Efficiency and Q for small antennas using Pareto optimality

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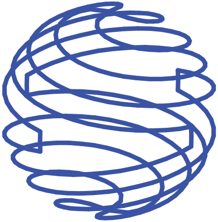
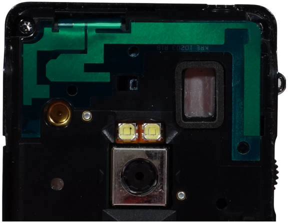
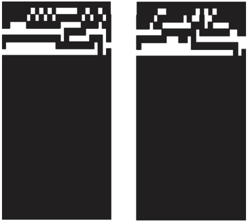
Design of small antennas



Folded spherical helix



SonyEricsson P1i



Fragmented patches

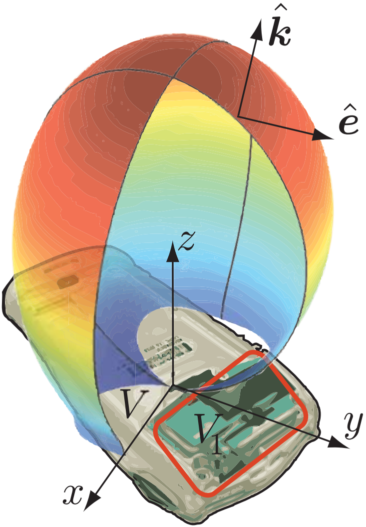
- There are many advanced methods to design small antennas.
- Often antennas embedded in structures.
- Performance in Q, bandwidth and efficiency.
- Fundamental tradeoff between Q and size (and bandwidth for passive matching).
- A figure of merit for performance.
- What about efficiency?

Tradeoff between performance and size

- Radiating (antenna) structure, V.
- Antenna volume, $V_1 \subset V$.
- Current density $\boldsymbol{J}_1$ in V_1.
- Radiated field, $\boldsymbol{F}(\hat{\boldsymbol{k}})$, in direction $\hat{\boldsymbol{k}}$ and polarization $\hat{\boldsymbol{e}}$.

Questions analyzed here, $\boldsymbol{J}_1$ for:

- maximum $G(\hat{\boldsymbol{k}}, \hat{\boldsymbol{e}})/Q$.
- superdirectivity.
- embedded antennas.
- efficiency.
- also minimum Q for given radiated fields, sidelobe levels, MIMO...



Background

- 1947 Wheeler: *Bounds based on circuit models.*
- 1948 Chu: *Bounds on Q and D/Q for spheres*.
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- 1999 Foltz & McLean, 2001 Sten, Koivisto, and Hujanen: *Attempts for bounds in spheroidal volumes.*
- 2006 Thal: *Bounds on Q for small hollow spherical antennas.*
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- 2010 Yaghjian & Stuart: *Bounds on Q for dipole antennas in the limit* $ka \to 0$.
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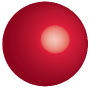

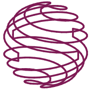

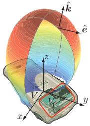

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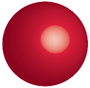
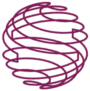
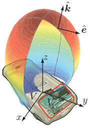
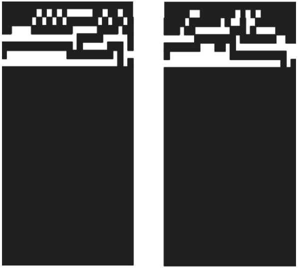
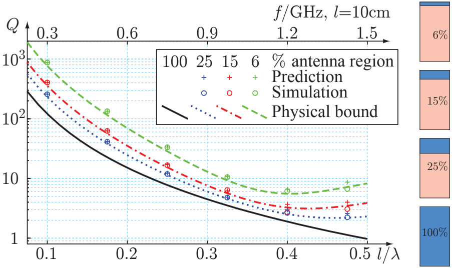





Antenna optimization





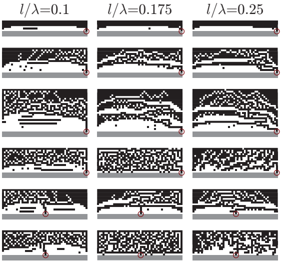
Optimization of structures

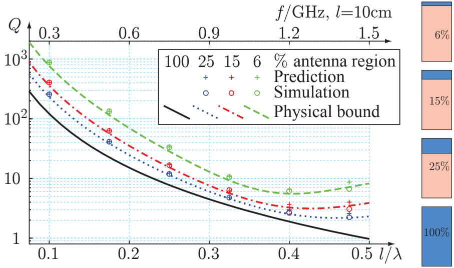
- global optimization.
- new non-intuitive designs.
- convergence?
- stopping criteria?
- optimal?

