Lecture 5: Diversity and more (chapter 5)

Diversity

Major topic in "Multiple antenna systems, EITN10". Briefly treated here

Assume signals of the form $$s_j(t) = \sum_{n=1}^{N} s_{j,n}\phi_n(t), \quad j = 0, \ , 1, \dots, M-1$$

Check: what information is contained above?

Lecture 5: Diversity and more (chapter 5)

Diversity

Major topic in "Multiple antenna systems, EITN10"

Assume signals of the form $$s_j(t) = \sum_{n=1}^{N} s_{j,n}\phi_n(t), \quad j = 0, \ , 1, \ldots, M-1$$

Check: what information is contained above?
M signal alternatives, N dimensional signal space

Lecture 5: Diversity and more (chapter 5)

Diversity

Major topic in "Multiple antenna systems, EITN10"

Assume signals of the form $$s_j(t) = \sum_{n=1}^{N} s_{j,n}\phi_n(t), \quad j = 0, \ , 1, \ldots, M-1$$

Assume a channel such that $$r(t) = z_j(t) + N(t)$$

$$= \sum_{n=1}^{N} \alpha_n s_{j,n}\phi_n(t), \quad j = 0, \ , 1, \ldots, M-1$$

Lecture 5: Diversity and more (chapter 5)

Diversity

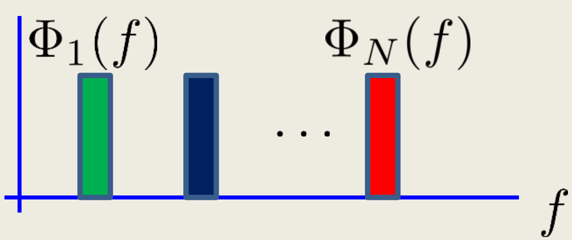
Major topic in "Multiple antenna systems, EITN10"

Assume signals of the form $$s_j(t) = \sum_{n=1}^{N} s_{j,n}\phi_n(t), \quad j = 0, \ , 1, \dots, M-1$$

Assume a channel such that $$r(t) = z_j(t) + N(t)$$

Can happen, e.g., if

$$= \sum_{n=1}^{N} \alpha_n s_{j,n}\phi_n(t), \quad j = 0, \ , 1, \dots, M-1$$



Lecture 5: Diversity and more (chapter 5)

Diversity

$$r(t) = \sum_{n=1}^{N} \alpha_n s_{j,n} \phi_n(t), \quad j = 0, \ ,1, \ldots, M-1$$

In signal space, we can write

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

$$r(t) = \sum_{n=1}^{N} \alpha_n s_{j,n} \phi_n(t), \quad j = 0, \ ,1, \ldots, M-1$$

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We are now interested in questions of the following form:

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \quad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

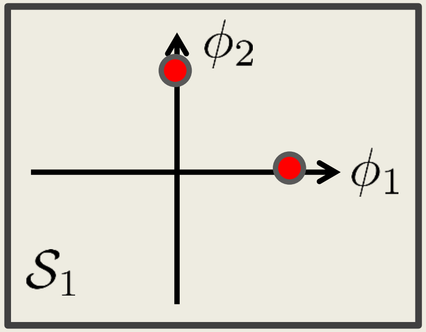
Lecture 5: Diversity and more (chapter 5)

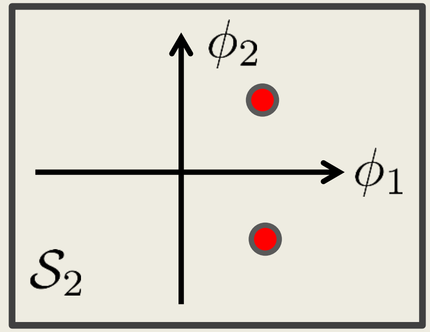
Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let's start with $\alpha_1 = \alpha_2 = 1$

Signal space
(Received)





Difference ?

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

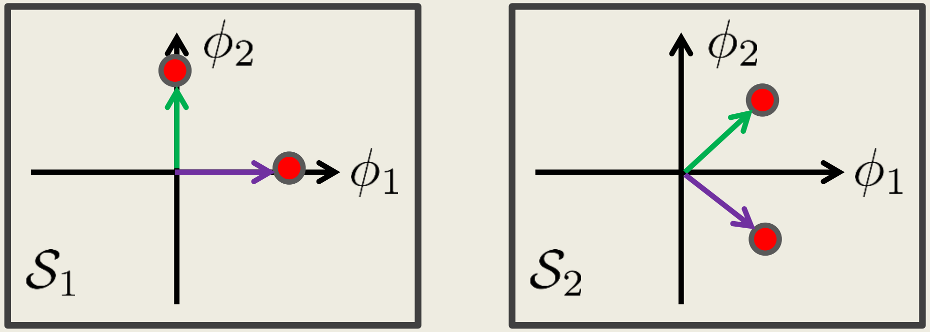
Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let's start with $\alpha_1 = \alpha_2 = 1$

Signal space



Difference ?

Same E_1/E_2

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

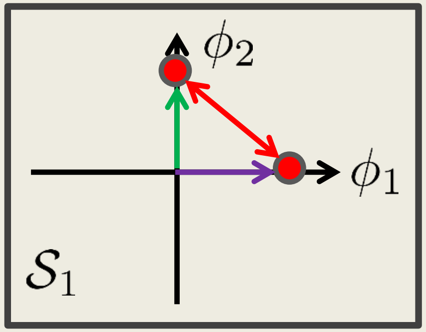
Lecture 5: Diversity and more (chapter 5)

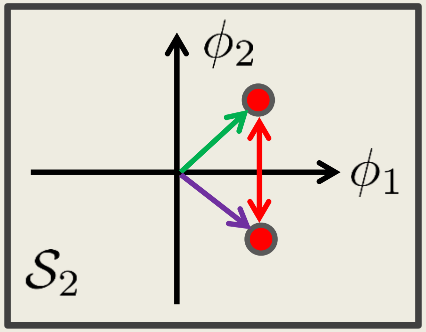
Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let's start with $\alpha_1 = \alpha_2 = 1$

Signal space





Difference ?

Same E_1/E_2

Same $D_{0,1}$

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \quad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

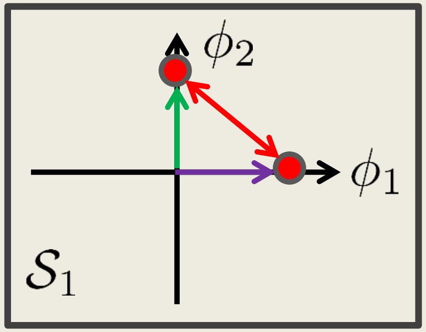
Lecture 5: Diversity and more (chapter 5)

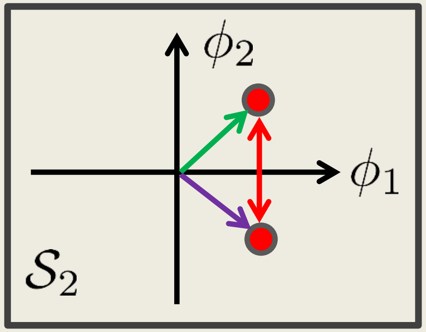
Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let's start with $\alpha_1 = \alpha_2 = 1$

Signal space





Difference ?
Same E_1/E_2
Same $D_{0,1}$

No difference

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \quad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

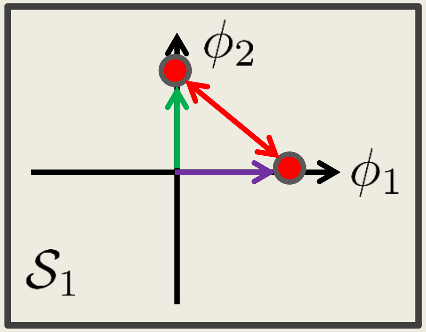
Lecture 5: Diversity and more (chapter 5)

