EM Modes for Model Order Reduction and Antenna Optimization

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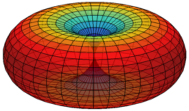
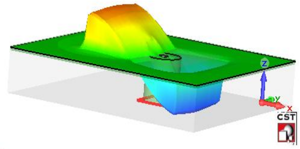
Modes

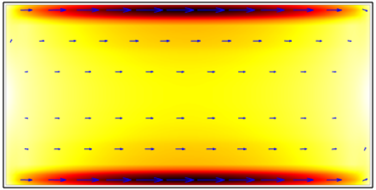
Expand fields or sources and often sufficient with the first dominant modes.

- Spherical modes: Mie series for scattering of spheres, expansion of far fields, near field measurements, probe compensation, Q-factor bounds (Chu), ...
- Cavity modes: resonance frequencies, losses, Q-factors, coupling, ...
- Characteristic modes: scattering properties, resonances, antenna feed placement, ...
- ...

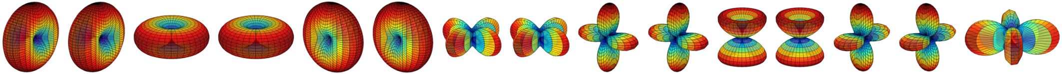
Determined from solutions of the Maxwell equations formed as eigenvalue problems or eigenvalue problems of integral equation.





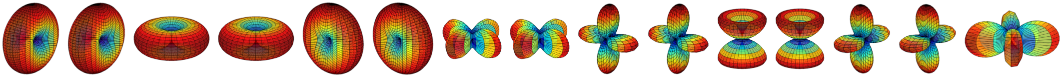


Spherical modes for spherical structures



- Orthogonal current densities on spheres, $\langle \boldsymbol{J}_m, \boldsymbol{J}_n \rangle \sim \mathbf{I}_m^{\mathsf{H}} \boldsymbol{\Psi} \mathbf{I}_n \sim \delta_{mn}$.
- Orthogonal radiated fields, $\langle \boldsymbol{F}_m, \boldsymbol{F}_n \rangle \sim \mathbf{I}_m^{\mathsf{H}} \mathbf{R}_{\mathrm{r}} \mathbf{I}_n \sim \delta_{mn}$
- Orthogonal reactance, $\mathbf{I}_m^{\mathsf{H}} \mathbf{X} \mathbf{I}_n \sim \delta_{mn}$
- Orthogonal stored energy, $\mathbf{I}_m^{\mathsf{H}} (\mathbf{X}_{\mathrm{m}} + \mathbf{X}_{\mathrm{e}}) \mathbf{I}_n \sim \delta_{mn}$
- Complete in L^2 on the sphere, $\boldsymbol{J} = \sum_m J_m \boldsymbol{J}_m$
- Monotonic Q-factor for TE and TM cases, $Q_n \geq Q_m$ if $n \geq m$
- Monotonic efficiency for TE and TM cases, $\eta_n \leq \eta_m$ if $n \geq m$

Spherical modes for spherical structures



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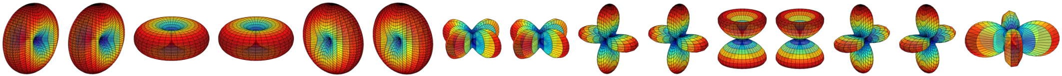
Properties preserved for arbitrary shaped objects using characteristic modes.

Spherical modes for spherical structures



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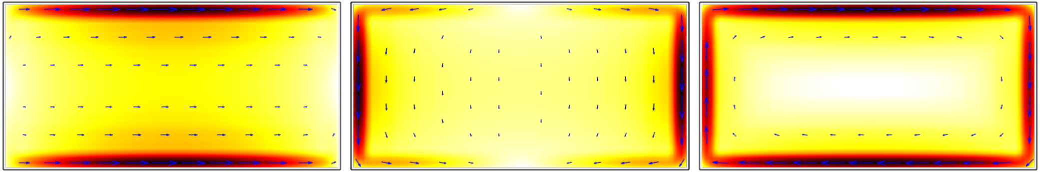
Properties preserved for arbitrary shaped objects using characteristic modes.

What about the other properties for arbitrary shapes.

