

Digital Communications, Advanced Course

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Previously researcher at Huawei (4G/5G)

Currently, part-time researcher (5G/6G) Sony Research

~ 250 patents in 4G-6G

~ 10000 Google Scholar citations

Current research interests: Large intelligent surfaces,
Distributed MIMO processing

Wireless communication

Steady evolution from 1G to 5G

6G already discussed and planned for

More engineers/researchers employed now than ever

Enormous industry, large amounts of money involved

Enormously much more work to do, healthy industry

- Incorporation of AI
- Positioning/localization
- Base stations replaced with surfaces of electromagnetic surfaces
- Higher data rates
- THz
- etc

- **All systems, 1G - 5G and the upcoming 6G system, based on the same principles**
- **Seen from a communication theory/mathematical perspective, they are almost the same system**
- **Important to understand said principles, no matter what part of the wireless industry one ends up in**

Project work in this course

Option 1

- 2 students/group.
- A communication application/technical problem/problem area, relevant for the course, is investigated.
- The choice of project is mainly done by the project group.
- Articles and conference papers from IEEE's database "IEEE Xplore"
<http://ieeexplore.ieee.org/Xplore/DynWel.jsp>
is recommended to get additional technical information.
- Written report, oral presentation, and be opponent to another group.

Project work in this course

Option 2

- 2 students/group.
- Matlab based project related to MAP receivers and iterative decoding

Some examples of applications/systems studied in previous projects:

- Mobile telephony/broadband (GSM, EDGE, LTE, 4G, 5G,...)
- Internet
- Modem (e.g., ADSL)
- WLAN (Wireless Local Area Network)
- Digital TV
- MIMO
- Massive MIMO
- OFDM+MIMO
- GPS (Global Positioning System)
- Bluetooth
- mmWave

Overview of course

The course contains a number of important concepts, not all that closely related

- MAP receiver Important for iterative decoding, used in most modern systems
- Concept of signal space Used everywhere to transform continuous to discrete time
- Capacity Super important for communication – deeper studied in information theory course
- Diversity Deeper studied in MIMO course
- Channels Deeper studied in channel modelling course
- OFDM Essential for WIFI, 4G and onwards
- Time varying channels Deeper studied in channel modelling course

In reality, systems are using error correction coding.

In this course we investigate (mostly) uncoded systems as they are the basic building block for coded systems.

The course Coding Theory deals with coded systems.

Lecture 1: MAP receiver and signal space

MAP receiver

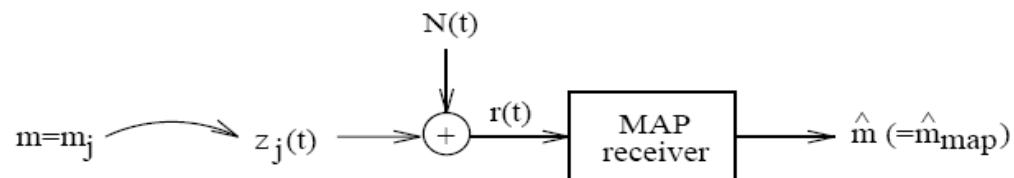


Figure 5.1: Reception of one of M possible waveforms $\{z_\ell(t)\}_{\ell=0}^{M-1}$ in AWGN.

In this course we study the MAP receiver in detail

Lecture 1: MAP receiver and signal space

MAP receiver

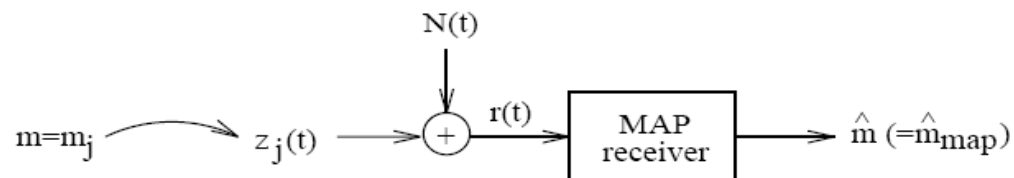


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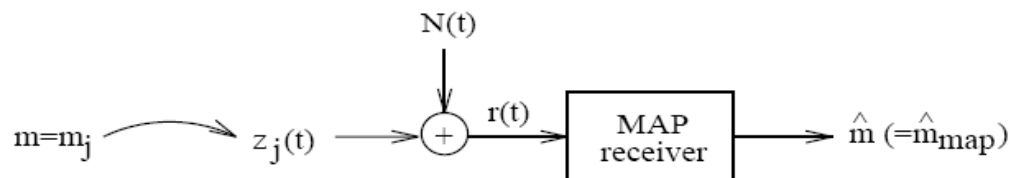