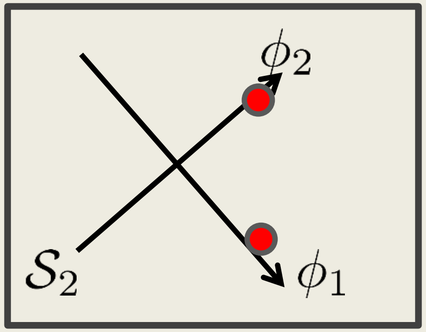
Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let's start with $\alpha_1 = \alpha_2 = 1$

Signal space





Alternative explanation

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

We now move to arbitrary values α_1, α_2

$$P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{\|\mathbf{z}_0 - \mathbf{z}_1\|^2}{2N_0}}\right)$$

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

We now move to arbitrary values α_1, α_2

$$P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{\|\mathbf{z}_0 - \mathbf{z}_1\|^2}{2N_0}}\right) = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}}\right)$$ For S_1

$$\mathbf{z_0} = \begin{bmatrix} \alpha_1\sqrt{2} \\ 0 \end{bmatrix} \quad \mathbf{z_1} = \begin{bmatrix} 0 \\ \alpha_2\sqrt{2} \end{bmatrix}$$

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \quad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

We now move to arbitrary values α_1, α_2

$$P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{\|\mathbf{z}_0 - \mathbf{z}_1\|^2}{2N_0}}\right) = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}}\right)$$ For S_1

$$= Q\left(\sqrt{\frac{2|\alpha_2|^2}{N_0}}\right)$$ For S_2

Assume N=M=2 and two signal sets S_1 and S_2 where

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \quad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

What is the difference between these two?

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let us now compare to a system with N=1

$$\mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

$$P_s = Q\left(\sqrt{\frac{D^2}{2N_0}} \right) = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}} \right) \qquad Q\left(\sqrt{\frac{2|\alpha_2|^2}{N_0}} \right)$$

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let us now compare to a system with N=1

$$\mathcal{S}_3 = \left\{-\sqrt{2},\ \sqrt{2}\right\} \qquad P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)$$

$$\mathcal{S}_1 = \left\{\begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}\right\} \qquad \mathcal{S}_2 = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$$

$$P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}}\right) \qquad = Q\left(\sqrt{\frac{2|\alpha_2|^2}{N_0}}\right)$$

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let us now compare to a system with N=1

$$\mathcal{S}_3 = \left\{-\sqrt{2},\ \sqrt{2}\right\} \qquad P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)$$

3dB better for $\alpha_1 = \alpha_2 = 1$

$$\mathcal{S}_1 = \left\{\begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}\right\} \qquad \mathcal{S}_2 = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$$

$$P_s = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}}\right) \qquad = Q\left(\sqrt{\frac{2|\alpha_2|^2}{N_0}}\right)$$

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let us now move to random channels (and ignore S_2)

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$P_s = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}} \right) \qquad P_s = Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}} \right)$$

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let us now move to random channels

Assume that

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\}$$

$$\mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$P_s = Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}} \right)$$

$$P_s = Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}} \right)$$

These are random

Lecture 5: Diversity and more (chapter 5)

Diversity

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \mathbf{0} \\ & \alpha_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \alpha_N \end{bmatrix} \begin{bmatrix} s_{j,1} \\ \vdots \\ s_{N,1} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

Let us now move to random channels

Assume that

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$P_s = E\left\{ Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}} \right) \right\} \qquad P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}} \right) \right\}$$

Take average

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

Let us now move to random channels

Assume that

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\}$$

$$\mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$P_s = E\left\{ Q\left(\sqrt{\frac{|\alpha_1|^2 + |\alpha_2|^2}{N_0}} \right) \right\}$$

$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}} \right) \right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} =$$

1. α_1 and α_2 are independent

2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{\left[\begin{array}{c}\sqrt{2}\\0\end{array}\right], \left[\begin{array}{c}0\\\sqrt{2}\end{array}\right]\right\} \qquad \mathcal{S}_3 = \left\{-\sqrt{2},\ \sqrt{2}\right\}$$

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} \qquad P_s = \left\{Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} = \Pr(\alpha_1 = \text{bad}, \alpha_2 = \text{bad})\frac{1}{2} + \Pr(\alpha_1 \text{ or } \alpha_2 = \text{good}) \times 0$$

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{\begin{bmatrix}\sqrt{2}\\0\end{bmatrix}, \begin{bmatrix}0\\\sqrt{2}\end{bmatrix}\right\} \qquad \mathcal{S}_3 = \left\{-\sqrt{2}, \sqrt{2}\right\}$$

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} \qquad P_s = \left\{Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} = \Pr(\alpha_1 = \text{bad}, \alpha_2 = \text{bad})\frac{1}{2}$$

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{\begin{bmatrix}\sqrt{2}\\0\end{bmatrix}, \begin{bmatrix}0\\\sqrt{2}\end{bmatrix}\right\} \qquad \mathcal{S}_3 = \left\{-\sqrt{2},\ \sqrt{2}\right\}$$

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} \qquad P_s = \left\{Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} = \Pr(\alpha_1 = \text{bad}, \alpha_2 = \text{bad})\frac{1}{2} = \frac{1}{2}P_B^2$$

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{\begin{bmatrix}\sqrt{2}\\0\end{bmatrix}, \begin{bmatrix}0\\\sqrt{2}\end{bmatrix}\right\} \qquad \mathcal{S}_3 = \left\{-\sqrt{2},\ \sqrt{2}\right\}$$

$$P_s = E\left\{Q\left(\sqrt{\frac{|\alpha_1|^2+|\alpha_2|^2}{N_0}}\right)\right\} \qquad P_s = \left\{Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right) \right\} = \Pr(\alpha_1 = \text{bad})\frac{1}{2} + \Pr(\alpha_1 = \text{good}) \times 0$$

1. α_1 and α_2 are independent

2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{ \left[\begin{array}{c} \sqrt{2} \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ \sqrt{2} \end{array} \right] \right\}$$

$$\mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$P_s = \frac{1}{2} P_B^2$$

$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right) \right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
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$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)\right\} = \Pr(\alpha_1 = \text{bad})\frac{1}{2}$$

1. α_1 and α_2 are independent

2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\}$$

$$P_s = \frac{1}{2}P_B^2$$

$$\mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right) \right\} = \Pr(\alpha_1 = \text{bad})\frac{1}{2} = \frac{1}{2}P_B$$

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

$$\mathcal{S}_1 = \left\{ \left[\begin{array}{c} \sqrt{2} \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ \sqrt{2} \end{array} \right] \right\}$$

$$P_s = \frac{1}{2}P_B^2$$

$$\mathcal{S}_3 = \left\{ -\sqrt{2}, \sqrt{2} \right\}$$

$$P_s = \frac{1}{2}P_B$$

Lecture 5: Diversity and more (chapter 5)

Diversity

Assume "good" = infinity
Assume "bad" = 0

$$P_s = \left\{ Q\left(\sqrt{\frac{4|\alpha_1|^2}{N_0}}\right) \right\} = \Pr(\alpha_1 = \text{bad})\frac{1}{2} = \frac{1}{2}P_B$$

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

Concept of "diversity"
By sending over many independent fading channels,
BER is heavily reduced

$$P_s = \frac{1}{2}P_B^2 \qquad P_s = \frac{1}{2}P_B$$

Result for S_2 ?

