Stored electromagnetic energy

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Stored EM energy (definitions and interpretations)

Stored electric W_{e} and magnetic W_{m} energies are instrumental in the analysis of small antennas. The stored energy is used to estimate the bandwidth [7, 22], determine physical bounds [4, 12, 21], antenna optimization [5]. Unfortunately, the stored energy is not uniquely defined and there are many different proposals in the literature [3, 10, 17, 22]:

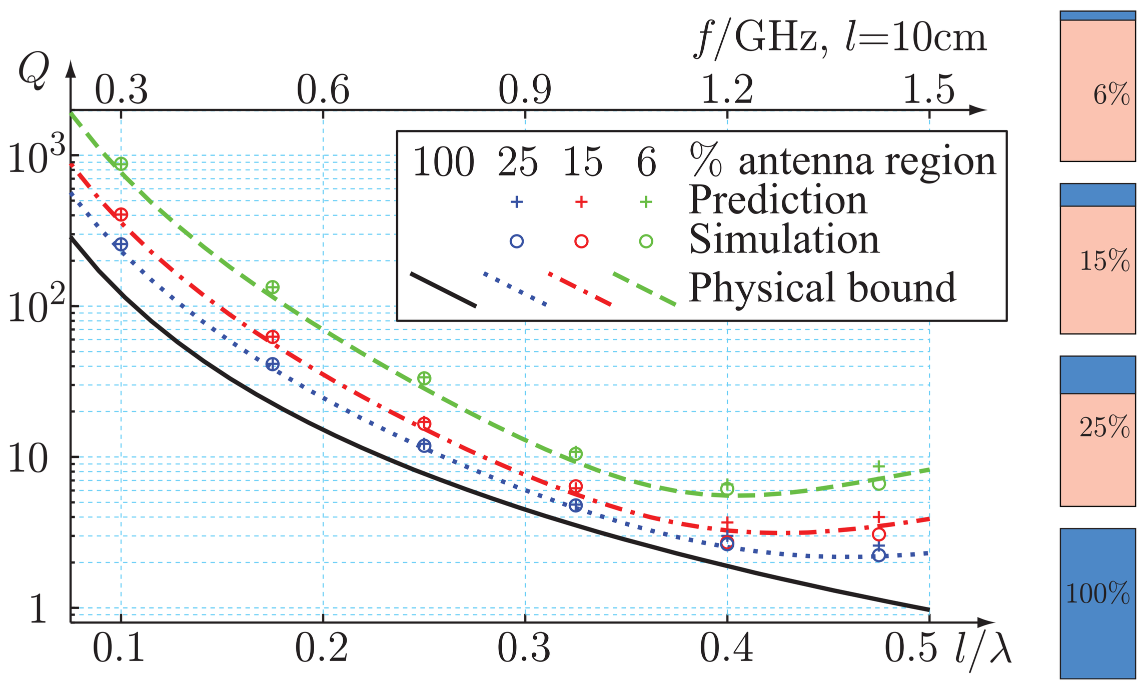
Fields with the difference between the energy density and the energy density of the radiated field as proposed by Collin & Rothschild 1964 [6, 7, 20, 22], see also [16]. The main problem is to define and evaluate the energy density of the radiated field and to perform the integration over $\mathbb{R}^3$.

Currents suggested by Harrington [15] based on MoM matrices, Geyi [8] for $ka \ll 1$, and Vandenbosch [18]. More general as a state-space representations [14]. Useful for antenna current optimization [12] and an intuitive interpretation of the stored energy (in the states). Problems with the time (phase) delay.

Input impedance and circuit models by Chu [4] for spherical modes and in general [10]. Also local approximations using differentiation [2, 13, 22]. Well defined stored energy for a rational input impedance but needs all frequencies. Differentiation is a good approximation for single resonances.

Physical bounds and optimization using stored energy expressed in the current density

Lower bounds on Q for a planar PEC rectangular plate with length $\ell = 10\,\mathrm{cm}$ and width $\ell/2 = 5\,\mathrm{cm}$, where the antenna region is constrained to 100, 25, 15, 6% of the rectangle [5, 12]. Optimized antennas using single frequency optimization [5].



