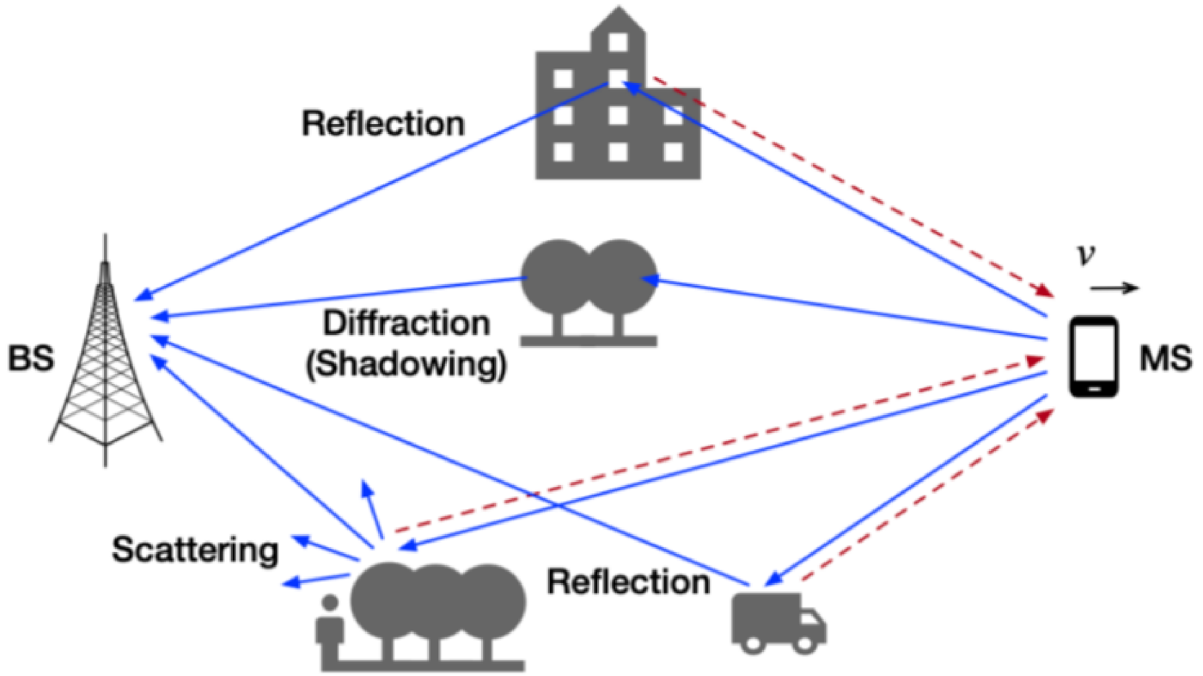
RIMAX Introduction

Junshi Chen

2019.09.30

Channel model



Reflector: dominating power, less amount, resolvable,
Scatter: weaker power, larger amount, not resolvable, and not ignorable

PDP of received signal vs PDP of DMC

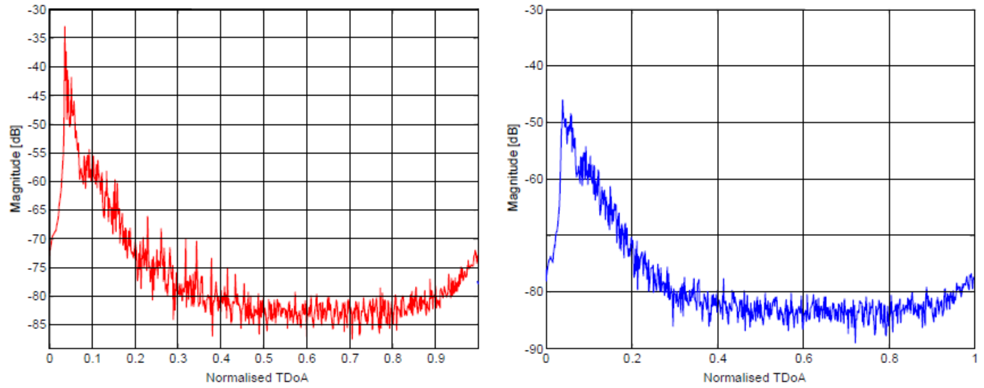


Figure 2-7: PDP of a complex SIMO-impulse response (LOS) averaged over all receive antennas (left) and the same impulse response after removing the concentrated propagation paths (right).

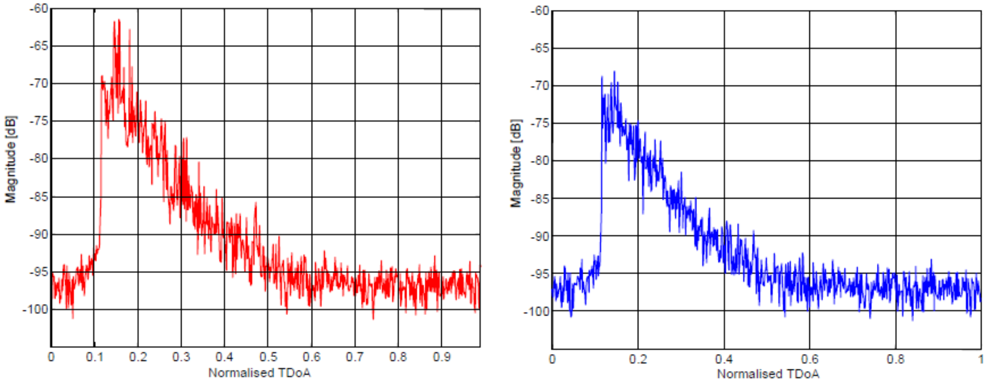


Figure 2-8: PDP of a SIMO-impulse response (NLOS) averaged over all receive antennas (left) and the same impulse response after removing the concentrated propagation paths (right).

What is RIMAX?

A Flexible Algorithm for Channel Parameter Estimation from Channel Sounding Measurements.

Please refer to the dissertation:

Andreas. Richter, "Estimation of radio channel parameters: Models and algorithms", Technische Universität, 2005, ISBN 3-938843-02-0.

