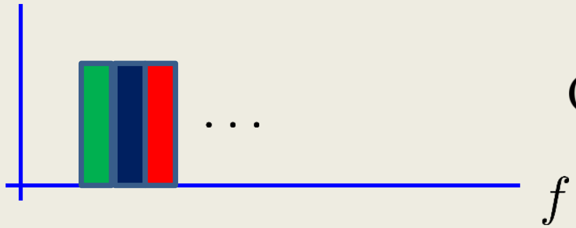
Lecture 7: OFDM

Recall: If $x(t) = A\cos(\omega_c t) - B\sin(\omega_c t),\ 0 \le t \le T_s$

Then $$z(t) = A_z \cos(\omega_c t) - B_z \sin(\omega_c t),\ T_h \le t \le T_s$$

$$A_z + iB_z = (A + iB)H(\omega_c)$$

We can transmit multiple signals at different sub-carriers: This is OFDM



We have

$$\text{OFDM signal} = g_{\text{rec}}(t) \sum_{k=0}^{K-1} \text{Re}\{a_k \exp(i2\pi f_k t)\}$$

Lecture 7: OFDM

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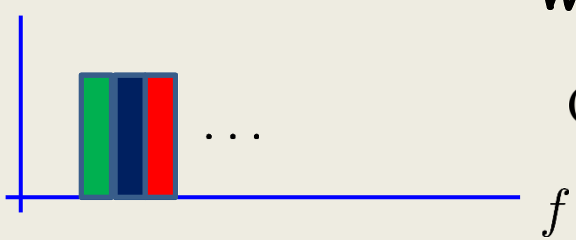

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Complex data (A+iB)

Lecture 7: OFDM

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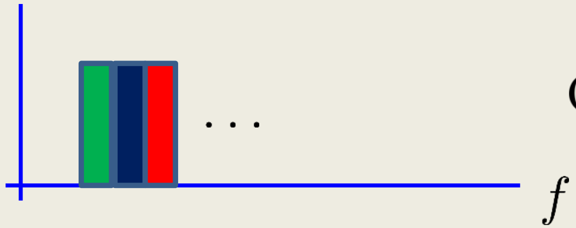

We have

Carrier

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Complex data (A+iB)

Lecture 7: OFDM

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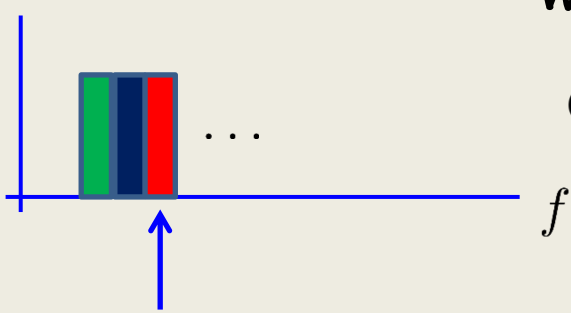
One signal

We have



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Lecture 7: OFDM

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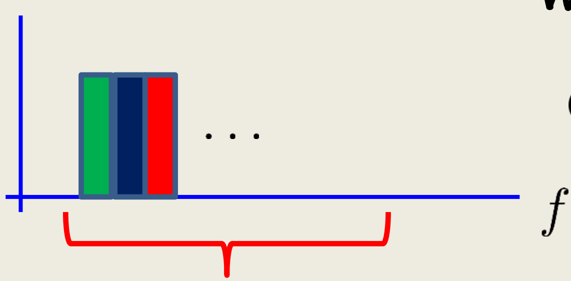

All signals

We have

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Lecture 7: OFDM

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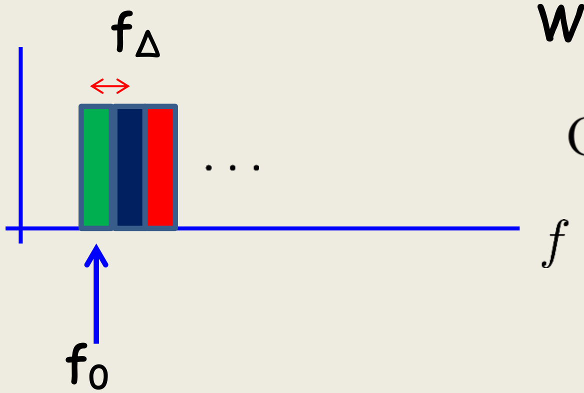

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Lecture 7: OFDM

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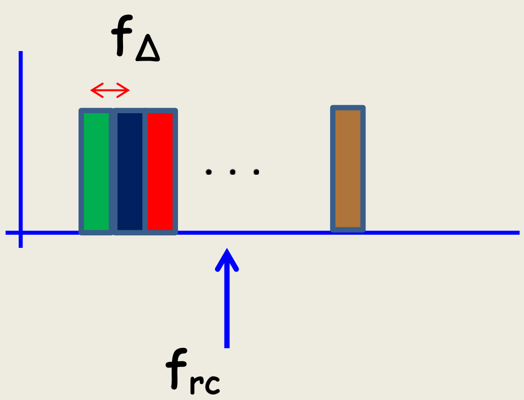

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g_k = …-2,-1,0,1,2,…

Lecture 7: OFDM

$$g_k: -\frac{K-1}{2} = g_0, \dots, -1,0,1, \dots, \frac{K-1}{2} = g_{K-1} \quad \text{if K is odd}$$

$$g_k: -\frac{K-2}{2} = g_0, \dots, -1,0,1, \dots, \frac{K}{2} = g_{K-1} \quad \text{if K is even}$$

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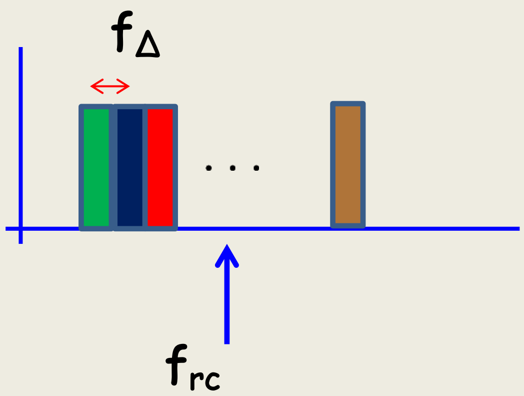

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Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \qquad 0 \le t \le T_s$$

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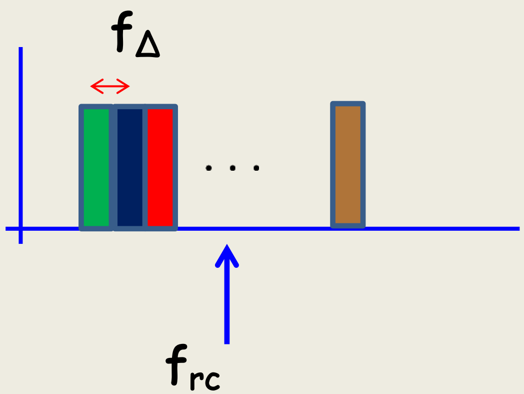

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$g_k = \ldots -2, -1, 0, 1, 2, \ldots$

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \qquad 0 \le t \le T_s$$

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Useful part at rx $\quad T_h \le t \le T_s$

Definition $\quad T_{\mathrm{obs}} = T_s - T_h$

Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \qquad 0 \le t \le T_s$$

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Useful part at rx $\quad T_h \le t \le T_s$

Definition $\quad T_{\text{obs}} = T_s - T_h$

So

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \qquad 0 \le t \le T_{\text{obs}} + T_h$$

We can also use (time-shift)

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \quad -T_h \le t \le T_{\text{obs}}$$

Lecture 7: OFDM

Objective of OFDM:

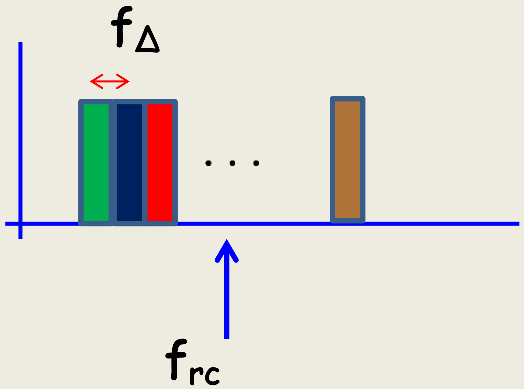
1. Create as many signal dimensions as there are symbols to be sent
2. Put one, and only one, symbol in each dimension (simple detection)

How many symbols? 2K

How long time is used at the receiver? T_{obs}

How much bandwidth? $K f_\Delta$

Shannon? Max #dims = 2 x Time x Bandwidth = $2K\underbrace{T_{\text{obs}} f_\Delta}_{=1}$



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Lecture 7: OFDM

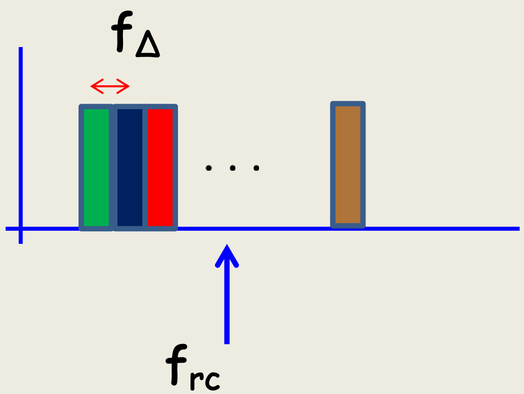
$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \quad \underbrace{0 \leq t \leq T_{\text{obs}}}$$

Let's begin here

$$x(t) = g_{\text{rec}}(t)\sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t)$$

Important: $f_\Delta T_{\text{obs}} > 1$

We can transmit multiple signals at different sub-carriers:
This is OFDM



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Lecture 7: OFDM

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To synthesize the signal, we

1. Sample the OFDM signal
2. Check how we can efficiently construct those samples
3. Perform D/A conversion

Lecture 7: OFDM

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How often do we need to sample?

Sampling theorem: Sample twice as fast the highest frequency component

Lecture 7: OFDM

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Lecture 7: OFDM

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Lecture 7: OFDM

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Lecture 7: OFDM

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Lecture 7: OFDM

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Lecture 7: OFDM

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$$x_n = x(nT_{obs}/N) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k f_\Delta n T_{obs}}{N}\right)$$

$$f_{samp} = N/T_{obs} = N f_\Delta$$

Lecture 7: OFDM

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$$= \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

The above gives a formula for how to compute the samples of the OFDM signal

Lecture 7: OFDM

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$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

Let us now compute the Fourier transform of the samples
(as of now, for no particular reason)

$$X(\nu) = \sum_{n=0}^{N-1} x_n \exp\left(-i2\pi\nu n\right)$$

Lecture 7: OFDM

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Take N samples of this Fourier transform
(as of now, for no particular reason)

$$X_m = X(m/N) = \sum_{n=0}^{N-1} x_n \exp\left(-\frac{i2\pi mn}{N}\right)$$

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\} \quad 0 \leq t \leq T_{\text{obs}}$$

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$$X_m = \sum_{n=0}^{N-1} x_n \exp\left(-\frac{i2\pi mn}{N}\right)$$

DFT

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

IDFT

Lecture 7: OFDM

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Let us now compute the Fourier transform of the samples (as of now, for no particular reason)

IDFT is <u>VERY</u> fast.
We can get x_n FAST if we know X_m

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Take N samples of this Fourier transform (as of now, for no particular reason)

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IDFT

Lecture 7: OFDM

Logics

We have $\{a_n\}$

We need $\{x_n\}$

We know that $$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

And that this can be computed FAST

Lecture 7: OFDM

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And that this can be computed FAST

Conclusion: We need to link $\{a_n\}$ and $\{X_m\}$

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi(g_0 + k)n}{N}\right)$$

Lecture 7: OFDM

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$$g_0 < 0$$

$$= \sum_{k=0}^{-g_0-1} a_k \exp\left(\frac{i2\pi (g_0 + k) n}{N}\right) + \sum_{k=-g_0}^{K-1} a_k \exp\left(\frac{i2\pi (g_0 + k) n}{N}\right)$$

Lecture 7: OFDM

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Lecture 7: OFDM

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Variable substitutions

$$[m = g_0 + k + N] \qquad [m = g_0 + k]$$

Lecture 7: OFDM

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Variable substitutions

$[m = g_0 + k + N]$ $\qquad$ $[m = g_0 + k]$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi(g_0+k)n}{N}\right)$$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi (g_0+k) n}{N}\right)$$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