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
~~Assume "bad" = 0~~

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G
3. good = α_G , bad = α_B
4. Binary antipodal signaling

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
~~Assume "bad" = 0~~

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G
3. good = α_G , bad = α_B
4. Binary antipodal signaling $\mathcal{E}_b = E\left\{\frac{E_{\mathrm{b,sent}}}{N}\sum_{n=1}^{N}|\alpha_n|^2\right\}$

$$\mathbf{s_0} = -\mathbf{s_1} = \begin{bmatrix} \sqrt{\frac{E_{\mathrm{b,sent}}}{N}} \\ \vdots \\ \sqrt{\frac{E_{\mathrm{b,sent}}}{N}} \end{bmatrix} \quad \mathbf{z_0} = -\mathbf{z_1} = \begin{bmatrix} \alpha_1\sqrt{\frac{E_{\mathrm{b,sent}}}{N}} \\ \vdots \\ \alpha_2\sqrt{\frac{E_{\mathrm{b,sent}}}{N}} \end{bmatrix}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
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1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G
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Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
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1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G
3. good = α_G , bad = α_B
4. Binary antipodal signaling

$$\mathcal{E}_b = E\left\{\frac{E_{\mathrm{b,sent}}}{N}\sum_{n=1}^{N}|\alpha_n|^2\right\} = E\left\{E_{\mathrm{b,sent}}|\alpha_n|^2\right\}$$
$$= E_{\mathrm{b,sent}}(P_G\alpha_G^2 + (1-P_G)\alpha_B^2)$$

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
~~Assume "bad" = 0~~

1. α_1 and α_2 are independent
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$$\mathcal{E}_b = E\left\{\frac{E_{\mathrm{b,sent}}}{N}\sum_{n=1}^{N}|\alpha_n|^2\right\} = E\left\{E_{\mathrm{b,sent}}|\alpha_n|^2\right\}$$
$$= E_{\mathrm{b,sent}}(P_G\alpha_G^2 + (1-P_G)\alpha_B^2)$$

$$P_s = E\left\{Q\left(\sqrt{2\frac{\mathcal{E}_b}{N_0}}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
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$$P_s = E\left\{Q\left(\sqrt{2\frac{\mathcal{E}_b}{N_0}}\right)\right\} = E\left\{Q\left(\sqrt{2\frac{E_{\mathrm{b,sent}}}{NN_0}\sum_{n=1}^{N}|\alpha_n|^2}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
~~Assume "bad" = 0~~

1. α_1 and α_2 are independent
2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G
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$$\mathcal{E}_b = E\left\{\frac{E_{\mathrm{b,sent}}}{N}\sum_{n=1}^{N}|\alpha_n|^2\right\} = E\left\{E_{\mathrm{b,sent}}|\alpha_n|^2\right\}$$
$$= E_{\mathrm{b,sent}}(P_G\alpha_G^2 + (1-P_G)\alpha_B^2)$$

$$P_s = E\left\{Q\left(\sqrt{2\frac{\mathcal{E}_b}{N_0}}\right)\right\} = E\left\{Q\left(\sqrt{2\frac{E_{\mathrm{b,sent}}}{NN_0}\sum_{n=1}^{N}|\alpha_n|^2}\right)\right\}$$
$$= E\left\{Q\left(\sqrt{\frac{2}{P_G\alpha_G^2+(1-P_G)\alpha_B^2}\frac{\mathcal{E}_b}{N_0}\frac{1}{N}\sum_{n=1}^{N}|\alpha_n|^2}\right)\right\}$$

Lecture 5: Diversity and more (chapter 5)

Diversity

~~Assume "good" = infinity~~
~~Assume "bad" = 0~~

1. α_1 and α_2 are independent

2. Prob(α_k ="good") = P_G , Prob(α_k ="bad") = P_B = 1-P_G

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$$\mathcal{E}_b = E\left\{\frac{E_{\text{b,sent}}}{N}\sum_{n=1}^{N}|\alpha_n|^2\right\} = E\left\{E_{\text{b,sent}}|\alpha_n|^2\right\}$$
$$= E_{\text{b,sent}}(P_G\alpha_G^2 + (1-P_G)\alpha_B^2)$$

$$P_s = E\left\{Q\left(\sqrt{2\frac{\mathcal{E}_b}{N_0}}\right)\right\} = E\left\{Q\left(\sqrt{2\frac{E_{\text{b,sent}}}{NN_0}\sum_{n=1}^{N}|\alpha_n|^2}\right)\right\}$$

$$= \sum_{n=0}^{N}\binom{N}{n}P_G^n(1-P_G)^{N-n}Q\left(\sqrt{\frac{2}{P_G\alpha_G^2+(1-P_G)\alpha_B^2}\frac{\mathcal{E}_b}{N_0}\frac{n+(N-n)\alpha_B^2/\alpha_G^2}{N}}\right)$$

Lecture 5: Diversity and more (chapter 5)

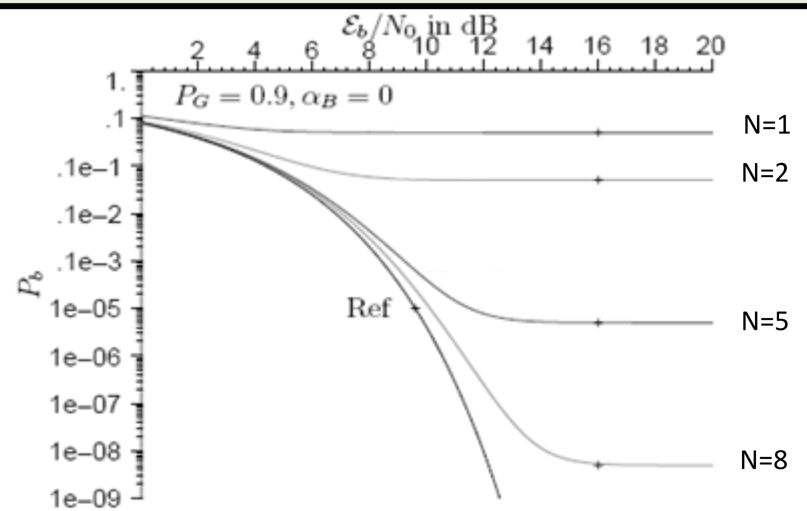


Figure 5.22: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $P_G = 0.9$ and $\alpha_B = 0$, with N = 1, 2, 5, 8.

$$= \sum_{n=0}^{N} \binom{N}{n} P_G^n (1-P_G)^{N-n} Q\left(\sqrt{\frac{2}{P_G\alpha_G^2 + (1-P_G)\alpha_B^2} \frac{\mathcal{E}_b}{N_0} \frac{n + (N-n)\alpha_B^2/\alpha_G^2}{N}}\right)$$

Lecture 5: Diversity and more (chapter 5)

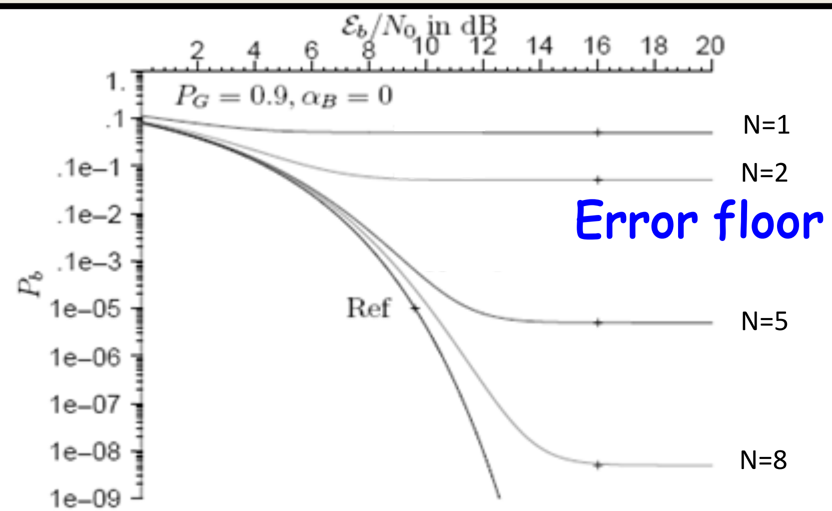


Figure 5.22: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $P_G = 0.9$ and $\alpha_B = 0$, with N = 1, 2, 5, 8.

