Minimum Q-factors for Antennas

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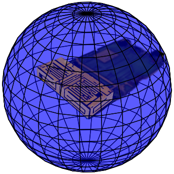
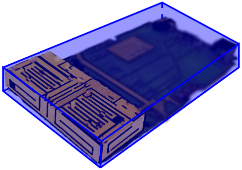
Minimum Q: some background



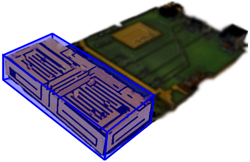
Spherical modes

- Chu 1948, TE,TM $Q \geq \frac{1}{k^3a^3} + \frac{1}{ka}$
- TE+TM
- Thal 2006, $\boldsymbol{J}$
- Antennas $\boldsymbol{J}$ and simulation $\boldsymbol{J}+\boldsymbol{M}$



Arbitrary shapes

- Gustafsson *etal* 2007, $D/Q \Rightarrow Q$ for small TM
- Yaghjian *etal* small $\boldsymbol{J}, \boldsymbol{M}, \boldsymbol{J}+\boldsymbol{M}$
- Jonsson& Gustafsson, Kim, $\boldsymbol{J}+\boldsymbol{M}$
- Designs for TM



Complex

- Gustafsson& Nordebo 2013, G/Q, Q s.t. $D \geq D_0$, Q s.t. $\boldsymbol{F}$,...
- Use Q for TE,TM
- small and large ground planes, losses, on/inbody, ...

Consensus for small and good understanding for larger TE,TM. What about for larger TE+TM?

Lower bound on the Q-factor for TE+TM for $\boldsymbol{J}$

Chu (and others) used spherical modes to show that a combination between TM and TE mode has the lowest Q-factor. Generalize to arbitrary shape

- Capek and Jelinek 2016, *Optimal composition of modal currents for minimal quality factor Q*, TAP2016. Combination of two characteristic (or similar) modes.
- Jelinek and Capek 2017, *Optimal currents on arbitrarily shaped surfaces*, TAP 2017. All modes and Lagrangian formulation.
- Gustafsson et al. 2016, *Antenna current optimization using MATLAB and CVX*, FERMAT, 2016. Relaxation to a dual convex problem.

Here, we follow Capek, Gustafsson, Schab, *Minimization of Antenna Quality Factor*, arXiv, 2016

Lower bound on the Q-factor

The lower bound on the Q-factor Q (MoM approximation) from

$$Q_{\mathrm{lb}} = \min_{\mathbf{I}} \frac{\max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}} = \frac{2\omega \max\{W_{\mathrm{e}}, W_{\mathrm{m}}\}}{P_{\mathrm{rad}}}$$

Use

1. for any antenna current $\mathbf{I}$, $Q(\mathbf{I}) \geq Q_{\mathrm{lb}}$, *i.e.*,

$$\frac{\max\{\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I}, \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}\}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}} = Q(\mathbf{I}) \geq Q_{\mathrm{lb}}$$

2. $\max\{Q_{\mathrm{e}}, Q_{\mathrm{m}}\} \geq \nu Q_{\mathrm{e}} + (1-\nu) Q_{\mathrm{m}}$ for $0 \leq \nu \leq 1$, *i.e.*,

$$Q_{\mathrm{lb}} \geq \min_{\mathbf{I}} \frac{\mathbf{I}^{\mathsf{H}}(\nu\mathbf{X}_{\mathrm{e}} + (1-\nu)\mathbf{X}_{\mathrm{m}})\mathbf{I}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}} = \widetilde{Q}(\mathbf{I}_\nu) = \widetilde{Q}_\nu$$

Totally, Q is in the range given by

$$Q(\mathbf{I}_\nu) \geq Q_{\mathrm{lb}} \geq \max_{\nu} \widetilde{Q}(\mathbf{I}_\nu)$$

Lower bound on the Q-factor II

Some observations

1. The Rayleigh quotient

$$\min_{\mathbf{I}} \frac{\mathbf{I}^{\mathsf{H}}(\nu\mathbf{X}_{\mathrm{e}} + (1-\nu)\mathbf{X}_{\mathrm{m}})\mathbf{I}}{\mathbf{I}^{\mathsf{H}}\mathbf{R}\mathbf{I}} = \widetilde{Q}_{\nu}$$

   is easily solved as a generalized eigenvalue problem.