This we know from the IDFT

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi (g_0 + k) n}{N}\right)$$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Thus,

$$\boxed{X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}}$$

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi (g_0 + k) n}{N}\right)$$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Thus,

$$\boxed{\begin{aligned} X_m &= N a_{m-g_0}, \quad 0 \le m \le g_{K-1} \\ X_m &= N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N-1 \end{aligned}}$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$

$$X_m = Na_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = Na_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below

$$\boxed{\begin{aligned} X_m &= N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1} \\ X_m &= 0, \quad g_{K-1}+1 \leq m \leq g_0 + N - 1 \\ X_m &= N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N-1 \end{aligned}}$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IDFT

$$\boxed{x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)}$$

$$\boxed{\begin{aligned} X_m &= N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1} \\ X_m &= 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1 \\ X_m &= N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1 \end{aligned}}$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IDFT

$$x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Speed: N^2 multiplications

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Altogether, FFT = "Fast Fourier transform"

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N=2^L \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IFFT

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Speed: N^2 multiplications

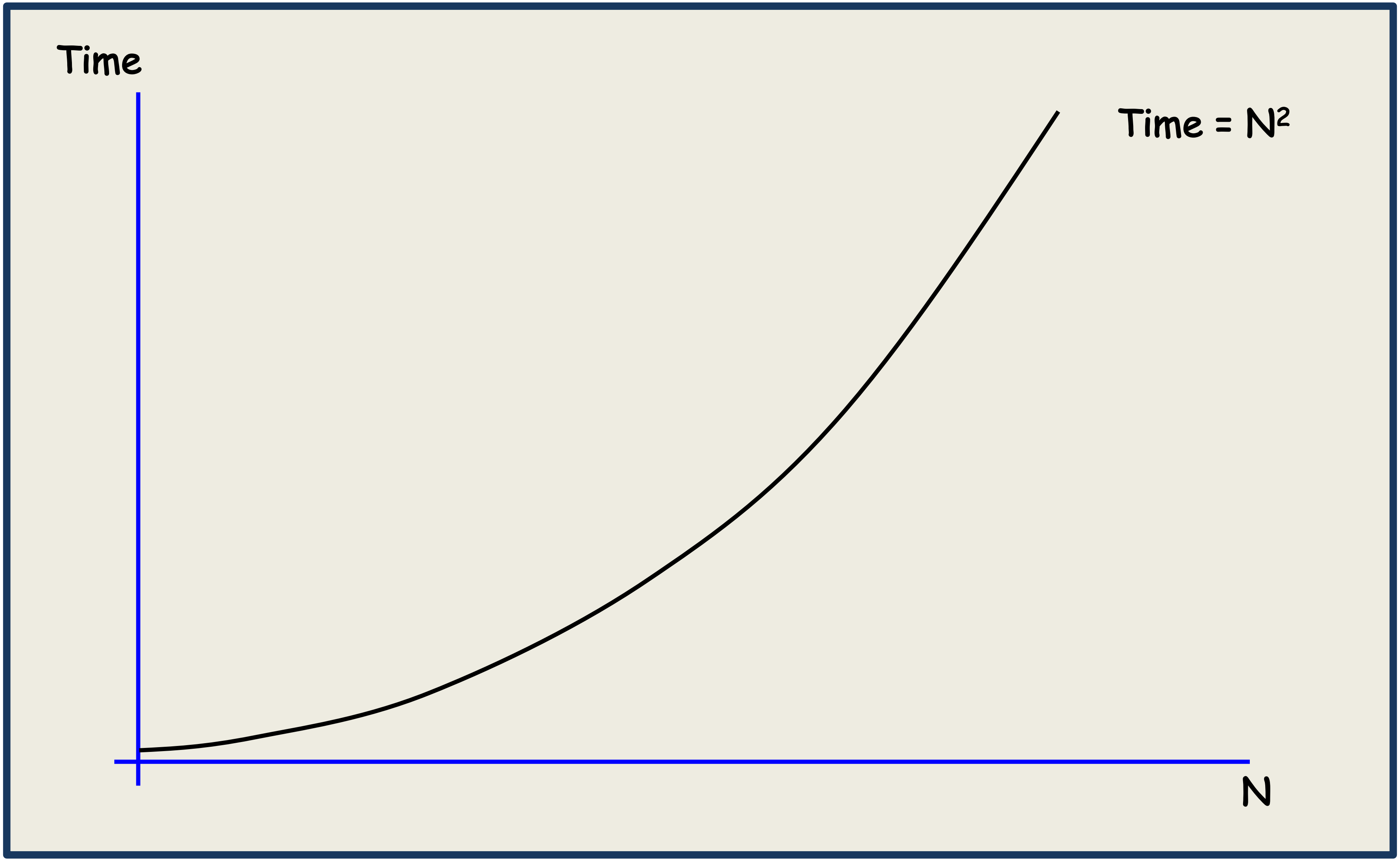
Speed: $N\log_2 N$

$$X_m = Na_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

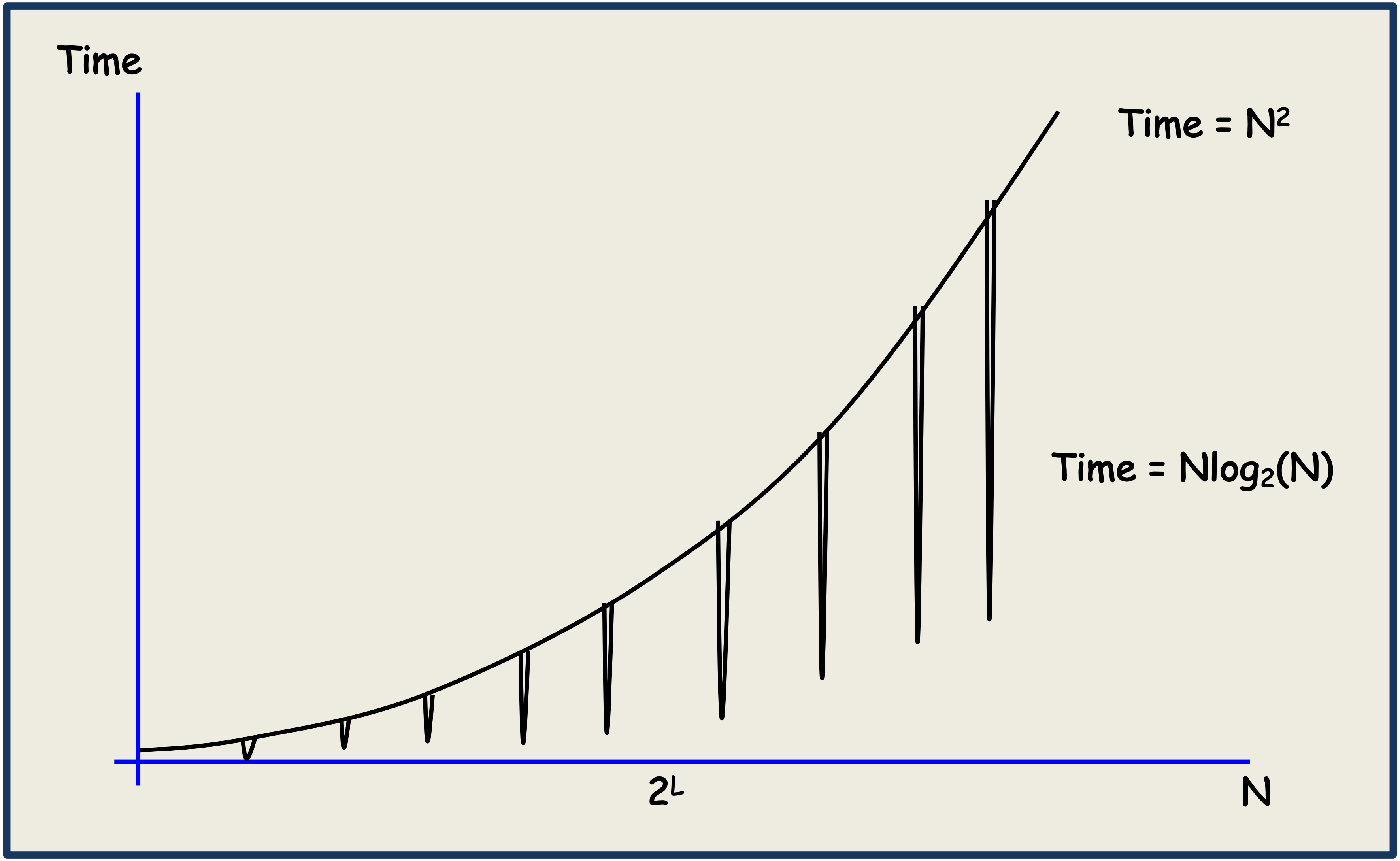
$$X_m = 0, \quad g_{K-1}+1 \leq m \leq g_0 + N - 1$$

$$X_m = Na_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N-1$$

Lecture 7: OFDM



Lecture 7: OFDM



Lecture 7: OFDM

Left to do

1.