Characteristic modes

- Developed in the 70s by Garbacz, Turpin, Harrington, Mautz [HM71].
- Provides physical understanding and complements simulation and optimization driven antenna design.
- Modes (electric current) determined by the geometry.
- Scattering properties, resonances, antenna feed placement, ...
- Generalized eigenvalue problem $\mathbf{X}\mathbf{I}_n = \lambda_n \mathbf{R}_\mathrm{r}\mathbf{I}_n$, where $\mathbf{Z} = \mathbf{R}_\mathrm{r} + \mathrm{j}\mathbf{X}$ denotes the MoM impedance matrix.
- Orthogonal far fields $\mathbf{I}_m^\mathrm{H}\mathbf{R}_\mathrm{r}\mathbf{I}_n = \delta_{mn}$ and reactance $\mathbf{I}_m^\mathrm{H}\mathbf{X}\mathbf{I}_n = \lambda_n\delta_{mn}$.

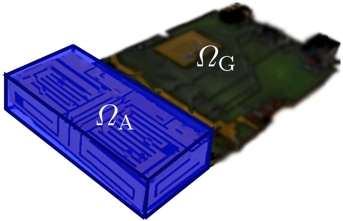


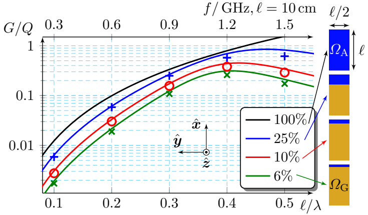
First three modes for a rectangle. Only a few dominant modes (small $|\lambda_n|$) for electrically small structures.

Antenna current optimization

Determine the optimal current distribution in a region.

- Physical bounds on Q, efficiency, gain, capacity, ..., [CG14; GN13; JC17]
- Convex optimization problems often reformulated in dual form as eigenvalue problems [CGS17].
- Expand in modes to reduce number of unknowns (model order reduction).
- Particularly important for the maximal capacity solved using semi-definite programming [EG18].
- Antenna synthesis.





Embedded antennas [CG14] compared with bounds.

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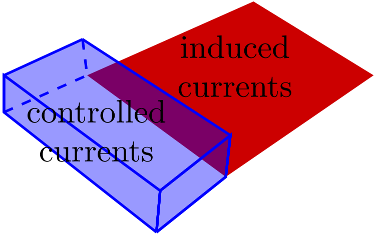

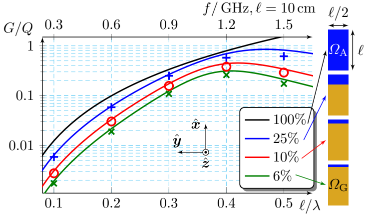

Embedded antennas [CG14] compared with bounds.

Used MoM matrix expressions

Method of Moments approximation [Gus+16] (expand $\boldsymbol{J}$ in basis functions)

$$W_{\mathrm{e}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \quad \text{stored E-energy, } \mathbf{X}_{\mathrm{e}} \text{ electric reactance [Van10]}$$

$$W_{\mathrm{m}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \quad \text{stored M-energy, } \mathbf{X}_{\mathrm{m}} \text{ magnetic reactance [Van10]}$$

$$P_{\mathrm{rad}} \approx \frac{1}{2}\mathbf{I}^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I} \quad \text{radiated power}$$

$$P_{\Omega} \approx \frac{1}{2}\mathbf{I}^{\mathsf{H}}\mathbf{R}_{\Omega}\mathbf{I} \quad \text{Ohmic losses, } \mathbf{R}_{\Omega} = R_{\mathrm{s}}\mathbf{\Psi} \text{ Gram matrix}$$

giving $\mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X} = \mathbf{R} + \mathrm{j}(\mathbf{X}_{\mathrm{m}} - \mathbf{X}_{\mathrm{e}})$.

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$$W_{\mathrm{m}} \approx \frac{1}{4\omega}\mathbf{I}^{\mathsf{H}}\boxed{\mathbf{X}_{\mathrm{m}}}\mathbf{I}$$ stored M-energy, $\mathbf{X}_{\mathrm{m}}$ magnetic reactance [Van10]

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Pre-computed matrices used in the optimization.