2. Self-resonance $\mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} = \mathbf{I}^{\mathsf{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I}$ implies $Q_{\mathrm{e}} = Q_{\mathrm{m}}$ and

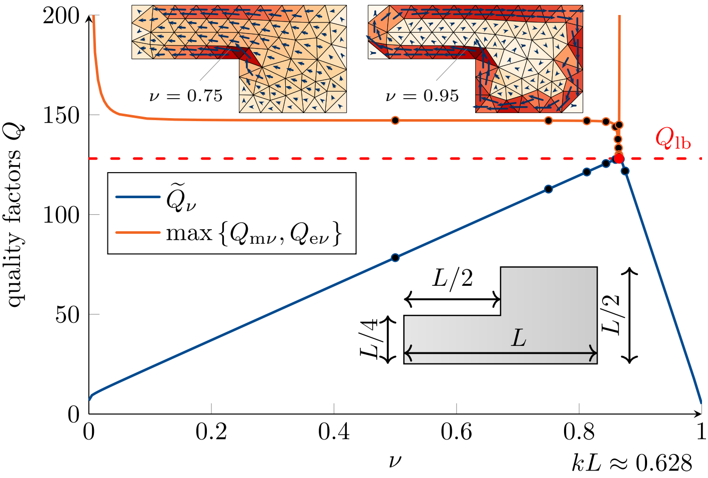
$$\max\{Q_{\mathrm{e}}, Q_{\mathrm{m}}\} = \nu Q_{\mathrm{e}} + (1-\nu)Q_{\mathrm{m}} \Rightarrow Q_{\mathrm{lb}} = \widetilde{Q}_{\nu}$$

Basic algorithm:

$$\begin{aligned} &\text{maximize}_{\nu} && \widetilde{Q}_{\nu} \\ &\text{subject to} && 0 \leq \nu \leq 1 \end{aligned}$$

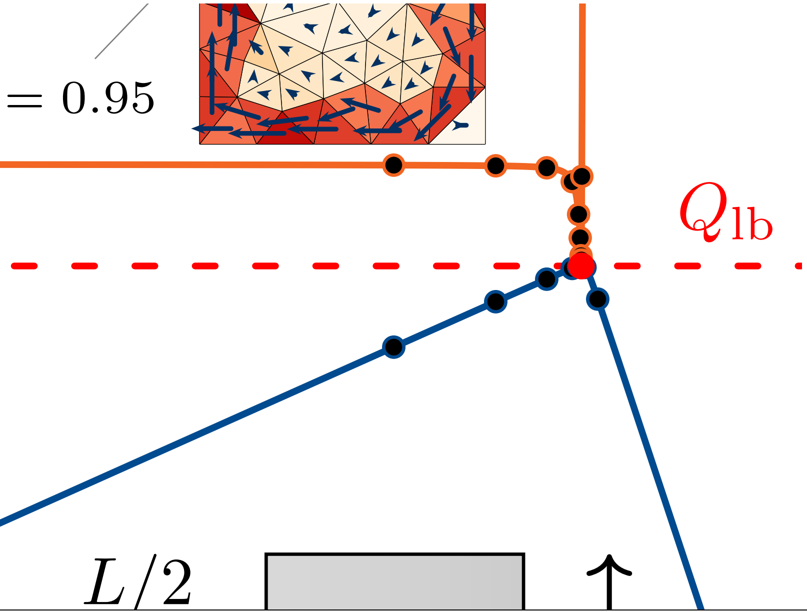
solve (bisection method) and verify that $Q(\boldsymbol{J}_{\nu}) = \widetilde{Q}(\boldsymbol{J}_{\nu})$ at max.