Multipath Components (MPC) Introduction

Propagation path

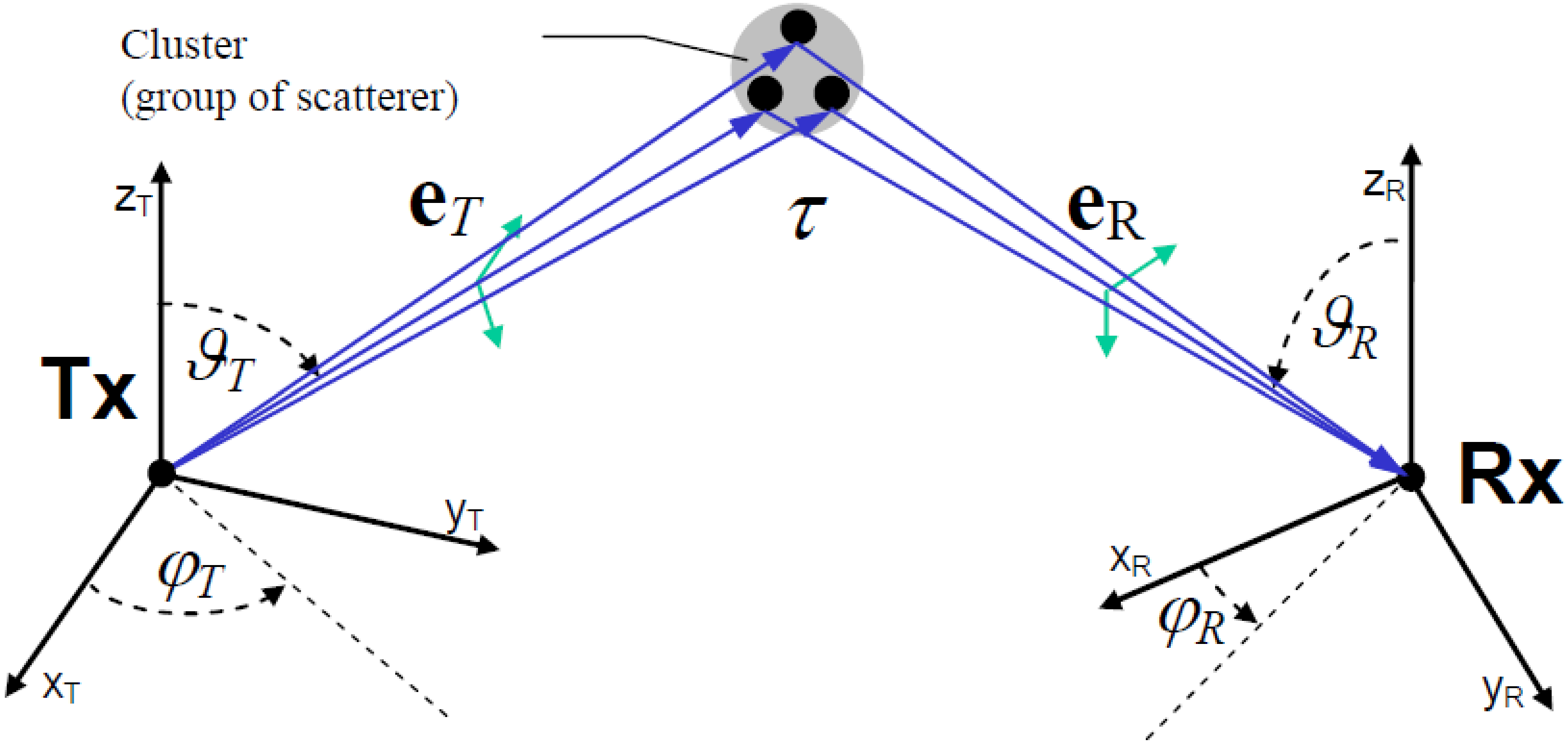


Figure 2-3: Definition of a propagation path

SISO signal and channel model in time domain with omni-directional antennas at Tx and Rx

$$y(t) = \gamma_p \cdot g_R(t) *_t g_T(t) *_t x\left(t - \frac{l_p}{C_0}\right) \cdot e^{-\frac{j2\pi f_c l_p}{C_0}}$$

$x(t)$: transmitted signal;

$*_t$: time domain convolution;

$g_R(t)$: receiver impulse response;

$g_T(t)$: transmitter impulse response;

f_c: carrier frequency;

C_0: speed of light;

l_p: propagation length;

γ_p: all effects which are frequency independent, e.g. free space loss, complex antenna gain, loss on scattering or reflection, etc.

SISO signal and channel model in frequency domain

$$Y(f) = X(f) \cdot \gamma_p \cdot G_{T_f}(f) \cdot G_{R_f}(f) \cdot e^{-j2\pi f \tau_p} \cdot e^{-\frac{j2\pi f_c l_p}{C_0}}$$

Where $\tau_p = \frac{l_p}{C_0}$

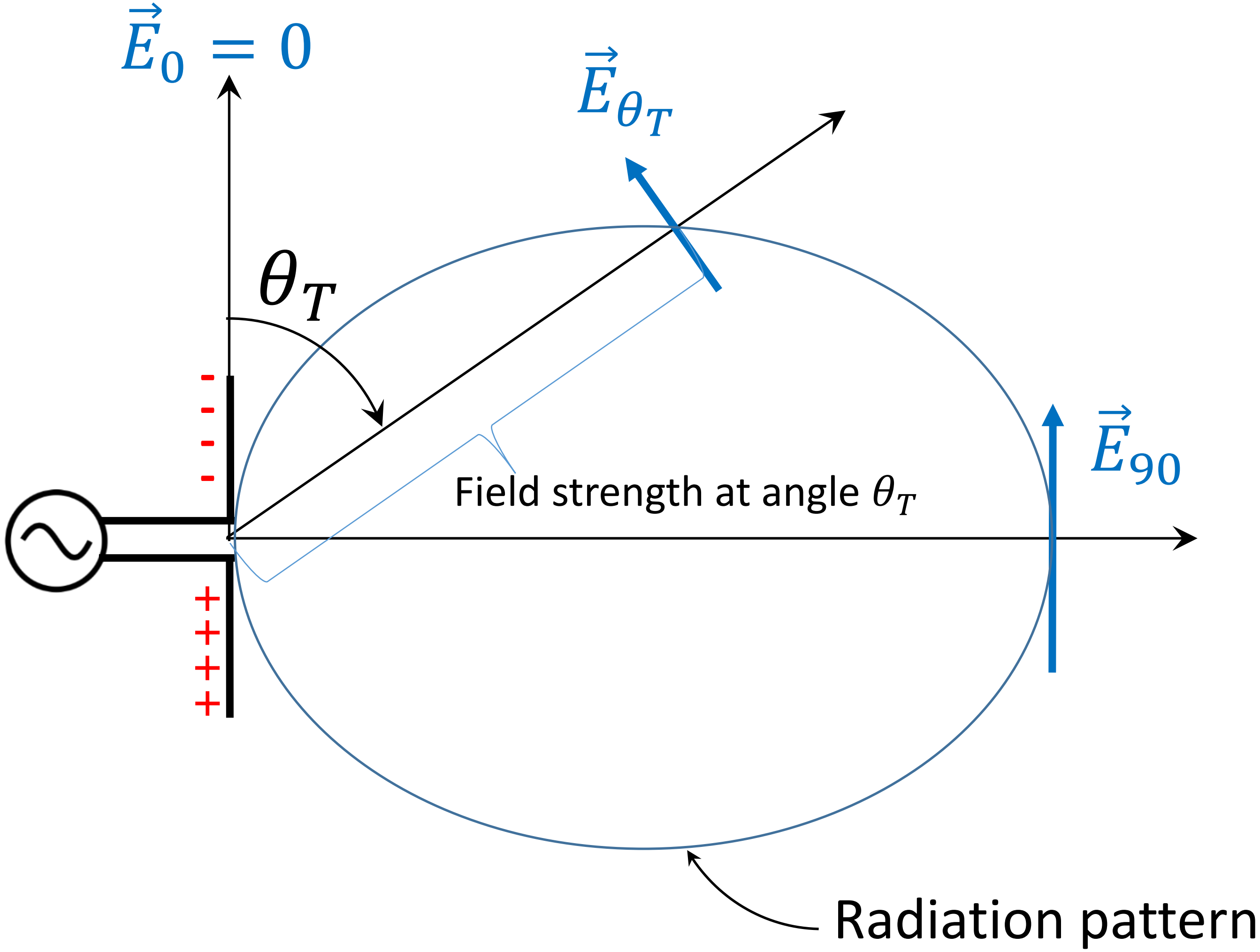
Time invariant frequency response:

$$H(f) = \gamma_p \cdot G_{T_f}(f) \cdot G_{R_f}(f) \cdot e^{-j2\pi f \tau_p} \cdot e^{-\frac{j2\pi f_c l_p}{C_0}}$$