$$= \sum_{n=0}^{N} \binom{N}{n} P_G^n (1-P_G)^{N-n} Q\left(\sqrt{\frac{2}{P_G\alpha_G^2 + (1-P_G)\alpha_B^2} \frac{\mathcal{E}_b}{N_0} \frac{n + (N-n)\alpha_B^2/\alpha_G^2}{N}}\right)$$

Lecture 5: Diversity and more (chapter 5)

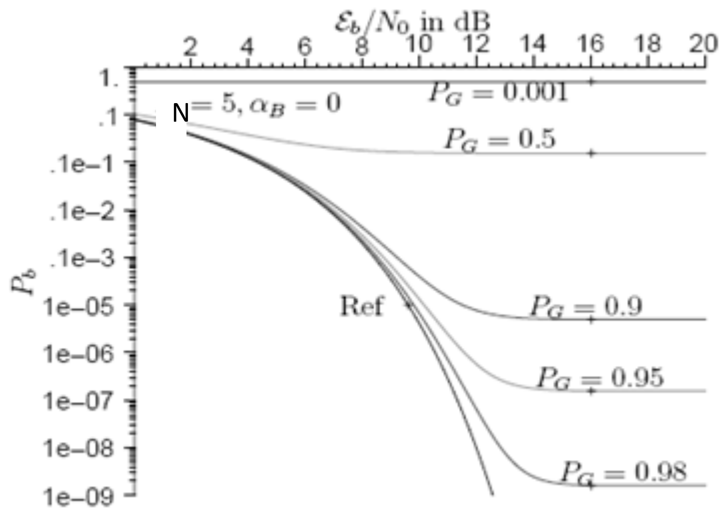


Figure 5.23: The bit error probability versus $\mathcal{E}_b/N_0$ for the case N $= 5$ and $\alpha_B = 0$, with $P_G = 0.001, 0.5, 0.9, 0.95, 0.98$.

$$\frac{0.02^5}{2}$$

$$= \sum_{n=0}^{N} \binom{N}{n} P_G^n (1-P_G)^{N-n} Q\left(\sqrt{\frac{2}{P_G\alpha_G^2 + (1-P_G)\alpha_B^2}\frac{\mathcal{E}_b}{N_0}\frac{n+(N-n)\alpha_B^2/\alpha_G^2}{N}}\right)$$

Lecture 5: Diversity and more (chapter 5)

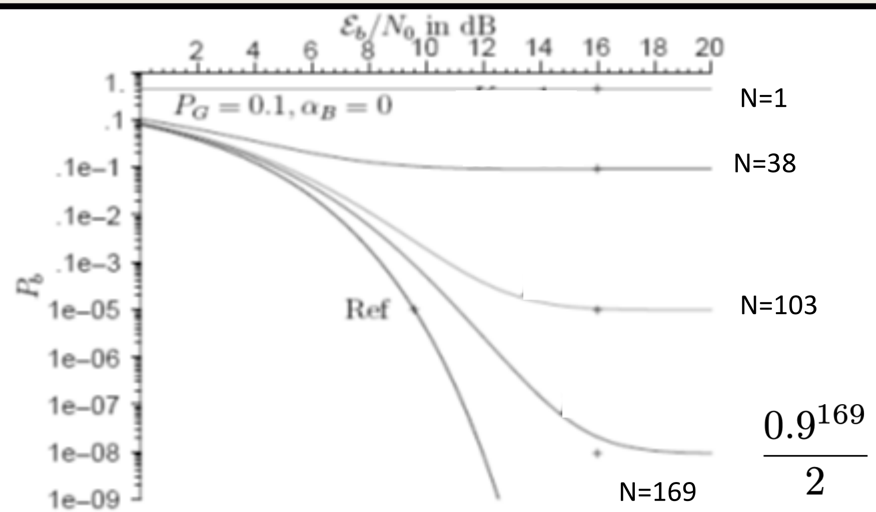


Figure 5.24: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $P_G = 0.1$ and $\alpha_B = 0$, with $K = 1, 38, 103, 169$.

$$= \sum_{n=0}^{N} \binom{N}{n} P_G^n (1-P_G)^{N-n} Q\left(\sqrt{\frac{2}{P_G\alpha_G^2 + (1-P_G)\alpha_B^2}\frac{\mathcal{E}_b}{N_0}\frac{n + (N-n)\alpha_B^2/\alpha_G^2}{N}}\right)$$

Lecture 5: Diversity and more (chapter 5)

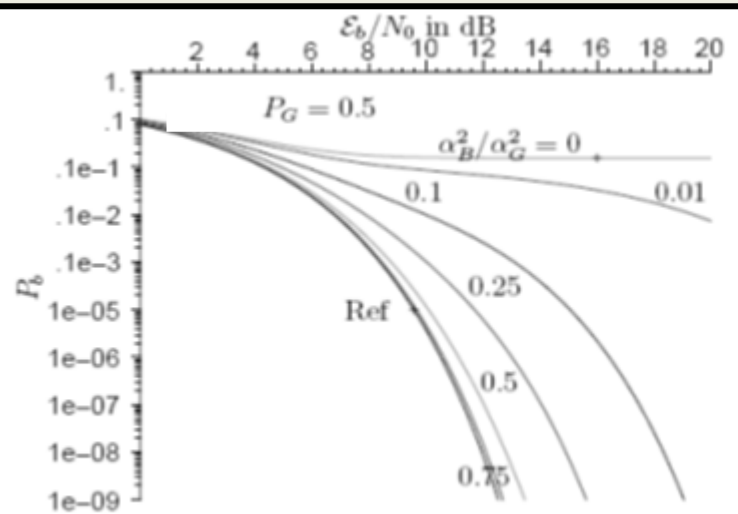


Figure 5.25: The bit error probability versus $\mathcal{E}_b/N_0$ for the case N=5 and $P_G = 0.5$, with $\alpha_B^2/\alpha_G^2 = 0, 0.01, 0.1, 0.25, 0.5, 0.75$.

No error floors if $\alpha_G > 0$

$$= \sum_{n=0}^{N} \binom{N}{n} P_G^n (1-P_G)^{N-n} Q\left(\sqrt{\frac{2}{P_G\alpha_G^2 + (1-P_G)\alpha_B^2}\frac{\mathcal{E}_b}{N_0}\frac{n + (N-n)\alpha_B^2/\alpha_G^2}{N}}\right)$$

Lecture 5: Diversity and more (chapter 5)

Price to pay for diversity?

Lecture 5: Diversity and more (chapter 5)

Price to pay for diversity?

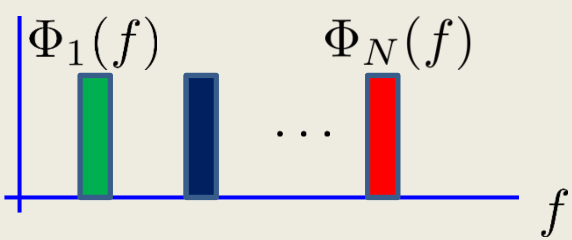
Cost of acquiring additional signal space basis function

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_3 = \left\{ -\sqrt{2},\ \sqrt{2} \right\}$$

Two basis functions needed
diversity order = 2

One basis function needed
diversity order = 1

Assume diversity through frequency



Consequences?

Lecture 5: Diversity and more (chapter 5)

Price to pay for diversity?