Characteristic and energy modes

Generalized eigenvalue problem

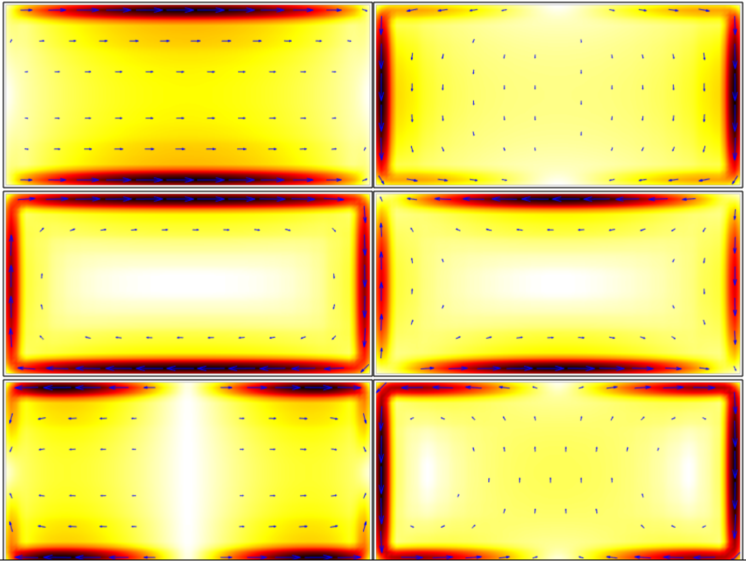
$$\mathbf{X}\mathbf{I}_n = \lambda_n \mathbf{R}_\mathrm{r}\mathbf{I}_n,$$

where $\mathbf{Z} = \mathbf{R}_\mathrm{r} + \mathrm{j}\mathbf{X}$ denotes the MoM impedance matrix. Orthogonal far fields $\mathbf{I}_m^\mathsf{H}\mathbf{R}_\mathrm{r}\mathbf{I}_n = \delta_{mn}$ and reactance $\mathbf{I}_m^\mathsf{H}\mathbf{X}\mathbf{I}_n = \lambda_n\delta_{mn}$.
Stored energy from $\mathbf{I}^\mathsf{H}(\mathbf{X}_\mathrm{m} + \mathbf{X}_\mathrm{e})\mathbf{I}$ is used to define stored energy modes from

$$(\mathbf{X}_\mathrm{m} + \mathbf{X}_\mathrm{e})\mathbf{I}_n = \lambda_n \mathbf{R}_\mathrm{r}\mathbf{I}_n,$$

with orthogonal far fields $\mathbf{I}_m^\mathsf{H}\mathbf{R}_\mathrm{r}\mathbf{I}_n = \delta_{mn}$ and reactance $\mathbf{I}_m^\mathsf{H}\mathbf{X}\mathbf{I}_n = \lambda_n\delta_{mn}$.

Characteristic modes: orthogonal far fields



Modes from eigenvalue problems

Generalized eigenvalue problems are used to define modes [HM71]

$$\mathbf{A}\mathbf{I}_n = \lambda_n \mathbf{R}_{\mathrm{r}} \mathbf{I}_n,$$

with characteristic ($\mathbf{A} = \mathbf{X}$), energy ($\mathbf{A} = \mathbf{X}_{\mathrm{e}} + \mathbf{X}_{\mathrm{m}}$), and efficiency modes ($\mathbf{A} = \mathbf{R}_{\Omega}$). Orthogonal with respect to $\mathbf{A}$ and $\mathbf{R}_{\mathrm{r}}$

$$\mathbf{I}_m^{\mathsf{H}} \mathbf{A} \mathbf{I}_n = \delta_{mn}\lambda_n \quad \text{and } \mathbf{I}_m^{\mathsf{H}} \mathbf{R}_{\mathrm{r}} \mathbf{I}_n = \delta_{mn}$$

Rewritten as a Rayleigh quotient ($\mathbf{A} \succeq \mathbf{0}$)

$$\lambda_n = \min_{\mathbf{I}_n^{\mathsf{H}} \mathbf{R}_{\mathrm{r}} \mathbf{I}_m = 0} \frac{\mathbf{I}_n^{\mathsf{H}} \mathbf{A} \mathbf{I}_n}{\mathbf{I}_n^{\mathsf{H}} \mathbf{R}_{\mathrm{r}} \mathbf{I}_n} \quad \text{for } n > m,$$