L shaped planar structure

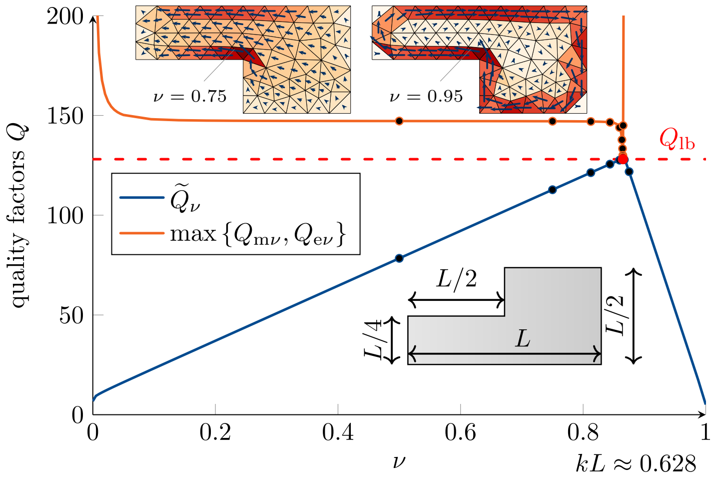


$\mathrm{maximize}_\nu\ \widetilde{Q}_\nu$ for $0 \leq \nu \leq 1$ and verify $Q(\boldsymbol{J}_\nu) = \widetilde{Q}(\boldsymbol{J}_\nu)$ at max.