Optimization of currents

- determine optimal currents for Q, G/Q, ...
- convex optimization.
- physical bounds.
- can we realize the currents?

Antenna optimization





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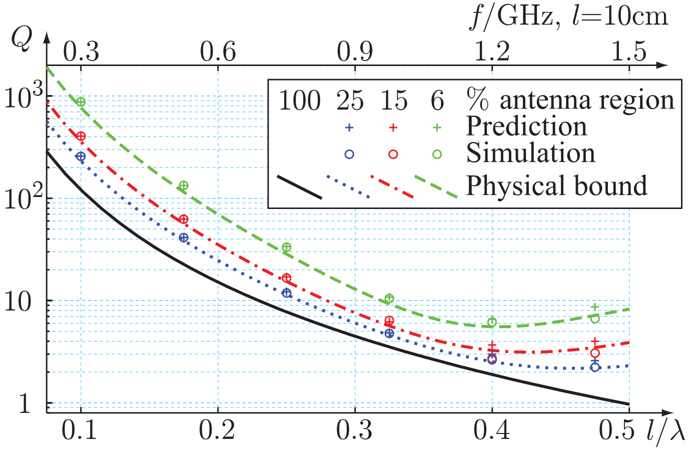
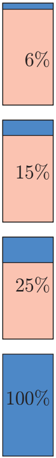
Q from G/Q for a planar PEC ground plane and $100, 25, 15, 6\%$ antenna region





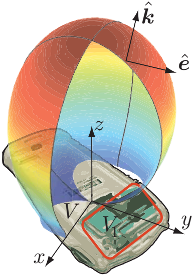
Optimization of antenna current

Gain over Q

$$\begin{array}{ll}\text{minimize} & \text{Stored energy}\\ \text{subject to} & \text{Radiation intensity} = P_0\end{array}$$

Q for superdirectivity $D \geq D_0$.

$$\begin{array}{ll}\text{minimize} & \text{Stored energy}\\ \text{subject to} & \text{Radiation intensity} = D_0 P_{\mathrm{rad}}/(4\pi)\\ & \text{Radiated power} \leq P_{\mathrm{rad}}\end{array}$$



Embedded structures

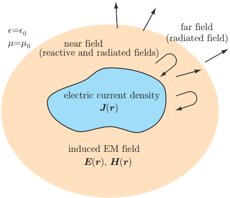
$$\begin{array}{ll}\text{minimize} & \text{Stored energy}\\ \text{subject to} & \text{Radiation intensity} = P_0\\ & \text{Correct induced currents}\end{array}$$

Stored EM energies from current densities $\boldsymbol{J}$ in V

Use the expressions by Vandenbosch (2010) (and Carpenter (1989), Geyi (2003) for small antennas). Stored electric energy $\widetilde{W}^{(\mathrm{e})}_{\mathrm{vac}} = \frac{\mu_0}{16\pi k^2} w^{(\mathrm{e})}$

$$w^{(\mathrm{e})} = \int_V \int_V \nabla_1 \cdot \boldsymbol{J}_1 \nabla_2 \cdot \boldsymbol{J}_2^* \frac{\cos(kR_{12})}{R_{12}} - \frac{k}{2}\big(k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* - \nabla_1 \cdot \boldsymbol{J}_1 \nabla_2 \cdot \boldsymbol{J}_2^*\big) \sin(kR_{12}) \,\mathrm{dV}_1 \,\mathrm{dV}_2,$$

where $\boldsymbol{J}_1 = \boldsymbol{J}(\boldsymbol{r}_1)$, $\boldsymbol{J}_2 = \boldsymbol{J}(\boldsymbol{r}_2)$, $R_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$. Stored magnetic energy $\widetilde{W}^{(\mathrm{m})}_{\mathrm{vac}} = \frac{\mu_0}{16\pi k^2} w^{(\mathrm{m})}$, where



$$w^{(\mathrm{m})} = \int_V \int_V k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* \frac{\cos(kR_{12})}{R_{12}} - \frac{k}{2}\big(k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* - \nabla_1 \cdot \boldsymbol{J}_1 \nabla_2 \cdot \boldsymbol{J}_2^*\big) \sin(kR_{12}) \,\mathrm{dV}_1 \,\mathrm{dV}_2.$$

