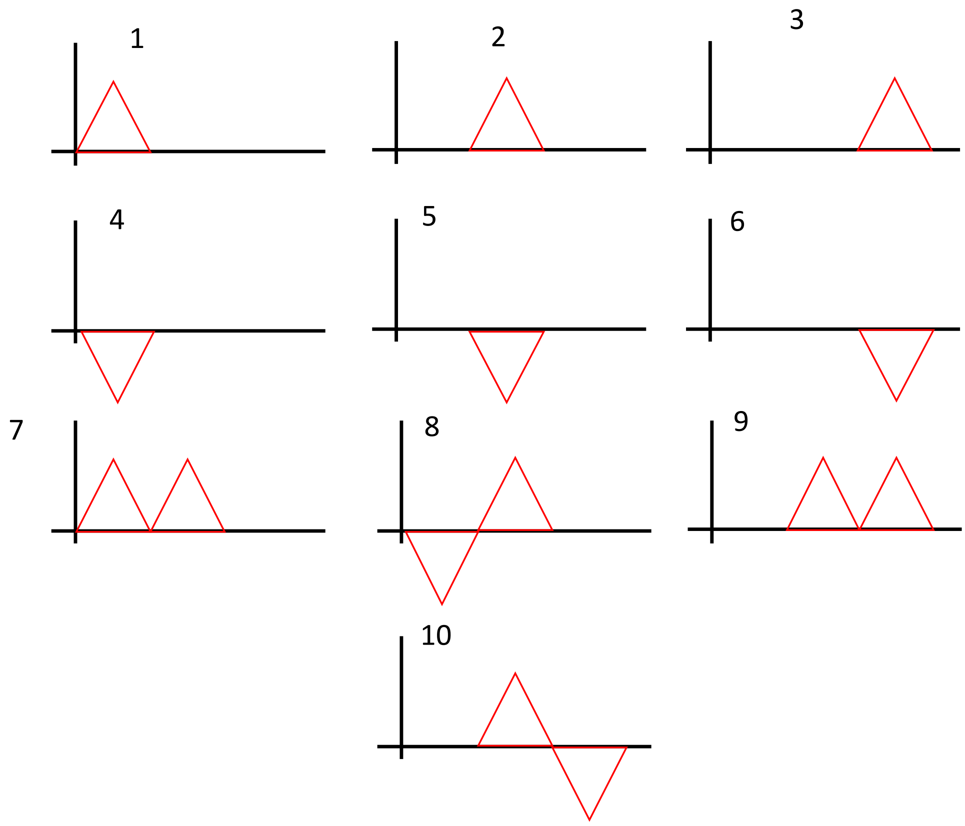
ETTN01 Advanced Digital Communications, (max: 50p)

Problem 1.

Consider the 10 pulse shapes given below. Assume that the energy of $S_1(t)$ is 1.



a) What is the dimensionality of signal space?

b) Select 3 pulses so that the signal space is equivalent to 3-FSK.

c) Sketch the signal space for 1, 2, 4 ,5, 7 ,8.

d) Sketch the decision regions for the pulses in c)

e) Select 4 pulses so that the signal space is equivalent to QPSK.

f) For 1,2, 4,5,7,8, determine the error probability if 5,7, or 8 are sent.

g) Is it true that an orthonormal set of basis functions can be created from 7,8, and 9?

h) Find the error probability of 1 and 9 (i.e.,the signal set is binary).

Problem 2.

Are the following statements true or false (provide motivation)

a) A system with the following specs can be designed and operate at very very small BER:
   - Bit-rate 100Mb/s
   - Bandwidth 400 MHz
   - Total transmit power 1W
   - 1024QAM per subcarrier
   - Noise density No=E-6 W/Hz

b) In OFDM, it seems reasonable to design the CP-length based on the delay-spread of the channel.

c) OFDM is vulnerable to time-variant channels. To have a robust system, T_{obs} should be smaller than the coherence time of the channel.

d) For a fading channel and a system design such that we have frequency-non-selective slow fading, one needs to design the system based on the average channel behaviour.

Problem 3.

Derive the spectral efficiency of OFDM. You need to introduce all relevant variables yourself.

Problem 4.

Assume a diversity system. Let the transmitted signal be $s_\ell(t) = \phi_1(t)a_\ell + \phi_2(t)a_\ell$ where $a_\ell \in \{\pm 1\}$. Let the received signal be $r(t) = \alpha_1\phi_1(t)a_\ell + \alpha_2\phi_2(t)a_\ell + N(t)$ where α_1, α_2 are random numbers that are either α_G (good) or α_B (bad).
For a)-b) assume probabilities of good and bad as P_G and P_B, respectively.

a) Assume α_1, α_2 to be independent. Discuss the BER behaviour.

b) Assume α_1, α_2 to be fully correlated, i.e., the same. Derive the BER behaviour.

c) Let α_1, α_2 have the following joint pdf

$$P(\alpha_1 = \alpha_G, \alpha_2 = \alpha_G) = P_{GG}$$
$$P(\alpha_1 = \alpha_G, \alpha_2 = \alpha_B) = P_{GB}$$
$$P(\alpha_1 = \alpha_B, \alpha_2 = \alpha_G) = P_{BG}$$
$$P(\alpha_1 = \alpha_B, \alpha_2 = \alpha_B) = P_{BB}$$

Derive the BER

d) In c), under what conditions are there an error floor?

Problem 5.

Assume a channel propagation environment according to:
- Doppler spread: 100Hz
- Delay spread: 0.1ms

Assume a system design according to

- 16QAM
- Rate 1/2 error control code
- Transmit power P according to P/No=10
- Carrier frequency: 20 GHz
- Single carrier
- Pulse shaping according to page 630 with $\beta = 0$

For what symbol rates is the system simultaneously offering frequency non-selective slow-fading and reliability according to Shannons formulas?