1

a/

$$E_0 = \int_0^T s_0^2(t)\,dt = \ldots = \frac{\alpha^2 T}{3} = E_1 = E_2 = E_3$$

$$\phi_1(t) = s_0(t)\,\frac{\sqrt{3}}{\alpha\sqrt{T}}$$

$$s_{1,1} = \int s_1(t)\,\phi_1(t)\,dt = \int_0^T \alpha\left(1-\frac{t}{T}\right)\frac{\alpha t}{T}\,\frac{1}{\alpha}\sqrt{\frac{3}{T}}\,dt$$

$$= \sqrt{\frac{3}{T}}\cdot\frac{\alpha}{T}\int_0^T t - \frac{t^2}{T}\,dt = \sqrt{\frac{3}{T}}\,\frac{\alpha}{T}\left(\frac{T^2}{2} - \frac{T^3}{3T}\right)$$

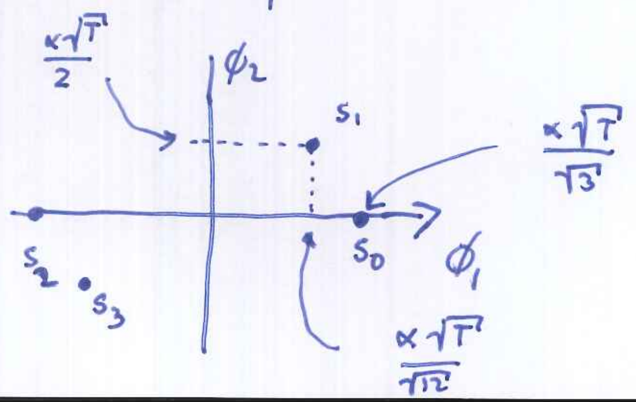
$$= \sqrt{\frac{3}{T}}\,\alpha\,T\,\frac{1}{6} = \alpha\,\frac{\sqrt{T}}{\sqrt{12}}$$

remaining energy of $s_1(t)$:

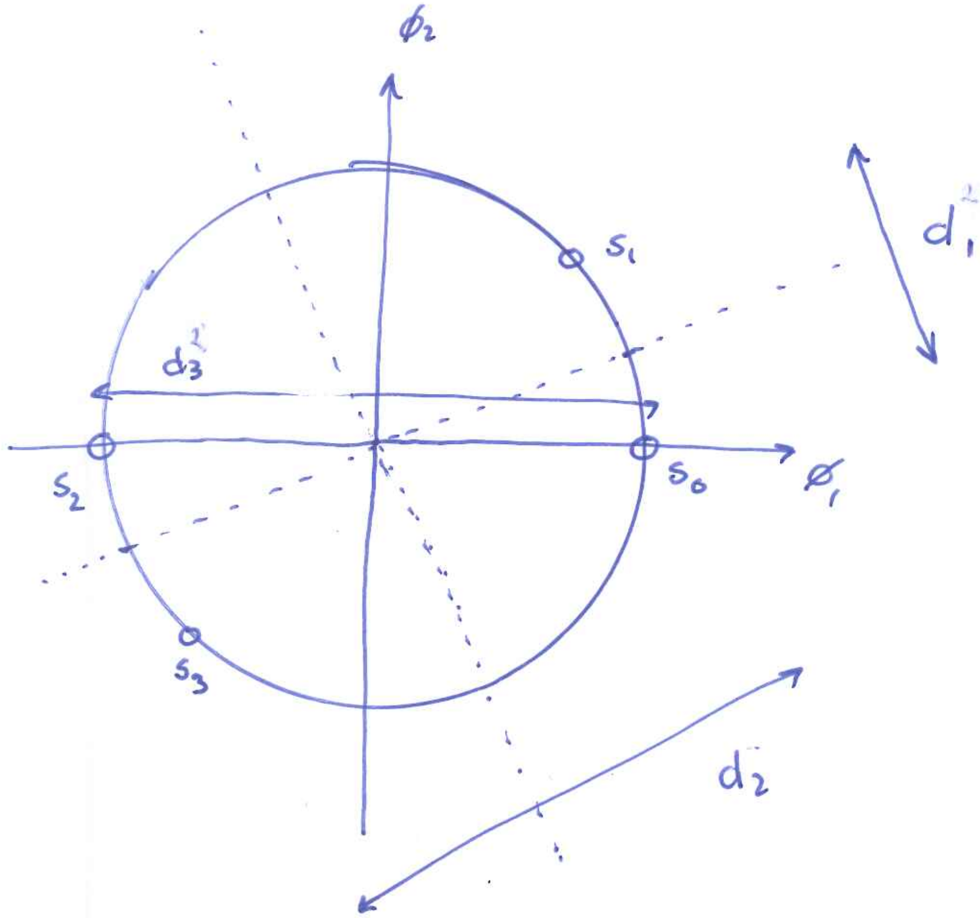
$$E_1 - s_{1,1}^2 = \frac{\alpha^2 T}{3} - \frac{\alpha^2 T}{12} = \frac{\alpha^2}{12}\left(4T - T\right)$$