L shaped planar structure

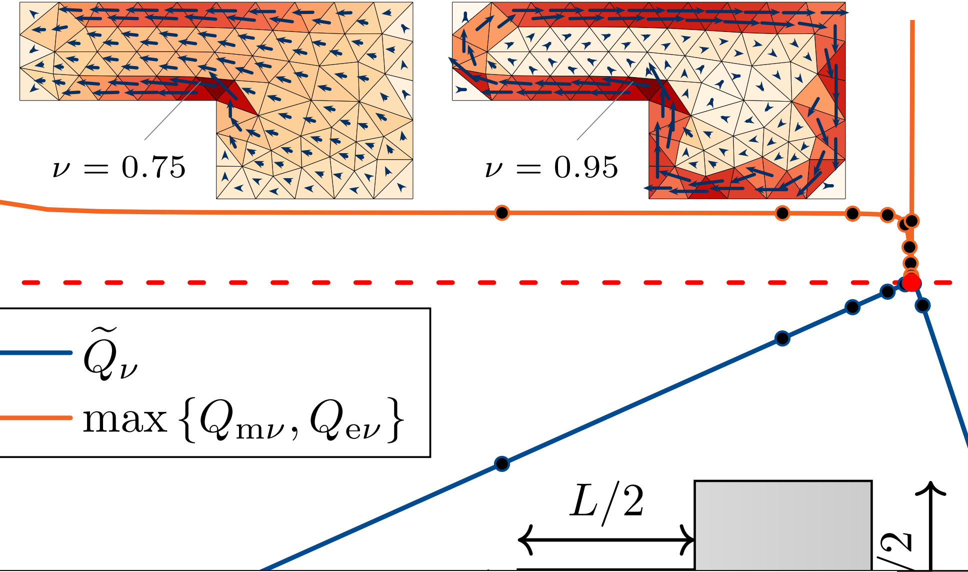


L shaped planar structure

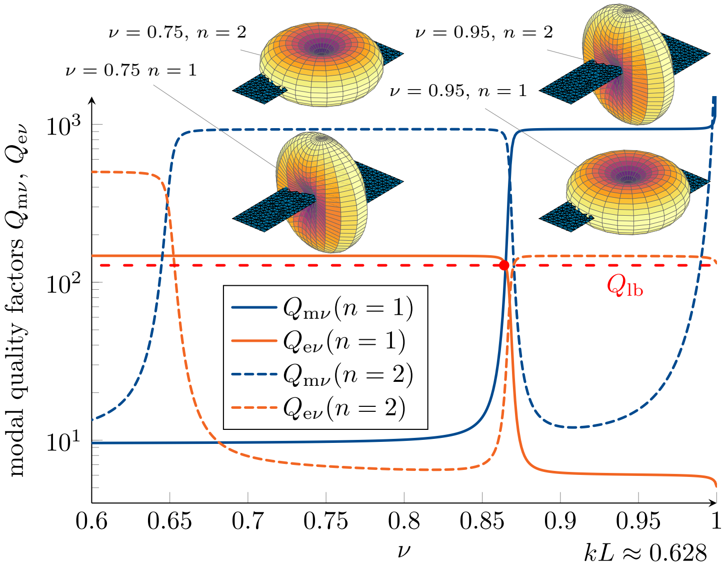


Mixture between dipole and loop currents

L shaped planar structure

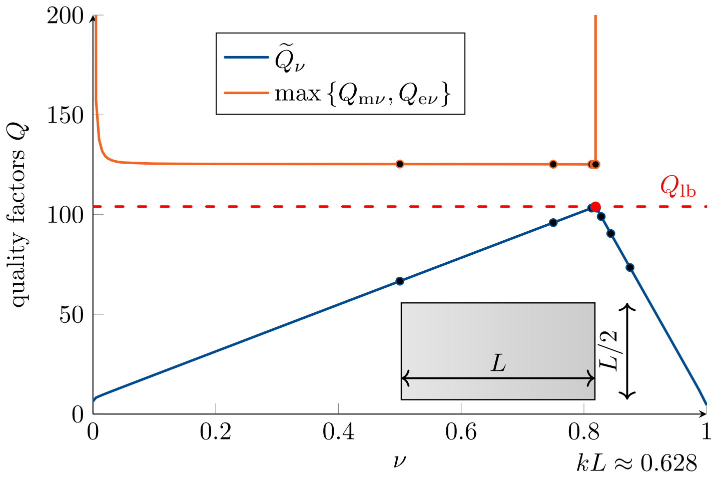


Modes

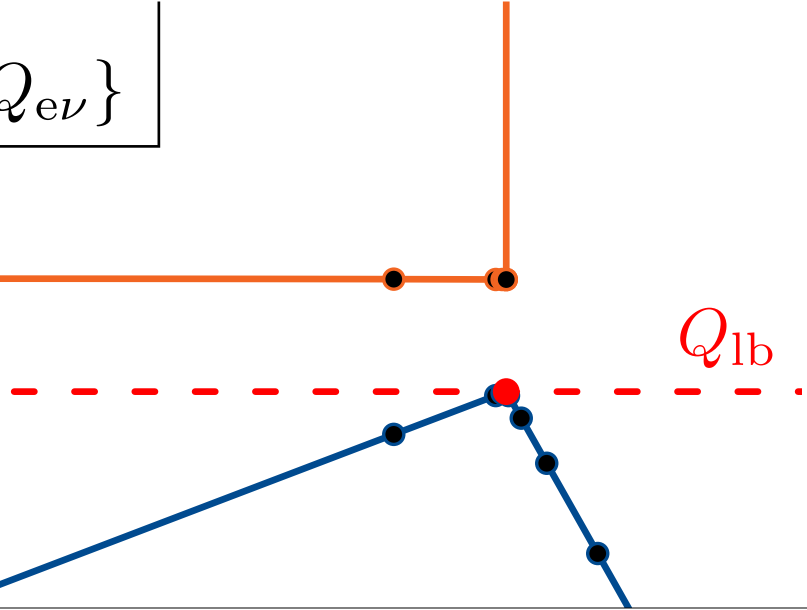


$\nu = 0.75,\ n = 2$
$\nu = 0.95,\ n = 2$
$\nu = 0.75\ n = 1$
$\nu = 0.95,\ n = 1$
modal quality factors $Q_{\mathrm{m}\nu},\ Q_{\mathrm{e}\nu}$
10^3
10^2
10^1
Q_{lb}
$Q_{\mathrm{m}\nu}(n = 1)$
$Q_{\mathrm{e}\nu}(n = 1)$
$Q_{\mathrm{m}\nu}(n = 2)$
$Q_{\mathrm{e}\nu}(n = 2)$
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
ν
$kL \approx 0.628$