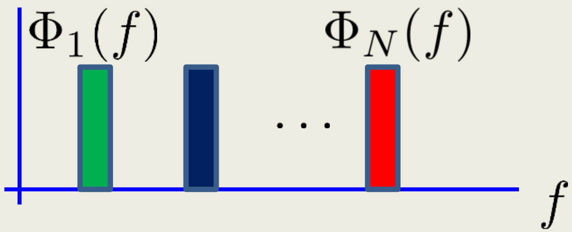
Cost of acquiring additional signal space basis function

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right\} \qquad \mathcal{S}_3 = \left\{ -\sqrt{2}, \ \sqrt{2} \right\}$$

Two basis functions needed
diversity order = 2

One basis function needed
diversity order = 1

Assume diversity through frequency



Consequences? Less bandwidth efficiency

Lecture 5: Diversity and more (chapter 5)

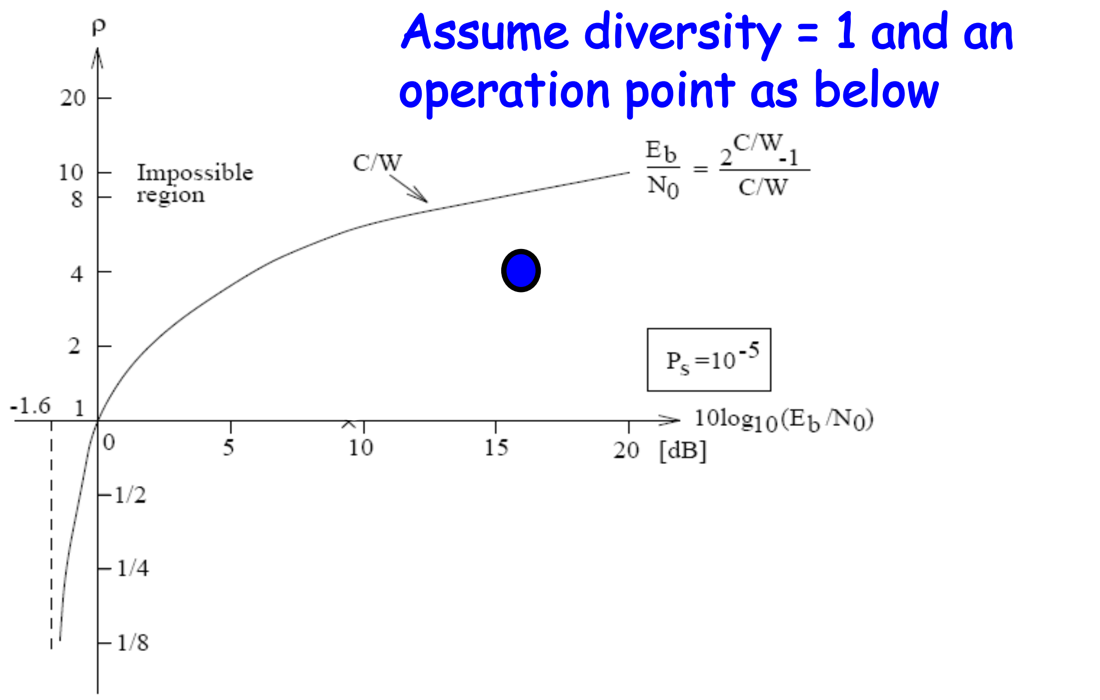


Figure 5.17: Sketch of the ρ versus $\mathcal{E}_b/N_0$ performance for some of the schemes studied in this section. Reliable communication is not possible above the capacity curve (see (5.64).

Lecture 5: Diversity and more (chapter 5)

What happens if we now use diversity = 2

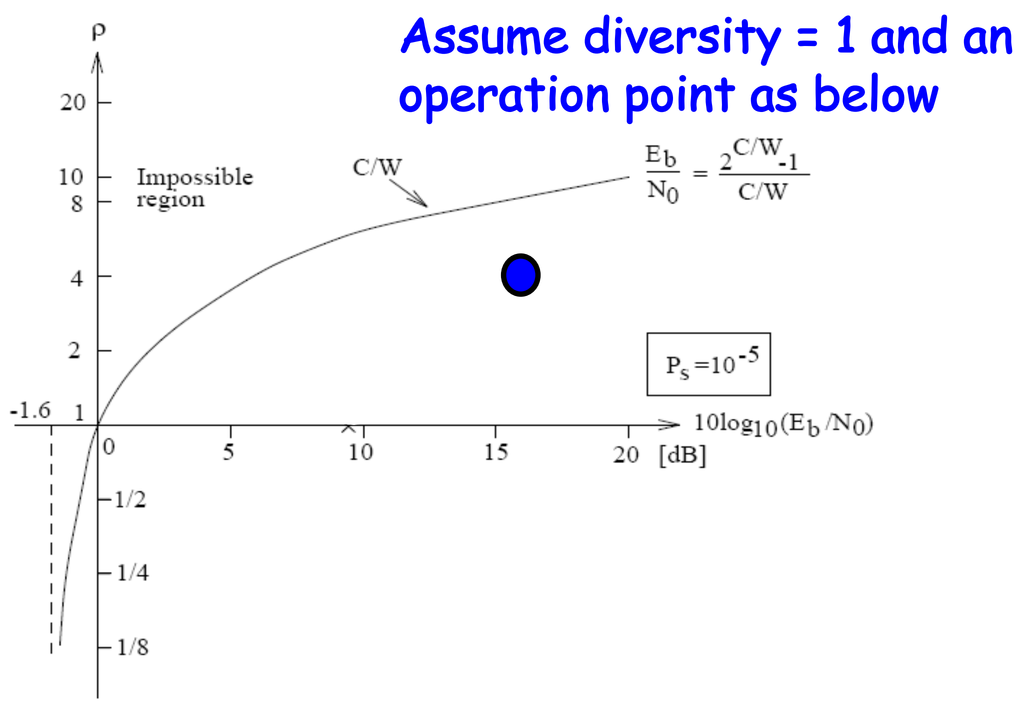


Figure 5.17: Sketch of the ρ versus $\mathcal{E}_b/N_0$ performance for some of the schemes studied in this section. Reliable communication is not possible above the capacity curve (see (5.64).

Lecture 5: Diversity and more (chapter 5)

What happens if we now use diversity = 2

Assume diversity = 1 and an operation point as below

Moves somewhere on this line

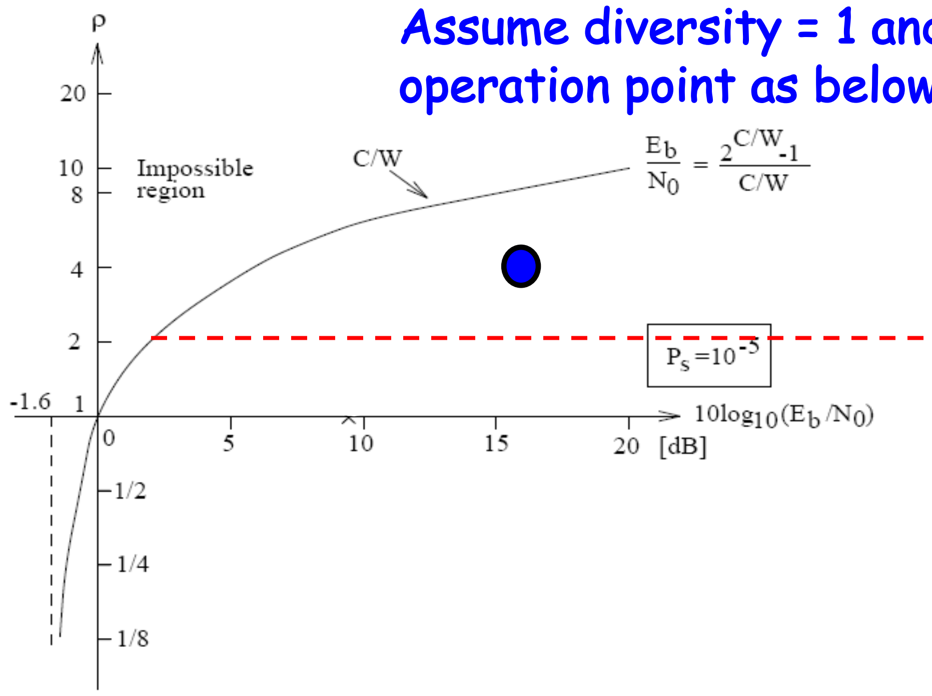


Figure 5.17: Sketch of the ρ versus $\mathcal{E}_b/N_0$ performance for some of the schemes studied in this section. Reliable communication is not possible above the capacity curve (see (5.64).