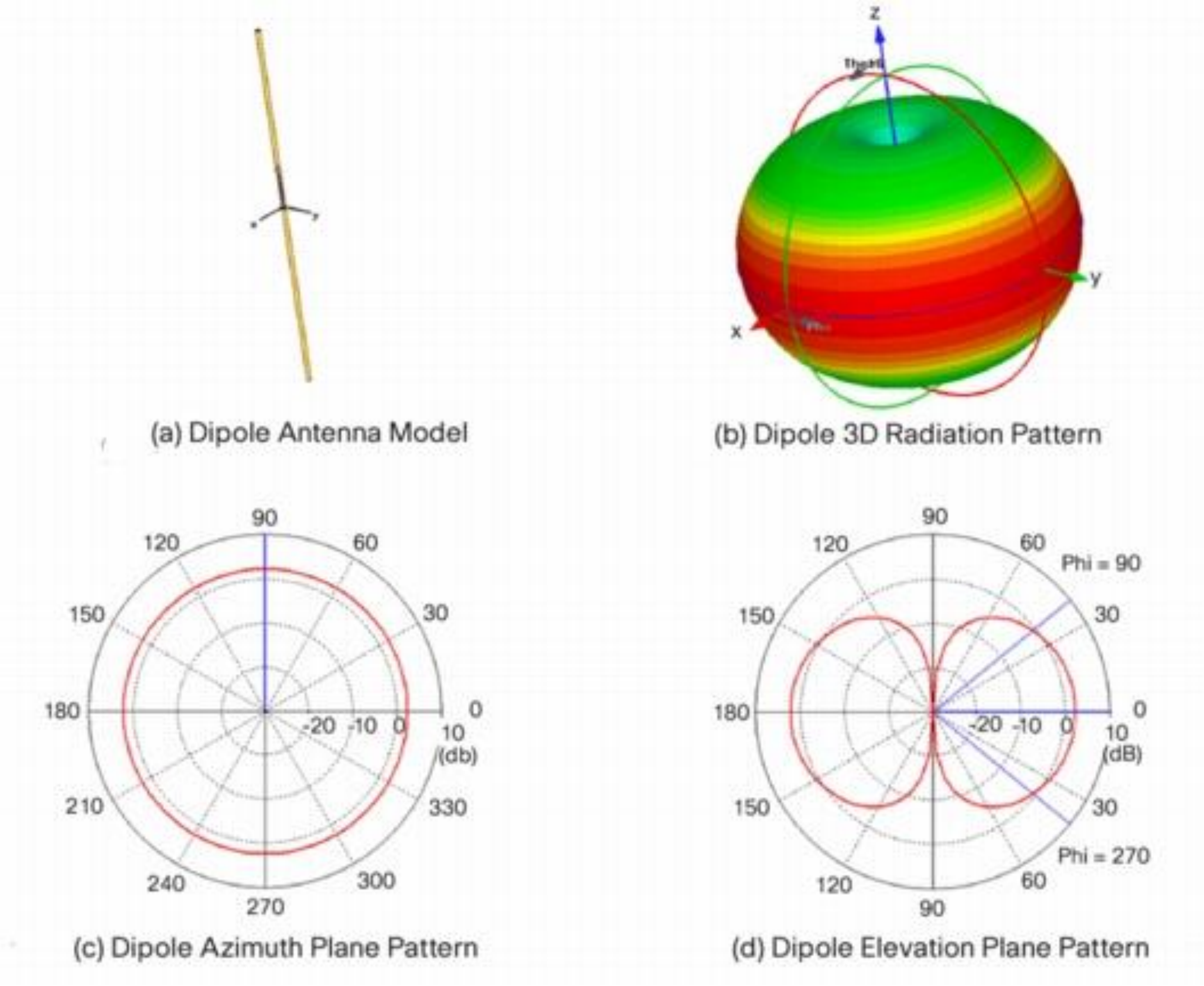
Time variant frequency response:

$$H(f,t) = \gamma_p \cdot G_{T_f}(f) \cdot G_{R_f}(f) \cdot e^{-j2\pi f \tau_p} \cdot e^{-\frac{j2\pi\,(l_p + v_p t)}{\lambda_c}}$$

Antenna Polarization



Antenna Radiation Pattern



(a) Dipole Antenna Model
(b) Dipole 3D Radiation Pattern
(c) Dipole Azimuth Plane Pattern
(d) Dipole Elevation Plane Pattern

[Link to Cisco site]

SISO signal and channel model in frequency domain with polarization

$$H(f,\varphi_T,\vartheta_T,\varphi_R,\vartheta_R,t)$$
$$= G_{R_f}(f)\cdot \boldsymbol{b}_R(\varphi_R,\vartheta_R)\cdot \boldsymbol{\Gamma}_p \cdot \boldsymbol{b}_T(\varphi_T,\vartheta_T)^T \cdot G_{T_f}(f)\cdot e^{-j2\pi f\tau_p}\cdot e^{-j2\pi\alpha_p t}$$

Where

$$b_T(\varphi_T,\vartheta_T,f) = \left[b_{T_H}(\varphi_T,\vartheta_T,f)\ \mathrm{b_{T_V}}(\varphi_T,\vartheta_T,f)\right] \in C^{1\times 2}$$
$$b_R(\varphi_R,\vartheta_R,f) = \left[b_{R_H}(\varphi_R,\vartheta_R,f)\ \mathrm{b_{R_V}}(\varphi_R,\vartheta_R,f)\right] \in C^{1\times 2}$$

$$\boldsymbol{\Gamma}_p = \begin{bmatrix}\gamma_{HH,p} & \gamma_{VH,p}\\ \gamma_{HV,p} & \gamma_{VV,p}\end{bmatrix} \in C^{2\times 2}$$

Extension to multipath and MIMO system

Extended to multipath channel, P is the multipath number.

$$H(f, \varphi_T, \vartheta_T, \varphi_R, \vartheta_R, t)$$
$$= G_{R_f}(f) \cdot G_{T_f}(f) \cdot \sum_{p=1}^{P} \left\{ \boldsymbol{b}_R\left(\varphi_{R,p}, \vartheta_{R,p}\right) \cdot \boldsymbol{\Gamma}_p \cdot \boldsymbol{b}_T(\varphi_T, \vartheta_T)^T \cdot e^{-j2\pi f \tau_p} \cdot e^{-j2\pi \alpha_p t} \right\} \in C^{1\times 1}$$

Extended to MIMO case:

$$\boldsymbol{H}(f, \varphi_T, \vartheta_T, \varphi_R, \vartheta_R, t)$$
$$= G_{R_f}(f) \cdot G_{T_f}(f) \cdot \sum_{p=1}^{P} \left\{ \boldsymbol{B}_R\left(\varphi_{R,p}, \vartheta_{R,p}\right) \cdot \boldsymbol{\Gamma}_p \cdot \boldsymbol{B}_T(\varphi_T, \vartheta_T)^T \cdot e^{-j2\pi f \tau_p} \cdot e^{-j2\pi \alpha_p t} \right\} \in C^{M_R \times M_T}$$

$$\boldsymbol{B}_R\left(\varphi_{R,p}, \vartheta_{R,p}\right) = \begin{bmatrix} \boldsymbol{b}_{R,1}\left(\varphi_{R,p}, \vartheta_{R,p}\right) \\ \vdots \\ \boldsymbol{b}_{R,M_R}\left(\varphi_{R,p}, \vartheta_{R,p}\right) \end{bmatrix} \quad \mathrm{R}^2 \rightarrow C^{M_R \times 2}$$

$$\boldsymbol{B}_{\boldsymbol{T}}\left(\varphi_{R,p}, \vartheta_{R,p}\right) = \begin{bmatrix} \boldsymbol{b}_{R,1}\left(\varphi_{R,p}, \vartheta_{R,p}\right) \\ \vdots \\ \boldsymbol{b}_{R,M_T}\left(\varphi_{R,p}, \vartheta_{R,p}\right) \end{bmatrix} \quad \mathrm{R}^2 \rightarrow C^{M_T \times 2}$$

Frequency domain sample

$$H_f(t) = \begin{bmatrix} H(-\frac{M_f - 1}{2} f_0, t) & 0 & & \cdots & 0 \\ 0 & & \ddots & \ddots & \vdots \\ & & & \ddots & 0 \\ \vdots & \ddots & & & \\ 0 & \cdots & & 0 & H(+\frac{M_f - 1}{2} f_0, t) \end{bmatrix}$$

$$\in C^{M_f M_R \times M_f M_T}$$

Representation in matrix form (SISO)

$$\boldsymbol{b}_R(\varphi_R,\vartheta_R)\cdot\boldsymbol{\Gamma}_p\cdot\boldsymbol{b}_T(\varphi_T,\vartheta_T)^T\cdot e^{-j2\pi f\tau_p}$$

$$=\left[b_{R_H}(\varphi_R,\vartheta_R,f)\ \mathrm{b}_{\mathrm{R_V}}(\varphi_R,\vartheta_R,f)\right]\cdot\begin{bmatrix}\gamma_{HH,p} & \gamma_{VH,p}\\ \gamma_{HV,p} & \gamma_{VV,p}\end{bmatrix}\cdot\begin{bmatrix}b_{T_H}(\varphi_T,\vartheta_T,f)\\ \mathrm{b}_{\mathrm{T_V}}(\varphi_T,\vartheta_T,f)\end{bmatrix}$$

$$= b_{R_H}(\varphi_R,\vartheta_R,f)\cdot\gamma_{HH,p}\cdot b_{T_H}(\varphi_T,\vartheta_T,f)$$

$$+b_{R_H}(\varphi_R,\vartheta_R,f)\cdot\gamma_{VH,p}\cdot b_{T_V}(\varphi_T,\vartheta_T,f)$$

$$+b_{R_V}(\varphi_R,\vartheta_R,f)\cdot\gamma_{HV,p}\cdot b_{T_H}(\varphi_T,\vartheta_T,f)$$

$$+b_{R_V}(\varphi_R,\vartheta_R,f)\cdot\gamma_{VV,p}\cdot b_{T_V}(\varphi_T,\vartheta_T,f)$$