Planar rectangle

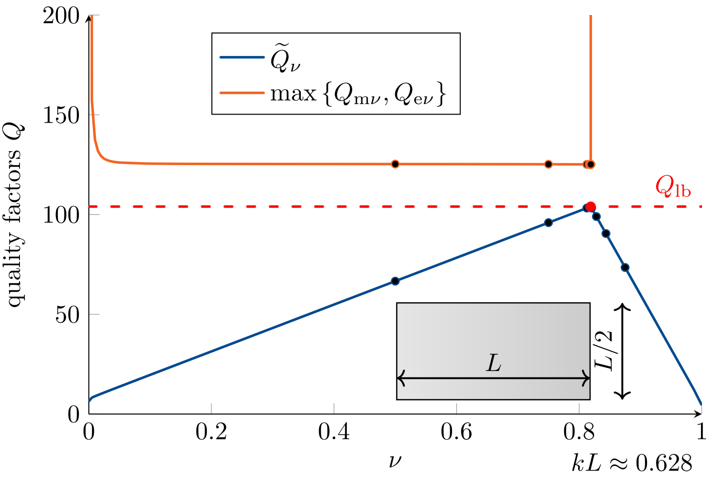


$\text{maximize}_\nu\ \widetilde{Q}_\nu$ for $0 \leq \nu \leq 1$ and verify $Q(\boldsymbol{J}_\nu) = \widetilde{Q}(\boldsymbol{J}_\nu)$ at max.



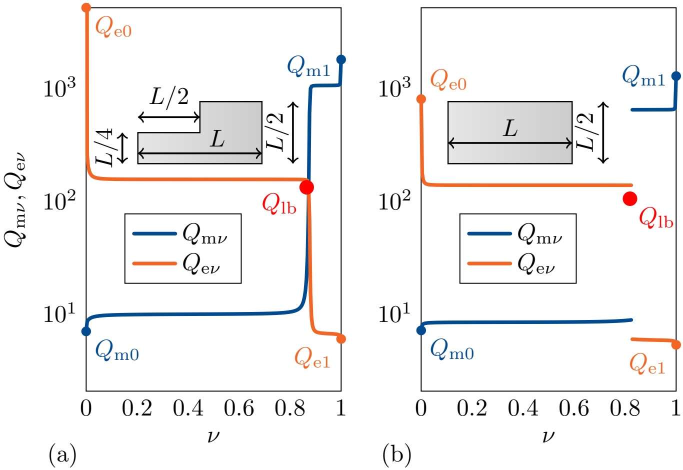
$Q_{\mathrm{e}\nu}\}$
Q_{lb}

Planar rectangle



Looks as a gap between $Q(\boldsymbol{J}_\nu)$ and $\widetilde{Q}(\boldsymbol{J}_\nu)$.

Comparison



Continuous for L-shaped and discontinuities for rectangle.

Symmetries and degenerate eigenvalues

- Discontinuities and the apparent duality gap can be explained by degenerate eigenvalues.
- Degenerate eigenvalues are related to symmetries of the object (and mesh), *cf.*, the von Neumann-Wigner theorem (Wigner and Von Neumann 1929) and its relation to crossing avoidance and group theory (Schab and Bernhard 2016).
- Any element in the degenerate eigenspace has the same $\widetilde{Q}_\nu$. Choose one that is self resonant.

For a two-dimensional eigenspace with basis vectors $\mathbf{I}_1$ and $\mathbf{I}_2$, the self-resonant eigenvector $\mathbf{I}_{\nu,\mathrm{sr}}$ is given by

$$\mathbf{I}_{\nu,\mathrm{sr}} = \mathbf{I}_1 + \chi \mathrm{e}^{\mathrm{j}\phi}\mathbf{I}_2,$$

where the real-valued coefficient χ is the solution to

$$\chi^2 \mathit{\Delta}_{22} + 2\chi\cos(\chi)\mathit{\Delta}_{12} + \mathit{\Delta}_{11} = 0, \quad \mathit{\Delta}_{mn} = \mathbf{I}_m^{\mathrm{T}}(\mathbf{X}_\mathrm{m} - \mathbf{X}_\mathrm{e})\mathbf{I}_n.$$