Convex optimization for the currents and GA for the antennas.

The results indicate that the physical bounds are tight.

Stored energy and Q-factor expressions based on fields, currents, and input impedance

Subtraction of the far field (FF)

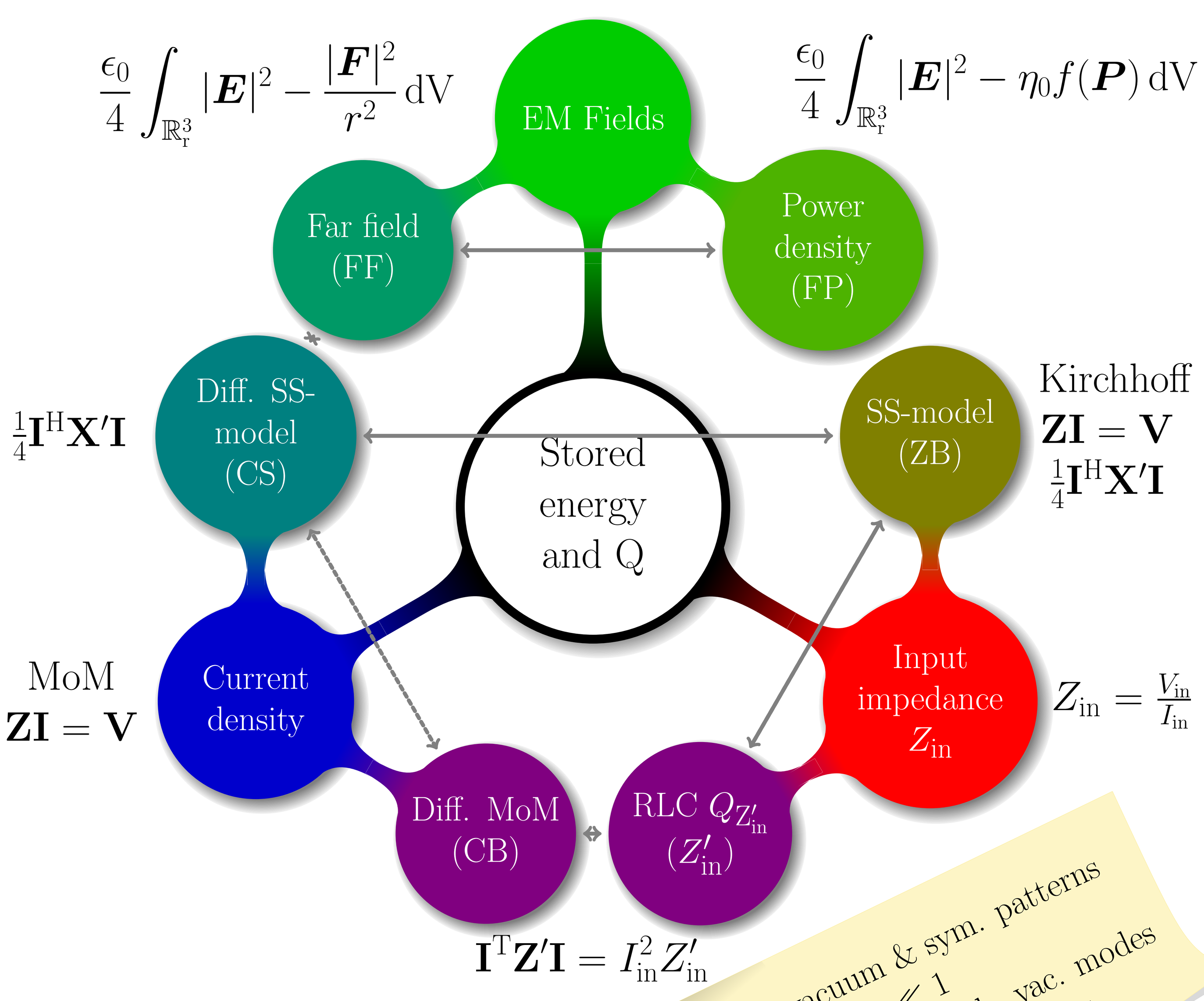
Subtraction of the energy density corresponding to the far field is common [7, 8, 22]

- Can be coordinate dependent [11, 22].
- Can be negative for large structures [10].
- Hard to generalize to lossy and dielectric media.
- Also expressed in the input reactance and far field.

