ETTN01 Advanced Digital Communications, (max: 30p)

Take home examination, August 19, 2020

Send scanned solutions (e.g., photos by mobile phone) to me (fredrik.rusek@eit.lth.se) no later than 08.00 August 20

Write clearly! If I cannot read what you write, it will count as 0 points.

It is important to show the intermediate steps in arriving an answer, otherwise you may lose points.

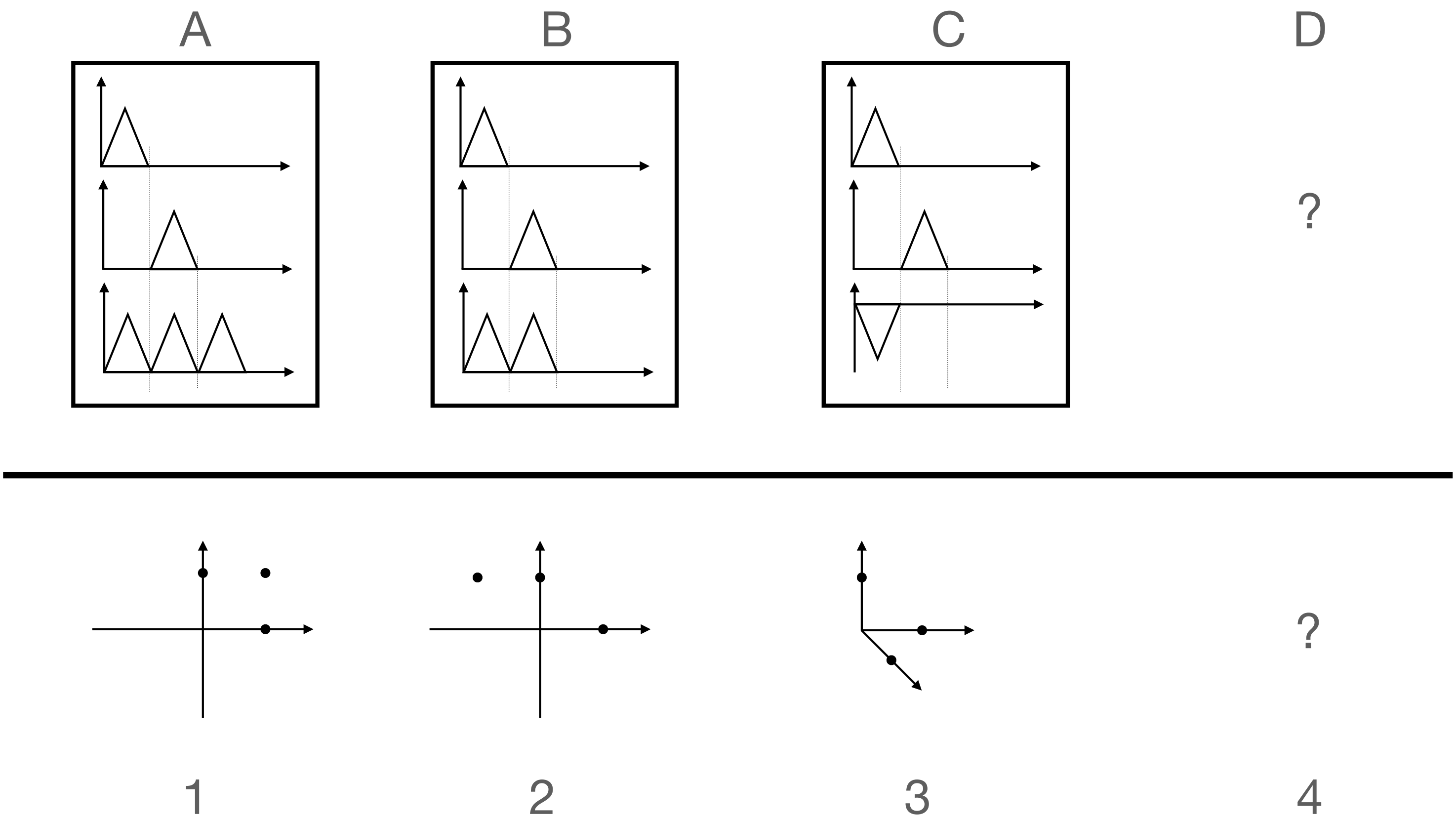
If you provide two answers to the same question, and one is wrong, you lose points. If you make side-comments or if you write too much about a problem, e.g., burying me in paper where you have written down everything you know about a topic with the goal that ”at least something must be correct”, you may lose points for everything that is wrong.

Passing grade: 15p.

Problem 1. 10p

Consider the Figure below. Three signal sets are provided (A,B, and C) and 3 signal space descriptions (1,2, and 3). Two of the signal sets correspond to 2 of the signal space descriptions, but one of the signal sets corresponds to signal space description 4, which is left unspecified. Similarly, One of the signal space descriptions corresponds to signal set D, which is also left unspecified.