Representation in matrix form (multipath)

$$\sum_{p=1}^{P}\left\{\boldsymbol{b}_R\left(\varphi_{R,p},\vartheta_{R,p}\right)\cdot\boldsymbol{\Gamma}_p\cdot\boldsymbol{b}_T(\varphi_T,\vartheta_T)^T\cdot e^{-j2\pi f\tau_p}\right\}=$$

$$\left[b_{R_H}\left(\varphi_{R,1},\vartheta_{R,1},f\right)\quad\ldots\quad b_{R_H}\left(\varphi_{R,p},\vartheta_{R,p},f\right)\right]\cdot diag\left\{\gamma_{HH,p}\right\}\cdot diag\{e^{-j2\pi f\tau_p}\}\begin{bmatrix}b_{T_H}\left(\varphi_{T,1},\vartheta_{T,1},f\right)\\ \vdots\\ b_{T_H}\left(\varphi_{T,p},\vartheta_{T,p},f\right)\end{bmatrix}$$

$$+\cdots$$

Representation in matrix form (multipath MIMO)

$$\sum_{p=1}^{P}\left\{\boldsymbol{B}_R\left(\varphi_{R,p},\vartheta_{R,p}\right)\cdot\boldsymbol{\Gamma}_p\cdot\boldsymbol{B}_T(\varphi_T,\vartheta_T)^T\cdot e^{-j2\pi f\tau_p}\right\}=$$

$$\begin{bmatrix} b_{R_H,1}\left(\varphi_{R,1},\vartheta_{R,1},f\right) & \dots & b_{R_H,1}\left(\varphi_{R,p},\vartheta_{R,p},f\right) \\ \vdots & \ddots & \vdots \\ b_{R_H,M_R}\left(\varphi_{R,1},\vartheta_{R,1},f\right) & \dots & b_{R_H,M_R}\left(\varphi_{R,p},\vartheta_{R,p},f\right) \end{bmatrix}\cdot diag\left\{\gamma_{HH,p}\right\}\cdot$$

$$diag\{e^{-j2\pi f\tau_p}\}\begin{bmatrix} b_{T_H,1}\left(\varphi_{T,1},\vartheta_{T,1},f\right) & \dots & b_{T_H,M_T}\left(\varphi_{T,1},\vartheta_{T,1},f\right) \\ \vdots & \ddots & \vdots \\ b_{T_H,1}\left(\varphi_{T,p},\vartheta_{T,p},f\right) & \dots & b_{T_H,M_T}\left(\varphi_{T,p},\vartheta_{T,p},f\right) \end{bmatrix}+\cdots$$

Representation in matrix form (multipath MIMO)

- $\sum_{p=1}^{P}\left\{\boldsymbol{B}_R\left(\varphi_{R,p},\vartheta_{R,p}\right)\cdot\boldsymbol{\Gamma}_p\cdot\boldsymbol{B}_T(\varphi_T,\vartheta_T)^T\cdot e^{-j2\pi f\tau_p}\right\}=$

$\boldsymbol{B}_{R_H}(\boldsymbol{\varphi}_R,\boldsymbol{\vartheta}_R)$

- $\begin{bmatrix} b_{R_H,1}\left(\varphi_{R,1},\vartheta_{R,1},f\right) & \cdots & b_{R_H,1}\left(\varphi_{R,p},\vartheta_{R,p},f\right) \\ \vdots & \ddots & \vdots \\ b_{R_H,M_R}\left(\varphi_{R,1},\vartheta_{R,1},f\right) & \cdots & b_{R_H,M_R}\left(\varphi_{R,p},\vartheta_{R,p},f\right) \end{bmatrix}\cdot diag\left\{\gamma_{HH,p}\right\}\cdot$

- $diag\left\{e^{-j2\pi f\tau_p}\right\}\begin{bmatrix} b_{T_H,1}\left(\varphi_{T,1},\vartheta_{T,1},f\right) & \cdots & b_{T_H,M_T}\left(\varphi_{T,1},\vartheta_{T,1},f\right) \\ \vdots & \ddots & \vdots \\ b_{T_H,1}\left(\varphi_{T,p},\vartheta_{T,p},f\right) & \cdots & b_{T_H,M_T}\left(\varphi_{T,p},\vartheta_{T,p},f\right) \end{bmatrix}+\cdots$

$\boldsymbol{B}_{T_H}^T(\boldsymbol{\varphi}_T,\boldsymbol{\vartheta}_T)$

Representation in matrix form (multipath MIMO)

$$H(0,0) = G_{T_f}(0) \cdot G_{R_f}(0) \cdot$$

$$\Big[\boldsymbol{B}_{R_H}(\boldsymbol{\varphi}_R, \boldsymbol{\vartheta}_R) \cdot diag\{\gamma_{HH,p}\} \cdot diag\{e^{-j2\pi f \tau_p}\} \cdot \boldsymbol{B}_{T_H}^T(\boldsymbol{\varphi}_T, \boldsymbol{\vartheta}_T) +$$

$$\boldsymbol{B}_{R_H}(\boldsymbol{\varphi}_R, \boldsymbol{\vartheta}_R) \cdot diag\{\gamma_{VH,p}\} \cdot diag\{e^{-j2\pi f \tau_p}\} \cdot \boldsymbol{B}_{T_V}^T(\boldsymbol{\varphi}_T, \boldsymbol{\vartheta}_T) +$$

$$\boldsymbol{B}_{R_V}(\boldsymbol{\varphi}_R, \boldsymbol{\vartheta}_R) \cdot diag\{\gamma_{HV,p}\} \cdot diag\{e^{-j2\pi f \tau_p}\} \cdot \boldsymbol{B}_{T_H}^T(\boldsymbol{\varphi}_T, \boldsymbol{\vartheta}_T) +$$

$$\boldsymbol{B}_{R_V}(\boldsymbol{\varphi}_R, \boldsymbol{\vartheta}_R) \cdot diag\{\gamma_{VV,p}\} \cdot diag\{e^{-j2\pi f \tau_p}\} \cdot \boldsymbol{B}_{T_V}^T(\boldsymbol{\varphi}_T, \boldsymbol{\vartheta}_T)\Big]$$

Representation in matrix form (multipath MIMO, frequency domain sample)

- $\boldsymbol{H}(\boxed{f}, \varphi_T, \vartheta_T, \varphi_R, \vartheta_R, t) =$
- $G_{R_f}(\boxed{f}) \cdot G_{T_f}(\boxed{f}) \cdot \sum_{p=1}^{P} \left\{ \boldsymbol{B}_R(\varphi_{R,p}, \vartheta_{R,p}) \cdot \boldsymbol{\Gamma}_p \cdot \boldsymbol{B}_T(\varphi_T, \vartheta_T)^T \cdot e^{-j2\pi \boxed{f} \tau_p} \cdot e^{-j2\pi\alpha_p t} \right\}$

$$H_f(0) = \left(\boldsymbol{G}_{R_f} \otimes \boldsymbol{B}_{R_H}\right) \cdot (\mathbf{I} \otimes diag\{\gamma_{HH}\}) \cdot diag\left\{vec\{\boldsymbol{A}_\tau^T\}\right\} \cdot \left(\boldsymbol{G}_{T_f} \otimes \boldsymbol{B}_{T_H}\right) + \cdots$$

$$\boldsymbol{G}_{R_f} = diag\left\{\left[G_{R_f}\left(-\frac{M_f - 1}{2} f_0\right) \quad \ldots \quad G_{R_f}\left(+\frac{M_f - 1}{2} f_0\right)\right]\right\} \in C^{M_f \times M_f}$$

$$\boldsymbol{A}_\tau(\tau) = \begin{bmatrix} e^{-j2\pi\left(-\frac{M_f-1}{2}\right) f_0 \tau_1} & e^{-j2\pi\left(-\frac{M_f-1}{2}\right) f_0 \tau_p} \\ e^{-j2\pi\left(+\frac{M_f-1}{2}\right) f_0 \tau_1} & e^{-j2\pi\left(+\frac{M_f-1}{2}\right) f_0 \tau_p} \end{bmatrix}$$

Kronecker product

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}\boldsymbol{B} & \ldots & a_{1n}\boldsymbol{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\boldsymbol{B} & \ldots & a_{mn}\boldsymbol{B} \end{bmatrix}$$