2. D/A conversion

3. Modulation to band-pass

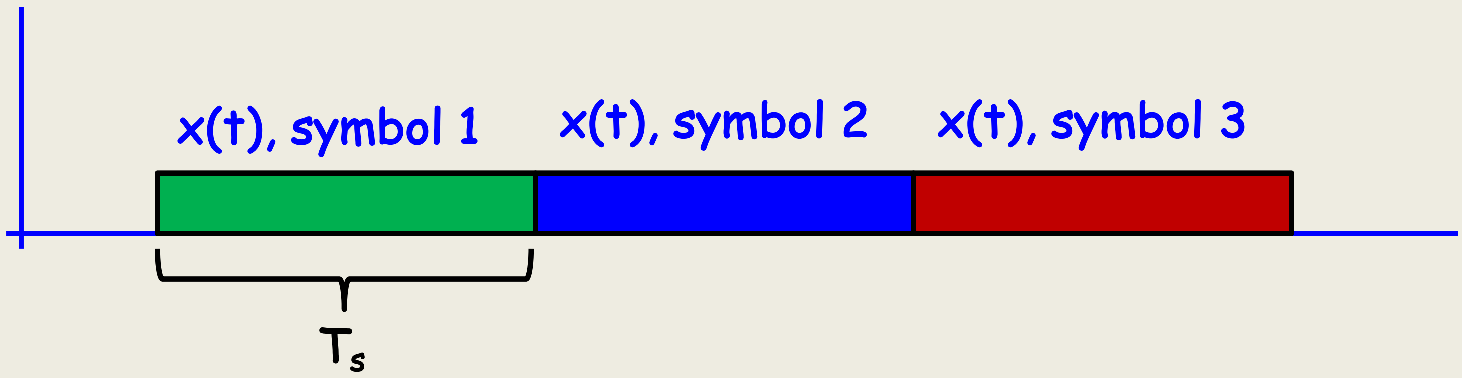
4. Recevier

Lecture 7: OFDM

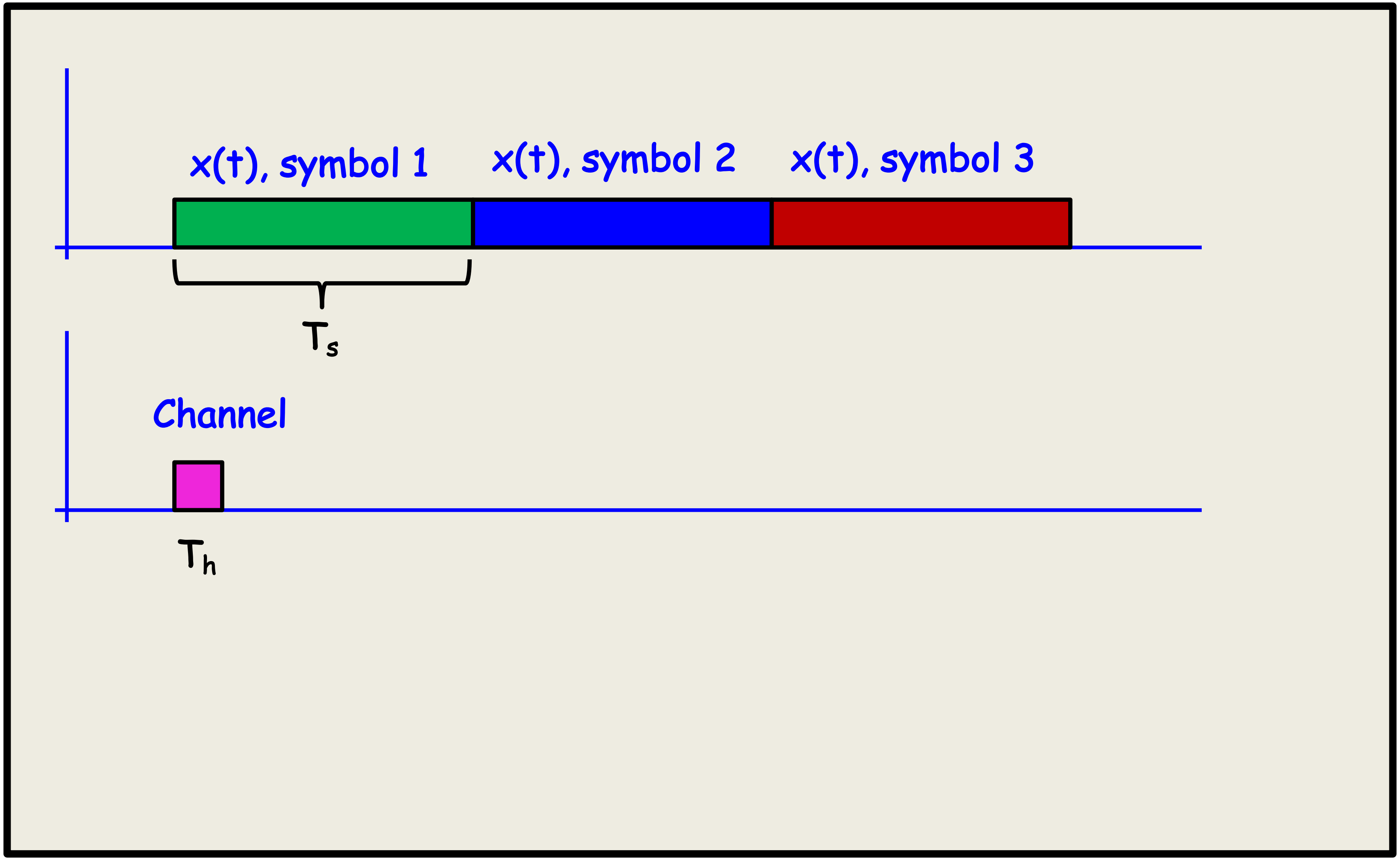
Left to do

1. Define signal in $-T_h < t < 0$ – The cyclic prefix
2. D/A conversion
3. Modulation to band-pass
4. Recevier