Stored EM energies from current densities $\boldsymbol{J}$ in V II

Also the total radiated power $P_{\mathrm{rad}} = \frac{\eta_0}{8\pi k} p_{\mathrm{rad}}$ with

$$p_{\mathrm{rad}} = \int_V \int_V \left(k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* - \nabla_1 \cdot \boldsymbol{J}_1 \nabla_2 \cdot \boldsymbol{J}_2^*\right) \frac{\sin(kR_{12})}{R_{12}} \,\mathrm{dV}_1 \,\mathrm{dV}_2.$$

Method of Moments approximation (expand $\boldsymbol{J}$ in basis functions)

$$w^{(\mathrm{e})} \approx \mathbf{J}^{\mathsf{H}} \mathbf{X}_{\mathrm{e}} \mathbf{J} \quad \text{stored E-energy}$$

$$w^{(\mathrm{m})} \approx \mathbf{J}^{\mathsf{H}} \mathbf{X}_{\mathrm{m}} \mathbf{J} \quad \text{stored M-energy}$$

$$p_{\mathrm{rad}} \approx \mathbf{J}^{\mathsf{H}} \mathbf{R}_{\mathrm{r}} \mathbf{J} \quad \text{radiated power}$$

We also use

$$\boldsymbol{F} \approx \mathbf{F}^{\mathsf{H}} \mathbf{J} \quad \text{far field}$$

$$\mathbf{J}_2 \approx \mathbf{Z}' \mathbf{J}_1 \quad \text{induced current on a PEC}$$

The normalized quantities $w^{(\mathrm{e})}, w^{(\mathrm{m})}$, and p_{rad} have dimensions given by volume, m^3, times the dimension of $|\boldsymbol{J}|^2$.

Convex optimization of antenna currents

Convex optimization offer many possibilities to analyze radiating structures. Quantities are:

linear near field, far field, and induced currents.

quadratic positive semidefinite radiation intensity, radiated power, absorbed power, stored energies.

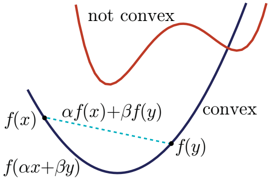
in the current density $\boldsymbol{J}$. In convex optimization, we can

- minimize convex quantities.
- maximize concave quantities.

The linear (affine) quantities are both convex and concave. Quadratic positive semidefinite forms are convex.

Convex optimization

$$\begin{aligned} &\text{minimize} && f_0(\mathbf{x}) \\ &\text{subject to} && f_i(\mathbf{x}) \leq 0, \quad i = 1, ..., N_1 \\ &&& \mathbf{A}\mathbf{x} = \mathbf{b} \end{aligned}$$



where $f_i(x)$ are convex, *i.e.*, $f_i(\alpha\mathbf{x} + \beta\mathbf{y}) \leq \alpha f_i(\mathbf{x}) + \beta f_i(\mathbf{y})$ for $\alpha, \beta \in \mathbb{R}$, $\alpha + \beta = 1$, $\alpha, \beta \geq 0$.

Solved with efficient standard algorithms. No risk of getting trapped in a local minimum. A problem is 'solved' if formulated as a convex optimization problem.

Can be used in many formulations for the antenna performance expressed in the current density $\boldsymbol{J}$.

Currents for maximal G/Q for embedded antennas

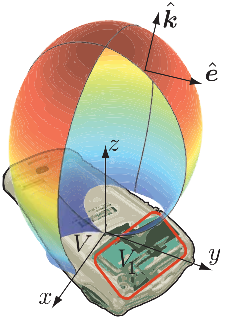
Determine an optimal current density $\boldsymbol{J}_1(\boldsymbol{r})$ in the volume V_1. Assume that V is PEC outside V_1.

