Detection Theory

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Quick facts

- **Textbook**
- **Lectures**
  - Given by students
  - Preparation assisted by seniors
  - **Tuesdays at 13.15 in E:2349 (Köket)**
- **Exercise classes**
  - Students present solutions
  - **Mondays at 15.15 in E:2349 (Köket)**
- **Examination requirements**
  - Giving one lecture
  - 80% of lecture attendance
  - 80% of exercise class attendance
  - Solving a small set of examination problems
- **Course web pages**
  - http://www.eit.lth.se/course/PHD009

Course start: **October 19**
Course end: **January 24**

9 ECTS

Lecture schedule

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Preparing lectures

- A PowerPoint template is available (can be downloaded from course web page)
- A few tips:
  - Concentrate on ideas, methodology and principles
  - Avoid excessive mathematical detail
  - Use graphical illustrations, whenever possible (be careful!)
  - Highlight main points
  - Use application examples to demonstrate main concepts, whenever possible
  - Provide external references, if appropriate
- Don’t forget to select exercises for the next exercise class
- Any slides should be available the day before the lecture (seniors put them on the course web page)
1. Introduction

2. Summary of Important PDFs

3. Statistical Decision Theory I

4. Deterministic Signals

5. Random Signals

6. Statistical Decision Theory II

7. Deterministic Signals with Unknown Parameters

8. Random Signals with Unknown Parameters

9. Unknown Noise Parameters

10. NonGaussian Noise
Textbook/course overview

11. Summary of Detectors
   Detection Approaches. Linear Model. Choosing a Detector. Other Approaches and Other Texts.

12. Model Change Detection

13. Complex/Vector Extensions, and Array Processing

What is the difference between Detection and Estimation?

Detection:
Discrete set of hypotheses.
One cares whether the decision is **right or wrong**

Estimation:
Infinite, or at least large, set of hypotheses.
The decision is almost always wrong – make **error as small as possible**.

Additional Material

Throughout the textbook a discrete time representation is assumed.

At the end we will give a lecture that treats the transition from continuous time to discrete time.

Material will be provided later.

Example

**Detection problem for the warship:**
To figure out whether there is an enemy submarine present or not (binary decision)

**Estimation problem for the warship:**
To find the location of the submarine (continuous decision)
Detection or estimation?

Finding out ...

• ... if an intruder is present (burglar alarm)
• ... if a car is speeding on a 90 km/h road (speed camera)
• ... the expected number of tanks in the enemy’s army, by observing their “serial” numbers (numbered from 1 to #of tanks)
• ... if the enemy has 0-9, 10-99, 100-999, or more than 1000 tanks (under the same conditions as above)

Different approaches

• Neyman-Pearson
  For a fixed probability of false alarm, find the decision rule that gives the maximal probability of detection.

• Bayesian
  Given a Bayes Risk (an expected “cost”)

\[ R = E(C) = \sum_{i,j} C_{i,j} P(H_i | H_j) P(H_j) \]

find the decision rule that gives the minimal R.

Signal Detection – the most basic example

Detection of binary signal 0/1 in additive Gaussian Noise.

It is inevitable that some mistakes will be made
Receiver Operating Characteristics (ROC)

The operating point depends on the application.

Operating Region of the Detection

The operating point depends on the application.
Neyman-Pearson: Maximize Prob(Hit) for fixed Prob(False alarm)

Computer detection of objects, not covered in this course

Increasing popularity in computer science etc. Falls within detection theory. Machines should identify certain objects from pictures (airport security, industrial applications etc)

Communication theory

And of course......a digital communication example.
Transmitted signal is $S(t)$, received is $Y(t)$, noise is $N(t)$

Hypothesis testing

$Y(t) = N(t)$ \hspace{1cm} H$_0$
$Y(t) = S(t) + N(t)$ \hspace{1cm} H$_1$

If $S(t)$ is a known signal
  -> Matched filter receiver
If $S(t)$ is a random signal
  -> Estimator-Correlator receiver
What if $N(t)$ is not Gaussian?
Detection of rare events is hard!

Suppose that a medical company devices a test for a rare disease. The test identifies the disease with 99% probability and gives false alarm with 1% probability, i.e.

\[
\begin{align*}
\text{Prob (positive test | sick person)} &= .99 \\
\text{Prob (negative test | healthy person)} &= .99
\end{align*}
\]

It appears that satisfactory detection of the disease is provided by the new test.

But is it really so….? 

Detection of rare events is hard!

Exercises

• The exercises for this lecture are only recommended as a “warm up” (no exercise class on Monday)
  • Chapter 1
    – 1.2 Detector performance, simple example
    – 1.3 Heuristic design of a test
    – 1.6 & 1.7 Calculation of deflection coefficient
  • Chapter 2
    – 2.2 Approximation of Q(x)
    – 2.4-2.6 Quadratic forms of random variables
    – 2.7 Bounds on eigenvalues of autocorrelation matrices
    – 2.9 Simple example regarding WSS processes
    – 2.10-2.11 DFTs and asymptotic eigenvectors of autocorrelation matrices
    – 2.12-2.13 Eigendecompositions of autocorrelation matrices

Apply Bayes’ rule and compute the posterior probability that a person is healthy, even though the test is positive (’+’)

\[
\text{Prob (healthy | +)} = \frac{\text{Prob (+ | healthy) } \times \text{Prob (healthy)}}{\text{Prob (+ | sick) } \times \text{Prob (sick)} + \text{Prob (+ | healthy) } \times \text{Prob (healthy)}}
\]

\[
= \frac{.99 \times .005}{.99 \times .005 + .01 \times .995}
\]

\[= 0.6678, \text{ Two thirds of all that test as sick are healthy!} \]