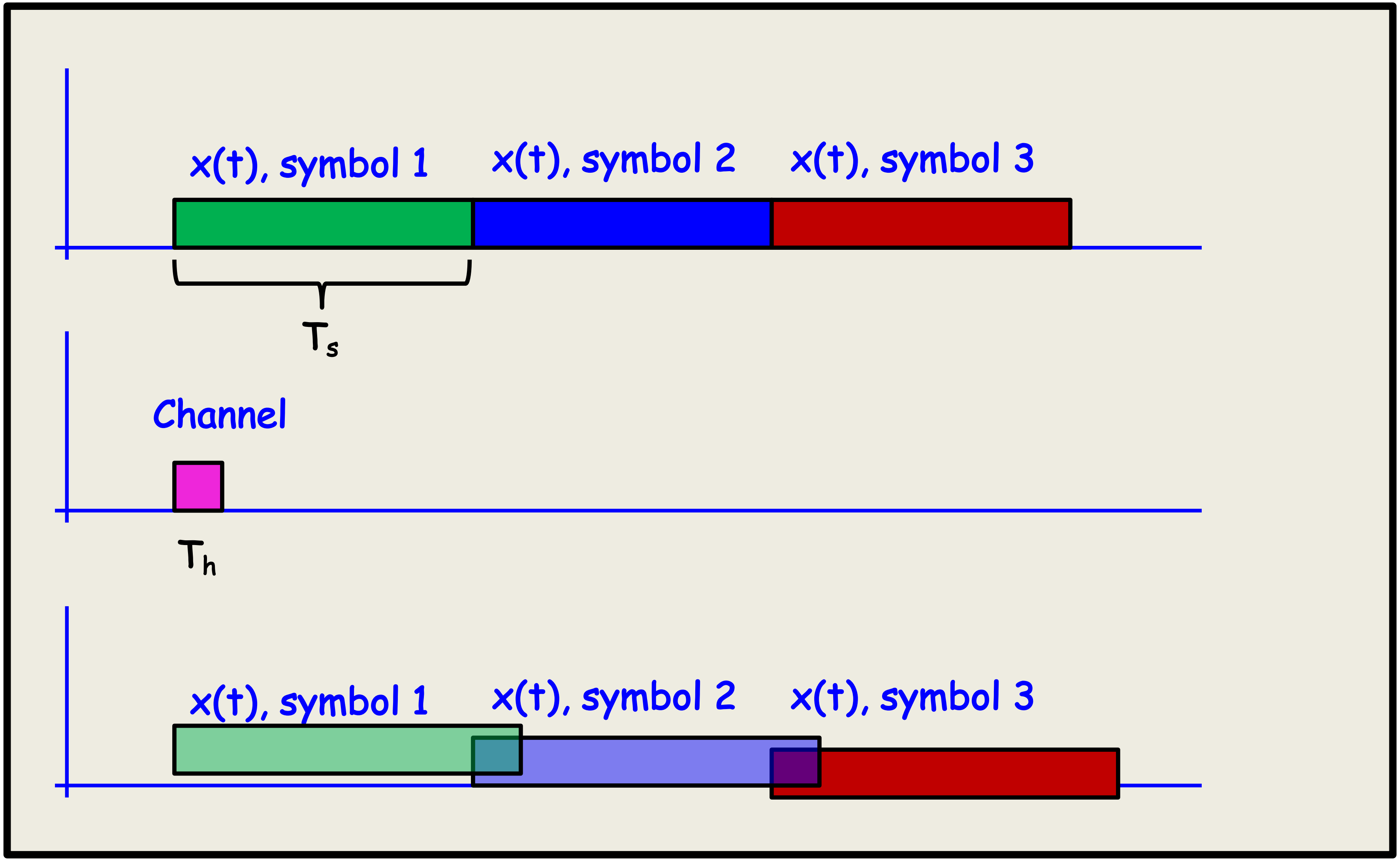
Lecture 7: OFDM



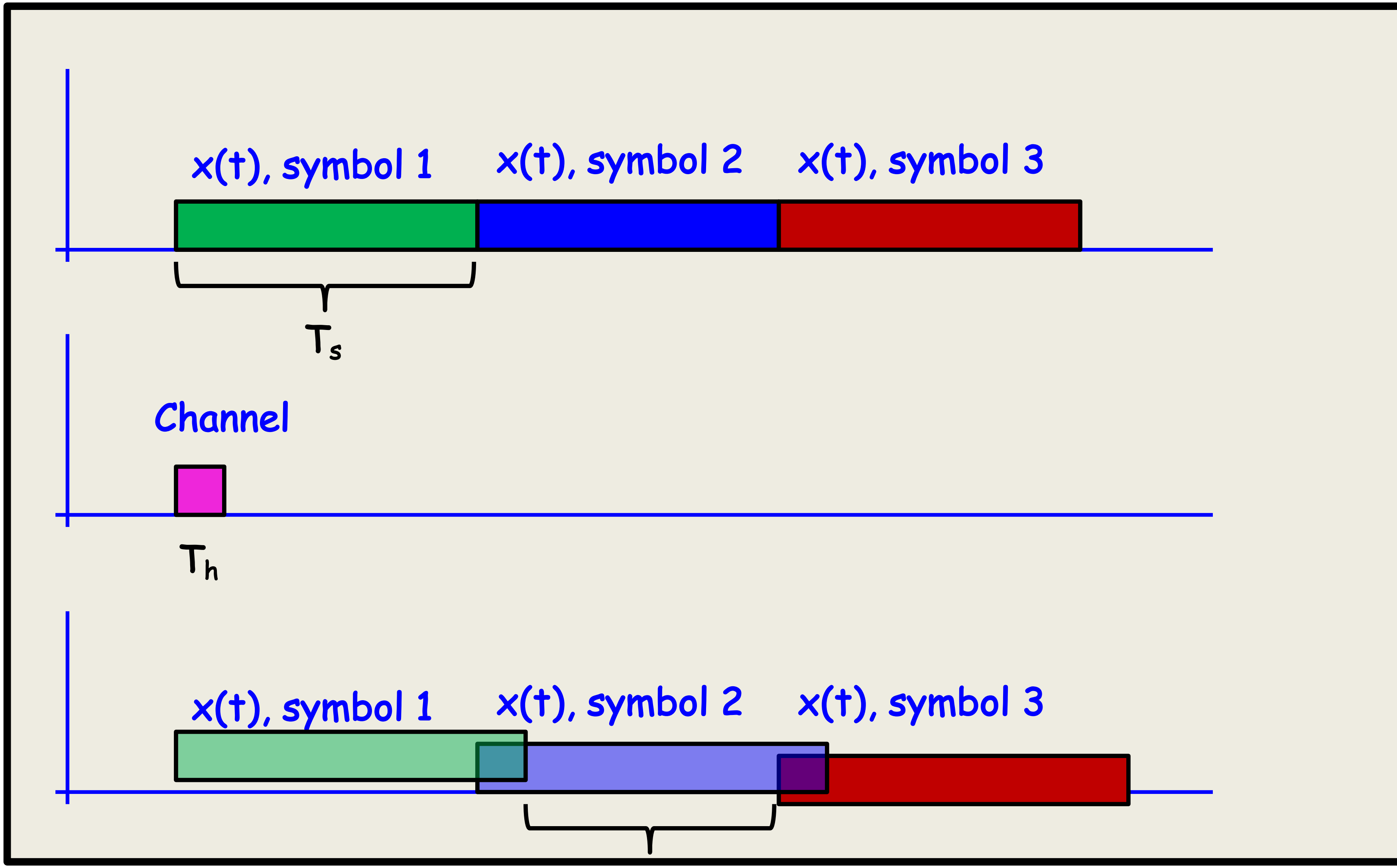
Lecture 7: OFDM



Lecture 7: OFDM

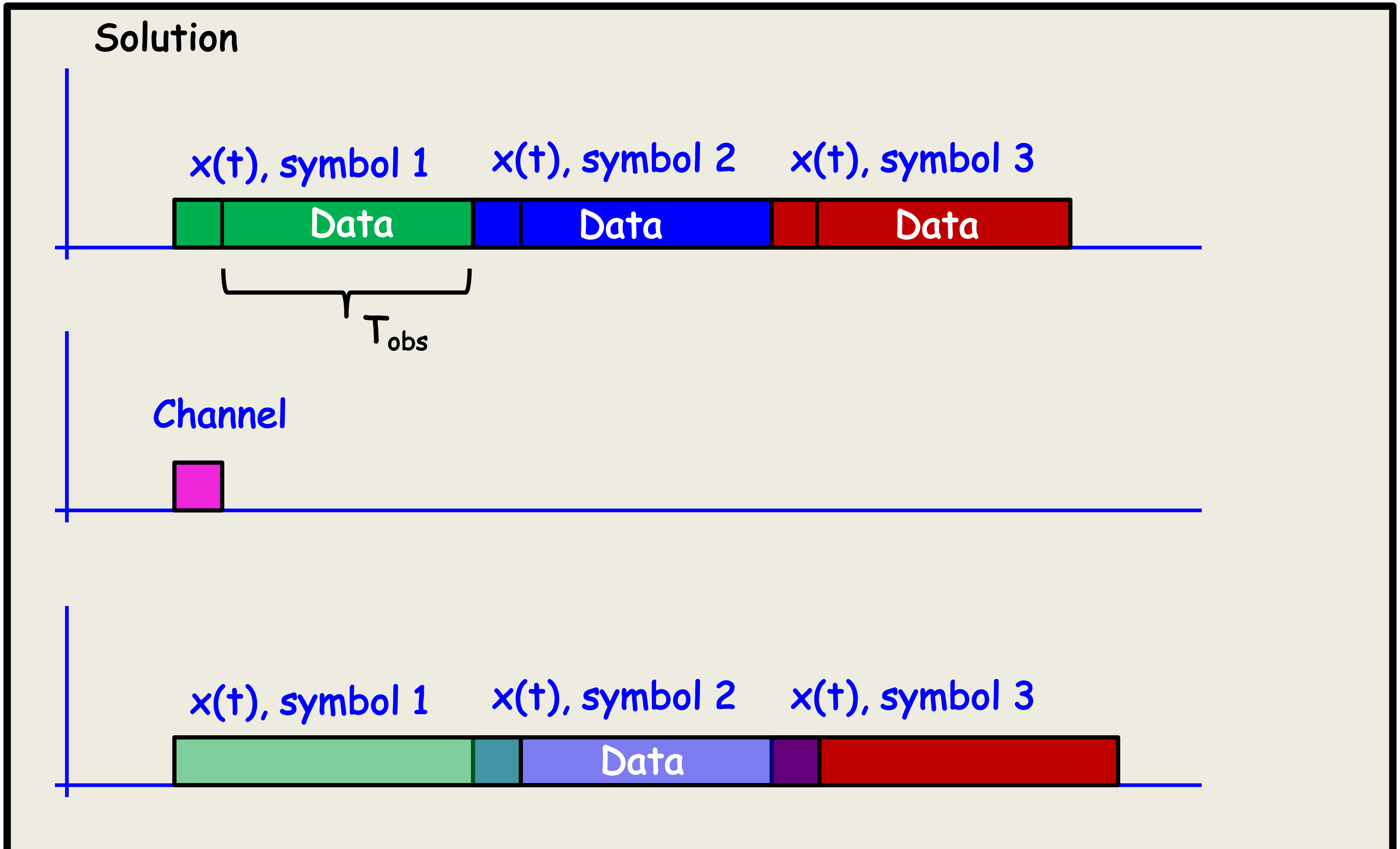


Lecture 7: OFDM

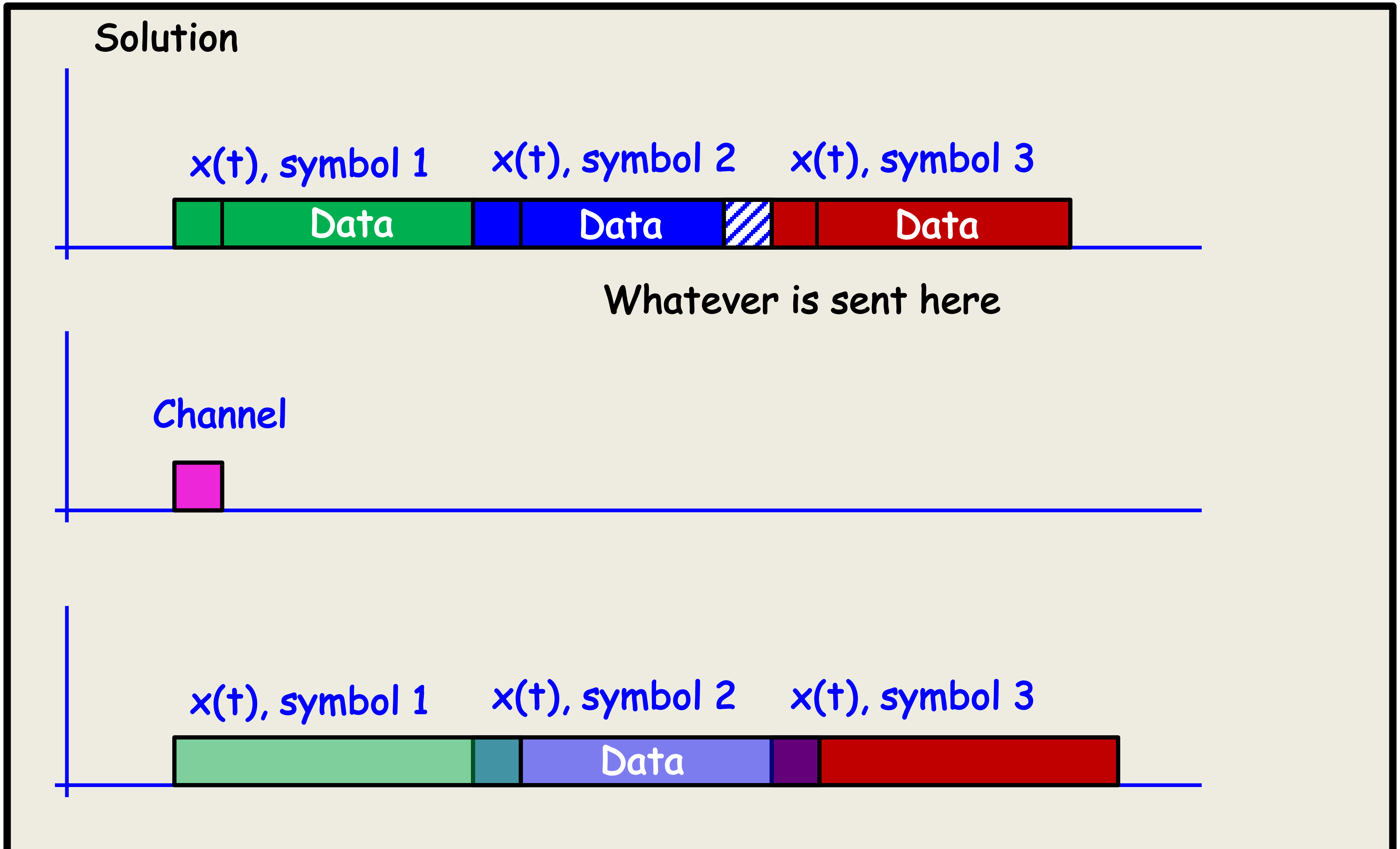


Useful part. T_{obs}

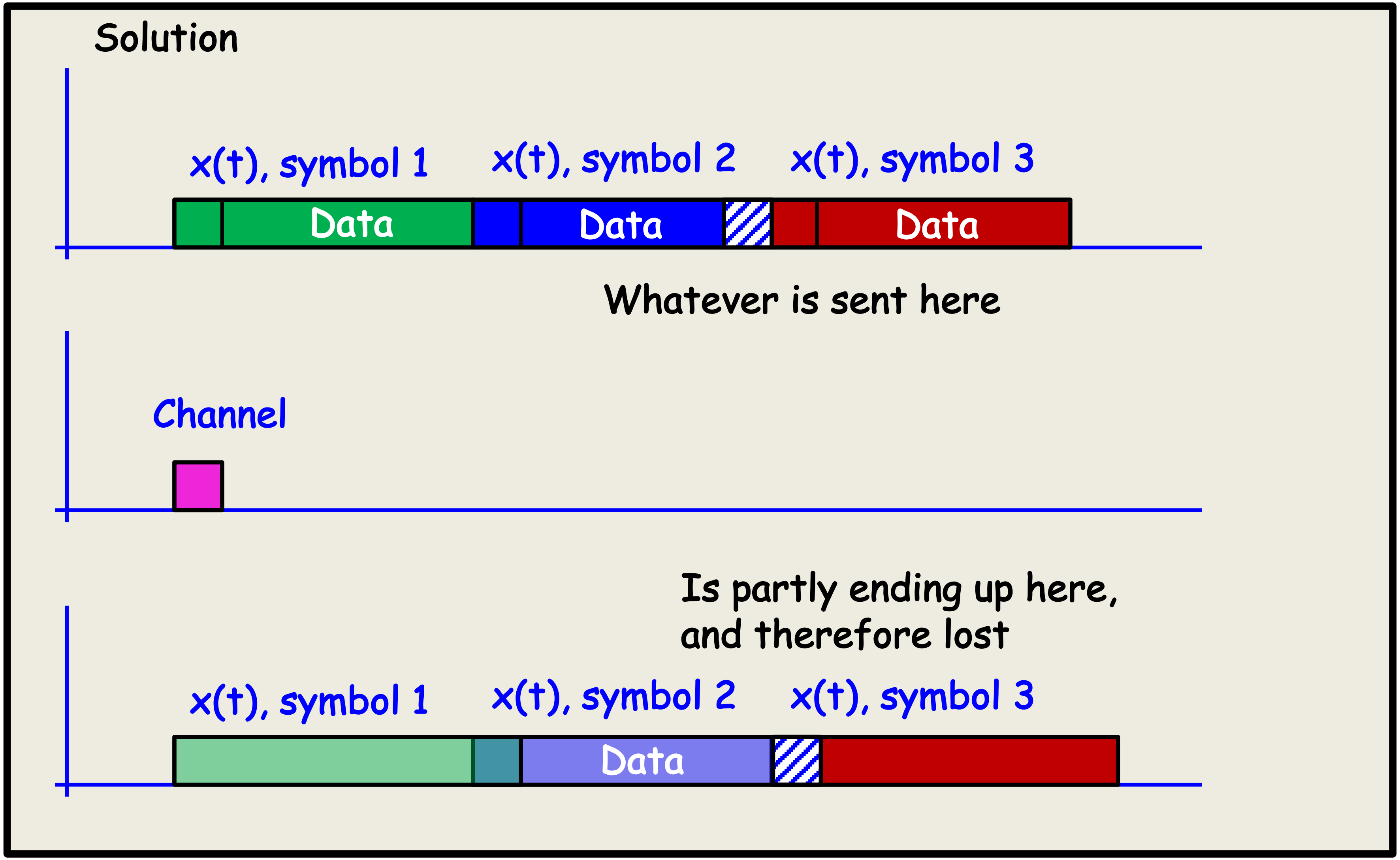
Lecture 7: OFDM



