EM Modes for Model Order Reduction and Antenna Optimization

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Modes

Expand fields or sources and often sufficient with the first dominant modes.

- Spherical modes: Mie series for scattering of spheres, expansion of far fields, near field measurements, probe compensation, Q-factor bounds (Chu), ...
- Cavity modes: resonance frequencies, losses, Q-factors, coupling, ...
- Characteristic modes: scattering properties, resonances, antenna feed placement, ...
- ...

Determined from solutions of the Maxwell equations formed as eigenvalue problems or eigenvalue problems of integral equation.
Orthogonal current densities on spheres, $\langle J_m, J_n \rangle \sim I_m^H \Psi I_n \sim \delta_{mn}$.

Orthogonal radiated fields, $\langle F_m, F_n \rangle \sim I_m^H R I_n \sim \delta_{mn}$

Orthogonal reactance, $I_m^H X I_n \sim \delta_{mn}$

Orthogonal stored energy, $I_m^H (X_m + X_e) I_n \sim \delta_{mn}$

Complete in $L^2$ on the sphere, $J = \sum_m J_m J_m$

Monotonic Q-factor for TE and TM cases, $Q_n \geq Q_m$ if $n \geq m$

Monotonic efficiency for TE and TM cases, $\eta_n \leq \eta_m$ if $n \geq m$
Spherical modes for spherical structures

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Properties preserved for arbitrary shaped objects using characteristic modes.
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Properties preserved for arbitrary shaped objects using characteristic modes.

What about the other properties for arbitrary shapes.
Characteristic modes

- Developed in the 70s by Garbacz, Turpin, Harrington, Mautz [HM71].
- Provides physical understanding and complements simulation and optimization driven antenna design.
- Modes (electric current) determined by the geometry.
- Scattering properties, resonances, antenna feed placement, ...
- Generalized eigenvalue problem $\mathbf{XI}_n = \lambda_n \mathbf{R}_r \mathbf{I}_n$, where $\mathbf{Z} = \mathbf{R}_r + j\mathbf{X}$ denotes the MoM impedance matrix.
- Orthogonal far fields $\mathbf{I}_m^H \mathbf{R}_r \mathbf{I}_n = \delta_{mn}$ and reactance $\mathbf{I}_m^H \mathbf{X} \mathbf{I}_n = \lambda_n \delta_{mn}$.

First three modes for a rectangle. Only a few dominant modes (small $|\lambda_n|$) for electrically small structures.
Antenna current optimization

Determine the optimal current distribution in a region.

- Physical bounds on $Q$, efficiency, gain, capacity, ..., [CG14; GN13; JC17]
- Convex optimization problems often reformulated in dual form as eigenvalue problems [CGS17].
- Expand in modes to reduce number of unknowns (model order reduction).
- Particularly important for the maximal capacity solved using semi-definite programming [EG18].
- Antenna synthesis.

Embedded antennas [CG14] compared with bounds.
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Method of Moments approximation [Gus+16] (expand $J$ in basis functions)

$$W_e \approx \frac{1}{4\omega} I^H X_e I$$ stored E-energy, $X_e$ electric reactance [Van10]

$$W_m \approx \frac{1}{4\omega} I^H X_m I$$ stored M-energy, $X_m$ magnetic reactance [Van10]

$$P_{\text{rad}} \approx \frac{1}{2} I^H R_r I$$ radiated power

$$P_\Omega \approx \frac{1}{2} I^H R_\Omega I$$ Ohmic losses, $R_\Omega = R_s \Psi$ Gram matrix

giving $Z = R + jX = R + j(X_m - X_e)$.
Used MoM matrix expressions

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giving $Z = R + jX = R + j(X_m - X_e)$.

Pre-computed matrices used in the optimization.
Characteristic and energy modes

Generalized eigenvalue problem

\[ X I_n = \lambda_n R_r I_n, \]

where \( Z = R_r + jX \) denotes the MoM impedance matrix. Orthogonal far fields \( I_m^H R_r I_n = \delta_{mn} \) and reactance \( I_m^H X I_n = \lambda_n \delta_{mn} \).

Stored energy from \( I^H (X_m + X_e) I \) is used to define stored energy modes from

\[ (X_m + X_e) I_n = \lambda_n R_r I_n, \]

with orthogonal far fields \( I_m^H R_r I_n = \delta_{mn} \) and reactance \( I_m^H X I_n = \lambda_n \delta_{mn} \).
Characteristic modes: orthogonal far fields
Generalized eigenvalue problems are used to define modes [HM71]

\[ \mathbf{A} \mathbf{I}_n = \lambda_n \mathbf{R}_\mathbf{r} \mathbf{I}_n, \]

with characteristic (\( \mathbf{A} = \mathbf{X} \)), energy (\( \mathbf{A} = \mathbf{X}_e + \mathbf{X}_m \)), and efficiency modes (\( \mathbf{A} = \mathbf{R}_\mathbf{\Omega} \)). Orthogonal with respect to \( \mathbf{A} \) and \( \mathbf{R}_\mathbf{r} \)

\[ \mathbf{I}_m^\mathsf{H} \mathbf{A} \mathbf{I}_n = \delta_{mn} \lambda_n \quad \text{and} \quad \mathbf{I}_m^\mathsf{H} \mathbf{R}_\mathbf{r} \mathbf{I}_n = \delta_{mn} \]

