ETTN01 Advanced Digital Communications, (max: 50p)

Examination (Zoom), January 13, 2022

Send scanned solutions (e.g., photos by mobile phone) to me (fredrik.rusek@eit.lth.se) no later than 13.15. You will receive a response mail once submitted.

Write clearly! If I cannot read what you write, it will count as 0 points.

It is important to show the intermediate steps in arriving at an answer, otherwise you may lose points. (Exception: Problem 2).

If you provide two answers to the same question, and one is wrong, you lose points. If you make side-comments or if you write too much about a problem, e.g., burying me in paper where you have written down everything you know about a topic with the goal that ”at least something must be correct”, you may lose points for everything that is wrong.

Passing grade (3): 20p; grade 4: 30p; grade 5: 40p. Thresholds may be adapted.

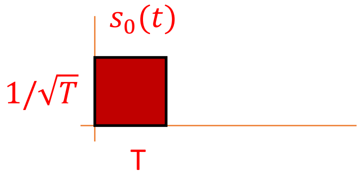
Problem 1 [9p=5+4]

In a 4-dimensional signal space, we have the following signals

$$\boldsymbol{s}_0 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \quad \boldsymbol{s}_1 = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \quad \boldsymbol{s}_2 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \quad \boldsymbol{s}_3 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \boldsymbol{s}_4 = \begin{bmatrix}1\\1\\-1\\1\end{bmatrix}$$

a) Provide an example signal set $\{s_m(t)\}_{m=0}^{4}$, i.e., draw 5 signal that would give this signal space. You may choose any set you want.

The simplest choice is, in my opinion,



and similar for the other 4. Note that the energy must be 1 in $s_0(t)$

b) Calculate $d_{\min}^2$

$$E_s = \frac{1}{5}(1+2+1+1+4) = \frac{9}{5}$$

$$E_b = \frac{E_s}{log_2(5)}$$

$$D_{min}^2 = D_{0,1}^2 = 1$$

$$d_{min}^2 = \frac{D_{min}^2}{2E_b} = \frac{5log_2(5)}{2\times 9} \approx 0.64$$

Problem 2 [12p]

Are the following statements true or false? Answer with

a) True, I am sure
b) True, but I am unsure
c) False, but I am unsure
d) False, I am sure.

A correct answer with a) or d) gives you +2 points.
A wrong answer with a) or d) gives you -2 points.
A correct answer with b) or c) gives you +1 points.
A wrong answer with b) or c) gives you 0 points.

A blank answer gives you 0 points.
The total score of the problem can never be less than 0 points.
MATLAB's random number generator was used when deciding if the answers should be yes or no.
No motivations are needed (or considered when grading) in this problem.

i) There are signal sets $\{s_m(t)\}_{m=0}^{M-1}$ for which the signal space description (on the transmitter side) MUST have more than M dimensions.
False. M signals need at most M dimensions

ii) Bandwidth properties are generally not so visible in a signal space description.
True. We get no information about the shape of the time-signals, which would be needed to obtain bandwidth properties.

iii) The bandwidth efficiency of OFDM is linearly proportional to the number of subcarriers.
False. We have carefully analyzed this in the exercises

iv) For a random channel, the bit error probability of the system at the average channel is not very representative of the actual bit error probability.
True. We have discussed this extensively during lectures.

v) The coherence time and coherence bandwidth are important parameters when designing a system. However, for OFDM this is not true. Said two parameters are not at all relevant as OFDM is orthogonal whenever $T_{obs}f_\Delta = 1$, no matter the coherence time and coherence bandwidth.
False. For example, the channel cannot change during one OFDM symbol.

vi) The decision regions for an ML and an MAP demodulator may be different.
True. We have calculated MAP decision regions during exercise classes

Problem 3 [10p=7+3]

In this problem we consider an OFDM system. Here are some parameters that are given/known:

- $t_{coh} = 10ms\ (coherence\ time)$
- $T_m = 10\mu s\ (delay\ spread)$
- $K = 1024\ (number\ of\ subcarriers)$
- $T_{obs} = 1ms$
- $P = 10\mu W\ (Transmit\ power)$
- $N_0 = 10^{-16}\ W/Hz$
- QPSK transmissions
- $|H(f_1)|^2 = 10^{-3}\ (Fourier\ transform\ of\ the\ impulse\ response\ at\ frequency\ f_1)$

a) For the QPSK data value that is transmitted at subcarrier f_1, derive the symbol error probability.

We need to calculate the received E_b. We know that $d_{min}^2 = 2$, and we make the approximation $P_s = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$. We have

$$E_b = \frac{|H(f_1)|^2 P T_{obs}}{K log_2(4)} \approx \frac{10^{-11}}{2000} = \frac{1}{2}10^{-14}$$

Wherefore,

$$P_s \approx Q(10)$$

b) For the QPSK data value that is transmitted at subcarrier f_2, can we estimate the symbol error probability?