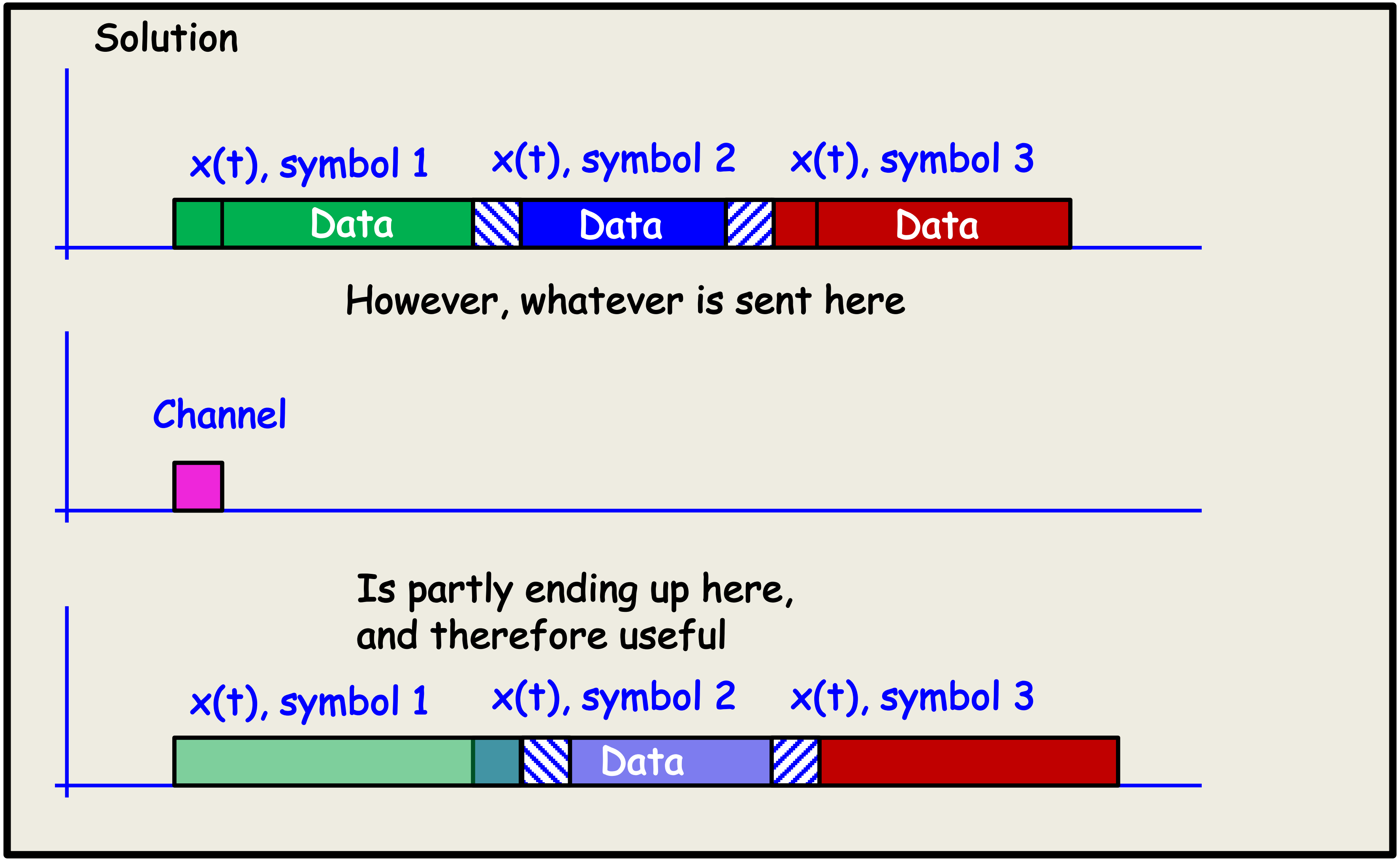
Lecture 7: OFDM



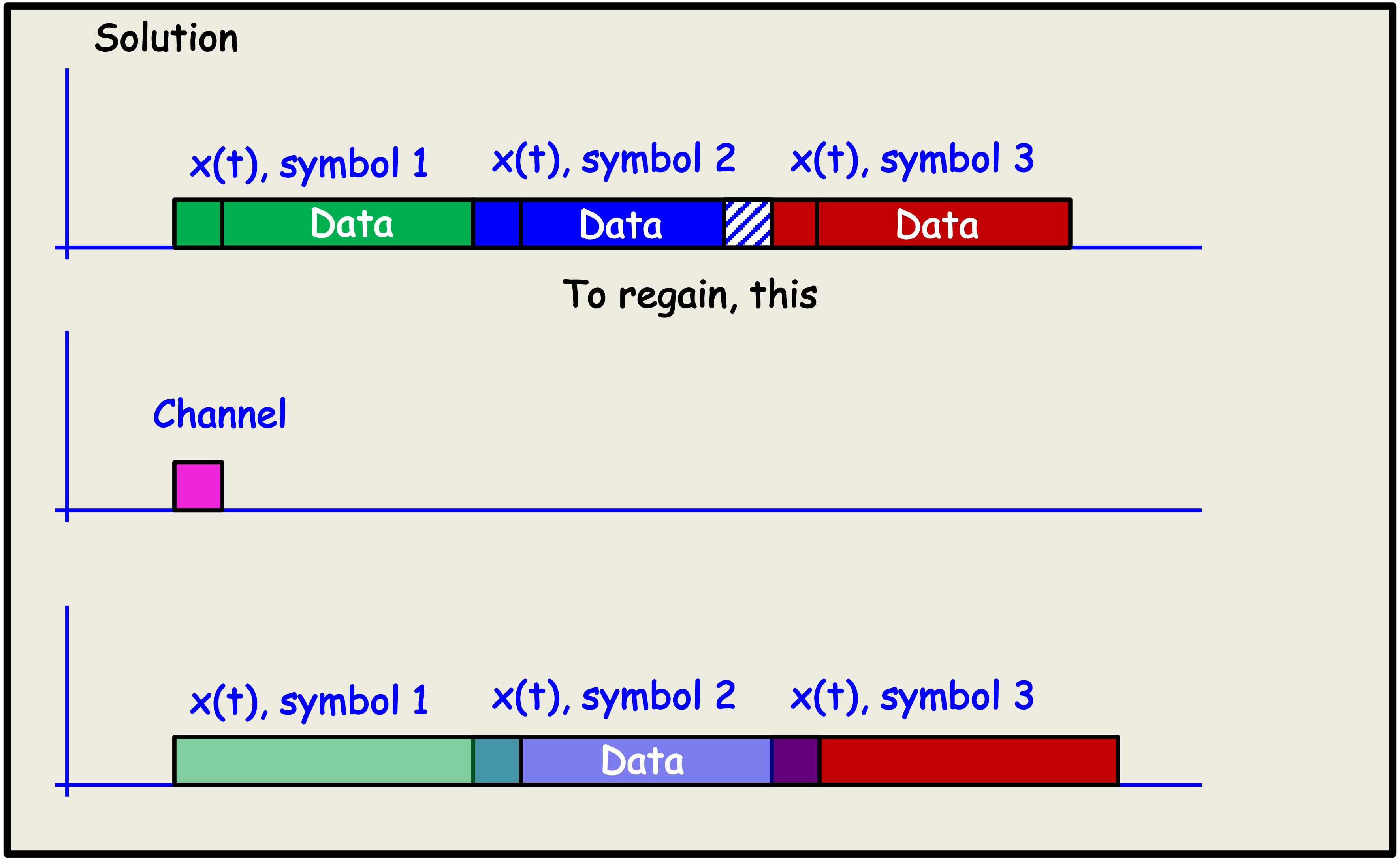
Lecture 7: OFDM



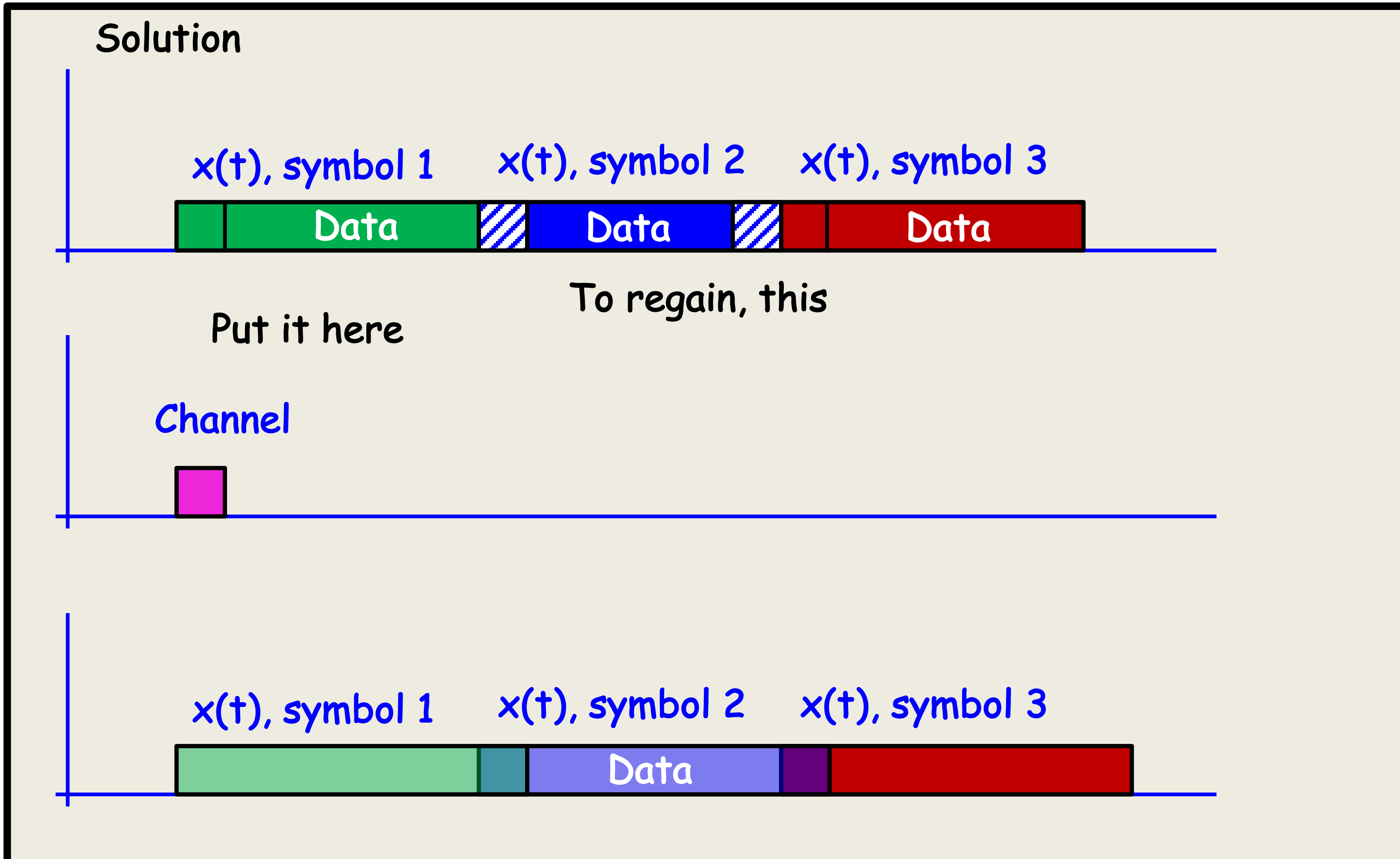
Lecture 7: OFDM



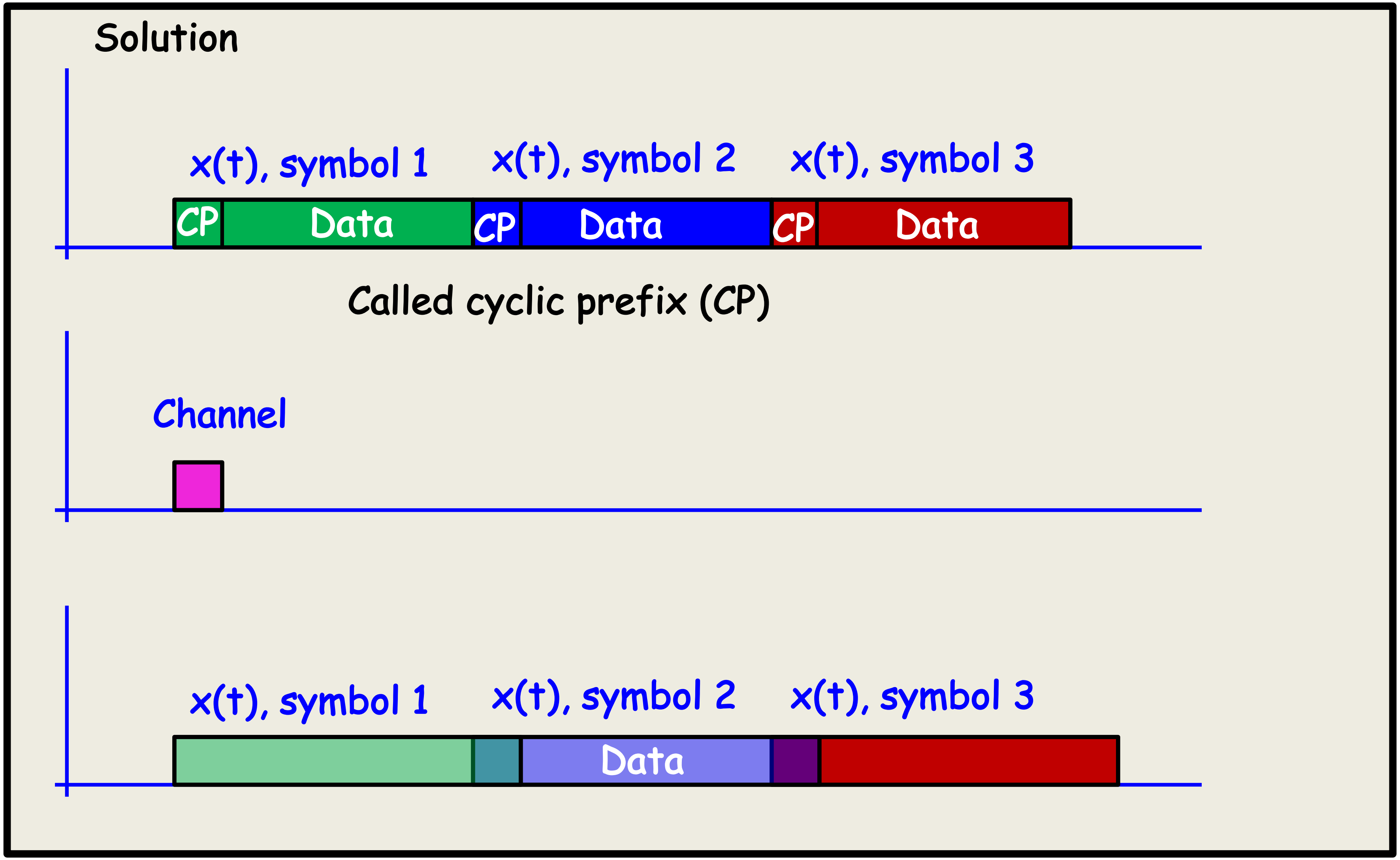
Lecture 7: OFDM



Lecture 7: OFDM

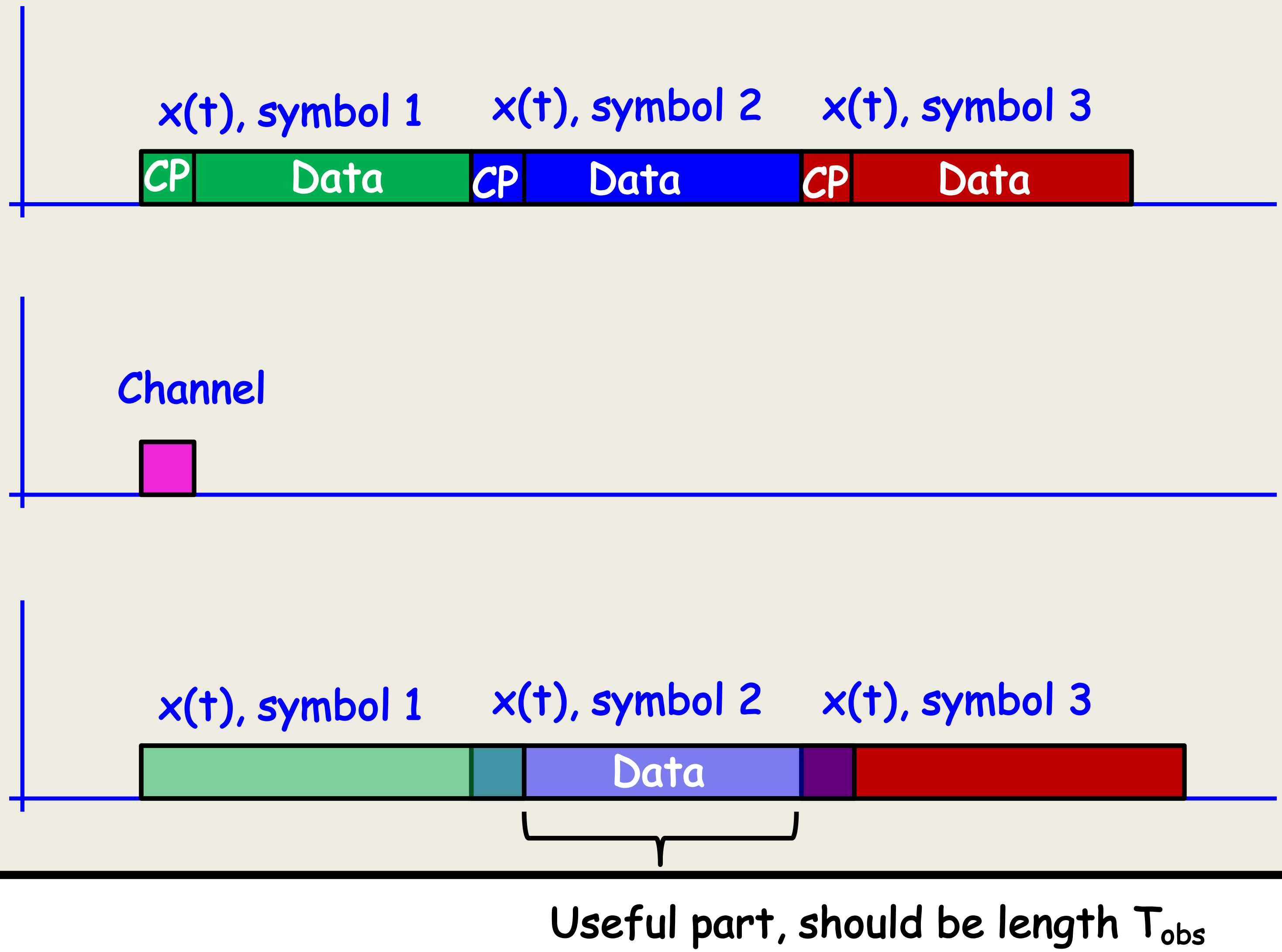