Representation in vector form (multipath MIMO)

$$vec\left(\sum_{p=1}^{P}\left\{\boldsymbol{B}_R\left(\varphi_{R,p},\vartheta_{R,p}\right)\cdot\boldsymbol{\Gamma}_p\cdot\boldsymbol{B}_T(\varphi_T,\vartheta_T)^T\cdot e^{-j2\pi f\tau_p}\right\}\right)\in C^{M_R M_T\times 1}$$

$$=\begin{bmatrix} b_{R_H,1}\left(\varphi_{R,1},\vartheta_{R,1},f\right) & \cdots & b_{R_H,1}\left(\varphi_{R,p},\vartheta_{R,p},f\right) \\ \vdots & \ddots & \vdots \\ b_{R_H,M_R}\left(\varphi_{R,1},\vartheta_{R,1},f\right) & \cdots & b_{R_H,M_R}\left(\varphi_{R,p},\vartheta_{R,p},f\right) \end{bmatrix}\diamond$$

$$\begin{bmatrix} b_{T_H,1}\left(\varphi_{T,1},\vartheta_{T,1},f\right) & \cdots & b_{T_H,1}\left(\varphi_{T,p},\vartheta_{T,p},f\right) \\ \vdots & \ddots & \vdots \\ b_{T_H,M_T}\left(\varphi_{T,1},\vartheta_{T,1},f\right) & \cdots & b_{T_H,M_T}\left(\varphi_{T,p},\vartheta_{T,p},f\right) \end{bmatrix}$$

$$\cdot\, diag\left\{\gamma_{HH,p}\right\}\cdot\ diag\{e^{-j2\pi f\tau_p}\}\ +\cdots$$

$$=\boldsymbol{B}_{R_H}(\boldsymbol{\varphi}_R,\boldsymbol{\vartheta}_R)\diamond\boldsymbol{B}_{T_H}(\boldsymbol{\varphi}_T,\boldsymbol{\vartheta}_T)\cdot diag\left\{\gamma_{HH,p}\right\}\cdot\ diag\{e^{-j2\pi f\tau_p}\}$$

here $\diamond$ means Khatri-Rao product,
i.e. column wise Kronecker product
$A\diamond B=[a_1\otimes b_1\quad\ldots\quad a_n\otimes b_n]$

Representation in vector form (multipath MIMO, frequency domain sample)

$$\boldsymbol{s}(\boldsymbol{\theta}_{sp}) = vec\left(H_f(0)\right)$$
$$= \left(B_{T_H} \diamond B_{R_H} \diamond \left(G_{s_f} \cdot A_\tau\right)\right) \cdot \gamma_{HH} + \left(B_{T_H} \diamond B_{R_V} \diamond \left(G_{s_f} \cdot A_\tau\right)\right) \cdot \gamma_{HV} +$$
$$\left(B_{T_V} \diamond B_{R_H} \diamond \left(G_{s_f} \cdot A_\tau\right)\right) \cdot \gamma_{VH} + \left(B_{T_V} \diamond B_{R_V} \diamond \left(G_{s_f} \cdot A_\tau\right)\right) \cdot \gamma_{VV}$$
$$= \left(B_{T_H} \diamond B_{R_H} \diamond B_f\right) \cdot \gamma_{HH} + \left(B_{T_H} \diamond B_{R_V} \diamond B_f\right) \cdot \gamma_{HV} + \left(B_{T_V} \diamond B_{R_H} \diamond B_f\right) \cdot \gamma_{VH} + \left(B_{T_V} \diamond B_{R_V} \diamond B_f\right) \cdot \gamma_{VV}$$
$$= \left[B_{T_H} \diamond B_{R_H} \diamond B_f \quad B_{T_H} \diamond B_{R_V} \diamond B_f \quad B_{T_V} \diamond B_{R_H} \diamond B_f \quad B_{T_V} \diamond B_{R_V} \diamond B_f\right] \begin{bmatrix} \gamma_{HH} \\ \gamma_{HV} \\ \gamma_{VH} \\ \gamma_{VV} \end{bmatrix}$$
$$= B(\boldsymbol{\mu}) \cdot \boldsymbol{\gamma}$$

Where , $B_f = G_{s_f} \cdot A_\tau = G_{R_f} \cdot G_{T_f} \cdot A_\tau$

Parameter vector to estimate:

$$\boldsymbol{\Theta}_{sp} = [\alpha^T \ \tau^T \ \varphi_T^T \ \ \vartheta_T^T \ \ \varphi_R^T \ \ \vartheta_R^T \ \ \Re\{\gamma_{HH}^T\} \ \ \Im\{\gamma_{HH}^T\} \ \Re\{\gamma_{HV}^T\} \ \ \Im\{\gamma_{HV}^T\} \ \Re\{\gamma_{VH}^T\} \ \ \Im\{\gamma_{VH}^T\} \ \Re\{\gamma_{VV}^T\} \ \ \Im\{\gamma_{VV}^T\}]$$

Dense Multipath Components (DMC) Introduction

DMC model

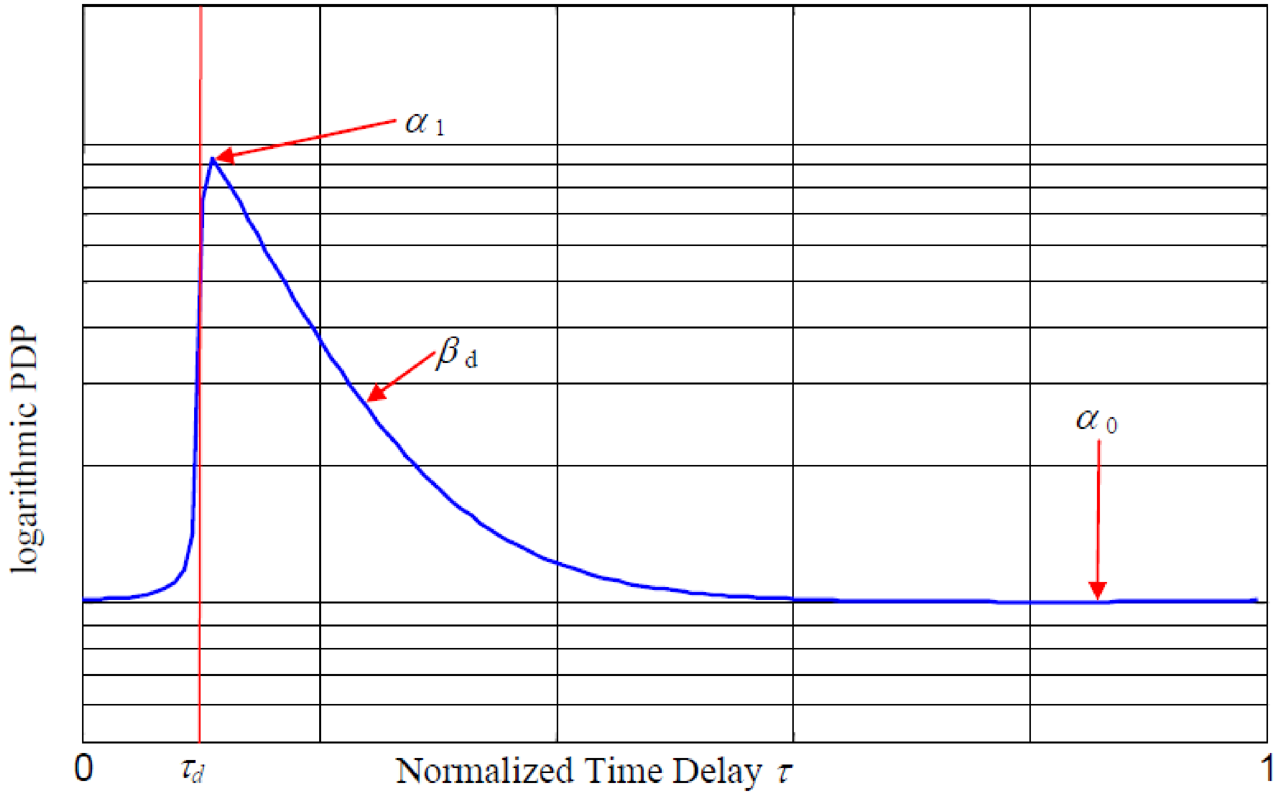


Figure 2-4: Dense multipath distribution model in the time delay domain $\psi_{xgg}(\tau,\tau)$.