Some observations

For a two-dimensional eigenspace with basis vectors $\mathbf{I}_1$ and $\mathbf{I}_2$, the self-resonant eigenvector $\mathbf{I}_{\nu,\mathrm{sr}}$ is given by

$$\mathbf{I}_{\nu,\mathrm{sr}} = \mathbf{I}_1 + \chi \mathrm{e}^{\mathrm{j}\phi}\mathbf{I}_2,$$

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- The current is equiphase (real valued) for non-degenerate eiganvalues (objects without symmetries), *i.e.*, elliptically polarized (often low directivity).
- Can use an arbitrary phase combination for degenerate eigenvalues (objects with symmetries), *e.g.*, equiphase with low directivity or j (90°) shifted for higher directivity (Huygens source).

Generalizations

The approach to minimize the Q-factor is valid for problems of the form ($\widetilde{\mathbf{X}}_{\mathrm{e}} \succeq \mathbf{0}$, $\widetilde{\mathbf{X}}_{\mathrm{m}} \succeq \mathbf{0}$, $\widetilde{\mathbf{R}} \succeq \mathbf{0}$)

$$\big(\nu\widetilde{\mathbf{X}}_{\mathrm{e}} + (1-\nu)\widetilde{\mathbf{X}}_{\mathrm{m}}\big)\mathbf{I}_{\nu} = \Upsilon_{\nu}\widetilde{\mathbf{R}}\mathbf{I}_{\nu}.$$

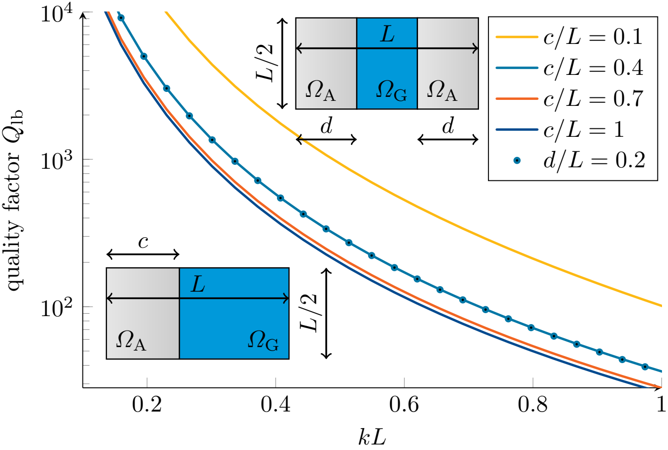
together with affine constraints $\mathbf{A}\mathbf{I} = \mathbf{B}$. Many possibilities

- Subdomain region

$$\begin{aligned}\mathbf{Z}_{\mathrm{AA}}\mathbf{I}_{\mathrm{A}} + \mathbf{Z}_{\mathrm{AG}}\mathbf{I}_{\mathrm{G}} &= \mathbf{V},\\ \mathbf{Z}_{\mathrm{GA}}\mathbf{I}_{\mathrm{A}} + \mathbf{Z}_{\mathrm{GG}}\mathbf{I}_{\mathrm{G}} &= \mathbf{0},\end{aligned}$$