Lecture 7: OFDM

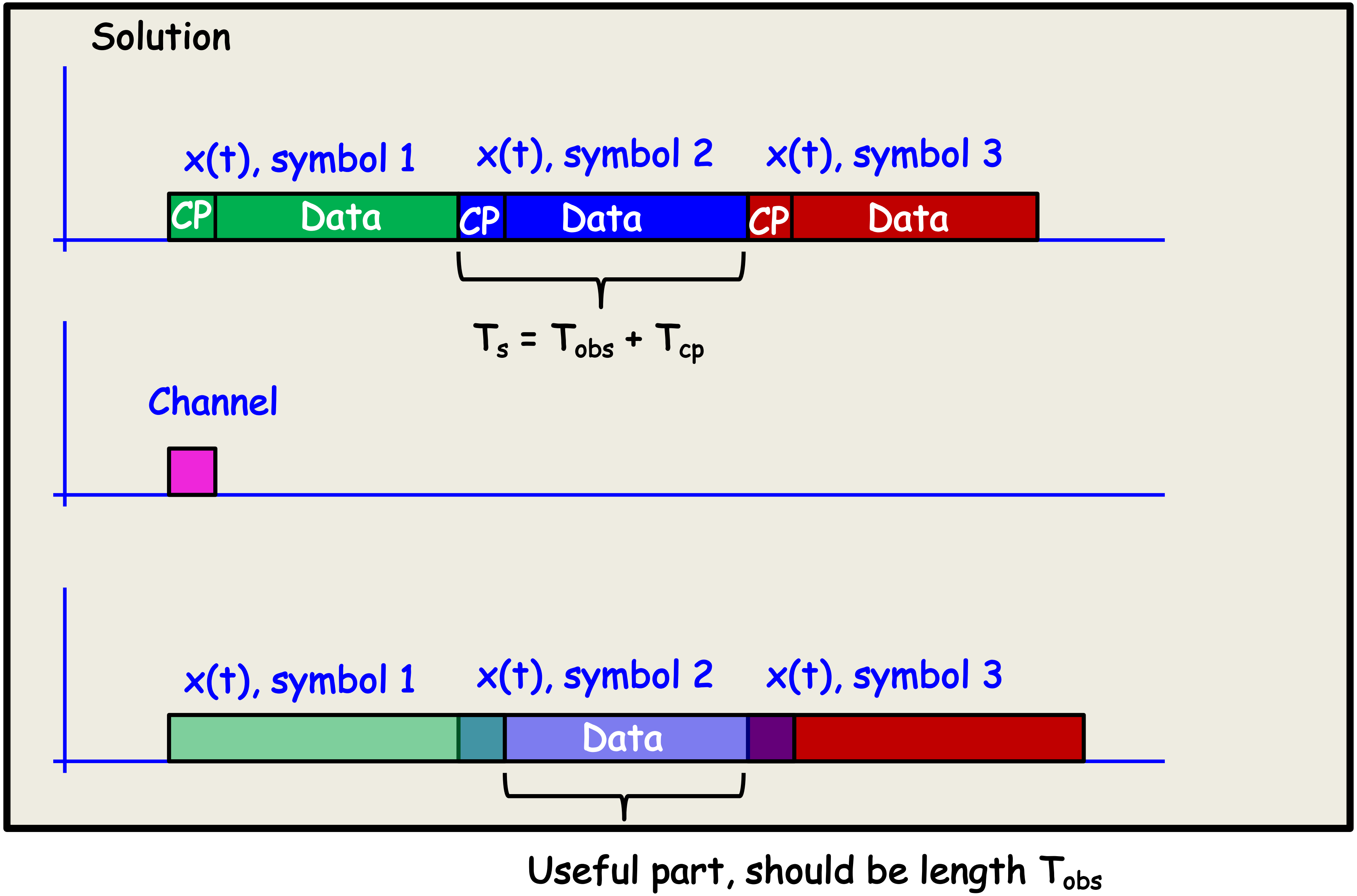


Lecture 7: OFDM

Solution

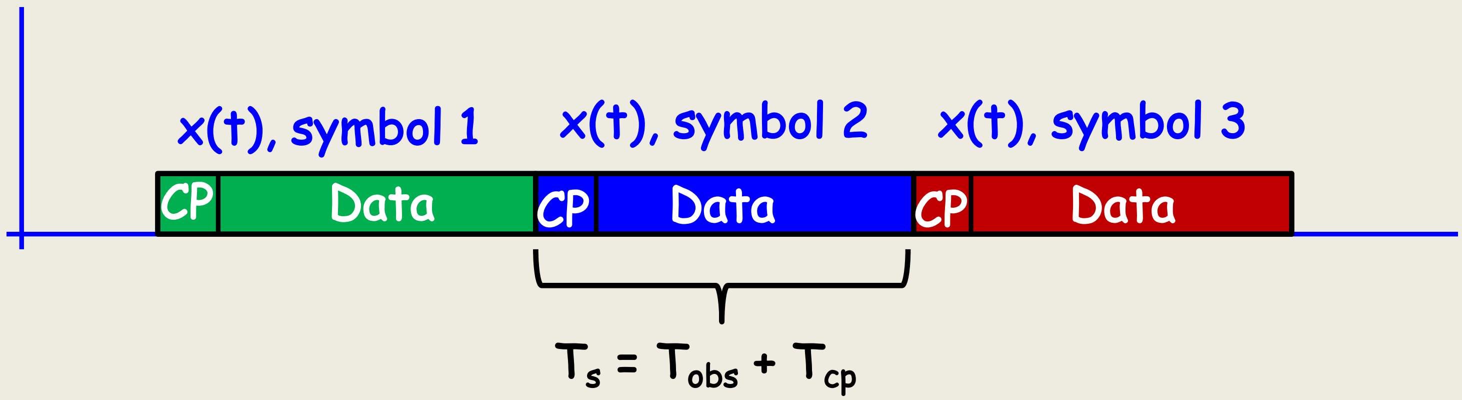


Lecture 7: OFDM



Lecture 7: OFDM

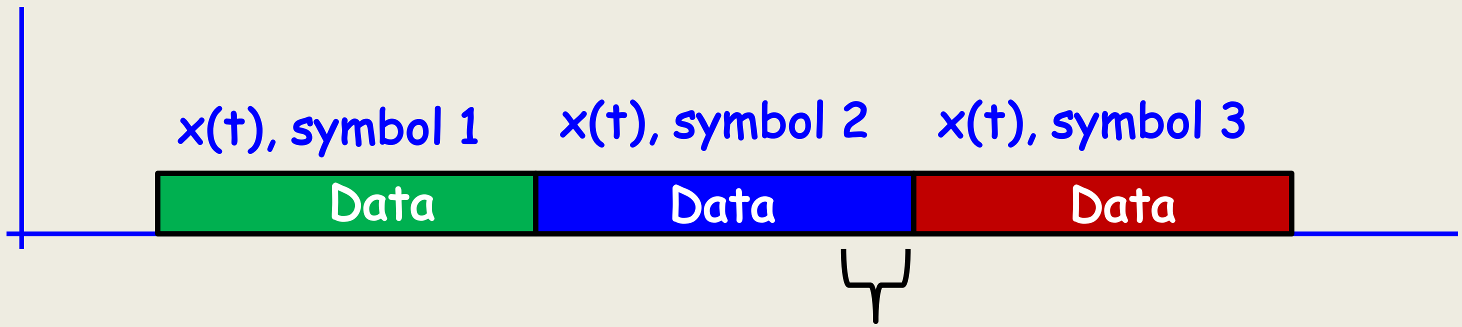
Solution



Summary:

Lecture 7: OFDM

Solution

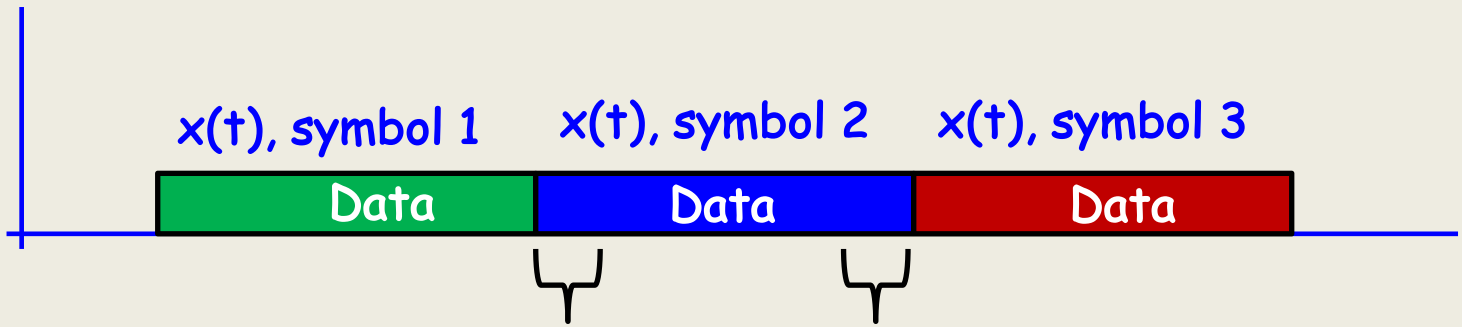


Summary:

1. Marked region is partly lost since it interferes with next block

Lecture 7: OFDM

Solution

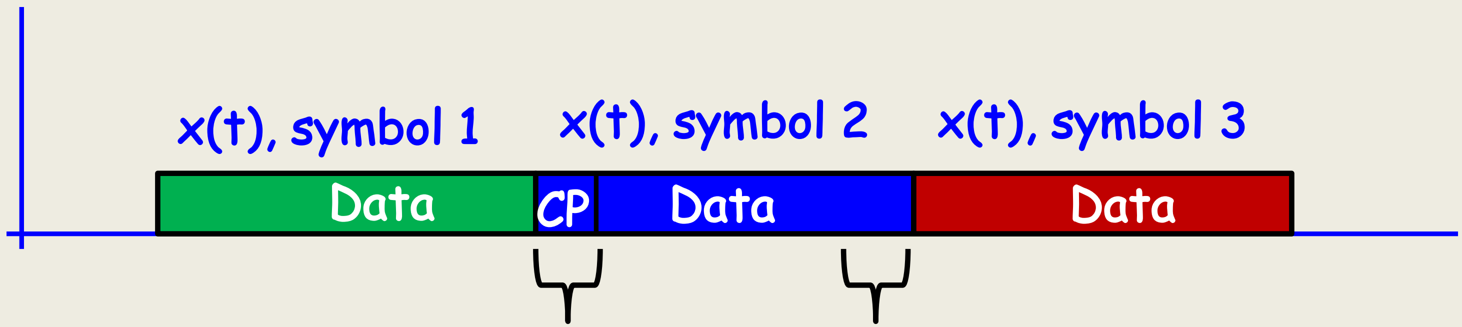


Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it

Lecture 7: OFDM

Solution

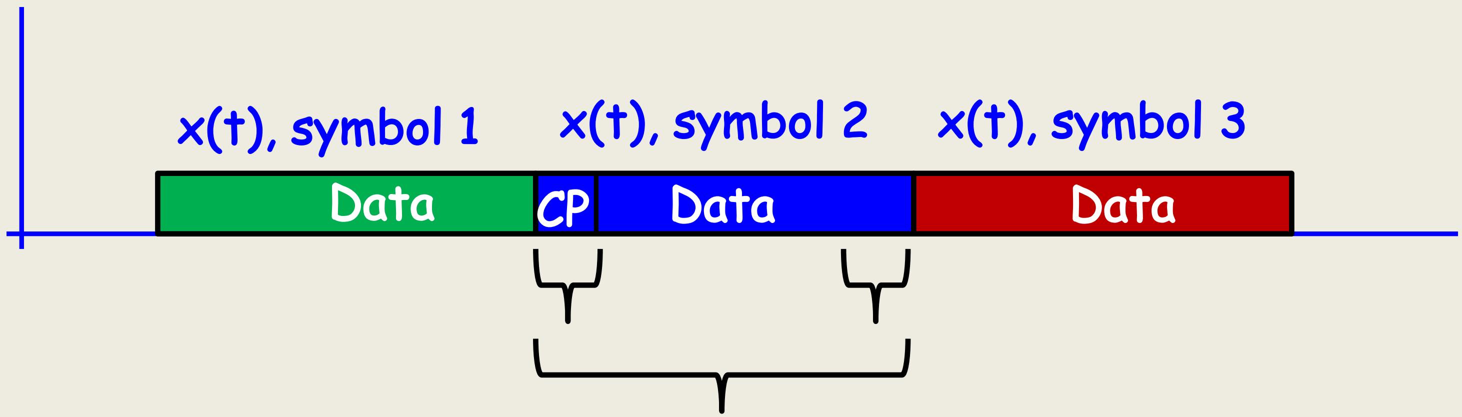


Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP

Lecture 7: OFDM

Solution



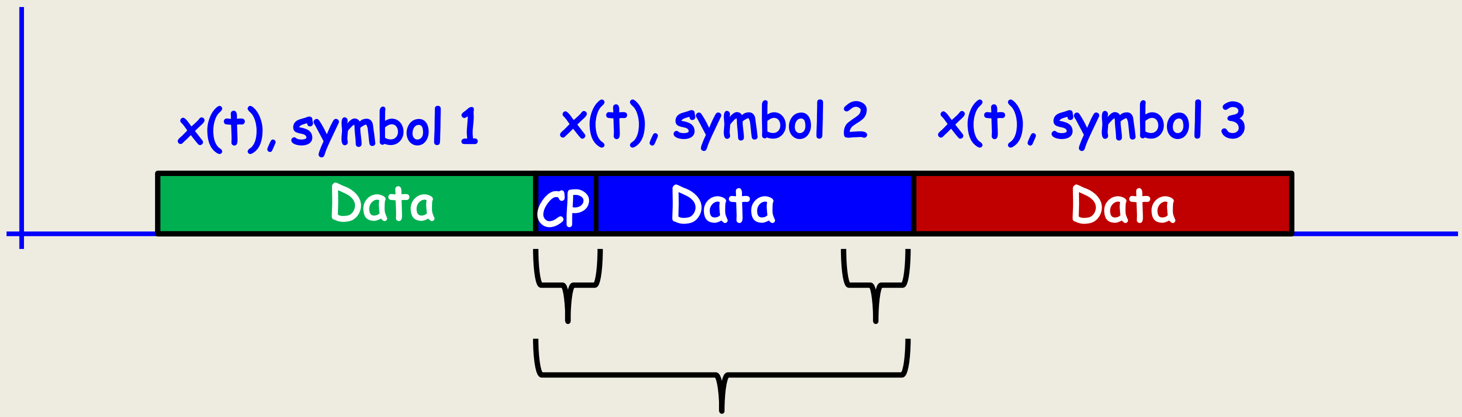
$T_s = T_{obs} + T_{cp}$

Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP
4. Length of data block should be $T_{obs}=1/f_\Delta$

Lecture 7: OFDM

Solution



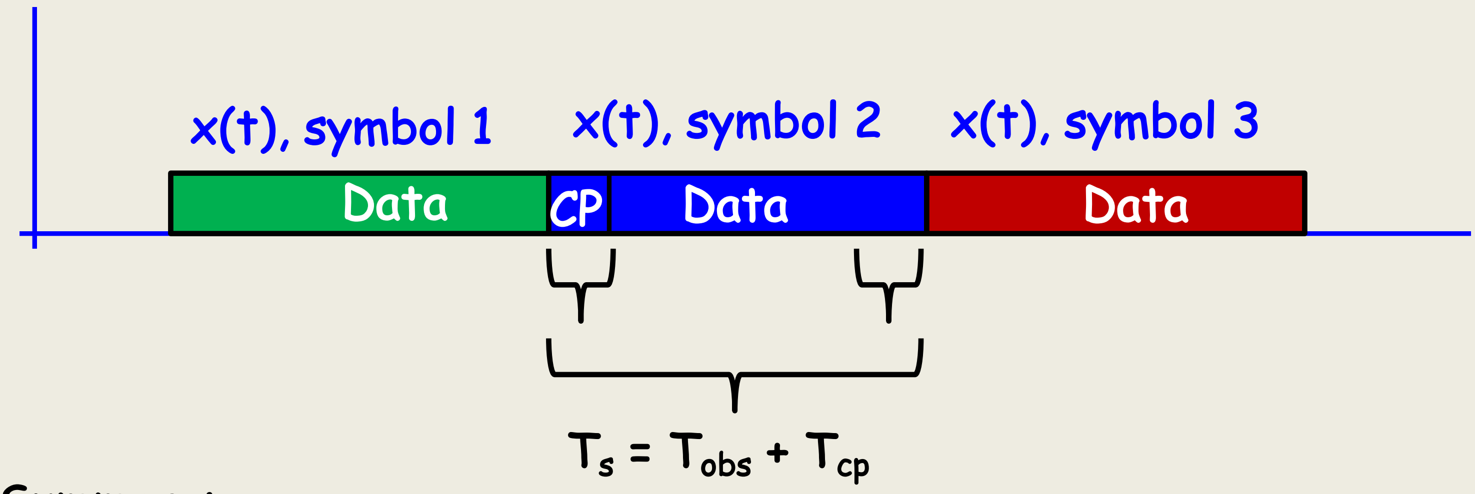
$T_s = T_{obs} + T_{cp}$

Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP
4. Length of data block should be $T_{obs}=1/f_{\Delta}$
5. No proof, yet, why this should be good, but it is plausible, since the lost part is present in the beginning

Lecture 7: OFDM

Solution

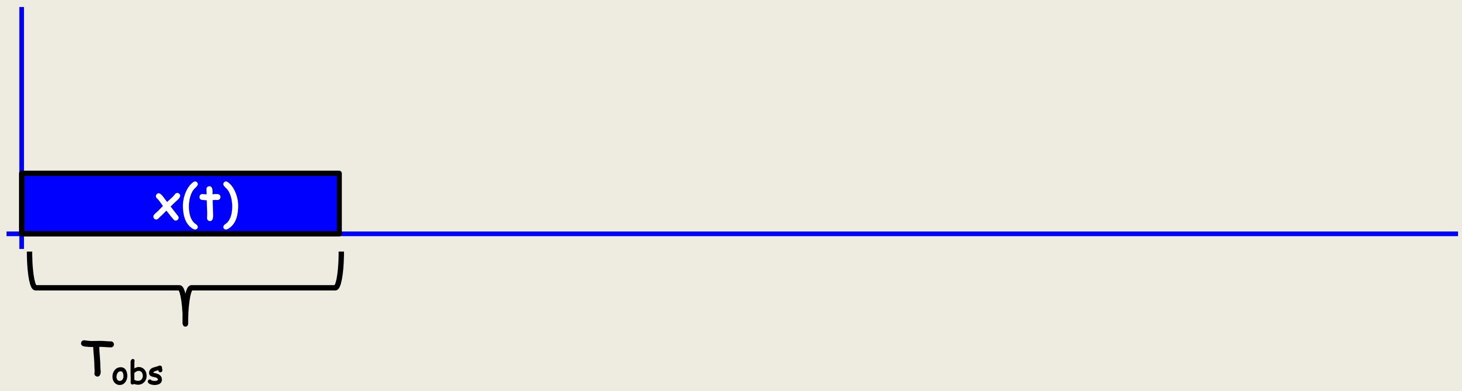


Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP
4. Length of data block should be $T_{obs}=1/f_\Delta$
5. No proof, yet, why this should be good, but it is plausible, since the lost part is present in the beginning
6. Spectral efficiency loss

Lecture 7: OFDM

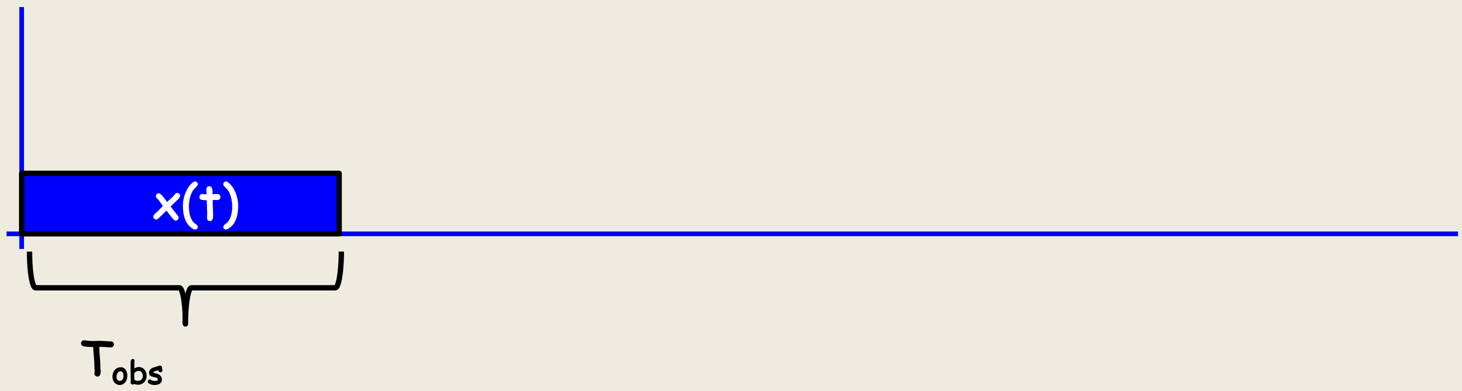
Formulas



$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t), \quad 0 \leq t \leq T_{obs}$$

Lecture 7: OFDM

Formulas

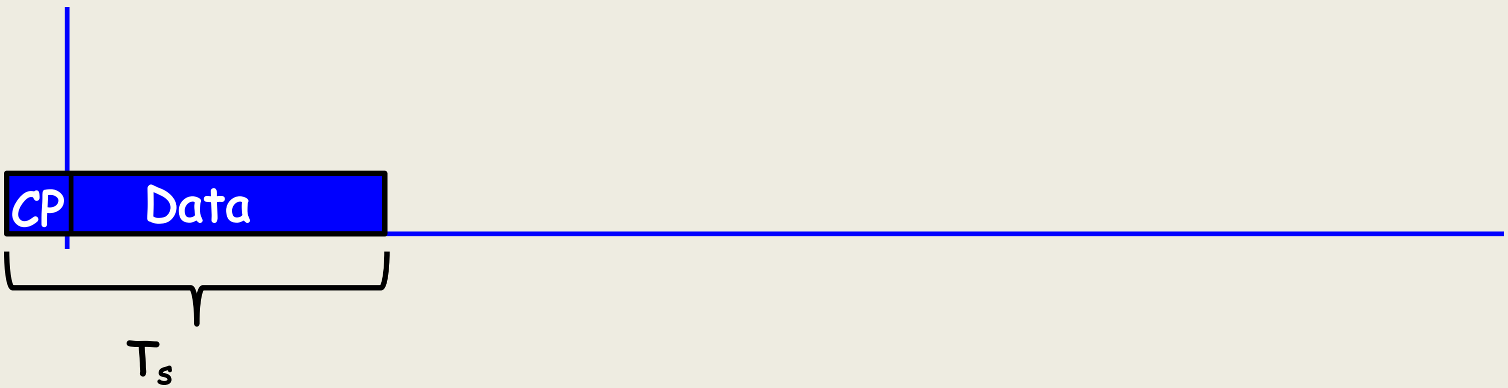


$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t)$$

Periodic with period $T_{obs} = 1/f_\Delta$

Lecture 7: OFDM

Formulas

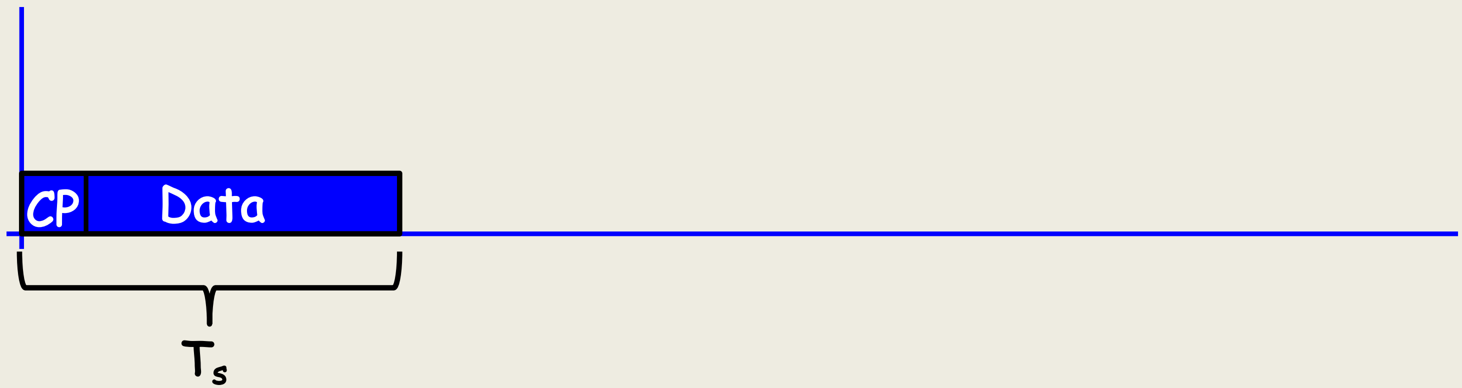


$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t), \quad -T_{cp} \leq t \leq T_{obs}$$