Lecture 5: Diversity and more (chapter 5)

What happens if we now use diversity = 2

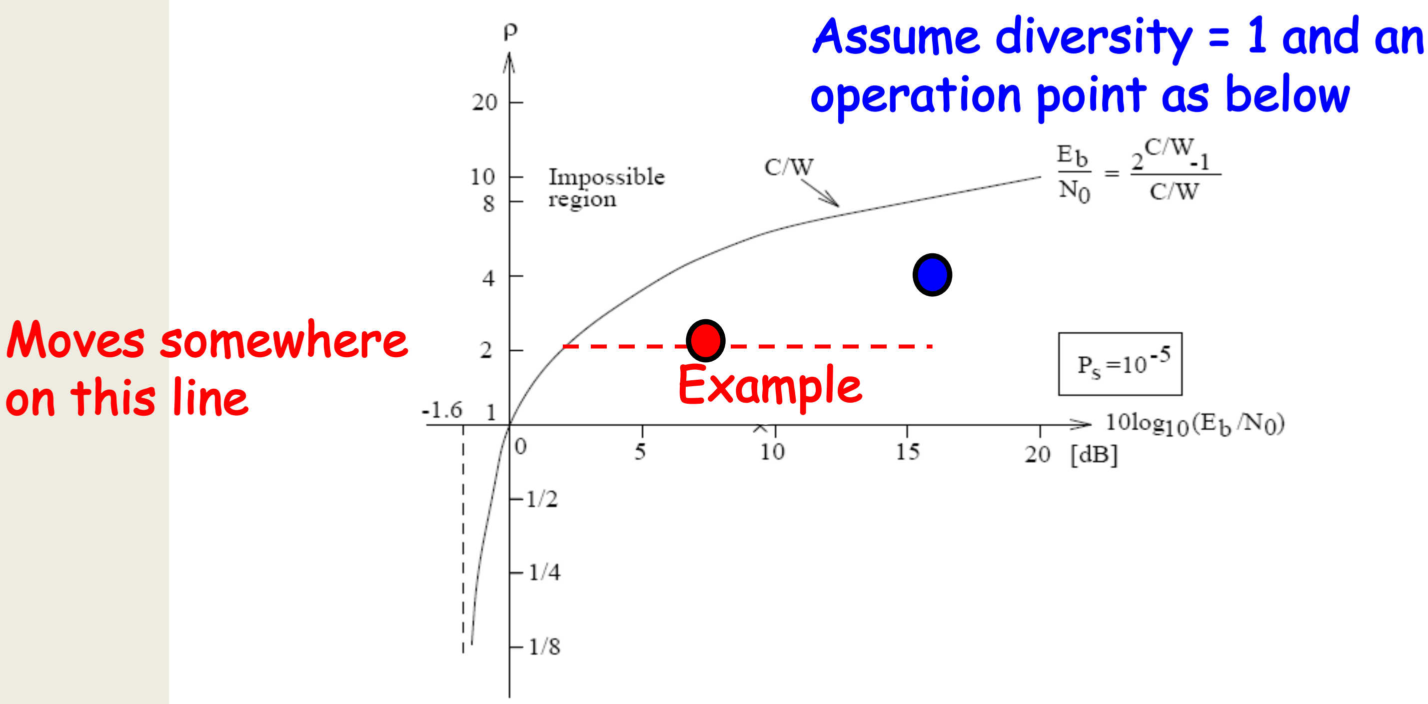


Figure 5.17: Sketch of the ρ versus $\mathcal{E}_b/N_0$ performance for some of the schemes studied in this section. Reliable communication is not possible above the capacity curve (see (5.64).

Lecture 5: Diversity and more (chapter 5)

Generating Basis function in time

Assume that we have a bandwidth W Hz available
Assume that we have a time duration of T seconds available

How many orthonormal basis functions can be obtain, and what do the look like?

Lecture 5: Diversity and more (chapter 5)

Generating Basis function in time

Assume that we have a bandwidth W Hz available
Assume that we have a time duration of T seconds available

How many orthonormal basis functions can be obtain, and what do the look like?

Classical result: Sampling Theorem, Shannon '48

We can obtain 2WT orthonormal functions. (Figure D.21, β=0)

Lecture 5: Diversity and more (chapter 5)

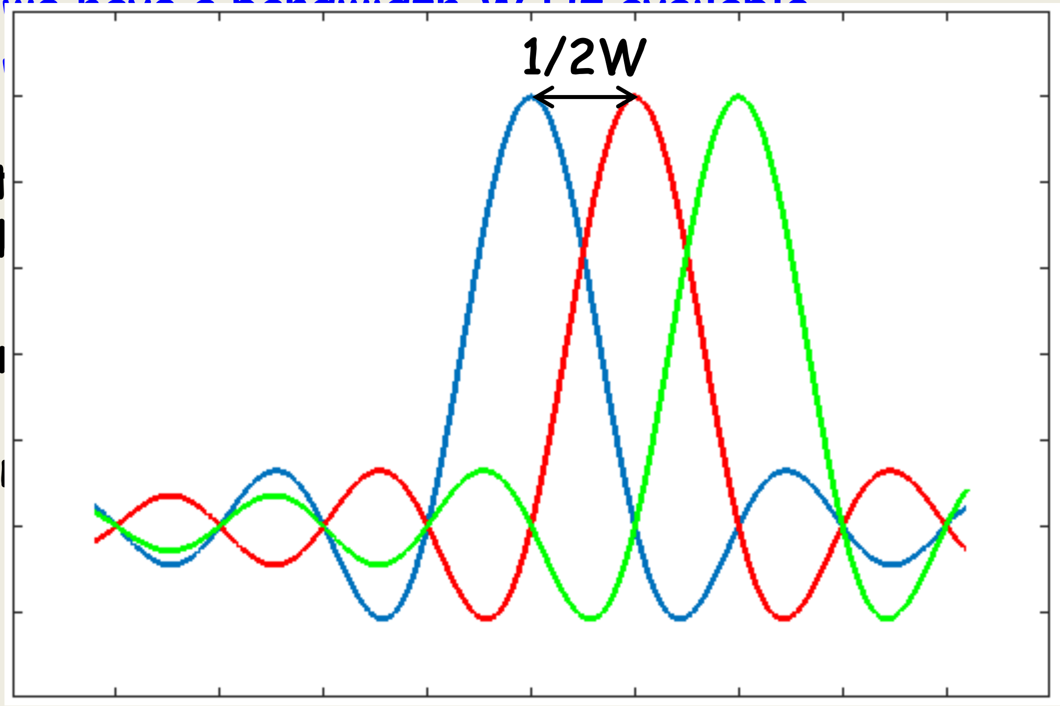
Generating Basis function in time

Assume that we have a bandwidth W Hz available
Assume that

How many ort at
do the look lil

Classical resu

We can obtai =0)



Sinc function (=sin(x)/x)

Lecture 5: Diversity and more (chapter 5)

Generating Basis function in time

Assume that we have a bandwidth W Hz available
Assume that we have a time duration of T seconds available

How many orthonormal basis functions can be obtain, and what do the look like?

