ETTN01 Advanced Digital Communications, (max: 50p)

Examination (Zoom), January 13, 2020

Send scanned solutions (e.g., photos by mobile phone) to me (fredrik.rusek@eit.lth.se) no later than 13.15. You will receive a response mail once submitted.

Problem 2 must be submitted before any toilet breaks are granted.

Write clearly! If I cannot read what you write, it will count as 0 points.

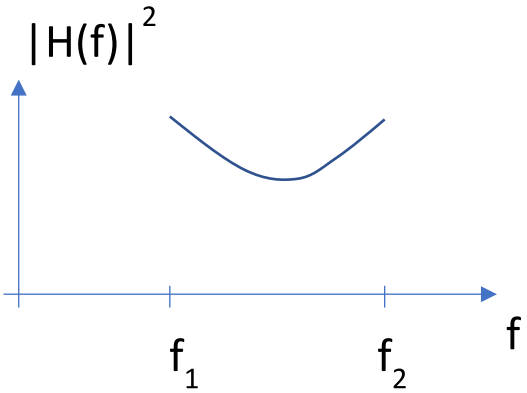
It is important to show the intermediate steps in arriving at an answer, otherwise you may lose points. (Exception: Problem 2).

If you provide two answers to the same question, and one is wrong, you lose points. If you make side-comments or if you write too much about a problem, e.g., burying me in paper where you have written down everything you know about a topic with the goal that ”at least something must be correct”, you may lose points for everything that is wrong.

Passing grade (3): 20p; grade 4: 30p; grade 5: 40p. Thresholds may be adapted.

Problem 1 [6p]

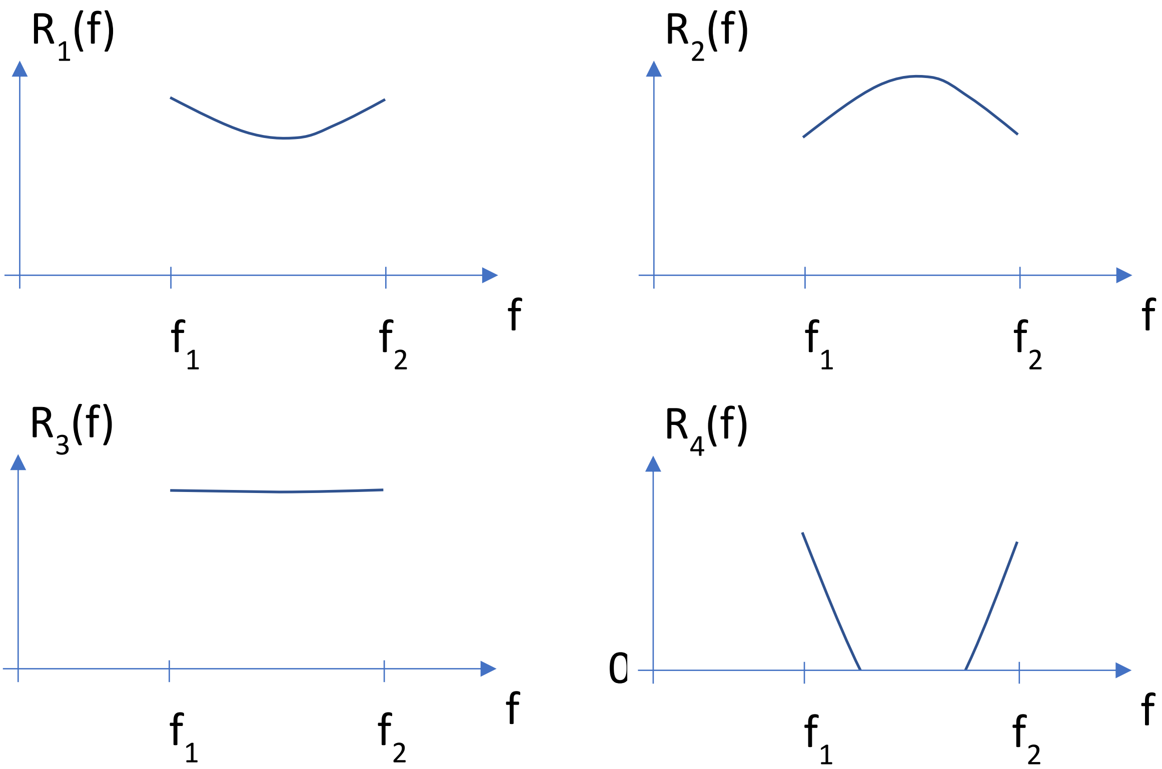
Below the transfer function, $|H(f)|^2$, of a channel is shown. Also shown is 4 transmit power spectral densities, $R_1(f)$ – $R_4(f)$. 3 of those power spectral densities have been found for the given $|H(f)|^2$ through the water filling algorithm for 3 different noise densities, while the fourth cannot possibly be the result of water filling for the given $|H(f)|^2$. The 3 noise densities are “very low”, “medium”, and “very high”, respectively. Your task is to determine which noise density that generated which power spectral density, and also to identify the power spectral density that is not found by water filling.



