 2011 IEEE International Symposium on*. IEEE, pp. 772–774.

Chen, Y. and C.-F. Wang (2015). *Characteristic Modes: Theory and Applications in Antenna Engineering*. John Wiley & Sons.

Chu, L. J. (1948). "Physical Limitations of Omni-directional Antennas". *J. Appl. Phys.* 19, pp. 1163–1175.

References II

Cismasu, M. and M. Gustafsson (2014a). "Antenna Bandwidth Optimization with Single Frequency Simulation". *IEEE Trans. Antennas Propag.* 62.3, pp. 1304–1311.

— (2014b). "Multiband Antenna Q Optimization using Stored Energy Expressions". *IEEE Antennas and Wireless Propagation Letters* 13.2014, pp. 646–649.

Collin, R. E. and S. Rothschild (1964). "Evaluation of Antenna Q". *IEEE Trans. Antennas Propag.* 12, pp. 23–27.

Foltz, H. D. and J. S. McLean (1999). "Limits on the radiation Q of electrically small antennas restricted to oblong bounding regions". In: *IEEE Antennas and Propagation Society International Symposium*. Vol. 4. IEEE, pp. 2702–2705.

Garbacz, R. J. and R. H. Turpin (1971). "A generalized expansion for radiated and scattered fields". *IEEE Trans. Antennas Propag.* 19.3, pp. 348–358.

Geyi, W. (2003a). "A method for the evaluation of small antenna Q". *IEEE Trans. Antennas Propag.* 51.8, pp. 2124–2129.

— (2003b). "Physical limitations of antenna". *IEEE Trans. Antennas Propag.* 51.8, pp. 2116–2123.

Gustafsson, M., M. Cismasu, and S. Nordebo (2010). "Absorption Efficiency and Physical Bounds on Antennas". *International Journal of Antennas and Propagation* 2010.Article ID 946746, pp. 1–7.

Gustafsson, M., J. Friden, and D. Colombi (2015). "Antenna Current Optimization for Lossy Media with Near Field Constraints". *Antennas and Wireless Propagation Letters, IEEE* 14, pp. 1538–1541.

Gustafsson, M. and B. L. G. Jonsson (2015a). "Antenna Q and stored energy expressed in the fields, currents, and input impedance". *IEEE Trans. Antennas Propag.* 63.1, pp. 240–249.

— (2015b). "Stored Electromagnetic Energy and Antenna Q". *Progress In Electromagnetics Research (PIER)* 150, pp. 13–27.

Gustafsson, M. and S. Nordebo (2013). "Optimal Antenna Currents for Q, Superdirectivity, and Radiation Patterns Using Convex Optimization". *IEEE Trans. Antennas Propag.* 61.3, pp. 1109–1118.

Gustafsson, M., C. Sohl, and G. Kristensson (2007). "Physical limitations on antennas of arbitrary shape". *Proc. R. Soc. A* 463, pp. 2589–2607.

— (2009). "Illustrations of New Physical Bounds on Linearly Polarized Antennas". *IEEE Trans. Antennas Propag.* 57.5, pp. 1319–1327.

Gustafsson, M., D. Tayli, C. Ehrenborg, M. Cismasu, and S. Nordebo (2016). "Antenna current optimization using MATLAB and CVX". *FERMAT* 15.5, pp. 1–29.

References III

Gustafsson, M., M. Cismasu, and B. L. G. Jonsson (2012). "Physical bounds and optimal currents on antennas". *IEEE Trans. Antennas Propag.* 60.6, pp. 2672–2681.

Gustafsson, M. and B. L. G. Jonsson (2012). *Stored Electromagnetic Energy and Antenna Q*. Tech. rep. LUTEDX/(TEAT-7222)/1–25/(2012). Lund University.

Gustafsson, M. and S. Nordebo (2006). "Bandwidth, Q factor, and resonance models of antennas". *Prog. Electromagn. Res.* 62, pp. 1–20.

Hansen, T. V., O. S. Kim, and O. Breinbjerg (2012). "Stored Energy and Quality Factor of Spherical Wave Functions–in Relation to Spherical Antennas With Material Cores". *IEEE Trans. Antennas Propag.* 60.3, pp. 1281–1290.

Harrington, R. F. and J. R. Mautz (1971). "Theory of characteristic modes for conducting bodies". *IEEE Trans. Antennas Propag.* 19.5, pp. 622–628.