Classical result: Sampling Theorem, Shannon '48

We can obtain 2WT orthonormal functions.

Thus, with fixed bandwidth, we can reduce R_b using diversity.

Same effect: Less bandwidth efficiency

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

We have:

1. FSK signals
2. Unknown phase
3. Thus, we don't know the signal set at the receiver
4. We want to decode nonetheless

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

We have:

1. FSK signals
2. Unknown phase
3. Thus, we don't know the signal set at the receiver
4. We want to decode nonetheless
5. From FSK-theory: Signals orthogonal if $w_j=2\pi n_j/T_s$ n_j integer

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

First attempt: Use $\phi_n(t) = \cos(\omega_j t)$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

First attempt: Use $\phi_n(t) = \cos(\omega_j t)$

If $v_j = 0$, $0 \le j \le M-1$

At position of transmitted symbol

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sqrt{2E/T_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \mathbf{w}$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

First attempt: Use $\phi_n(t) = \cos(\omega_j t)$

If $\nu_j = 0$, $0 \le j \le M-1$

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sqrt{2E/T_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \mathbf{w}$$

However

$$\cos(\omega_j t + \nu_j) = \cos(\omega_j t)\cos(\nu_j) - \sin(\omega_j t)\sin(\nu_j)$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \leq t \leq T_s$$

First attempt: Use $\phi_n(t) = \cos(\omega_j t)$

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sqrt{2E/T_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \mathbf{w}$$

However If $\nu_j = \pi/2$

$$\cos(\omega_j t + \nu_j) = \cos(\omega_j t)\cos(\nu_j) - \sin(\omega_j t)\sin(\nu_j)$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

First attempt: Use $\phi_n(t) = \cos(\omega_j t)$

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sqrt{2E/T_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \mathbf{w}$$

However If $\nu_j = \pi/2$

$$\cos(\omega_j t + \nu_j) = \cos(\omega_j t)\cos(\nu_j) - \sin(\omega_j t)\sin(\nu_j)$$
$$= -\sin(\omega_j t)$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

Assume unknown

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

First attempt: Use $\phi_n(t) = \cos(\omega_j t)$

However If $\nu_j = \pi/2$

$$\cos(\omega_j t + \nu_j) = \cos(\omega_j t)\cos(\nu_j) - \sin(\omega_j t)\sin(\nu_j)$$
$$= -\sin(\omega_j t)$$

$$\mathbf{r} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} + \mathbf{w}$$

Signal is lost!

$$\int_0^{T_s} \cos(\omega_j t)\sin(\omega_j t)\mathrm{d}t = 0$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s$$

Second attempt: Use $\phi_n(t) = \cos(\omega_j t)$ $\tilde{\phi}_n(t) = \sin(\omega_j t)$

Since sin(x) and cos(x) are orthogonal we can expand the signal space

$$\int_0^{T_s} \cos(\omega_j t) \sin(\omega_j t) \mathrm{d}t = 0$$

Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

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Lecture 5: Diversity and more (chapter 5)

Non-coherent FSK

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Envelope detector

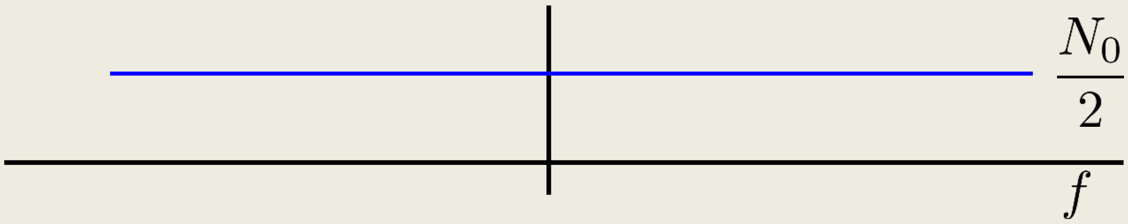
$$\hat{m} = \arg\max_j |r_j|^2 + |\tilde{r}_j|^2$$

See book for error rate

Lecture 5: Diversity and more (chapter 5)

Non-white noise

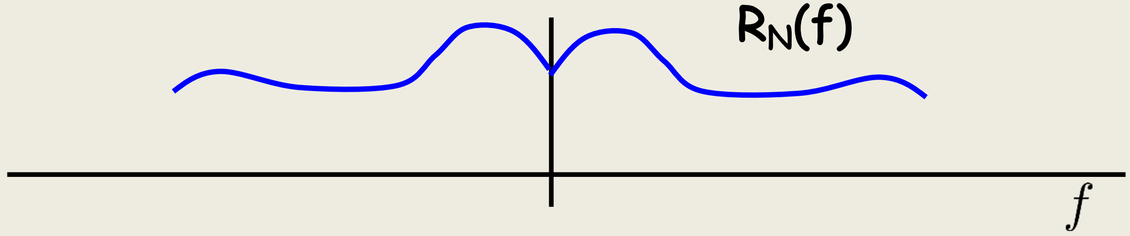
ML and MAP and signal space derived under condition of flat noise PSD



Lecture 5: Diversity and more (chapter 5)

Non-white noise

In many cases, we have non-white noise. What to do?



Lecture 5: Diversity and more (chapter 5)

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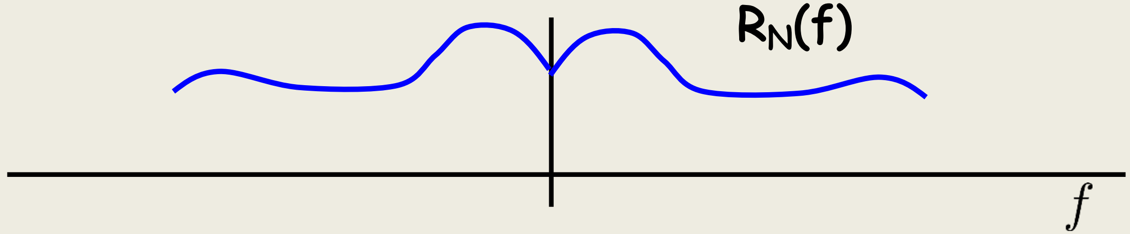
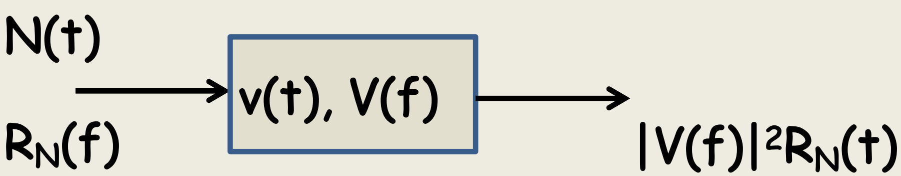


Figure 3.20, p.177



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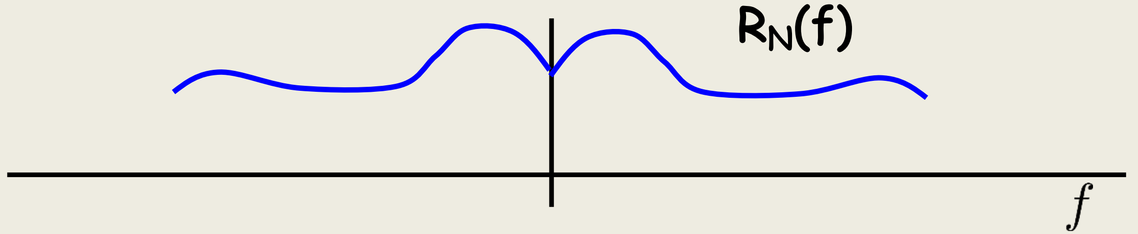
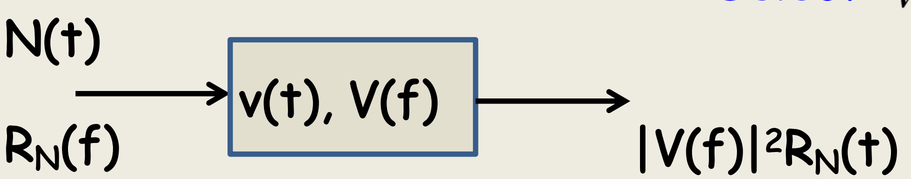