$N_{0,1}$ = very low

$N_{0,2}$ = medium

$N_{0,3}$ = very high



Problem 2 [12p]

Answer the following questions with

a) Yes, I am sure
b) Yes, but I am unsure
c) No, but I am unsure
d) No, I am sure.

A correct answer with a) or d) gives you +2 points.
A wrong answer with a) or d) gives you -2 points.
A correct answer with b) or c) gives you +1 points.
A wrong answer with b) or c) gives you -1 points.
A blank answer gives you 0 points.

The total score of the problem can never be less than 0 points.
MATLAB's random number generator was used when deciding if the answers should be yes or no.
No motivations are needed (or considered when grading) in this problem.

i) Consider Shannon's capacity formula $C=W \log_2(1+P/WN_0)$. For a fixed value of N_0, does a doubling of bandwidth give exactly the same capacity gain as a doubling of transmit power?

ii) Is it true that the signal space corresponding to a set of N signal alternatives $s_j(t)$ always has dimension less than or equal to N?

iii) In a diversity system, the error probability is in general heavily reduced by transmitting data across multiple channels with equal power allocation across all channels. However, if the channels are known to the transmitter, is it better to transmit data with full power over the best channel only and don't use the weaker channels?

iv) With the signal space tool, is it possible to, fairly easily, calculate exact bit error rates for M-PAM, M-QAM, and M-PSK?

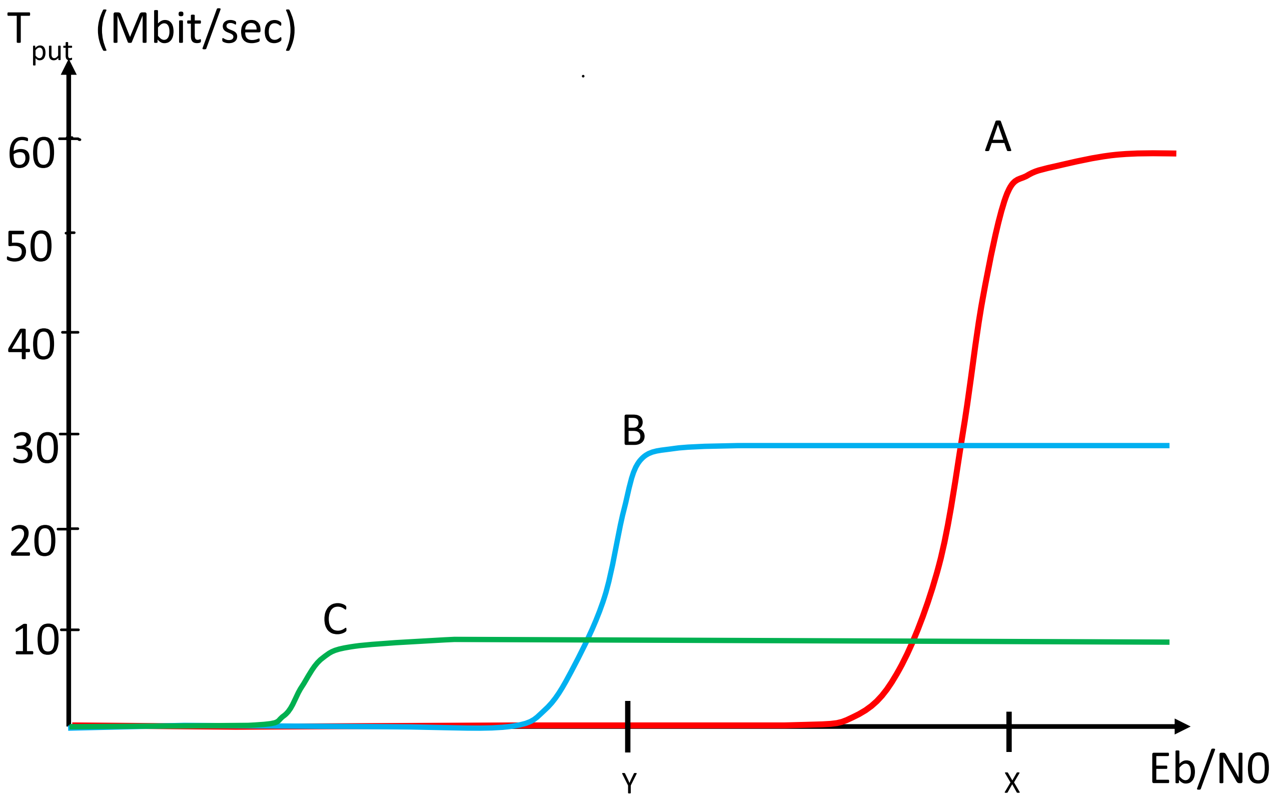
v) In OFDM, is it true that for a fixed total bandwidth, the spectral efficiency is linearly proportional to the number of sub-carriers? (That is, a doubling of the number of subcarriers implies a doubling of spectral efficiency).