Orthogonal $\mathbf{I}_n^{\mathsf{H}} \mathbf{R}_{\mathrm{r}} \mathbf{I}_m = 0$ for $n > m$ can be written $\mathbf{K}_n \mathbf{I}_n$, where $\mathbf{K}_n$ is a matrix formed by the modes $\mathbf{I}_m$ for $m = 1, .., n-1$. We focus on orthogonal far fields and orthogonal current densities induced by the matrices

$$\mathbf{R}_{\mathrm{r}} \quad \text{and } \boldsymbol{\Psi}$$

Modes from optimization problems

Optimal currents can *e.g.*, be determined from

$$
\begin{aligned}
\text{maximize} \quad & \mathbf{I}^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I} \\
\text{subject to} \quad & \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \leq 2\bar{P}_{\mathrm{w}} \\
& \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \leq 2\bar{P}_{\mathrm{w}} \\
& \mathbf{I}^{\mathsf{H}}\mathbf{R}_{\Omega}\mathbf{I} \leq 2\bar{P}_{\Omega}.
\end{aligned}
$$

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Orthogonal to the previous modes

$$\begin{aligned} \text{maximize} \quad & \mathbf{I}^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I} \\ \text{subject to} \quad & \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \leq 2\bar{P}_{\mathrm{w}} \\ & \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \leq 2\bar{P}_{\mathrm{w}} \\ & \mathbf{I}^{\mathsf{H}}\mathbf{R}_{\Omega}\mathbf{I} \leq 2\bar{P}_{\Omega} \\ & \mathbf{K}_{p}\mathbf{I} = \mathbf{0}. \end{aligned}$$

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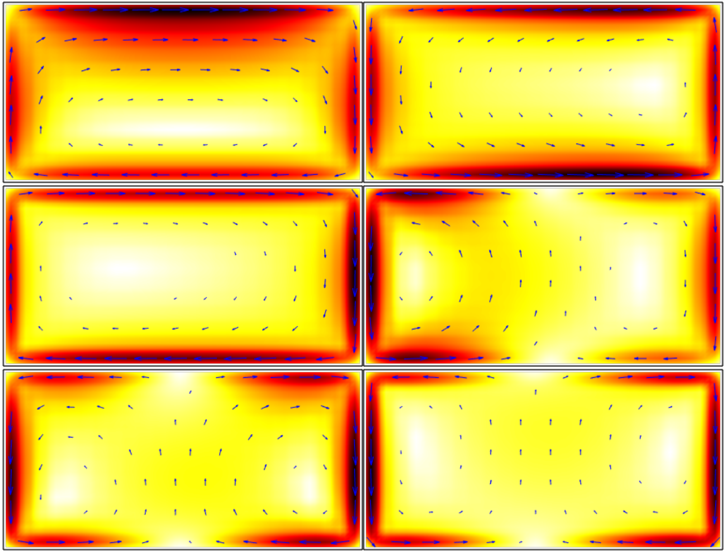
These problems can be reformulated in convex form [CGS17; Gus+16; JC17].
Solutions as eigenvalue problems [CGS17], *e.g.*, minimum Q-factor ($\mathbf{X}_\nu = \nu\mathbf{X}_{\mathrm{e}} + (1-\nu)\mathbf{X}_{\mathrm{m}}$) and maximum self-resonant efficiency ($\mathbf{X}_\nu = \nu\mathbf{X} + \mathbf{R}_\Omega$)

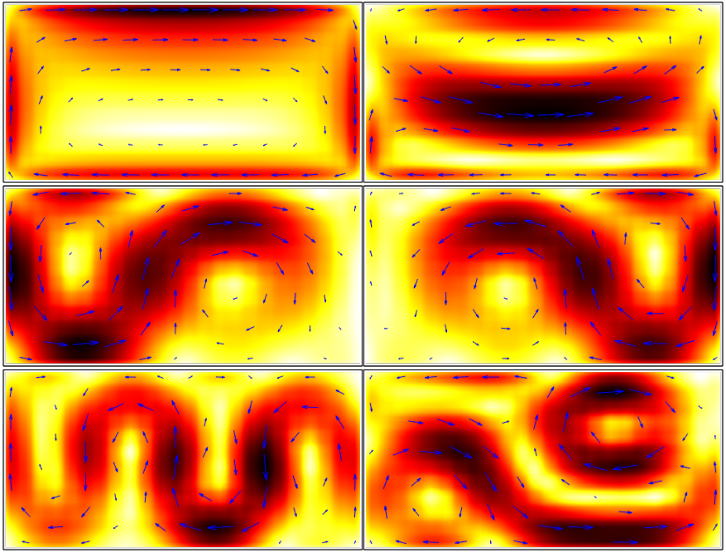
$$\lambda_n = \max_{\nu} \min_{\mathbf{I}_n^{\mathsf{H}}\mathbf{A}\mathbf{I}_m = \delta_{mn}} \frac{\mathbf{I}_n^{\mathsf{H}}\mathbf{X}_\nu\mathbf{I}_n}{\mathbf{I}_n^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{I}_n}$$