Can minimize the stored energy for given radiated field

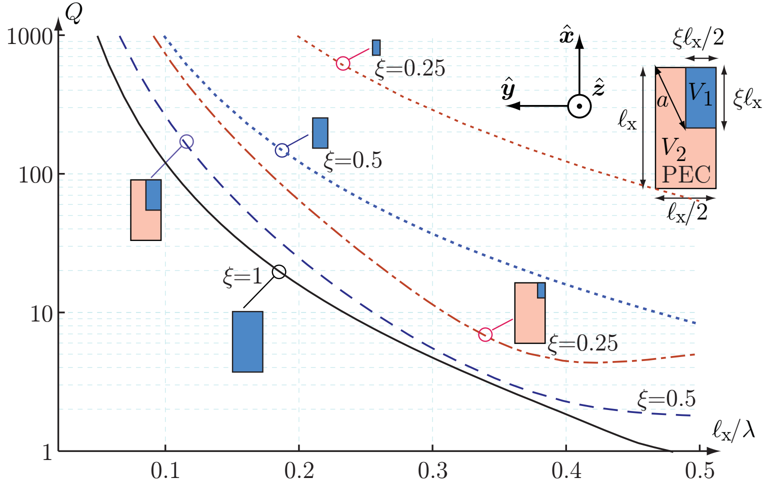
$$\begin{aligned} \text{minimize} \quad & \max\{\mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J}, \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J}\} \\ \text{subject to} \quad & \mathrm{Re}\{\mathbf{F}^{\mathsf{H}}\mathbf{J}\} = 1 \\ & \mathbf{J}_2 = \mathbf{Z}'\mathbf{J}_1 \end{aligned}$$

or maximize the radiated field for given stored energy

$$\begin{aligned} \text{maximize} \quad & \mathrm{Re}\{\mathbf{F}^{\mathsf{H}}\mathbf{J}\} \\ \text{subject to} \quad & \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J} \le 1 \\ & \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J} \le 1 \\ & \mathbf{J}_2 = \mathbf{Z}'\mathbf{J}_1 \end{aligned}$$



Embedded antennas in planar PEC rectangles



D/Q (or G/Q) bounds

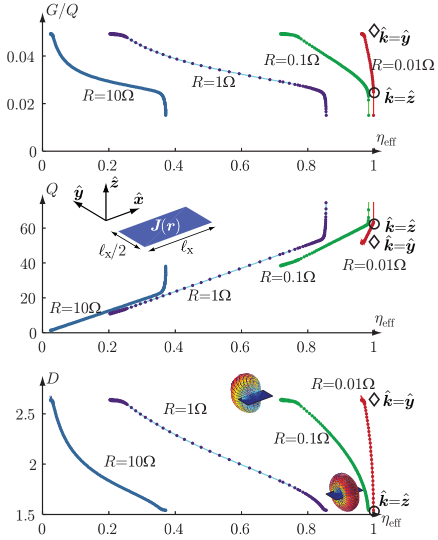
Typical (but not optimal) matlab code using CVX

```
cvx_begin
variable J(n) complex;
dual variables We Wm
maximize(real(F'*J))
subject to
  We: quad_form(J,Xe) <= 1;
  Wm: quad_form(J,Xm) <= 1;
cvx_end
```

We can reformulate the complex optimization problem to analyze superdirectivity, antennas with a prescribed radiation pattern, ... Now we generalize the approach to analyze efficiency.

Efficiency, η_{eff}

- For lossy structures, it is desired to minimize the stored energy and the ohmic losses simultaneously.
- The ohmic losses is positive-semidefinite quadratic form in the current density $\boldsymbol{J}$.
- MoM approximation $p_\Omega \approx \mathbf{J}^{\mathsf{H}}\mathbf{R}_\Omega\mathbf{J}$.
- We consider resistive sheets for planar structures with resistance $\{10, 1, 0.1, 0.01\}\,\Omega/\square$.



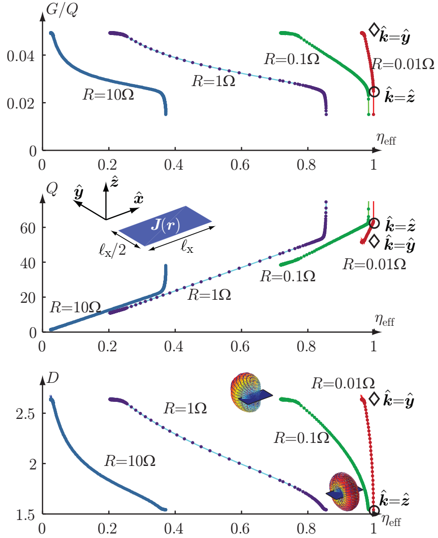
Multi-objective optimization and Pareto optimality to minimize the stored energy and ohmic losses.