Determine which signal sets that correspond to which signal space descriptions, and sketch the missing signal set D and signal space description 4.



Problem 2. 10p

Assume a system with a design according to:

- An OFDM system,with
  - 1000 subcarriers spaced 15kHz apart
  - negligible length of the CP (i.e., you may assume it is 0)
- A transmit power of P = 3W is available at the transmitter side
- 4QAM modulation
- At each subcarrier, and time slot, the channel is Rayleigh distributed with parameter b=1 (see (9.41)).

Determine the noise density N_0 so that the bit error probability is 10^{-5}

Problem 3. 10p

Assume two equally likely signal alternatives $s_0(t)$ and $s_1(t)$ where $\int s_0(t)s_1(t)\mathrm{d}t = 0$, $\int s_0^2(t)\mathrm{d}t = 1$, and $\int s_1^2(t)\mathrm{d}t = 2$. Let $N_0 = 0.1$. Assume that the received signal is $r(t) = \alpha s_j(t) + N(t)$ where $\mathrm{Prob}(\alpha = 1) = \mathrm{Prob}(\alpha = 10) = 0.5$. Determine the bit error probability.

Appendix B

Question

Given

Solution

Since $s_0(t)$ and $s_1(t)$ are orthogonal, signal space is at least 2-dimensional

Now find representation of $s_2(t)$ in $\phi_1(t)$ and $\phi_2(t)$

$S = \{ S_0(t), \quad S_1(t), \quad S_2(t) \}$

$\boxed{\text{Is } S \text{ 2-dimensianal ?}}$

$$\int S_0(t)\, S_1(t)\, dt = \ldots = 0$$

$$\int S_0^2(t)\, dt = \ldots = 2 \quad = E_0$$

$$\int S_1^2(t)\, dt = \ldots = 3 \quad = E_1$$

$$\phi_1(t) = S_0(t) \qquad \phi_2(t) = S_1(t)$$

Not orthonormal, but I don't care....

$$S_{2,1} = \int S_2(t)\, \phi_1(t)\, dt = 4$$

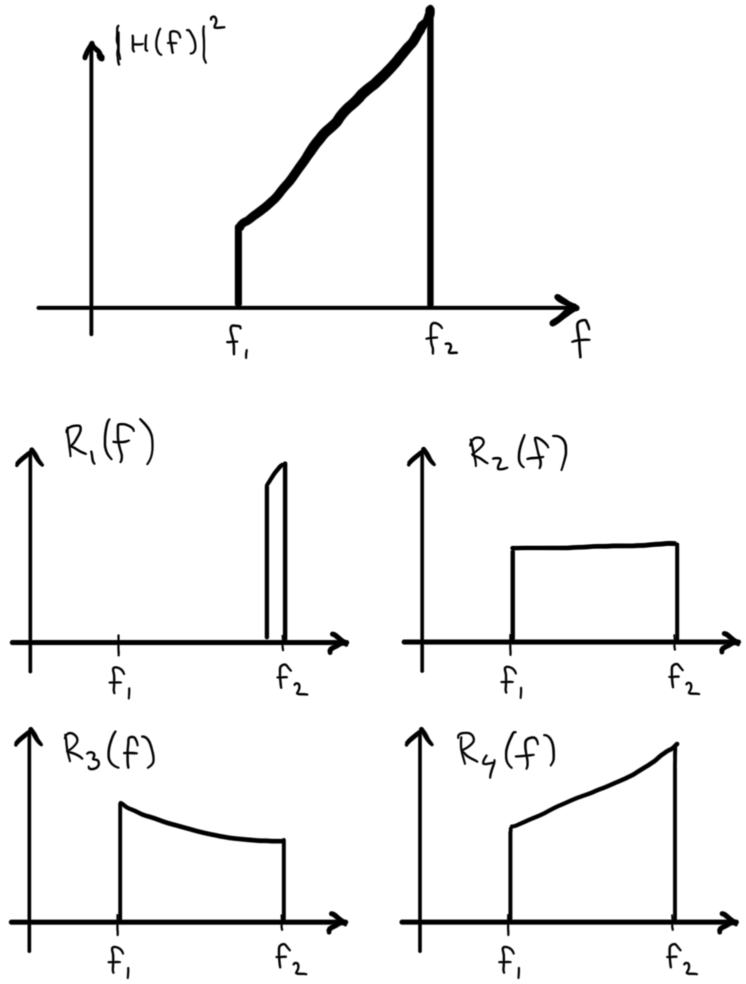
$$S_{2,2} = \int S_2(t)\, \phi_2(t)\, dt = 3$$

$$E_2 = \int S_2^2(t)\, dt = 12$$

Since $\dfrac{S_{2,1}^2}{E_0} + \dfrac{S_{2,2}^2}{E_1} = 11 < 12$

the signal set is 3-dimensianal.

Appendix C



Amplitudes of $R_k(f)$ not specified