with $\mathbf{A} = \mathbf{R}_{\mathrm{r}}$ and $\mathbf{A} = \mathbf{\Psi}$ for orthogonal far fields and current densities, respectively.

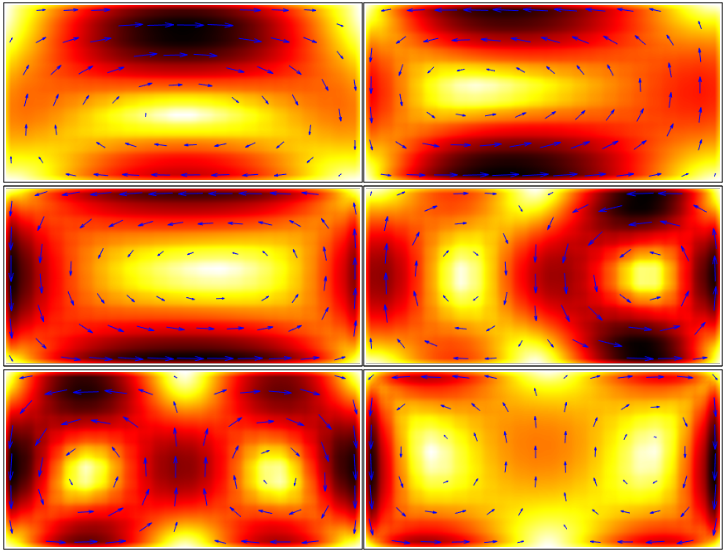
min Q and dissipation modes: orthogonal far fields



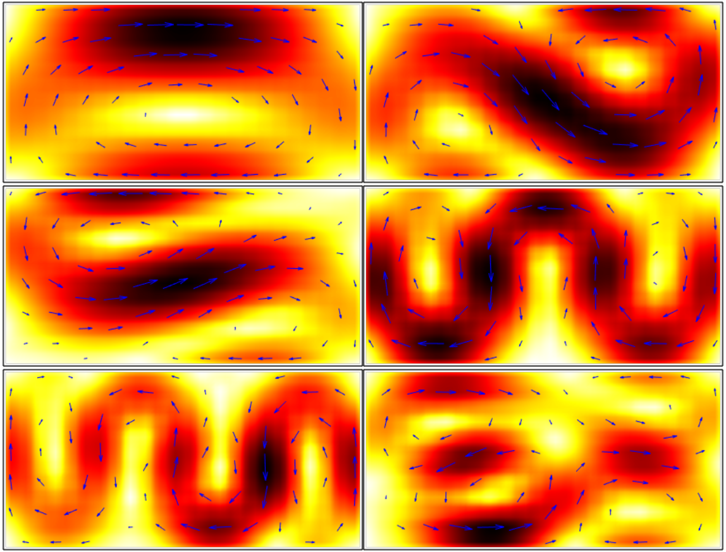
min Q and dissipation modes: orthogonal current