Efficiency, η_{eff}, using Pareto optimality

Linear combination of the stored energy and ohmic losses

$$\begin{aligned}\text{min.}\quad & \alpha \max\{\mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J}, \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J}\} \\ & \qquad + (1-\alpha)\mathbf{J}^{\mathsf{H}}\mathbf{P}_{\Omega}\mathbf{J} \\ \text{s.t.}\quad & \mathrm{Re}\{\mathbf{F}^{\mathsf{H}}\mathbf{J}\} = 1\end{aligned}$$

where $+ \leq \alpha \leq 1$. A planar rectangle with side lengths ℓ_{x} and $\ell_{\mathrm{x}}/2$ modeled as a resistive sheet with $R = 1/(\sigma d) = \{10, 1, 0.1, 0.01\}\,\Omega/\square$ is used to illustrate the tradeoff between Q, D, and η_{eff} for $\ell_{\mathrm{x}}/\lambda \approx 0.13$ (or $ka \approx 0.44$).

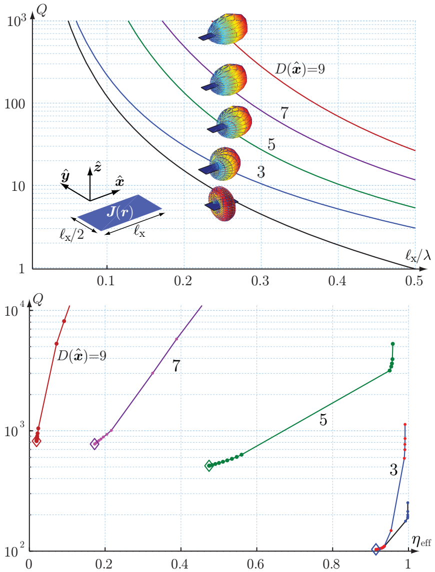


Superdirectivity: $\min.\ G/Q$ s.t. $D \geq D_0$

A superdirective antenna has a directivity that is much larger than for a typical reference antenna. Add the constraint $P_{\mathrm{rad}} \leq 4\pi D_0^{-1}$ the get the convex optimization problem

$$\begin{aligned} \min. \quad & \max\{\mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J}, \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J}\} \\ \text{s.t.} \quad & \mathrm{Re}\{\mathbf{F}^{\mathsf{H}}\mathbf{J}\} = 1 \\ & \mathbf{J}^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{J} \leq k^3 D_0^{-1} \end{aligned}$$

Example for current densities confined to planar rectangles with side lengths ℓ_{x} and $\ell_{\mathrm{y}} = 0.5\ell_{\mathrm{x}}$.

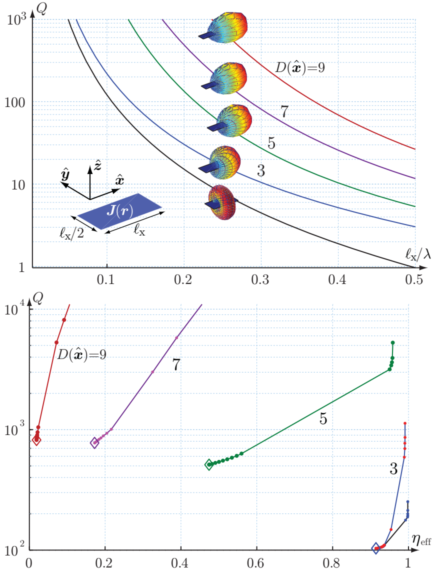


Superdirectivity: $\min. G/Q$ s.t. $D \geq D_0$

Linear combination for losses:

$$\begin{aligned} \min. \quad & \alpha W + (1-\alpha)\mathbf{J}^{\mathsf{H}}\mathbf{P}_{\Omega}\mathbf{J} \\ \text{s.t.} \quad & \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J} \leq W \\ & \mathbf{J}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J} \leq W \\ & \mathrm{Re}\{\mathbf{F}^{\mathsf{H}}\mathbf{J}\} = 1 \\ & \mathbf{J}^{\mathsf{H}}\mathbf{R}_{\mathrm{r}}\mathbf{J} \leq k^3 D_0^{-1} \end{aligned}$$

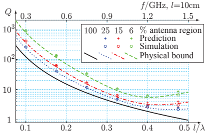
Example for current densities confined to planar rectangles with side lengths ℓ_{x} and $\ell_{\mathrm{y}} = 0.5\ell_{\mathrm{x}}$, $R = 0.01\,\Omega/\square$, and $ka = 0.44$.

