

Nanoscale MOSFETs – Equations

Planck's Constant	$h = 6.602 \times 10^{-34} \text{ Js}; \hbar = h/2\pi$	
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$	
Electron Charge	$q = 1.602 \times 10^{-19} \text{ C}$	
Electron Mass	$m_0 = 9.11 \times 10^{-31} \text{ kg}$	
Boltzmann's Constant	$k = 1.38 \times 10^{-23} \text{ J/K}$	
1D DOS / Effective DOS	$D_{1D} = \frac{\sqrt{2m^*}}{\pi\hbar} \frac{1}{\sqrt{E - E_C}}$	$N_{1D} = \sqrt{\frac{2m^*kT}{\pi}} \frac{1}{\hbar}$
2D DOS / Effective DOS	$D_{2D} = \frac{m^*}{\pi\hbar^2}$	$N_{2D} = \frac{m^*kT}{\pi\hbar^2}$
Thermal Velocity	$v_T = \sqrt{\frac{2kT}{\pi m^*}}$	

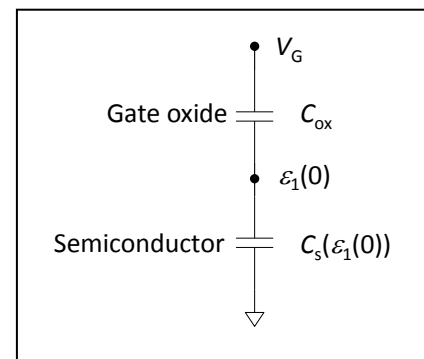
Fermi-Dirac Integrals: $F_j(\eta_F) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{\eta^j}{1+e^{\eta-\eta_F}} d\eta$

Normalized Fermi Energy:	$\eta = \frac{E}{kT}, \eta_F = (E_F - \epsilon(0))/kT$
Normalized Drain Energy	$U_D = \frac{qV_D}{kT}$
Non-degenerate	$F_j(\eta_F) \rightarrow e^{\eta_F}$
Degenerate	$F_j(\eta_F) \rightarrow \frac{\eta_F^{j+1}}{\Gamma(j+2)}$
Derivative	$\frac{dF_j(\eta_F)}{d\eta_F} = F_{j-1}(\eta_F)$

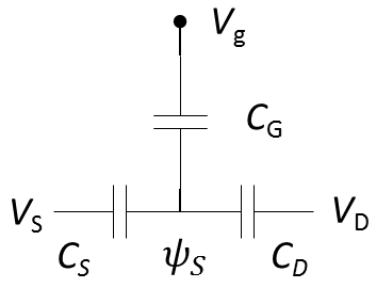
x	0.5	1	1.5	2	2.5	3
$\Gamma(x)$	$\sqrt{\pi}$	1	$\sqrt{\pi}/2$	1	$3\sqrt{\pi}/4$	2

1D Electrostatics:

Bias self-consistency	Threshold Voltage
$\epsilon_1(0) = -qV_G + \frac{q^2n}{C_{ox}}$	$V_T = -\frac{E_{FS}}{q}$
2D Degenerate Quantum Capacitance	
$C_q = \frac{q^2m^*}{\pi\hbar^2}$	



3D Electrostatics



2D Ballistic FET Expressions

	General	Degenerate
Sheet-Charge	$n^+ = \frac{N_{2D}}{2} F_0(\eta_F)$	$n^+ \approx \frac{m}{\pi \hbar^2} [E_{FS} - \varepsilon(0)]$
Current	$I^+ = \frac{qN_{2D}}{2} v_T F_{1/2}(\eta_F)$	$I^+ \approx \frac{qW\sqrt{8m^*}}{3\pi^2 \hbar^2} [E_{FS} - \varepsilon(0)]^{1.5}$

For I : $\eta_F \rightarrow \eta_F - U_D$

1D Ballistic FET

Line Charge	$n_L = \frac{N_{1D}}{2} \left[F_{-\frac{1}{2}}(\eta_F) + F_{-\frac{1}{2}}(\eta_F - U_D) \right]$
Current	$I = \frac{2qkT}{h} [(F_0(\eta_F) + F_0(\eta_F - U_D))]$

General FET Model

Level Broadening	$\gamma = \frac{\hbar}{\tau}$
Current	$I = \frac{2q}{h} \int dE T(E) [f_1(E) - f_2(E)]$
Transmission	$T(E) = \frac{\pi\gamma_1\gamma_2}{\gamma_1 + \gamma_2} D(E)$

Scattering:

Transmission	$T = \frac{\lambda_0}{L + \lambda_0}$
Linear and QCL Current	$I_{scatt} = T \times I_{Ballistic}$
Mean free path/mobility relationship	$\frac{\lambda_0 v_T}{2} = \left(\frac{kT}{q}\right) \mu_{eff}$