Parameters characterize DMC $\boldsymbol{\theta}_{dmc} = [\tau_d \quad \alpha_1 \quad \beta_d \quad \alpha_0]$

DMC model

The proposed model (2.60) describes the power delay profile (PDP) of the dense multipath components, which is an exponential decay over the time-delay.

$$\psi_h(\tau) = \mathrm{E}\left\{|h(\tau)|^2\right\} = \begin{cases} 0 & \tau < \tau'_d \\ \alpha_1 \cdot \frac{1}{2} & \tau = \tau'_d \\ \alpha_1 \cdot \mathrm{e}^{-B_d(\tau-\tau'_d)} & \tau > \tau'_d \end{cases} \tag{2.60}$$

The related power spectrum density, i.e., the Fourier-transform of (2.60) is

$$\Psi_H(\Delta f) = \frac{\alpha_1}{\beta_d + \mathrm{j}2\pi\Delta f} \cdot \mathrm{e}^{-\mathrm{j}2\pi\Delta f \tau_d}\ , \tag{2.61}$$

where $\beta_d = \frac{B_d}{B_m} = \frac{B_d}{M_f \cdot f_0}$ is the normalized coherence bandwidth of the DMC.

Frequency domain covariance matrix

$$\Phi_{hh}(\tau_1,\tau_2) = E\{h(\tau_1)h(\tau_2)^*\}$$

Covariance function of the channel impulse response

$$\Psi_{HH}(f_1,f_2) = E\{H(f_1)H(f_2)^*\}$$

Covariance function of the channel transfer function

Discrete version:

$$\mathbf{R}_f = \begin{bmatrix} \Psi_H(0) & \Psi_H(-f_0) & \cdots & \Psi_H((M_f-1)f_0) \\ \Psi_H(f_0) & \Psi_H(0) & \ddots & \\ \vdots & \ddots & \ddots & \Psi_H(-f_0) \\ \Psi_H((M_f-1)f_0) & \cdots & \Psi_H(f_0) & \Psi_H(0) \end{bmatrix}.$$

This is a Toeplitz matrix

$$\boldsymbol{R}_f = toeplitz\left(k(\boldsymbol{\theta}), k^H(\boldsymbol{\theta})\right)$$

$$\boldsymbol{\kappa}(\boldsymbol{\theta}_{dan}) = \frac{\alpha_1}{M_f}\left[\frac{1}{\beta_d} \quad \frac{e^{-j2\pi\tau_d}}{\beta_d + j2\pi\frac{1}{M_f}} \quad \cdots \quad \frac{e^{-j2\pi(M_f-1)\tau_d}}{\beta_d + j2\pi\frac{M_f-1}{M_f}}\right]^T + \alpha_0\mathbf{e}_0, \qquad (2.68)$$

Overall channel model

Complete radio channel model

$$\boldsymbol{h} = \boldsymbol{s}\left(\boldsymbol{\theta}_{sp}\right) + \boldsymbol{d}_{dmc} = \boldsymbol{B}(\boldsymbol{\mu}) \cdot \boldsymbol{\gamma} + \boldsymbol{d}_{dmc}$$

$\boldsymbol{d}_{dmc} \sim \mathcal{N}_c(0, \boldsymbol{R}(\boldsymbol{\theta}_{dmc}))$, complex circular symmetric Gaussian process.

$$\boldsymbol{h} \sim \mathcal{N}_c(\boldsymbol{s}(\boldsymbol{\theta}_{sp}), \boldsymbol{R}(\boldsymbol{\theta}_{dmc}))$$

Global maximization algorithm

$$\begin{bmatrix} \hat{\boldsymbol{\theta}}_{sp} \\ \hat{\boldsymbol{\theta}}_{dan} \end{bmatrix} = \arg\max\left(-\ln(\det(\mathbf{R}(\boldsymbol{\theta}_{dac}))) - \left(\mathbf{x} - \mathbf{s}\left(\boldsymbol{\theta}_{sp}\right)\right)^{\mathrm{H}} \cdot \mathbf{R}^{-1}(\boldsymbol{\theta}_{dan}) \cdot \left(\mathbf{x} - \mathbf{s}\left(\boldsymbol{\theta}_{sp}\right)\right)\right). \qquad (5.1)$$

How RIMAX works

RIMAX: an alternating way of channel estimation

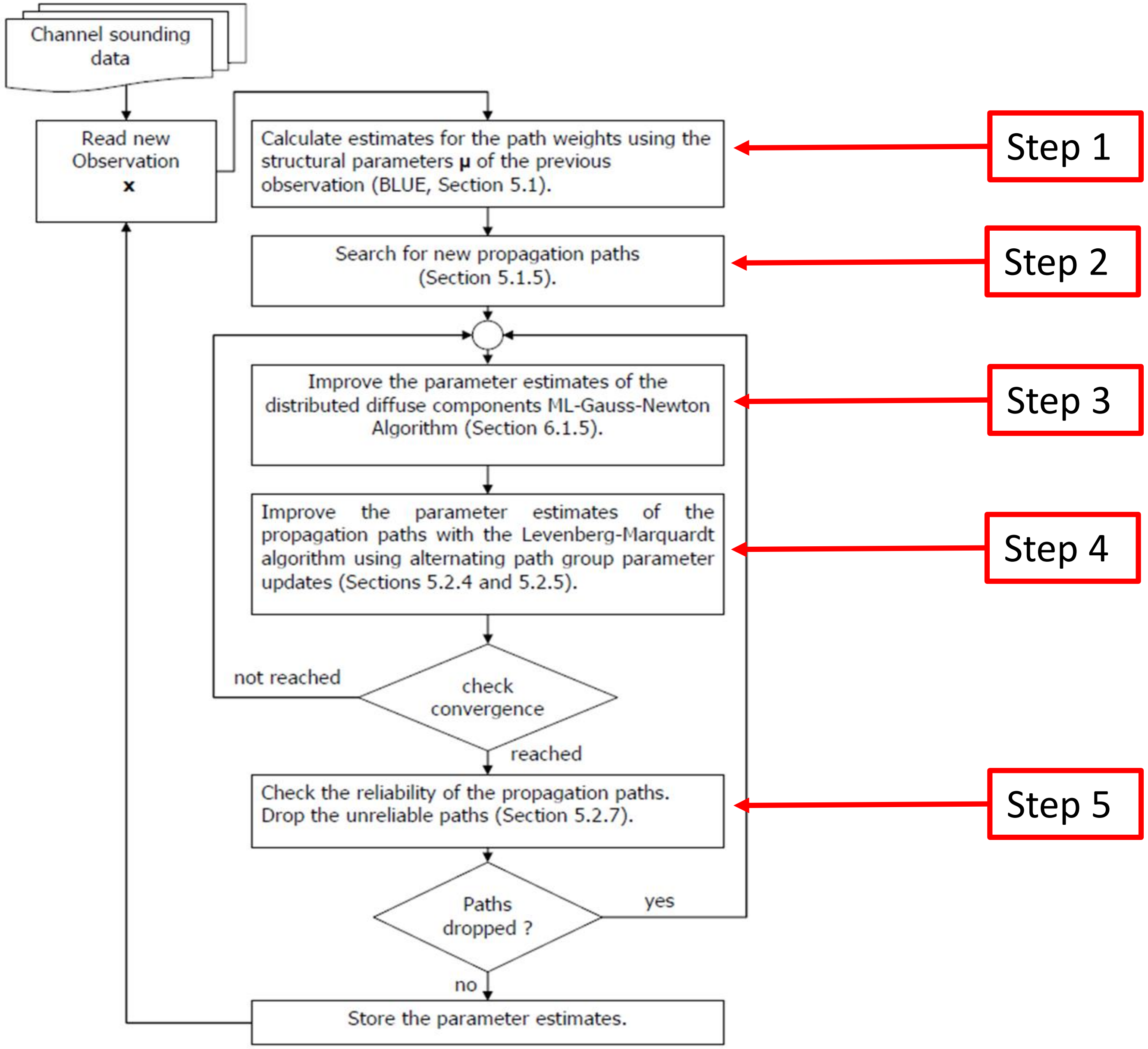


Figure 6-13: Outline of the RIMAX structure

Step 1: BLUE (Best Linear Unbiased Estimator)