Current density and state-space models

Differentiated EFIE MoM impedance matrix (vacuum) and state-space models (dispersive media)

- Sesquilinear form in $\mathbf{I}$ for the stored energy (CS).
  - Can be negative for large structures [10].
  - Useful for antenna current optimization [5, 12].
  - Vacuum case [15, 18] is identical to the subtracted far field (FF) for coordinate independent cases [10].
- Bilinear form in $\mathbf{I}$ (analytic in s) for Z_{in}' (CB).
  - Indefinite and hence $\min Q_{Z_{\mathrm{in}}'} = 0$ [13].
  - $Q_{Z_{\mathrm{in}}'} \leq Q$ for small antennas [14].



FF=CS, vacuum & sym. patterns
CS≈ ZB for $ka \ll 1$
$Q_{\mathrm{FF}} = Q_{\mathrm{FPa}} - ka$, sph. vac. modes
CS=CB for real-valued states

Subtraction of power density (FP)

Subtraction of the energy related to the power density (Poynting vector) [6, 11, 16]. Use $\boldsymbol{P} = \mathrm{Re}\{\boldsymbol{E} \times \boldsymbol{H}^*\}$

- $f(\boldsymbol{P}) = \hat{\boldsymbol{r}}\cdot\boldsymbol{P}$ coordinate dependent. Q differ with ka from the subtracted far field for spherical modes [11].
- $f(\boldsymbol{P}) = |\boldsymbol{P}|$ hard to evaluate but can be generalized to lossy and dielectric media.

Input impedance (ZB, Z_{in}')

- Circuit synthesis (Brune) or state-space model (ZB)
  - Rational PR input impedance (approximation).
  - Energy stored in the constructed states [19] (or lumped circuit elements [1]).
- Differentiated input impedance [22] (Z_{in}')
  - Easy to evaluate.
  - Padé approximation with a resonance model [13].
  - Inversely proportionality to the fractional bandwidth (*) as $\varGamma_0 \to 0$ if $Q_{Z_{\mathrm{in}}'} > 0$.

Q-factor Q and fractional bandwidth B

The Q-factor is defined by the stored energy and inversely proportional to B, *i.e.*,

$$Q = \frac{2\omega \max\{W_{\mathrm{e}}, W_{\mathrm{m}}\}}{P_{\mathrm{d}}}, \quad B \approx \frac{2}{Q}\frac{\varGamma_0}{\sqrt{1-\varGamma_0^2}} \text{ single self-resonance } Q \approx Q_{Z_{\mathrm{in}}'} = \frac{\omega|Z_{\mathrm{in}}'|}{2R_{\mathrm{in}}} \qquad (*)$$

Stored energy and state-space models

The EFIE impedance matrix is

$$\mathbf{Z} = s\mu_{\mathrm{r}}\mathbf{L} + \frac{1}{s\epsilon_{\mathrm{r}}}\mathbf{C}_{\mathrm{i}} \quad \text{and} \quad \mathbf{ZI} = (s\mu_{\mathrm{r}}\mathbf{L} + \frac{1}{s\epsilon_{\mathrm{r}}}\mathbf{C}_{\mathrm{i}})\mathbf{I} = \mathbf{V}v_{\mathrm{in}}$$

where the matrices $\mathbf{L}$ and $\mathbf{C}_{\mathrm{i}}$ depend on the frequency $s = \mathrm{j}\omega$ and

$$L_{mn} = \int_V\!\!\int_V \boldsymbol{\psi}_{m1}\cdot\boldsymbol{\psi}_{n2}\frac{\mathrm{e}^{-snc_0R_{12}}}{4\pi R_{12}}\,\mathrm{dV}_1\,\mathrm{dV}_2, \quad C_{imn} = \int_V\!\!\int_V \nabla_1\cdot\boldsymbol{\psi}_{m1}\nabla_2\cdot\boldsymbol{\psi}_{n2}\frac{\mathrm{e}^{-snc_0R_{12}}}{4\pi R_{12}}\,\mathrm{dV}_1\,\mathrm{dV}_2,$$