Has properties that we want: first part (left of 0) equals last part

Lecture 7: OFDM

Formulas



$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp}), \quad 0 \leq t \leq T_s$$

Make causal

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IDFT
5. Add last L samples to the beginning

$$x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Left to do

1. Define signal in $-T_h < t < 0$ – The cyclic prefix
2. D/A conversion
3. Modulation to band-pass
4. Receiver

Explanation of 1
comes in 4

Warning: numbering of steps is not the same as in the compendium

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp})), \quad 0 \le t \le T_s$$

Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp})), \quad 0 \le t \le T_s$$

$$u_n = \text{IFFT}(\mathbf{X})$$ Temporary variable

$$\begin{cases} X_m = N a_{m-g_0}, & 0 \le m \le g_{K-1} \\ X_m = 0, & g_{K-1}+1 \le m \le g_0 + N - 1 \\ X_m = N a_{m-(g_0+N)}, & g_0 + N \le m \le N-1 \end{cases}$$

$$\mathbf{x} = [\underbrace{u_{N-L}\; u_{N-L+1}\; \ldots u_{N-1}}_{\text{CP}}\; u_0\; \ldots u_{N-1}]$$

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp})), \quad 0 \le t \le T_s$$

$$u_n = \mathrm{IFFT}(\mathbf{X})$$

$$\begin{cases} X_m = N a_{m-g_0}, & 0 \le m \le g_{K-1} \\ X_m = 0, & g_{K-1}+1 \le m \le g_0 + N - 1 \\ X_m = N a_{m-(g_0+N)}, & g_0 + N \le m \le N-1 \end{cases}$$

D/A

$$\mathbf{x} = [\underbrace{u_{N-L}\ u_{N-L+1}\ \ldots u_{N-1}}_{CP}\ u_0\ \ldots u_{N-1}]$$

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$u_n = \mathrm{IFFT}(\mathbf{X})$$

$$\begin{cases} X_m = Na_{m-g_0}, & 0 \le m \le g_{K-1} \\ X_m = 0, & g_{K-1}+1 \le m \le g_0 + N - 1 \\ X_m = Na_{m-(g_0+N)}, & g_0 + N \le m \le N-1 \end{cases}$$

$$\mathbf{x} = [\underbrace{u_{N-L}\ u_{N-L+1}\ \ldots u_{N-1}}_{CP}\ u_0\ \ldots u_{N-1}]$$

D/A

Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$\boxed{g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}}$$

$$u_n = \text{IFFT}(\mathbf{X})$$

$$\begin{cases} X_m = N a_{m-g_0}, & 0 \le m \le g_{K-1} \\ X_m = 0, & g_{K-1} + 1 \le m \le g_0 + N - 1 \\ X_m = N a_{m-(g_0+N)}, & g_0 + N \le m \le N - 1 \end{cases}$$

$$\mathbf{x} = [\underbrace{u_{N-L}\ u_{N-L+1}\ \ldots u_{N-1}}_{CP}\ u_0\ \ldots u_{N-1}]$$

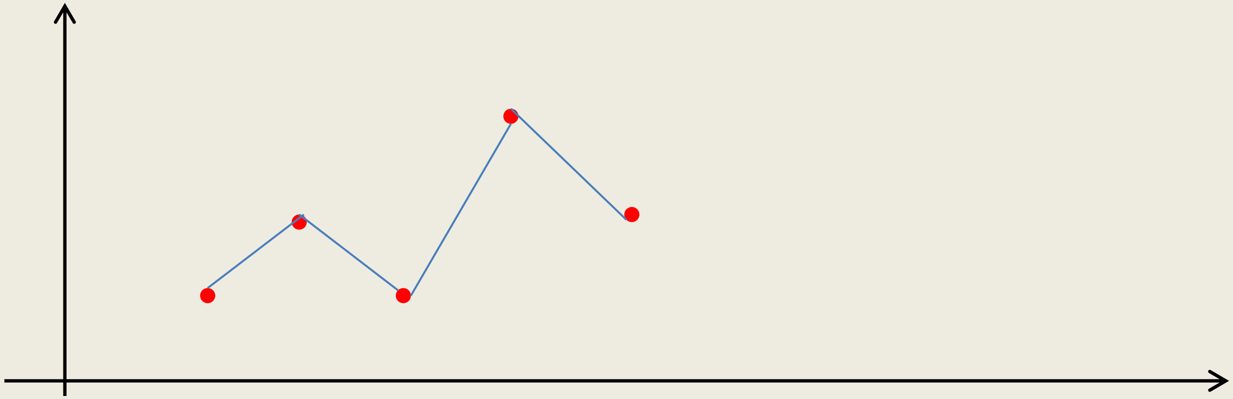
Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{\ell} x_{\ell} g(t - \ell/f_{samp})$$

$$g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}$$

Alternative (suboptimal)



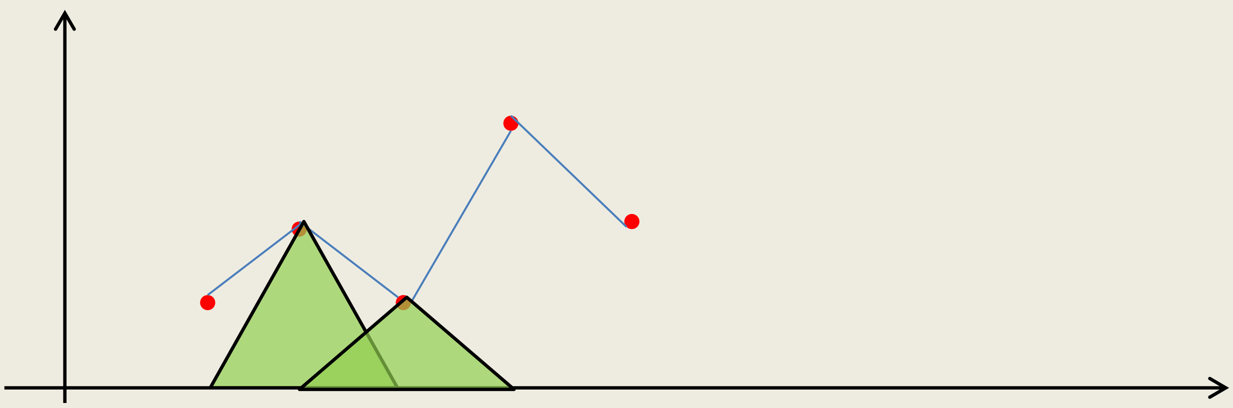
Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})$$

$$g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}$$

Alternative (suboptimal)



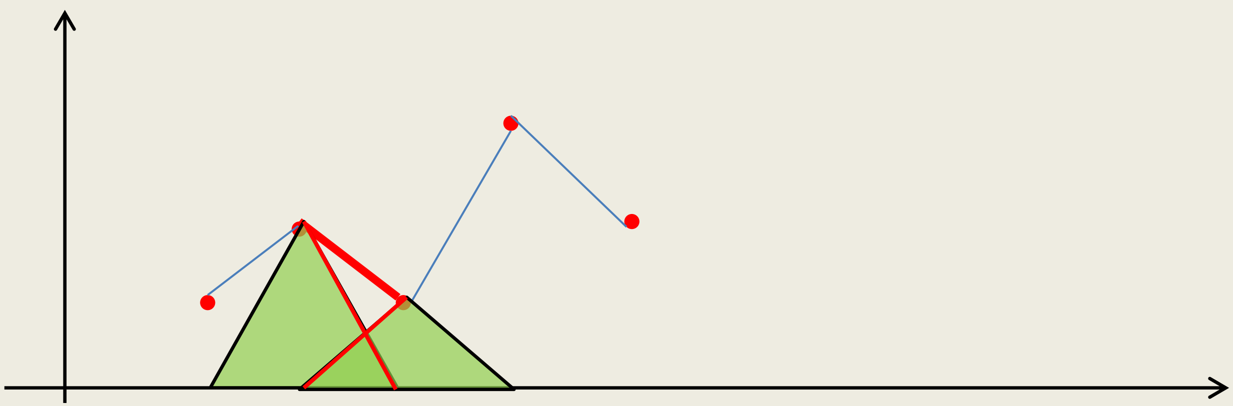
Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$\boxed{g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}}$$

Alternative (suboptimal)



Lecture 7: OFDM

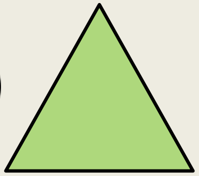
$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

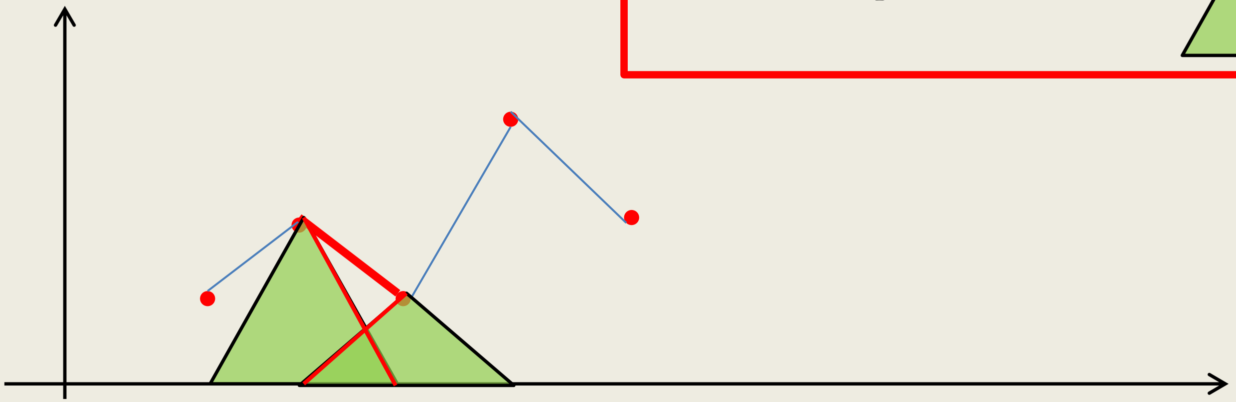
$$x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})$$

$$g_{ideal}(t) = \frac{\sin(\pi f_{samp}t)}{\pi f_{samp}t}$$

Alternative (suboptimal)

$$g_{simple}(t) = \text{tri}(t)$$





Lecture 7: OFDM

Left to do

1. Define signal in $-T_h < t < 0$ – The cyclic prefix
2. D/A conversion
3. Modulation to band-pass
4. Receiver

Explanation of 1
comes in 4

Warning: numbering of steps is not the same as in the compendium

Lecture 7: OFDM

Left to do

1. Define signal in $-T_h < t < 0$ – The cyclic prefix
2. D/A conversion
3. Modulation to band-pass

   Very simple

   $$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

4. Receiver

Warning: numbering of steps is not the same as in the compendium

Lecture 7: OFDM

Left to do

1. Define signal in $-T_h < t < 0$ – The cyclic prefix
2. D/A conversion
3. Modulation to band-pass $\quad \text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$
4. Receiver Next lecture

Warning: numbering of steps is not the same as in the compendium