Jelinek, L and M Capek (2017). "Optimal Currents on Arbitrarily Shaped Surfaces". *IEEE Trans. Antennas Propag.* 65.1, pp. 329–341.

Jonsson, B. L. G. and M. Gustafsson (2015). "Stored energies in electric and magnetic current densities for small antennas". *Proc. R. Soc. A* 471.2176, p. 20140897.

Kim, O. (2010). "Low-Q Electrically Small Spherical Magnetic Dipole Antennas". *IEEE Trans. Antennas Propag.* 58.7, pp. 2210–2217.

McLean, J. S. (1996). "A Re-Examination of the Fundamental Limits on the Radiation Q of Electrically Small Antennas". *IEEE Trans. Antennas Propag.* 44.5, pp. 672–676.

Shahpari, M., D. Thiel, and A Lewis (2014). "An Investigation Into the Gustafsson Limit for Small Planar Antennas Using Optimization". *IEEE Trans. Antennas Propag.* 62.2, pp. 950–955.

Sohl, C. and M. Gustafsson (2008). "A priori estimates on the partial realized gain of Ultra-Wideband (UWB) antennas". *Quart. J. Mech. Appl. Math.* 61.3, pp. 415–430.

Sten, J. C.-E., P. K. Koivisto, and A. Hujanen (2001). "Limitations for the Radiation Q of a Small Antenna Enclosed in a Spheroidal Volume: Axial Polarisation". *AEÜ Int. J. Electron. Commun.* 55.3, pp. 198–204.

Stuart, H. R. and C. Tran (2007). "Small spherical antennas using arrays of electromagnetically coupled planar elements". *IEEE Antennas and Wireless Propagation Letters* 6, pp. 7–10.

Tayli, D. and M. Gustafsson (2016). "Physical Bounds for Antennas Above a Ground Plane". *Antennas and Wireless Propagation Letters, IEEE* 15, pp. 1281–1284.

References IV

Thal, H. L. (2006). "New Radiation Q Limits for Spherical Wire Antennas". *IEEE Trans. Antennas Propag.* 54.10, pp. 2757–2763.

Thal, H. L. (2012). "Q Bounds for Arbitrary Small Antennas: A Circuit Approach". *IEEE Trans. Antennas Propag.* 60.7, pp. 3120–3128.

Vandenbosch, G. A. E. (2010). "Reactive Energies, Impedance, and Q Factor of Radiating Structures". *IEEE Trans. Antennas Propag.* 58.4, pp. 1112–1127.

— (2011). "Simple procedure to derive lower bounds for radiation Q of electrically small devices of arbitrary topology". *IEEE Trans. Antennas Propag.* 59.6, pp. 2217–2225.

Vandenbosch, G. A. E. (2013a). "Radiators in time domain, part I: electric, magnetic, and radiated energies". *IEEE Trans. Antennas Propag.* 61.8, pp. 3995–4003.

— (2013b). "Radiators in time domain, part II: finite pulses, sinusoidal regime and Q factor". *IEEE Trans. Antennas Propag.* 61.8, pp. 4004–4012.

Volakis, J., C. C. Chen, and K. Fujimoto (2010). *Small Antennas: Miniaturization Techniques & Applications*. McGraw-Hill.

Wheeler, H. A. (1947). "Fundamental limitations of small antennas". *Proc. IRE* 35.12, pp. 1479–1484.

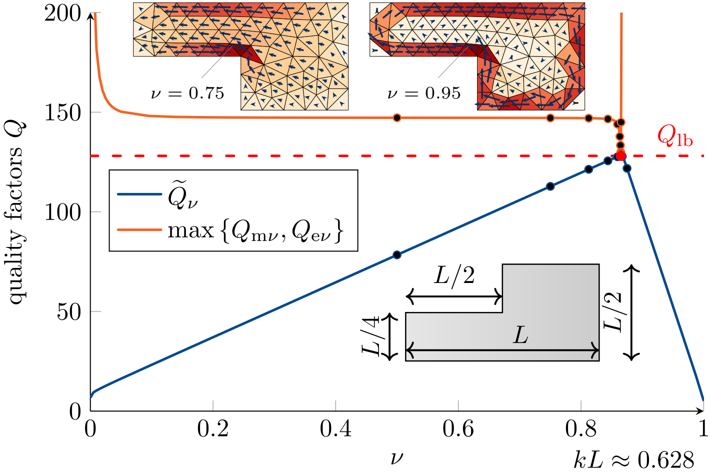
Yaghjian, A. D., M. Gustafsson, and B. L. G Jonsson (2013). "Minimum Q for Lossy and Lossless Electrically Small Dipole Antennas". *Progress In Electromagnetics Research* 143, pp. 641–673.