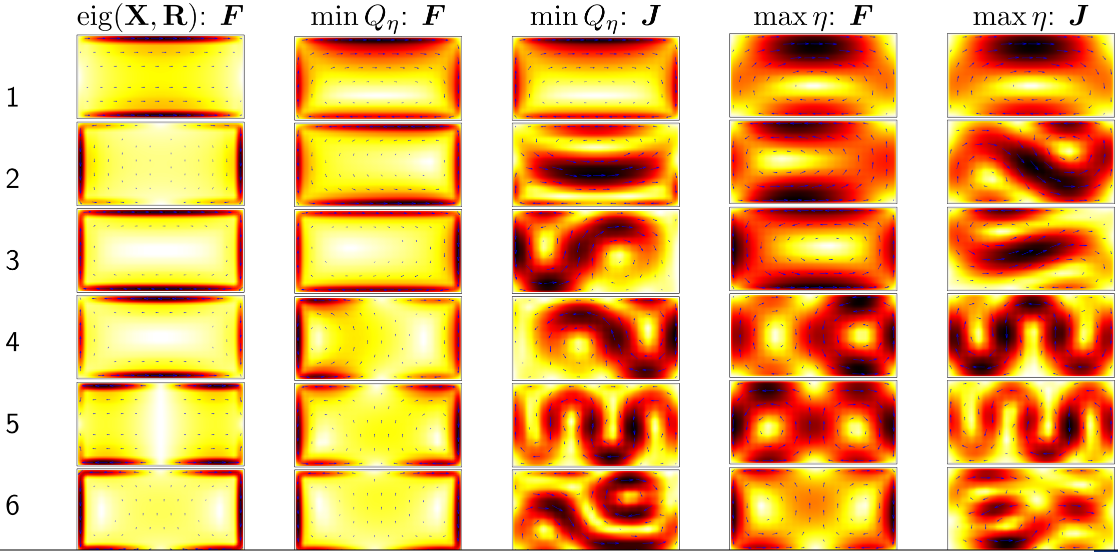
Self-resonant maximum efficiency: orthogonal far fields



Self-resonant maximum efficiency: orthogonal current



Comparison

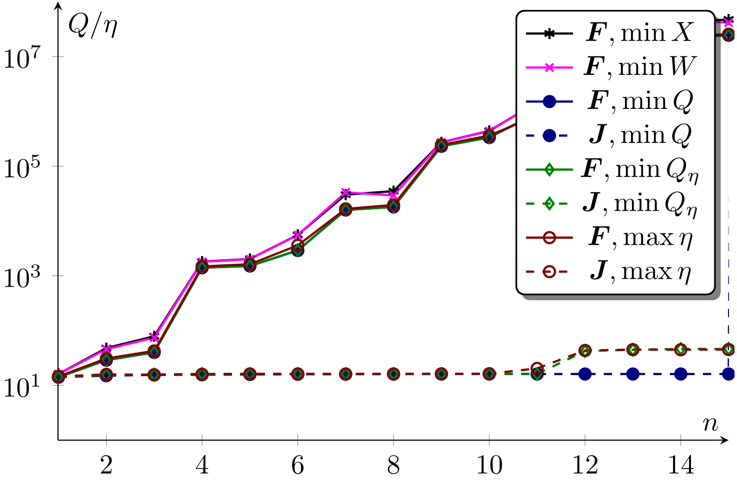


$\mathrm{eig}(\mathbf{X},\mathbf{R})$: $\boldsymbol{F}$
$\min Q_\eta$: $\boldsymbol{F}$
$\min Q_\eta$: $\boldsymbol{J}$
$\max \eta$: $\boldsymbol{F}$
$\max \eta$: $\boldsymbol{J}$
1
2
3
4
5
6

Q-factors for different modes

Q-factors for the first 15 modes on a planar rectangle for the $\min Q$, $\min Q_\eta$, and $\max \eta$ cases. The modes are orthogonal with respect to

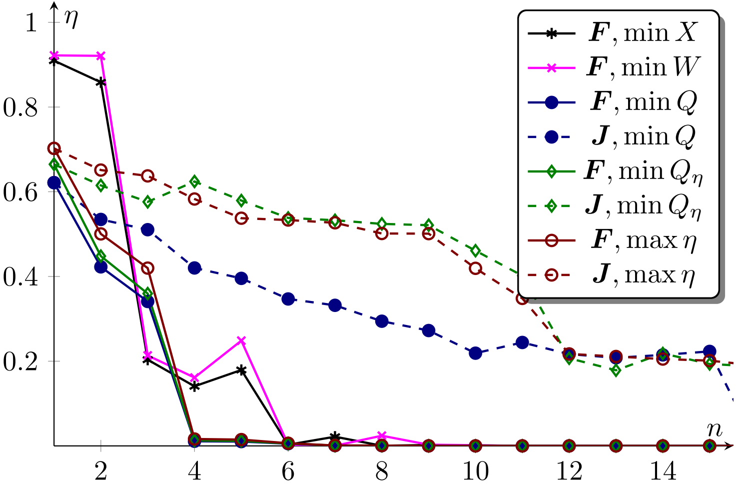
- $\boldsymbol{F}$: far field
- $\boldsymbol{J}$: current density.