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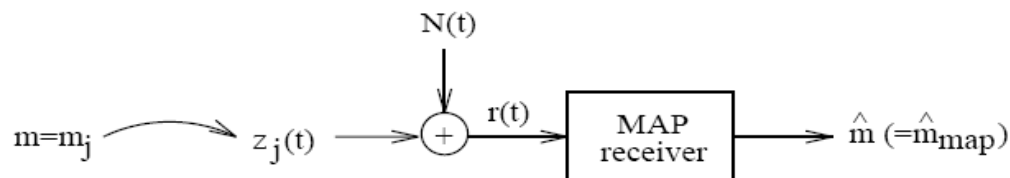


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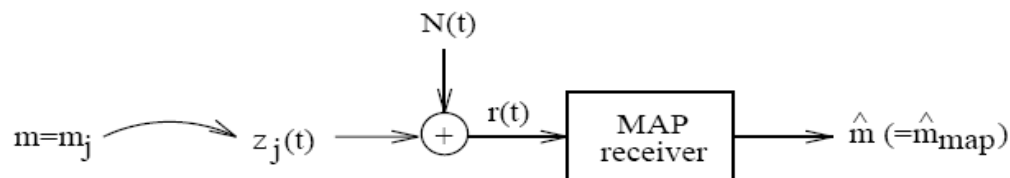


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Which one is more logical?

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MAP receiver

Suppose that on the next lecture, I will not show up

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MAP receiver

Suppose that on the next lecture, I will not show up Observation $r(t) = \text{"fredrik is not here"}$

Why am I not here?

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Why am I not here?

Explanation $m = \text{"Fredrik is....."}$

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Why am I not here?

Explanation $m = \text{"Fredrik is....."}$

ML-rule

$p(r(t)|m_1) = 0.9$
 $m_1 = \text{"Fredrik is sick"}$

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Explanation $m = \text{"Fredrik is....."}$

ML-rule

$$p(r(t)|m_1) = 0.9$$

$m_1 = \text{"Fredrik is sick"}$

$$p(r(t)|m_2) = ?$$

$m_2 = \text{"Fredrik is dead"}$

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$m_1 = \text{"Fredrik is sick"}$

$$p(r(t)|m_2) = 1$$

$m_2 = \text{"Fredrik is dead"}$

$$\text{Fredrik is dead} = \operatorname{argmax}_m p(r(t)|m)$$

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$$= \operatorname{argmax}_m p(r(t)|m)p(m)$$

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MAP receiver

Suppose that on the next lecture, I will not show up Observation $r(t) = \text{"fredrik is not here"}$

Why am I not here?

Explanation $m = \text{"Fredrik is....."}$

MAP-rule

$$p(m_1|r(t)) \propto p(r(t)|m_1)p(m_1)$$

$m_1 = \text{"Fredrik is sick"}$

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MAP receiver

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Why am I not here?

Explanation $m = \text{"Fredrik is....."}$

MAP-rule

$$p(m_1|r(t)) \propto p(r(t)|m_1)p(m_1) \approx 0.9 \times 0.1 = 0.09$$

$m_1 = \text{"Fredrik is sick"}$

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MAP-rule

$$p(m_1|r(t)) \propto p(r(t)|m_1)p(m_1) \approx 0.9 \times 0.1 = 0.09$$

$m_1 = \text{"Fredrik is sick"}$

$$p(m_2|r(t)) \propto p(r(t)|m_2)p(m_2)$$

$m_2 = \text{"Fredrik is dead"}$

MAP is defined as

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$m_1 = \text{"Fredrik is sick"}$

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MAP receiver

$$\text{Fredrik is sick} = \operatorname{argmax}_m p(r(t)|m)$$

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$$p(m_1|r(t)) \propto p(r(t)|m_1)p(m_1) \approx 0.9 \times 0.1 = 0.09$$

m_1 = "Fredrik is sick"

$$p(m_2|r(t)) \propto p(r(t)|m_2)p(m_2) \approx 1 \times 0.0001 = 0.0001$$

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MAP receiver

The ML rule is, in general, totally crazy

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The ML rule is, in general, totally crazy

- According to what rule do we do everyday decisions?
- According to which rule should a court make their decisions?

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MAP receiver

The ML rule is, in general, totally crazy

- According to what rule do we do everyday decisions? **MAP**
- According to which rule should a court make their decisions? **MAP**

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MAP receiver

True court case: Sally Clark case, England 1998

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Observation: Sally Clark, mother of two, had two babies that died in infancy

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True court case: Sally Clark case, England 1998

Observation: Sally Clark, mother of two, had two babies that died in infancy

$P(\text{observation}|\text{natural causes}) = 1/10000$ according to expert in child deaths

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Implication (to us):

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Implication (to us): NONE Implication to court:

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Implication (to us): NONE

Implication to court: Lifetime jail sentence

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True court case: Sally Clark case, England 1998

CLEARLY ML

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MAP ?