The Lorentz model

$$\epsilon_{\mathrm{r}}(s) = \epsilon_\infty + \frac{\alpha^2}{\beta^2 + \gamma s + \delta s^2}$$

gives the state-space representation [14]

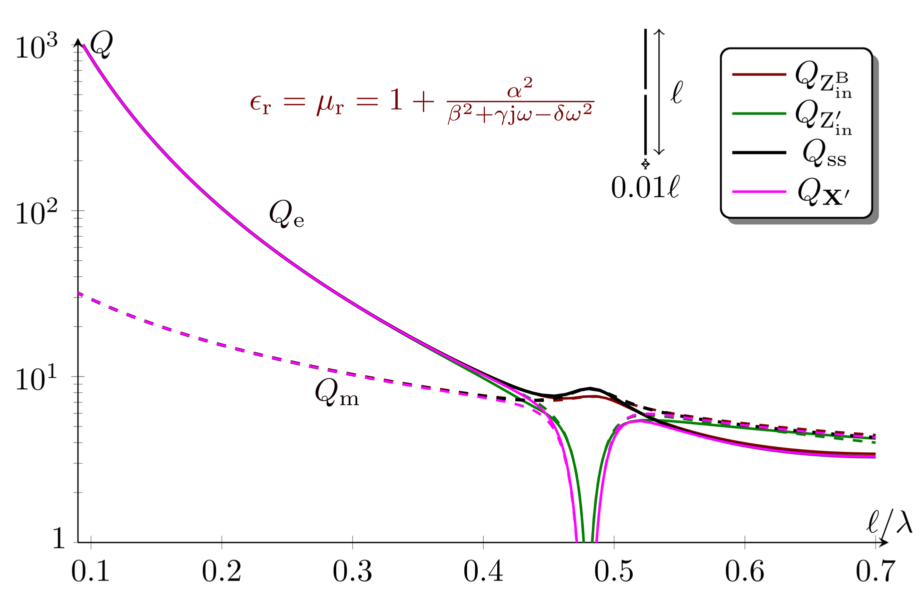
$$s\begin{pmatrix} \mu_{\mathrm{r}}\mathbf{L} & 0 & 0 & 0\\ 0 & \epsilon_\infty\mathbf{C} & 0 & 0\\ 0 & 0 & \mathbf{C}_{\mathrm{i}} & 0\\ 0 & 0 & 0 & \beta^2\delta\mathbf{C}_{\mathrm{i}} \end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\\ \mathbf{P}\\ \dot{\mathbf{P}}\end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0\\ 1 & 0 & 0 & -\alpha\mathbf{1}\\ 0 & 0 & 0 & \beta^2\mathbf{C}_{\mathrm{i}}\\ 0 & \alpha\mathbf{1} & -\beta^2\mathbf{C}_{\mathrm{i}} & -\gamma\beta^2\mathbf{C}_{\mathrm{i}} \end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\\ \mathbf{P}\\ \dot{\mathbf{P}}\end{pmatrix} + \begin{pmatrix}\mathbf{V}\\ 0\\ 0\\ 0\end{pmatrix}v_{\mathrm{in}}$$

with $i_{\mathrm{in}} = \mathbf{YI}$ and $y_{\mathrm{in}} = \mathbf{YI}/v_{\mathrm{in}}$. Reciprocal system (with internal symmetry $\mathrm{diag}(1,-1,-1,1)$ if $\mathbf{V} = \mathbf{Y}^{\mathrm{T}}$, see [19].

Strip dipole in ϵ, μ Lorentz media

Stored energy expressions are compared for a strip dipole in an electric and magnetic Lorentz medium [14]. The resulting Q-factors are depicted.

- State-space results Q_{ss} (currents and polarizations) agree with the circuit synthesized values[14].
- The results from the differentiated MoM matrices vanish at the resonance wavelength $\ell \approx 0.48\lambda$.



Also physical bounds using current optimization with near-field constraints for antennas in lossy and temporally dispersive media [9].

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