1

general WF rule : More power to better Channels

$\Rightarrow$ $R_2(f)$ not possible

High SNR : All channels get same power (since $\frac{N_0}{|H(f)|^2} \approx 0$)

$\Rightarrow$ $R_3(f) \leftrightarrow N_{0,1}$ = very low

Low SNR : Best channels get all power (since $\frac{N_0}{H(f)} \gg 0$)

$\Rightarrow$ $R_4(f) \leftrightarrow N_{0,3}$ = very high

$\Rightarrow$ $R_1(f) \leftrightarrow N_{0,2}$ = medium

2

i) Question : $2W \log\left(\frac{P}{2N_0}\right) \stackrel{?}{=} W \log\left(\frac{2P}{W N_0}\right)$

Numerical check $\Rightarrow$ <u>No</u>

ii) <u>Yes</u> N signals can always be represented in at most N dimensions.

iii/ **Yes.** Let $y_k(t) = \alpha_k \phi_k(t) \cdot s_{\ell,k} + N_k(t)$

and $\ell = 0, 1$ $\quad k = 1 \ldots N$

In signal space:

$$y_k = \alpha_k s_{\ell,k} + n_k$$

equal power allocation:

$$s_{0,k} = \frac{\sqrt{E}}{\sqrt{N}} \quad \forall k$$

$$s_{1,k} = -\frac{\sqrt{E}}{\sqrt{N}} \quad \forall k$$

$$D_{0,1}^2 = \sum_{k=1}^{N} \left| \alpha_k \left[s_{0,k} - s_{1,k} \right] \right|^2$$

$$= \sum_{k=1}^{N} \alpha_k^2 \cdot 4 \frac{E}{N} = 4 E \frac{1}{N} \sum_{k=1}^{N} \alpha_k^2$$

best channel only

assume $|\alpha_1| > |\alpha_k| \quad k \geq 2$

$$s_{0,1} = \sqrt{E} \qquad s_{1,1} = \sqrt{E}$$

$$s_{j,k} = 0 \qquad k \geq 2$$

$$D_{0,1}^2 = 4 |\alpha_1|^2 E \qquad \alpha_1^2 > \frac{1}{N} \sum_{k=1}^{N} \alpha_k^2$$

↓

(since α_i^2 is largest and $\frac{1}{N}\sum_{k=1}^{N}\alpha_k^2$ is an average value)

iv/ No M-PSK was hard and required Numerical integration

v/ No

$$R_b = \frac{K \cdot \log_2(M)}{T_{obs} + T_{cp}}$$

$$BW = K f_\Delta = \frac{K}{T_{obs}} \rightarrow$$

$$T_{obs} = \frac{K}{BW} \rightarrow$$

$$R_b = \frac{K \log_2(M)}{\frac{K}{BW} + T_{cp}}$$

$$\rho = \frac{R_b}{BW} = \frac{K \log_2(M)}{\left(\frac{K}{BW} + T_{cp}\right) BW} =$$

$$= \frac{\log_2(M)}{1 + \frac{T_{cp} \cdot BW}{k}}$$

ρ not linearly prop. to k

i.e. $\rho \neq k \cdot$ something

vi/ No. channel must be constant during one OFDM symbol.

CP protects against multipath spread.

3

$$R_b = \frac{R \log_2(M)}{T_s} = 10^7 R \cdot \log_2(M)$$

a/ $R = 1$, $R_b = 6 \cdot 10^7 \rightarrow M = 64$

b/ $P_s \approx Q\left(\sqrt{d_{min}^2 \frac{E_b}{N_0}}\right)$ $P_b \approx \frac{P_s}{6}$ ← very loose

64-QAM $\rightarrow d_{min}^2 = \frac{2}{7}$

Make approximation at $P_e \approx 10^{-3}$

$$Q\left(\sqrt{\frac{2}{7}\frac{E_b}{N_0}}\right) = 10^{-3} \qquad \frac{E_b}{N_0} = \ldots \text{ [check table]}$$

c/

$$R_b = 3 \cdot 10^7 \qquad M = 64 \rightarrow R = \frac{1}{2}$$

$$R_b = 1 \cdot 10^7 \qquad M = 64 \rightarrow R = \frac{1}{6}$$

d/ optimal code → capacity applies

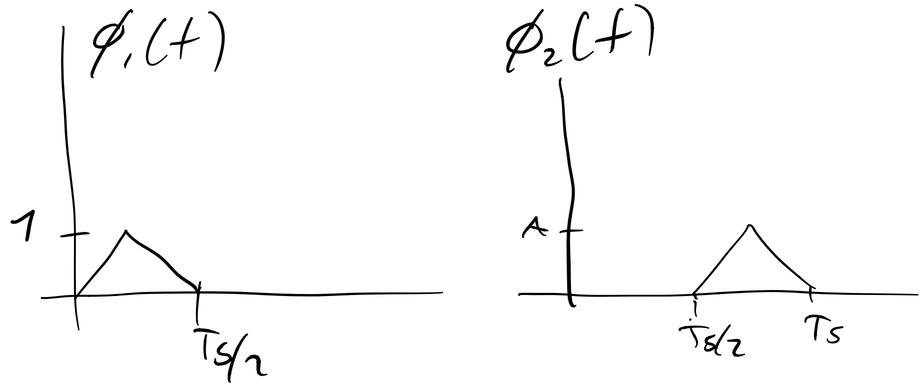
$$\frac{E_b}{N_0} > \frac{2^{\rho} - 1}{\rho}$$