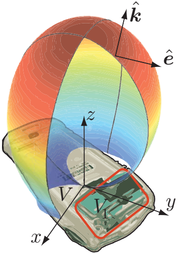


Summary

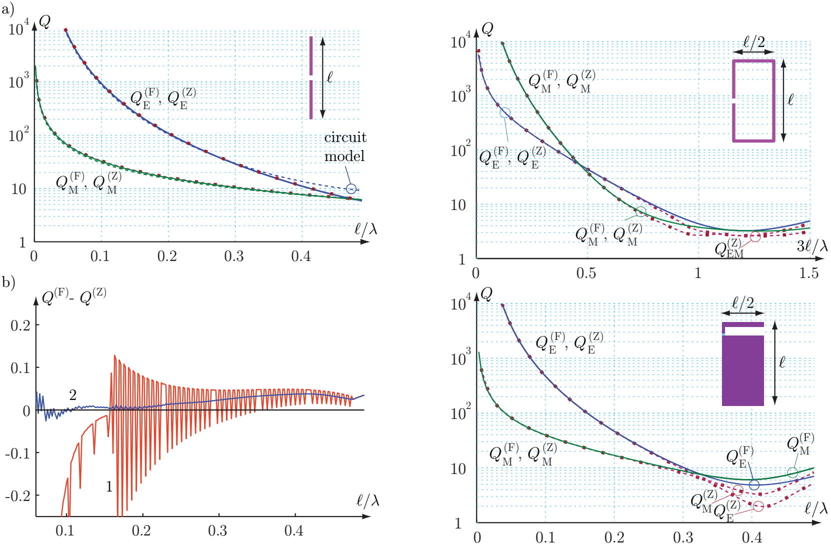
- Convex optimization to determine bounds and optimal currents:
  - D/Q and G/Q.
  - Q for superdirective antennas.
  - Embedded antennas in PEC structures.
  - Q for antennas with prescribed far fields.
  - Multi-objective optimization for efficiency.
- Closed form solution for small antennas.
- Non-Foster to overcome $B \sim 1/Q$.
- Initial results for efficiency. Self resonance?
- More realistic geometries.
- MIMO.

Gustafsson and Nordebo, *Optimal antenna currents for Q, superdirectivity, and radiation patterns using convex optimization*, IEEE-TAP, 61(3), 1109-1118, 2013





Antenna examples



Optimal current distributions on small spheres

- The optimization problem for small dipole antennas show that the charge distribution is the most important quantity.
- On a sphere, we have

$$\rho(\theta,\phi) = \rho_0 \cos\theta$$

for optimal antennas with polarization $\hat{\boldsymbol{e}} = \hat{\boldsymbol{z}}$.

- The current density satisfies

$$\nabla \cdot \boldsymbol{J} = -\mathrm{j}k\rho$$

Many solutions, *e.g.*, all surface currents

$$\boldsymbol{J} = J_{\theta 0}\hat{\boldsymbol{\theta}}\big(\sin\theta - \frac{\beta}{\sin\theta}\big) + \frac{1}{\sin\theta}\frac{\partial A}{\partial\phi}\hat{\boldsymbol{\theta}} - \frac{\partial A}{\partial\theta}\hat{\boldsymbol{\phi}}$$

where $J_{\theta 0} = -\mathrm{j}ka\rho_0$, β is a constant, and $A = A(\theta,\phi)$

Optimal current distributions on small spheres

Some solutions:

- Spherical dipole,
  $\beta = 0, A = 0.$
- Capped dipole,
  $\beta = 1, A = 0.$
- Folded spherical helix,
  $\beta = 0, A \neq 0.$

They all have almost identical charge distributions

$$\rho(\theta, \phi) = \rho_0 \cos\theta$$

Can mathematical solutions suggest antenna designs?