Efficiency for different modes

Efficiencies for the first 15 modes on a planar rectangle for the $\min Q$, $\min Q_\eta$, and $\max \eta$ cases. The modes are orthogonal with respect to

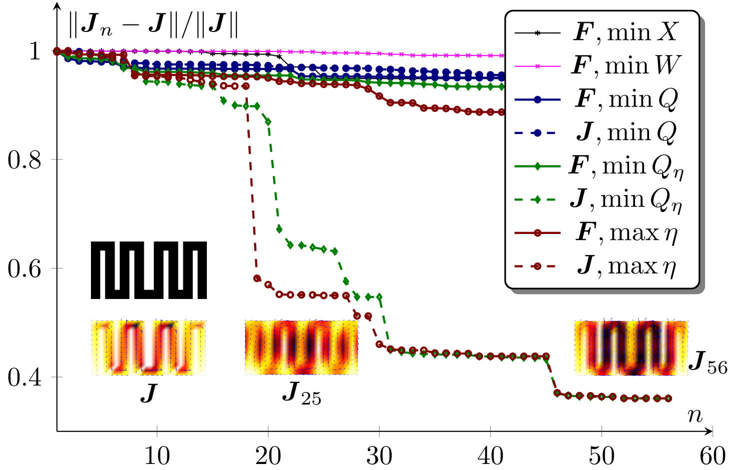
- $\boldsymbol{F}$: far field
- $\boldsymbol{J}$: current density.



Expansion of $\boldsymbol{J}$ for a self-resonant meander line antenna

Expansion using the n first modes defined on the circumscribing rectangle.

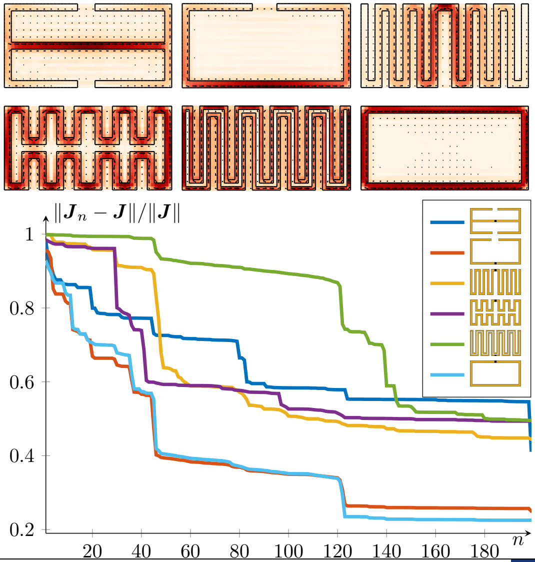
- Modes with orthogonal far fields $\boldsymbol{F}$ approximate far fields very well but not current densities.
- Modes with orthogonal current densities $\boldsymbol{J}$ can approximate the current density much better.



Meander line, capped dipole, SRR, loop antennas

Expansion using the n first modes defined on the circumscribing rectangle.

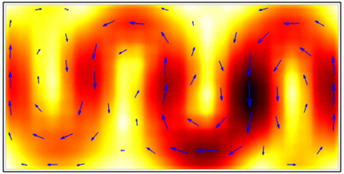
- Self-resonant maximum efficiency modes with orthogonal current densities.
- Resonant and non-resonant antennas.
- 4000 basis functions.
- Increasing number of expansion functions for fine details, see *e.g.*, the folded meander line (———) with strip width $\ell/64$.

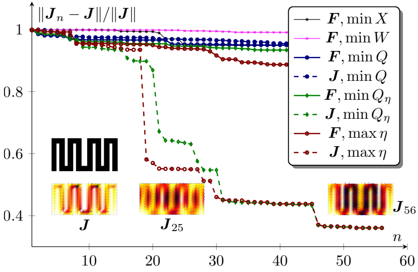


Conclusions

- Dedicated mode expansions for specific applications
- Orthogonal far fields for expansions targeting radiation
- Orthogonal current densities for expansions targeting sources
- Construction from eigenvalue and optimization problems
- Easy to compute

- Convergence properties
- Model order reduction
- Synthesis





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