Assume $BW = \frac{1}{T_s} = 10^7$ Hz

$$R_b = 3 \cdot 10^7 \qquad \rho = \frac{R_b}{BW} = 3$$

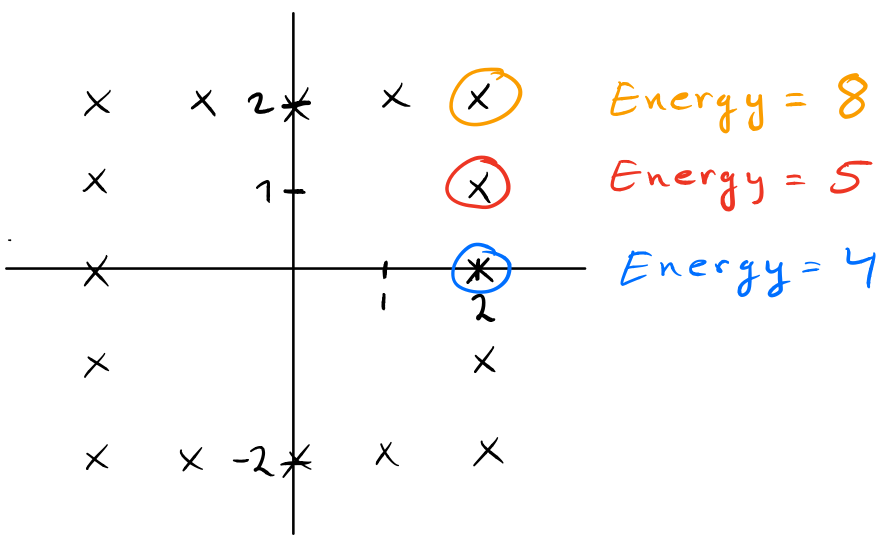
$$\boxed{\frac{E_b}{N_0} > \frac{7}{3}}$$

4



$$\int \phi_1^2(t)\,dt = \ldots = \frac{A T_s^2}{2} = 1$$

a/



b/

$$E_s = \frac{1}{16}\left[4 \cdot 8 + 8 \cdot 5 + 4 \cdot 4\right]$$

$$= \frac{1}{16}\left[32 + 40 + 16\right] = \frac{1}{16}\left[88\right]$$

$$= \frac{22}{4}$$

$$E_b = \frac{E_s}{4} = \frac{22}{16}$$

c/ $D^2_{min} = 7$ (from figure)

$$d^2_{min} = \frac{D^2_{min}}{2E_b} = \frac{1}{\frac{44}{16}} = \frac{16}{44} = \frac{4}{11}$$

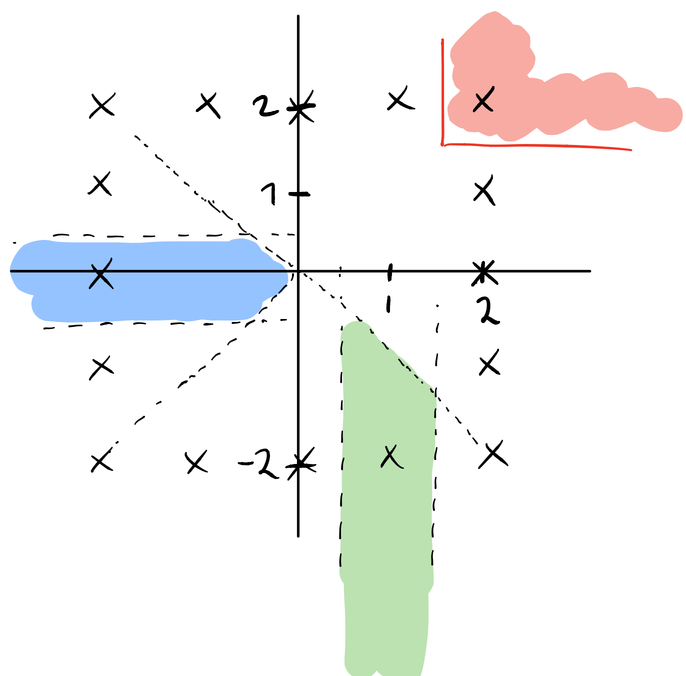
$$P_e \approx Q\left(\sqrt{\frac{4}{11}\frac{E_b}{N_0}}\right)$$

d/ 16-PSK (see table) $d^2_{min} = 0.6090$

$$10\log_{10}\left(\frac{0.6090}{0.3636}\right) = 2.24$$

16 PSK 2.24 dB better

e/



Decision regions

ONLY the red region can be used to obtain closed form solutions. c.f. M-PSK

5

$$E_b = \frac{P \cdot T_{obs}}{\log_2(M) \cdot K}$$

← Tot energy in 1 OFDM symbol

← # bits in one symbol

$M = 4$

$$= \frac{T_{obs}}{4000}$$

$$P_e \approx \frac{1}{2 d_{min}^2 \frac{E_b}{N_0}} = \frac{1000}{T_{obs}} \cdot N_0$$

($d_{min}^2 = 2$)

$$\frac{1000 \cdot N_0}{T_{obs}} = \frac{10^{-9}}{T_{obs}} < 10^{-5} \Rightarrow$$

$$T_{obs} > 10^{-4} \Rightarrow$$

$$\frac{1}{f_\Delta} > 10^{-4} \Rightarrow$$

$$\boxed{f_\Delta < 10^4}$$

Also $T_{obs} < t_{coh} \Rightarrow$

$$\frac{1}{f_\Delta} < t_{coh} \Rightarrow \boxed{f_\Delta > \frac{1}{t_{coh}} = 100}$$

$\Rightarrow$ $100 < f_\Delta < 10000$ (Hz)