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$P(\text{mother is murderer}) = 1/1000...000$

$P(\text{mother is not murderer}) = 0.9999...999$

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$P(\text{observation}|\text{murder}) = 1$ according to common sense

$P(\text{mother is murderer}) = 1/1000 \dots 000$

$P(\text{mother is not murderer}) = 0.9999 \dots 999$

$P(\text{natural causes}|\text{observation}) \propto 0.999 \dots 999 / 10000$

MAP is defined as

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$P(\text{mother is murderer}) = 1/1000\dots000$

$P(\text{mother is not murderer}) = 0.9999\dots999$

$P(\text{natural causes}|\text{observation}) \propto 0.999\dots999/10000$

$P(\text{murder}|\text{observation}) \propto 1/1000\dots000$

MAP is defined as

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Which one is more logical?

Lecture 1: MAP receiver and signal space

MAP receiver

True court case: Sally Clark case, England 1998

MAP: NOT GUILTY

Observation: Sally Clark, mother of two, had two babies that died in infancy

$P(\text{observation}|\text{natural causes}) = 1/10000$ according to expert in child deaths

$P(\text{observation}|\text{murder}) = 1$ according to common sense

$P(\text{mother is murderer}) = 1/1000\dots000$

$P(\text{mother is not murderer}) = 0.9999\dots999$

$P(\text{natural causes}|\text{observation}) \propto 0.999\dots999/10000$

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MAP is defined as

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MAP receiver

True court case: Sally Clark case, England 1998

MAP: NOT GUILTY

Aftermath: Released in 2003, after some math professors took a look at the case.

Sally died from alcoholism somewhat later

MAP is defined as

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Which one is more logical?

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MAP receiver

Hypothetical case: Lottery with 1000000 tickets

MAP is defined as

$$\begin{aligned}\hat{m} &= \operatorname{argmax}_m p(m|r(t)) \\ &= \operatorname{argmax}_m p(r(t)|m)p(m)\end{aligned}$$

ML is defined as

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Which one is more logical?

Lecture 1: MAP receiver and signal space

MAP receiver

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$$P(\text{someone presents the winning ticket} | \text{person bought a ticket}) = 1/1000000$$

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ML: ??

MAP: ??

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ML: Jail

MAP: Prior probability of fraud must be evaluated

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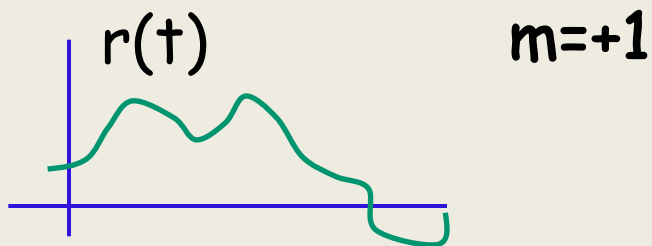
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MAP receiver

To analyze a digital communications system, it is difficult to work with a continuous time signal

It is hard to evaluate a probability of the form $p(r(t)|m)$

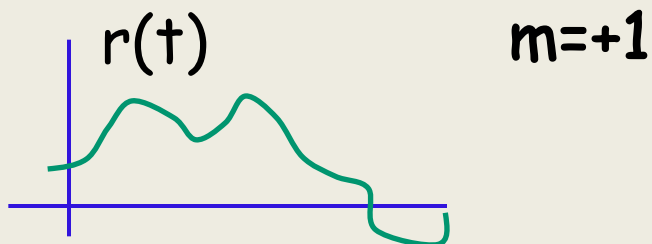


Lecture 1: MAP receiver and signal space

MAP receiver

To analyze a digital communications system, it is difficult to work with a continuous time signal

It is hard to evaluate a probability of the form $p(r(t)|m)$



We are used to evaluate probabilities of the form $p(r|m)$

$$r=1.4312$$

$$m=+1$$

Lecture 1: MAP receiver and signal space

MAP receiver

To analyze a digital communications system, it is difficult to work with a continuous time signal

Concept of signal space:

- Transfer all continuous signals into discrete vectors
- Transformation should be such that no information is lost
- Transformation is done via a set of basis functions
- For two systems with identical signal spaces, all properties (E_b , BER, etc) are identical
- However, bandwidth - properties are not. They depend on the basis functions
- Allows for a simpler description and analysis of the system.