Figure 3.20, p.177



Select $V(f) = \frac{1}{\sqrt{R_N(f)}}$

Lecture 5: Diversity and more (chapter 5)

Non-white noise

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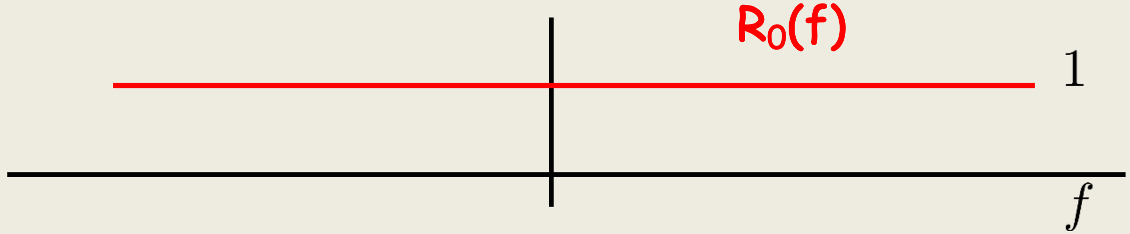
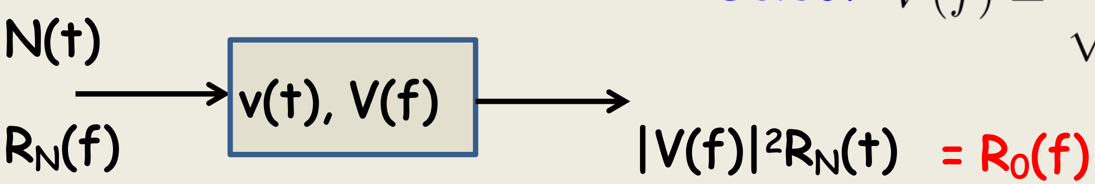


Figure 3.20, p.177

Select $V(f) = \dfrac{1}{\sqrt{R_N(f)}}$



Lecture 5: Diversity and more (chapter 5)

Non-white noise

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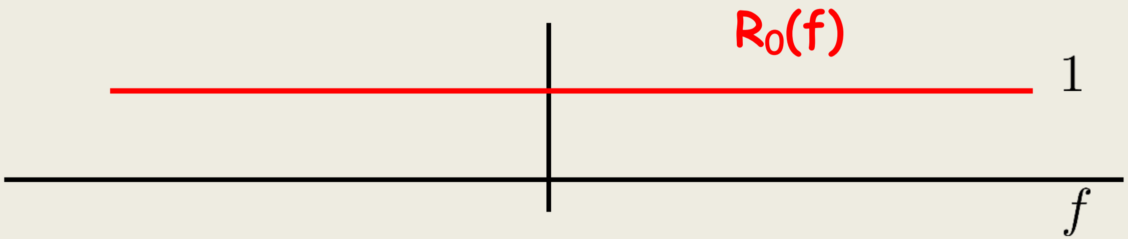
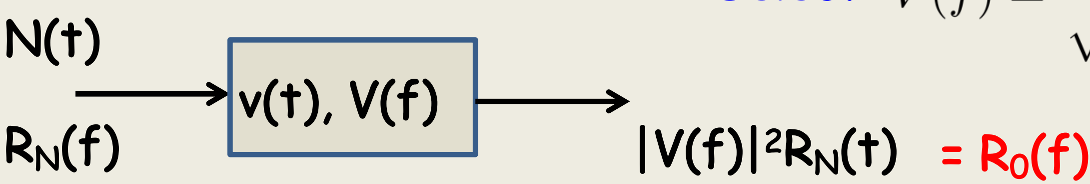


Figure 3.20, p.177

Select $V(f) = \dfrac{1}{\sqrt{R_N(f)}}$



Impact on basis functions?

Lecture 5: Diversity and more (chapter 5)

Non-white noise

In many cases, we have non-white noise. What to do?

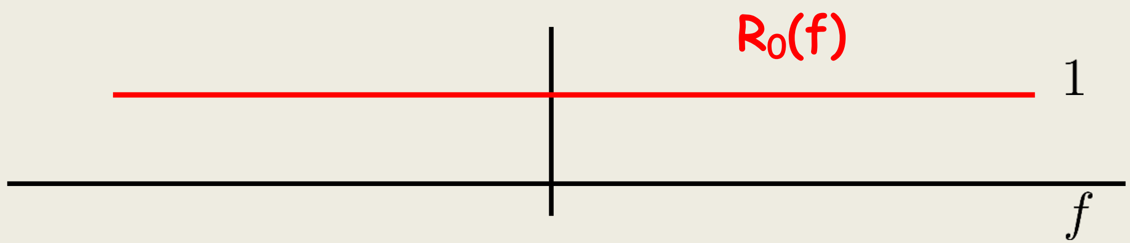
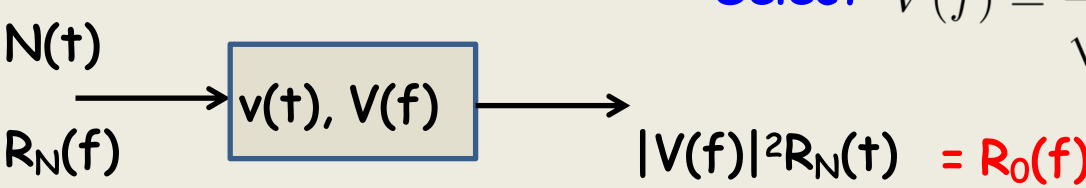


Figure 3.20, p.177

Select $V(f) = \frac{1}{\sqrt{R_N(f)}}$



Impact on basis functions? Basis for $\{s_j(t)\}$ does not work

Lecture 5: Diversity and more (chapter 5)

Non-white noise

In many cases, we have non-white noise. What to do?

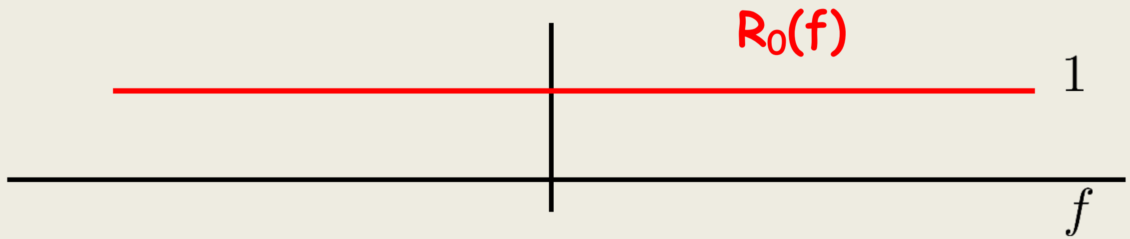
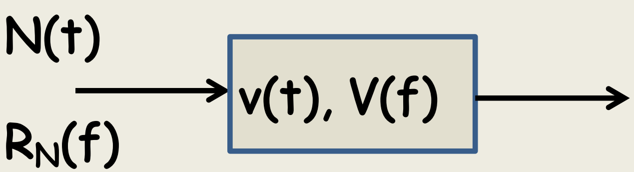


Figure 3.20, p.177

Select $V(f) = \frac{1}{\sqrt{R_N(f)}}$



$|V(f)|^2 R_N(t) = R_0(f)$

Need to derive basis for $\{z_j(t) = s_j(t) * v(t)\}$