Yaghjian, A. D. and H. R. Stuart (2010). "Lower Bounds on the Q of Electrically Small Dipole Antennas". *IEEE Trans. Antennas Propag.* 58.10, pp. 3114–3121.

Yaghjian, A. D. and S. R. Best (2005). "Impedance, Bandwidth, and Q of Antennas". *IEEE Trans. Antennas Propag.* 53.4, pp. 1298–1324.

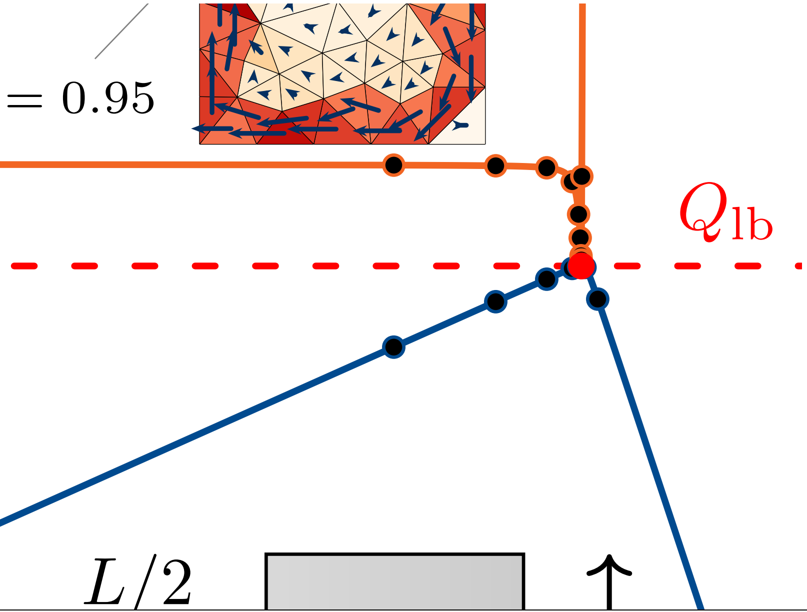
Ziolkowski, R. W., M.-C. Tang, and N. Zhu (2013). "An efficient, broad bandwidth, high directivity, electrically small antenna". *Microwave and Optical Technology Letters* 55.6, pp. 1430–1434.

Q_{lb} for a planar structure

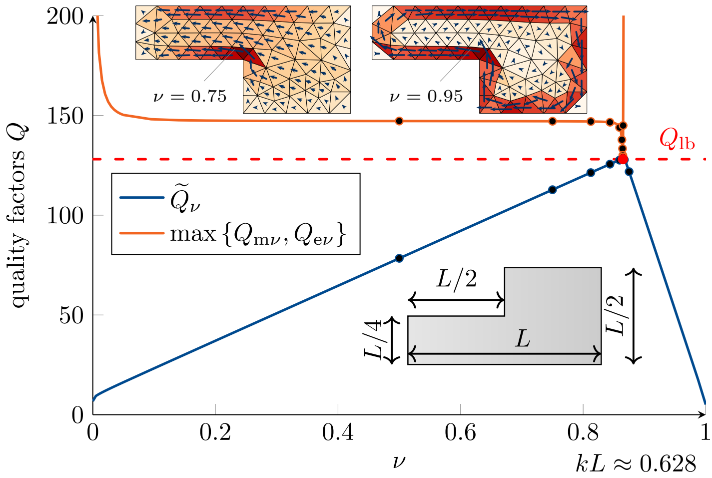


maximize$_\nu$ $\widetilde{Q}_\nu$ for $0 \le \nu \le 1$ and verify $Q(\boldsymbol{J}_\nu) = \widetilde{Q}(\boldsymbol{J}_\nu)$ at max.

Q_{lb} for a planar structure



Q_{lb} for a planar structure



Mixture between dipole and loop currents

$Q_{\rm lb}$ for a planar structure