$$= \frac{\alpha^2}{4}\,T$$

Thus



b/



$d_1 = \sqrt{E_0}$

$d_2 = \sqrt{3E_0}$

$d_3 = 2\sqrt{E_0}$

$\text{Prob}(\text{error} \mid s_0 \text{ sent}) =$

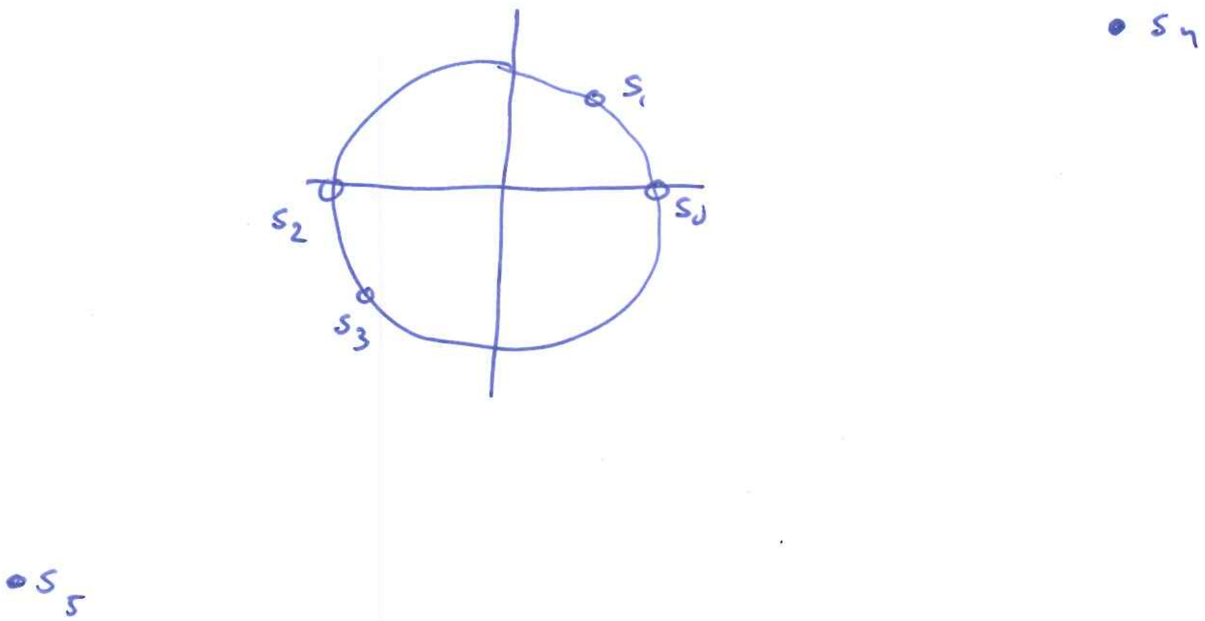
$= 1 - \text{Prob}(\text{correct} \mid s_0 \text{ sent}) =$

$$= 1 - \left[1 - Q\left(\sqrt{\frac{d_1^2}{2N_0}}\right)\right]\left[1 - Q\left(\sqrt{\frac{d_2^2}{2N_0}}\right)\right]$$

$$= \boxed{Q\left(\sqrt{\frac{d_1^2}{2N_0}}\right) + Q\left(\sqrt{\frac{d_2^2}{2N_0}}\right) - Q\left(\sqrt{\frac{d_1^2}{2N_0}}\right)Q\left(\sqrt{\frac{d_2^2}{2N_0}}\right)}$$