If the noise parameter $\boldsymbol{\theta}_{dan}$ is known, then maximization problem reduces to

$$\widehat{\boldsymbol{\theta}} = arg\ \min_{\boldsymbol{\theta}} \left(\boldsymbol{x} - \boldsymbol{s}(\boldsymbol{\theta})\right)^H \boldsymbol{R}_{nn}^{-1} \left(\boldsymbol{x} - \boldsymbol{s}(\boldsymbol{\theta})\right)$$

$$= arg\ \min_{\boldsymbol{\theta}} (\boldsymbol{x} - \boldsymbol{B}(\boldsymbol{\mu}) \cdot \boldsymbol{\gamma})^H \boldsymbol{R}_{nn}^{-1} (\boldsymbol{x} - \boldsymbol{B}(\boldsymbol{\mu}) \cdot \boldsymbol{\gamma})$$

With BLUE

$$\widehat{\boldsymbol{\gamma}} = \left(\boldsymbol{B}^H(\widehat{\boldsymbol{\mu}}) \cdot \boldsymbol{R}_{nn}^{-1} \cdot \boldsymbol{B}(\widehat{\boldsymbol{\mu}})\right)^{-\mathbf{1}} \cdot \boldsymbol{B}^H(\widehat{\boldsymbol{\mu}}) \cdot \boldsymbol{R}_{nn}^{-1} \cdot \boldsymbol{x}$$

Replacing $\boldsymbol{\gamma}$ by its estimator $\widehat{\boldsymbol{\gamma}}$ yields

$$\widehat{\boldsymbol{\theta}} = arg\ \min_{\boldsymbol{\theta}} \left(\boldsymbol{x}^H \boldsymbol{R}_{nn}^{-1} \boldsymbol{x} - \left(\boldsymbol{x}^H \boldsymbol{R}_{nn}^{-1} \boldsymbol{B}(\boldsymbol{\mu})\right) \cdot \left(\boldsymbol{B}^H(\boldsymbol{\mu}) \boldsymbol{x}^H \boldsymbol{R}_{nn}^{-1} \boldsymbol{B}(\boldsymbol{\mu})\right)^{-1} \left(\boldsymbol{B}^H(\boldsymbol{\mu}) \boldsymbol{x}^H \boldsymbol{R}_{nn}^{-1} \boldsymbol{x}\right)\right)$$

Match filter tries all the possible parameters combination of $\boldsymbol{\mu}$, and find the best one.

Step 2: to find new MPC

Remove the contribution of the tracked MPC, we get $\boldsymbol{x}_r$ and search fro new MPC iteratively, stop until the signal power is lower enough.

$$\boldsymbol{x}_r = \boldsymbol{x} - \sum_{p=1}^{P} s\left(\widehat{\boldsymbol{\theta}}_{sp,p}\right)$$

Step 3: Gauss-Newton algorithm introduction

Given m functions $\boldsymbol{r} = (\boldsymbol{r}_1, \ldots, \boldsymbol{r}_m)$ of n variables $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$, with $m \geq n$, the Gauss-Newton algorithm iteratively finds the value of the variables that minimizes the sum of squares

$$S(\boldsymbol{\beta}) = \sum_{i=1}^{m} r_i^2(\beta)$$

Starting with an initial guess $\boldsymbol{\beta}^{(0)}$ for the minimum, the method proceeds by the iterations

$$\boldsymbol{\beta}^{(s+1)} = \boldsymbol{\beta}^{(s)} - (\boldsymbol{J}_r^T \boldsymbol{J}_r)^{-1} \boldsymbol{J}_r^T \boldsymbol{r}\left(\boldsymbol{\beta}^{(s)}\right)$$

Where $\boldsymbol{r}$ and $\boldsymbol{\beta}$ are column vectors, the entries of the Jacobian matrix are

$$(J_r)_{i,j} = \frac{\partial r_i\left(\beta^{(s)}\right)}{\partial \beta_j}$$

Step 3: Gauss-Newton algorithm

Gaussian-Newton algorithm

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n + \Delta\boldsymbol{\theta}$$

Where,

$$\Delta\boldsymbol{\theta} = (\boldsymbol{D}^H\boldsymbol{D})^{-1}\boldsymbol{D}^H \cdot vec\left(\boldsymbol{R}_{xx} - \boldsymbol{R}(\boldsymbol{\theta}_n)\right)$$

Step 3: Jacobian of DMC

$$\frac{\partial}{\partial\theta_i}\boldsymbol{R}(\boldsymbol{\theta}_{dmc}) = toep\left(\frac{\partial}{\partial\theta_i}k(\boldsymbol{\theta}_{DMC}), \frac{\partial}{\partial\theta_i}k^H(\boldsymbol{\theta}_{DMC})\right)$$

$$\frac{\partial}{\partial\alpha_0}\boldsymbol{\kappa}(\boldsymbol{\theta}) = \mathbf{e}_0, \tag{4.93}$$

$$\frac{\partial}{\partial\alpha_1}\boldsymbol{\kappa}(\boldsymbol{\theta}) = \frac{1}{M}\begin{bmatrix}\frac{1}{\beta_d} & \frac{e^{-j2\pi\tau_0}}{\beta_d + j2\pi\frac{1}{M}} & \cdots & \frac{e^{-j2\pi(M-1)\tau_0}}{\beta_d + j2\pi\frac{M-1}{M}}\end{bmatrix}^T, \tag{4.94}$$

$$\frac{\partial}{\partial\beta}\boldsymbol{\kappa}(\boldsymbol{\theta}) = -\frac{\alpha_1}{M}\begin{bmatrix}\frac{1}{(\beta_d)^2} & \frac{e^{-j2\pi\tau_0}}{(\beta_d + j2\pi\frac{1}{M})^2} & \cdots & \frac{e^{-j2\pi(M-1)\tau_0}}{(\beta_d + j2\pi\frac{M-1}{M})^2}\end{bmatrix}^T, \tag{4.95}$$

$$\frac{\partial}{\partial\tau_0}\boldsymbol{\kappa}(\boldsymbol{\theta}) = \frac{\alpha_1}{M}\begin{bmatrix}0 & \frac{-j2\pi\cdot e^{-j2\pi\tau_0}}{\beta_d + j2\pi\frac{1}{M}} & \cdots & \frac{-j2\pi(M-1)\cdot e^{-j2\pi(M-1)\tau_0}}{\beta_d + j2\pi\frac{M-1}{M}}\end{bmatrix}^T. \tag{4.96}$$

$$\mathbf{D}(\boldsymbol{\theta}) = \left[\operatorname{vec}\left\{\mathbf{L}^{-1}(\boldsymbol{\theta})\left(\frac{\partial}{\partial\theta_1}\mathbf{R}(\boldsymbol{\theta})\right)\mathbf{L}^{-H}(\boldsymbol{\theta})\right\} \;\cdots\; \operatorname{vec}\left\{\mathbf{L}^{-1}(\boldsymbol{\theta})\left(\frac{\partial}{\partial\theta_L}\mathbf{R}(\boldsymbol{\theta})\right)\mathbf{L}^{-H}(\boldsymbol{\theta})\right\}\right]. \tag{4.88}$$

Step 3: Estimation of DMC parameters with Gauss-Newton algorithm

Table 6-1: Iterative optimisation of the parameters $\boldsymbol{\theta}_{dmc}$ using the direct approach (Gauß-Newton algorithm)

Input data: Data matrix $\mathbf{X}$, initial solution $\boldsymbol{\theta}^{\{0\}}$

Preprocessing: Compute the estimate of the non-parametric covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{N}\mathbf{X}\mathbf{X}^{\mathrm{H}}$$

1) Compute the first order derivatives to the parameters $\boldsymbol{\theta}$ of $\boldsymbol{\kappa}(\boldsymbol{\theta})$ using equations (4.93) - (4.96).
2) Compute the Jacobian $\mathbf{D}(\boldsymbol{\theta}^{\{i\}})$.
3) Compute $\Delta\boldsymbol{\theta}^{\{i\}} = \arg\min_{\Delta\boldsymbol{\theta}} \left\| \mathbf{D}(\boldsymbol{\theta}^{\{i\}}) \cdot \Delta\boldsymbol{\theta} - \mathrm{vec}\left\{ \mathbf{R}^{-1}(\boldsymbol{\theta}^{\{i\}})\hat{\mathbf{R}} - \mathbf{I} \right\} \right\|_F^2$. Set $\lambda^{\{i\}} = 1$
4) Compute the update $\boldsymbol{\theta}^{\{i+1\}} = \boldsymbol{\theta}^{\{i\}} + \lambda^{\{i\}} \cdot \Delta\boldsymbol{\theta}^{\{i\}}$.
5) Check strict maximization $\mathcal{L}(\mathbf{X}|\boldsymbol{\theta}^{\{i+1\}}) > \mathcal{L}(\mathbf{X}|\boldsymbol{\theta}^{\{i\}})$; yes: go to 1. , no: reduce $\lambda^{\{i\}}$ go to 4.
6) Check convergence; not converged: go to 1.