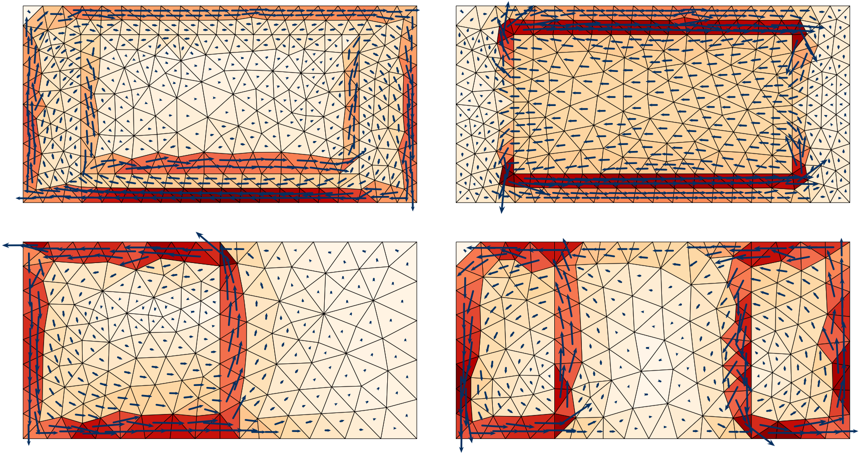
- Gain Q-factor quotient (G/Q)
- Radiation in specific regions.
- See Capek, Gustafsson, and Schab 2016 for additional examples.

Sub region with controllable currents

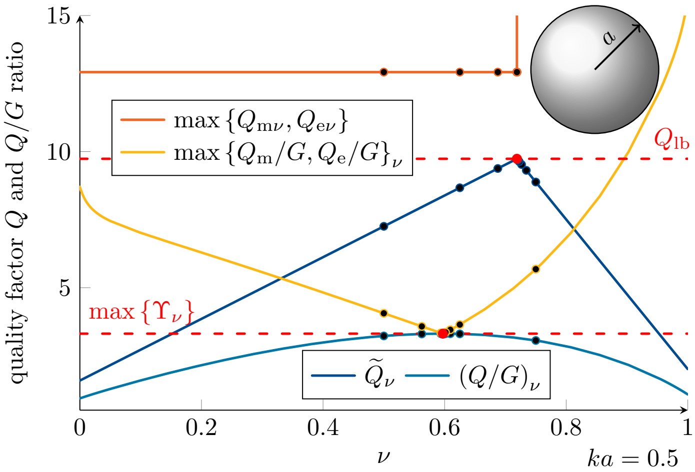


Controllable currents in Ω_A and induced currents on Ω_G (PEC).

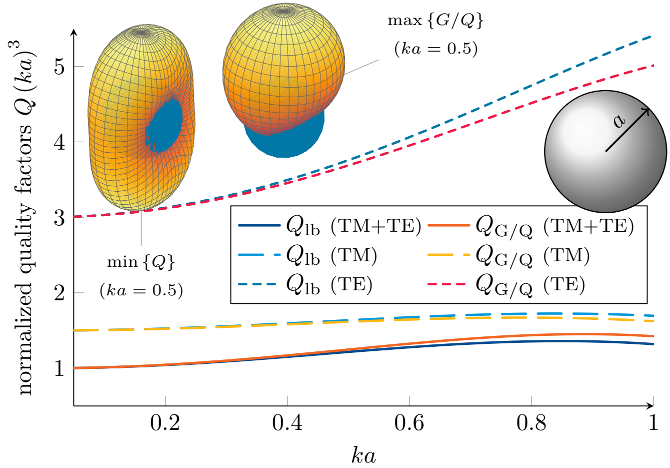
Corresponding currents



Q and G/Q for a spherical region



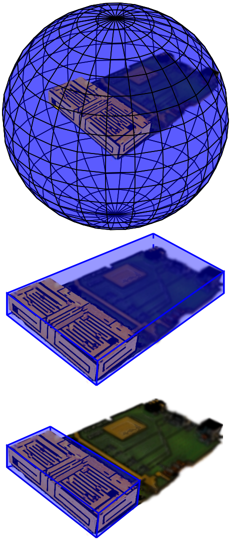
Q and G/Q for a spherical region



Q_{lb} and Q-factors from G/Q are similar. Degenerate eigenvalues (symmetries) Q_{lb} same for the Huygens source (G/Q) as for the equiphase current.

Summary

- Physical bounds on the Q-factor.
- Generalized eigenvalue problems.
- Easy to implement and computationally efficient.
- Equiphase currents.
- Symmetries and degenerate eigenvalues.
- Sub regions, G/Q, ...
- Closed form solutions for small antennas.



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Slides: `http://www.eit.lth.se/staff/mats.gustafsson`

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