vi) Due to the presence of the CP, the symbol duration (i.e., $T_{obs}+T_{CP}$ using notation from the compendium) may exceed the coherence time of the channel without losing orthogonality among the subcarriers.

Problem 3 [10p]

Let a coded system, with coding rate R, have information bit rate R_b (bits/sec) and a bit error rate of P_e. Define throughput as $T_{put} = R_b(1-P_e)$; this corresponds to the number of correctly decoded information bits per second. A symbol time of $T_s = 10^{-7}$ is used and the modulation type is QAM.

In the below figure you find throughput vs. SNR results for three systems (A-C). All three systems are based on the same constellation size, i.e., the "M" in M-QAM is the same in all three cases.



a) The system A is uncoded. What is the constellation size M? **[2p]**

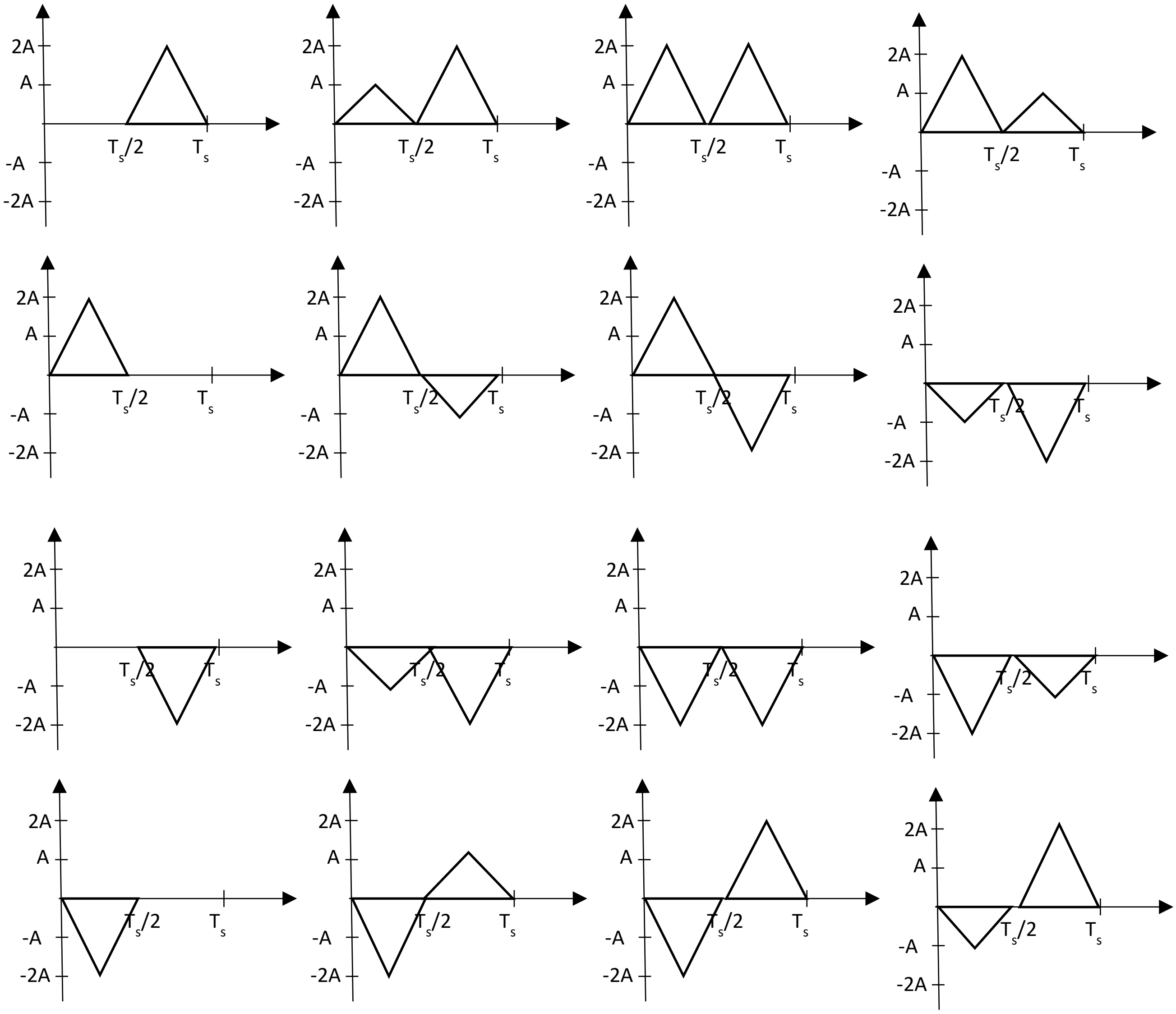
b) Provide a rough estimate for X, i.e., the point where the throughput of system A is almost 60Mbit/s. Hint: Rough means rough; I care about how you reach your answer, not so much about the precision of the approximation. **[3p]**

c) What is the coding rate R for C and B? If you did not solve a) then you may assume an M. **[2p]**

d) Under assumption that the coding system is capacity optimal, provide a rough estimate for Y, i.e., the point where the throughput of system B is almost 30Mbit/s. Your estimate may be very rough, but the basis upon which you form your estimate must be clear. **[3p]**

Problem 4 [12p]

Consider the following signal set comprising 16 signal alternatives:



Clarification: All triangles are equally wide ($T_s/2$) and has amplitudes in the set {-2A, -A, 0, A, 2A}
Assume that $AT_s^2/6 = 1$.

a) Draw the signal space diagram for the signal set. **[5p]**

b) Determine the average energy per transmitted bit, E_b. **[2p]**

c) Provide an approximation of the symbol error probability based on the minimum distance. **[1p]**

d) Compare the symbol error rate to that of 16PSK with the same average energy per bit. **[2p]**

e) Determine all signals for which you can determine the exact symbol error probability conditioned on that symbol being transmitted. **[2p]**

Problem 5 [10p]

Assume an OFDM system with 1000 subcarriers and 4-QAM in each subcarrier. The channel model in each subcarrier is assumed to be a Rayleigh fading channel with b=1 (its second moment, or, average transmit power = average receive power). In this problem we can neglect the effect of the CP, or, in other words, we can assume that the duration of the CP is 0.

Assume $N_0=10^{-12}$ and that a total transmit power of 0.5W is available. We have a requirement on the symbol error probability which is 10^{-5}. Further, the coherence time of the channel is 10ms.

What range of subcarrier spacings f_Δ is possible?