Step 4: MPC parameters estimation

Probability dense function of MPC

$$p(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn}) = \frac{1}{\pi^M \det(\mathbf{R}_{nn}(\boldsymbol{\theta}))} \mathrm{e}^{-(\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}))^{\mathrm{H}} \cdot \mathbf{R}_{nn}(\boldsymbol{\theta})^{-1} \cdot (\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}))}. \quad (4.4)$$

Take the logarithm form yields the log-likelihood function

$$\mathcal{L}(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn}) = \ln(p(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn})) = -M \cdot \ln(\pi) - \ln(\det(\mathbf{R}_{nn})) - (\mathbf{x}-\mathbf{s}(\boldsymbol{\theta}))^{\mathrm{H}} \mathbf{R}_{nn}^{-1} (\mathbf{x}-\mathbf{s}(\boldsymbol{\theta})). \quad (4.5)$$

The first order partial derivative with respect to the parameters $\boldsymbol{\theta}_{sp}$ of the log-likelihood function is called score function

$$\mathbf{q}(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn}) = \frac{\partial}{\partial \boldsymbol{\theta}} \mathcal{L}(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn}) = \left[\frac{\partial}{\partial \theta_1} \mathcal{L}(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn}) \ldots \frac{\partial}{\partial \theta_L} \mathcal{L}(\mathbf{x}|\boldsymbol{\theta},\mathbf{R}_{nn}) \right]^{\mathrm{T}} \in \mathbb{C}^{L\times 1}. \quad (4.6)$$

Step 4: MPC parameters estimation

The first order derivation to specific parameter θ_i is as follow:

$$\frac{\partial}{\partial \theta_i} \mathcal{L}(\mathbf{x}|\boldsymbol{\theta}, \mathbf{R}_{nn}) = 2 \cdot \Re\left\{ \left(\frac{\partial}{\partial \theta_i} \mathbf{s}^{\mathrm{H}}(\boldsymbol{\theta}) \right) \mathbf{R}_{nn}^{-1} (\mathbf{x} - \mathbf{s}(\boldsymbol{\theta})) \right\}. \qquad (4.8)$$

If we define the Jacobian of $\boldsymbol{s}(\boldsymbol{\theta})$ as

$$\mathbf{D}(\boldsymbol{\theta}) = \frac{\partial}{\partial \boldsymbol{\theta}^{\mathrm{T}}} \mathbf{s}(\boldsymbol{\theta}) = \left[\left(\frac{\partial}{\partial \theta_1} \mathbf{s}(\boldsymbol{\theta}) \right) \cdots \left(\frac{\partial}{\partial \theta_L} \mathbf{s}(\boldsymbol{\theta}) \right) \right] \in \mathbb{C}^{M \times L}. \qquad (4.9)$$

Then we can get score function as

$$\mathbf{q}(\mathbf{x}|\boldsymbol{\theta}, \mathbf{R}_{nn}) = 2 \cdot \Re\{\mathbf{D}^{\mathrm{H}}(\boldsymbol{\theta}) \mathbf{R}_{nn}^{-1} (\mathbf{x} - \mathbf{s}(\boldsymbol{\theta}))\}. \qquad (4.10)$$

And the second order derivation of log-likelihood function (Fisher information) as:

$$\mathcal{J}(\boldsymbol{\theta}, \mathbf{R}_{nn}) = 2 \cdot \Re\{\mathbf{D}^{\mathrm{H}}(\boldsymbol{\theta}) \mathbf{R}_{nn}^{-1} \mathbf{D}(\boldsymbol{\theta})\}. \qquad (4.16)$$

Step 4: MPC parameters estimation

For Gauss-Newton algorithm

$$\hat{\boldsymbol{\theta}}^{\{i+1\}} = \hat{\boldsymbol{\theta}}^{\{i\}} + \varsigma\, \mathcal{J}^{-1}\left(\hat{\boldsymbol{\theta}}^{\{i\}}, \mathbf{R}_{nn}\right) \mathbf{q}\left(\mathbf{x} \middle| \hat{\boldsymbol{\theta}}^{\{i\}}, \mathbf{R}_{nn}\right).$$

Levenberg-Marquardt Method

$$\hat{\boldsymbol{\theta}}_m^{\{i+1\}} = \hat{\boldsymbol{\theta}}^{\{i\}} + \left(\mathcal{J}\left(\hat{\boldsymbol{\theta}}^{\{i\}}, \mathbf{R}_{nn}\right) + \varsigma\, \mathbf{I} \circ \mathcal{J}\left(\hat{\boldsymbol{\theta}}^{\{i\}}, \mathbf{R}_{nn}\right)\right)^{-1} \mathbf{q}\left(\mathbf{x} \middle| \hat{\boldsymbol{\theta}}^{\{i\}}, \mathbf{R}_{nn}\right). \qquad (5.61)$$

Step 4: MPC parameters estimation

Table 5-3: Iterative optimisation of the parameters $\boldsymbol{\theta}_{sp}$ using the Levenberg-Marquardt algorithm.

Input: observation $\mathbf{x}$, initial solution $\hat{\boldsymbol{\theta}}^{\{0\}}$, covariance matrix $\mathbf{R}_{nn}$, ς.

1) Compute the component matrices of the Jacobian matrix Table 4-2 - Table 4-4.
2) Compute the score function
$$\mathbf{q}\left(\mathbf{x}\middle|\boldsymbol{\theta}^{\{i\}},\mathbf{R}_{nn}\right)=2\cdot\Re\left\{\mathbf{D}^{\mathrm{H}}\left(\boldsymbol{\theta}^{\{i\}}\right)\mathbf{R}_{nn}^{-1}\left(\mathbf{x}-\mathbf{s}\left(\boldsymbol{\theta}^{\{i\}}\right)\right)\right\}.$$
3) Compute the approximation of the Hessian using equations (4.70), (4.75), or (4.78)
$$\mathcal{J}\left(\hat{\boldsymbol{\theta}}^{\{i\}},\mathbf{R}_{nn}\right).$$
4) Compute the parameter update
$$\hat{\boldsymbol{\theta}}_{m}^{\{i+1\}}=\hat{\boldsymbol{\theta}}^{\{i\}}+\left(\mathcal{J}\left(\hat{\boldsymbol{\theta}}^{\{i\}},\mathbf{R}_{nn}\right)+\varsigma\,\mathbf{I}\circ\mathcal{J}\left(\hat{\boldsymbol{\theta}}^{\{i\}},\mathbf{R}_{nn}\right)\right)^{-1}\mathbf{q}\left(\mathbf{x}\middle|\hat{\boldsymbol{\theta}}^{\{i\}},\mathbf{R}_{nn}\right).$$
5) Check strict maximization $\mathcal{L}\left(\mathbf{x}\middle|\boldsymbol{\theta}^{\{i+1\}}\right)>\mathcal{L}\left(\mathbf{x}\middle|\boldsymbol{\theta}^{\{i\}}\right)$; yes: set $\varsigma=\frac{\varsigma}{4}$ go to 6. , no: set $\varsigma=8\varsigma$ go to 4.
6) Check convergence; not converged: set $i=i+1$ and go to 1.

Step 5: check the reliability of MPC

The problem is to check how reliable the MPC is, and decide if we want to keep it or discard it.

Since the estimation is asymptotic distribution of

$$\hat{\boldsymbol{\theta}} \overset{as.d.}{\sim} \mathcal{N}\left(\hat{\boldsymbol{\theta}}, \mathcal{J}^{-1}\left(\hat{\boldsymbol{\theta}}, \mathbf{R}_{nn}\right)\right). \qquad (5.63)$$

The valid MPC should fulfill the following criteria:

$$\frac{\operatorname{var}\{|\boldsymbol{\gamma}|\}}{|\boldsymbol{\gamma}|^2} < \varepsilon_{|\boldsymbol{\gamma}|}^2 < 1 . \qquad (5.64)$$

Which means we require that the certainty of the estimated path magnitude must be larger than its uncertainty.