Rewritten as a Rayleigh quotient (\( \mathbf{A} \succeq 0 \))

\[ \lambda_n = \min_{\mathbf{I}_m^\mathsf{H} \mathbf{R}_\mathbf{r} \mathbf{I}_m = 0} \frac{\mathbf{I}_n^\mathsf{H} \mathbf{A} \mathbf{I}_n}{\mathbf{I}_n^\mathsf{H} \mathbf{R}_\mathbf{r} \mathbf{I}_n} \quad \text{for} \quad n > m, \]

Orthogonal \( \mathbf{I}_n^\mathsf{H} \mathbf{R}_\mathbf{r} \mathbf{I}_m = 0 \) for \( n > m \) can be written \( \mathbf{K}_n \mathbf{I}_n \), where \( \mathbf{K}_n \) is a matrix formed by the modes \( \mathbf{I}_m \) for \( m = 1, .., n - 1 \). We focus on orthogonal far fields and orthogonal current densities induced by the matrices

\[ \mathbf{R}_\mathbf{r} \quad \text{and} \quad \Psi \]
Optimal currents can e.g., be determined from

\[
\begin{align*}
\text{maximize} & \quad I^H R_r I \\
\text{subject to} & \quad I^H X_m I \leq 2\bar{P}_w \\
& \quad I^H X_e I \leq 2\bar{P}_w \\
& \quad I^H R_\Omega I \leq 2\bar{P}_\Omega.
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Orthogonal to the previous modes

\[
\begin{align*}
\text{maximize} & \quad I^H R_r I \\
\text{subject to} & \quad I^H X_m I \leq 2 \bar{P}_w \\
& \quad I^H X_e I \leq 2 \bar{P}_w \\
& \quad I^H R_\Omega I \leq 2 \bar{P}_\Omega \\
& \quad K_p I = 0.
\end{align*}
\]
Modes from optimization problems

\[
\begin{align*}
\text{maximize} & \quad \mathbf{I}^\mathsf{H} \mathbf{R}_r \mathbf{I} \\
\text{subject to} & \quad \mathbf{I}^\mathsf{H} \mathbf{X}_m \mathbf{I} \leq 2 \bar{P}_w \\
& \quad \mathbf{I}^\mathsf{H} \mathbf{X}_e \mathbf{I} \leq 2 \bar{P}_w \\
& \quad \mathbf{I}^\mathsf{H} \mathbf{R}_\Omega \mathbf{I} \leq 2 \bar{P}_\Omega.
\end{align*}
\]

These problems can be reformulated in convex form [CGS17; Gus+16; JC17]. Solutions as eigenvalue problems [CGS17], e.g., minimum Q-factor \((\mathbf{X}_\nu = \nu \mathbf{X}_e + (1 - \nu) \mathbf{X}_m)\) and maximum self-resonant efficiency \((\mathbf{X}_\nu = \nu \mathbf{X} + \mathbf{R}_\Omega)\)

\[
\lambda_n = \max_{\nu} \min_{\mathbf{I}^\mathsf{H} \mathbf{A} \mathbf{I} = \delta_{mn}} \frac{\mathbf{I}^\mathsf{H} \mathbf{X}_\nu \mathbf{I}_n}{\mathbf{I}^\mathsf{H} \mathbf{R}_r \mathbf{I}_n}
\]

with \(\mathbf{A} = \mathbf{R}_r\) and \(\mathbf{A} = \mathbf{\Psi}\) for orthogonal far fields and current densities, respectively.
min Q and dissipation modes: orthogonal far fields
min Q and dissipation modes: orthogonal current
Self-resonant maximum efficiency: orthogonal far fields
Self-resonant maximum efficiency: orthogonal current
Comparison

\[ \text{eig}(X, R): F \quad \text{min } Q_\eta: F \quad \text{min } Q_\eta: J \quad \max \eta: F \quad \max \eta: J \]

1
2
3
4
5
6

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Q-factors for the first 15 modes on a planar rectangle for the $\min Q$, $\min Q_\eta$, and $\max \eta$ cases. The modes are orthogonal with respect to

- $F$: far field
- $J$: current density.
Efficiencies for the first 15 modes on a planar rectangle for the \( \min Q \), \( \min Q_\eta \), and \( \max \eta \) cases. The modes are orthogonal with respect to:

- \( F \): far field
- \( J \): current density.
Expansion of $J$ for a self-resonant meander line antenna

Expansion using the $n$ first modes defined on the circumscribing rectangle.

- Modes with orthogonal far fields $F$ approximate far fields very well but not current densities.
- Modes with orthogonal current densities $J$ can approximate the current density much better.
Meander line, capped dipole, SRR, loop antennas

Expansion using the $n$ first modes defined on the circumscribing rectangle.

- Self-resonant maximum efficiency modes with orthogonal current densities.
- Resonant and non-resonant antennas.
- 4000 basis functions.
- Increasing number of expansion functions for fine details, see e.g., the folded meander line (bold) with strip width $\ell/64$. 

\[
\frac{\|J_n - J\|}{\|J\|}
\]
Conclusions

▶ Dedicated mode expansions for specific applications
▶ Orthogonal far fields for expansions targeting radiation
▶ Orthogonal current densities for expansions targeting sources
▶ Construction from eigenvalue and optimization problems
▶ Easy to compute
▶ Convergence properties
▶ Model order reduction
▶ Synthesis
References I