Intention here was that f_2 is the subcarrier next to f_1. The spacing is $f_\Delta = \frac{1}{T_{obs}} = 1000\ Hz$ The coherence bandwidth is $\frac{1}{T_m} = 100000 Hz$, so the next subcarrier has the same channel (roughly). Therefore, the same error performance.

Problem 4 [11p=5+3+1+2]

Assume a total bandwidth of 1 MHz and $P/N_0 = 3\text{x}10^6$.

a) Assume that the bandwidth is available between 0 and W Hz, i.e., the signaling is taking place around baseband. Assume single carrier transmissions, i.e., not OFDM. Select a normal constellation (e.g., PAM, QAM, FSK, PSK etc) and a coding rate so that the system operates close to capacity.

We have $C = 10^6 log_2(1+3) = 2\times10^6$. Assume $W = \frac{1}{2T_s}$. Since we are at baseband, we use M-PAM. Baseband also explains the factor 2 in $W = \frac{1}{2T_s}$. We have

$$R_b = r_c log_2(M) R_s = r_c log_2(M) 2W < C = 2\times10^6$$

Which simplifies into

$$r_c log_2(M) < 1$$

We can for example take 4PAM ($M = 4$) and a 5/6 code, i.e., $r_c = 5/6$.

b) Assume that the bandwidth is available between 10 GHz and 10.001 GHz. Repeat part A. Show a block diagram of how the system which you carefully annotate.

Capacity remains the same, but we need to use M-QAM. Further, $W = \frac{1}{T_s}$. Thus,

$$R_b = r_c log_2(M) R_s = r_c log_2(M) W < C = 2\times10^6$$

Which simplifies into

$$r_c log_2(M) < 2$$

We can for example take 16QAM ($M = 16$) and a 5/6 code, i.e., $r_c = 5/6$.

Now assume that $P/N_0 = 3\text{x}10^6$ (i.e., no change), but that an unlimited bandwidth is available.

c) What is the Shannon capacity?

Use the standard limit for capacity with infinite bandwidth to obtain $C \approx 4.3281\times10^6$.

d) With a normal constellation, what code rate should be used to reach this asymptotic capacity in practice?

As bandwidth is infinite, we treat this as being around baseband. Thus,

$$R_b = r_c log_2(M)2W < 4.3281 \times 10^6$$

But since W is infinite, some variable on the left-hand-side must tend to 0. M cannot do this, so it must be the code-rate r_c. So, the answer is 0. (More strictly, as the bandwidth increases, the code-rate decreases to 0).

Problem 5 [8p=4+4]

We have seen in the course that for random channels, the error probability is grossly affected. We have also seen that with diversity, the situation improves.

In this problem we consider a Rayleigh fading channel with coherence time t_{coh}. We assume that the symbol duration $T_s \ll t_{coh}$.

To protect the data from deep fades (i.e., bad channels), the designer decides that the same symbol should be transmitted K times. We assume QPSK data.

In general, for Rayleigh fading with $K = 1$, the error rate is, at high SNR, $\approx {}^{1}/_{SNR}$. We say this on the very last lecture.

a) The first attempt by the designer is to transmit the K copies in a sequence. That is, if the QPSK symbols are denoted by $a_0, a_1, a_2, \ldots$, then the transmitted signal is

$$\underbrace{a_0\ a_0 \ldots a_0}_{K\ copies}\ \underbrace{a_1\ a_1 \ldots a_1}_{K\ copies}\ \underbrace{a_2\ a_2 \ldots a_2}_{K\ copies} \ldots \ldots \ldots.$$

Analyze the gain of the K repetitions, i.e., compare the error probability to K=1.

The K copies experience the same channel. Therefore, there is only a power gain, not a diversity gain. The error performance becomes, roughly, ${}^{1}/_{K \times SNR}$

b) In a second attempt, the designer spreads the K copies by a time separation of at least t_{coh}. The transmitted signal is

$$\overset{>t_{coh}}{\longleftrightarrow}$$

$$a_0\ a_1\ a_2 \ldots a_N\ a_0\ a_1\ a_2 \ldots a_N a_0\ a_1\ a_2 \ldots a_N$$

K copies

Analyze the gain of the K repetitions, i.e., compare the error probability to K=1.

(The time between 2 copies of the same symbol exceeds the channel coherence time, and there are K copies transmitted of each symbol).

The K copies experience different channels. At high SNR, to get an error all channels must be bad at the same time. As we have seen in the course, the error probability becomes $\approx {}^{1}/_{SNR^K}$