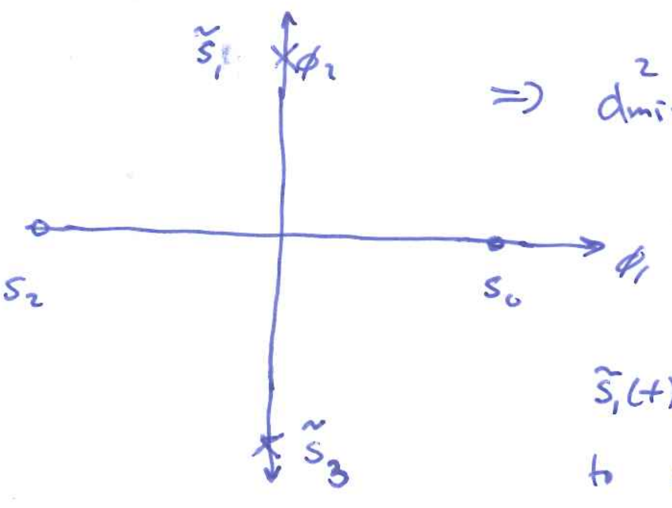
c/ $d_{min} = d_1 = \sqrt{E_0}$

d/ $s_4(t) = 3 s_0(t) + 2 s_1(t)$



e/ same as b/ signal space does not depend on prior probabilities

F/



$\Rightarrow d_{min}^2 = 2$ (maximum)

$\tilde{s}_1(t)$ any signal orthogonal to $s_1(t)$ with energy E_0

e.g.

g/ We must first establish $\phi_2(t)$

step 1/ $s_{1,1} = \int s_1(t)\phi_1(t)\,dt = I_1$

2/ $\tilde{\phi}_2(t) = s_1(t) - I_1\phi_1(t)$

3/ $\phi_2(t) = \dfrac{\tilde{\phi}_2(t)}{\sqrt{I_2}}$ where $I_2 = \int \tilde{\phi}_2^2(t)\,dt$

check energy in $s_4(t)$ projected to basis

1/ $s_{4,1} = \int s_4(t)\,\phi_1(t)\,dt = I_3\beta_1 + I_4\beta_2$

$s_{4,2} = \int s_4(t)\,\phi_2(t)\,dt = I_5\beta_1 + I_6\beta_2$

2/ Energy of $s_4(t)$ in basis $= s_{4,1}^2 + s_{4,2}^2$

total energy of s_4

$$E_4 = \int \beta_1^2 + \beta_1\beta_2\frac{2t^2}{T} + \beta_2^2\frac{t^4}{T^2}\,dt$$

$$= \beta_1^2 I_7 + \beta_2^2 I_8 + \beta_1\beta_2 I_9$$

remaing energy

$$E_4 - s_{4,1}^2 - s_{4,2}^2 = \beta_1^2 I_7 + \beta_2^2 I_8 + \beta_1\beta_2 I_9 - \beta_1^2(I_3^2 + I_5^2)$$
$$- \beta_2^2(I_4^2 + I_6^2) - 2\beta_1\beta_2\left[I_3 I_4 + I_5 I_6\right]$$

g cont./

This must be zero for $s_4(t)$ to be in the basis.

So,

check if the following problem has any solutions

$$0 = \beta_1^2 \underbrace{\left[I_7 - I_3^2 - I_5^2\right]}_{A_1} + \beta_2^2 \underbrace{\left[I_8 - I_4^2 - I_6^2\right]}_{A_2} + \beta_1 \beta_2 \underbrace{\left[I_9 - 2I_3I_4 - 2I_5I_6\right]}_{A_3/2}$$

$$\beta_1^2 A_1 + \beta_2^2 A_2 + 2\beta_1\beta_2 A_3 = 0$$

can be shown that $A_1 A_2 - A_3^2 = 0$ for this to happen.

2

a/

$$\int z_1(t) z_2(t) dt = \ldots = \int \underbrace{x_1(t) y_2(t)}\, dt + \int \underbrace{x_2(t) y_1(t)}\, dt$$

we dont know anything about

so : No, not in general

b/

$$\int v^2(t) dt = \sum_{n=1}^{N} v_n^2 A_n$$

c/

$$\int v^2(t) dt = v_1^2 + v_2^2 + 2 v_1 v_2 \int \psi_1(t) \psi_2(t) dt$$

$$= v_1^2 + v_2^2 + v_1 v_2$$

d/

$$f(\bar{x}) = \int \left[y(t) - x(t)\right]^2 dt = \int y^2(t) + x^2(t) - 2\sum y(t) x_n \phi_n(t)\, dt$$

$$= E_y + \sum_{n=1}^{N} x_n^2 - 2 \sum x_n \underbrace{\int y(t) \phi_n(t) dt}_{y_n}$$

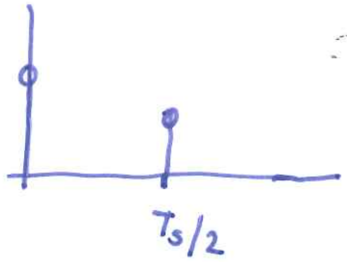
$$= E_y + \sum_{n=1}^{N} x_n^2 - 2 \sum x_n y_n$$

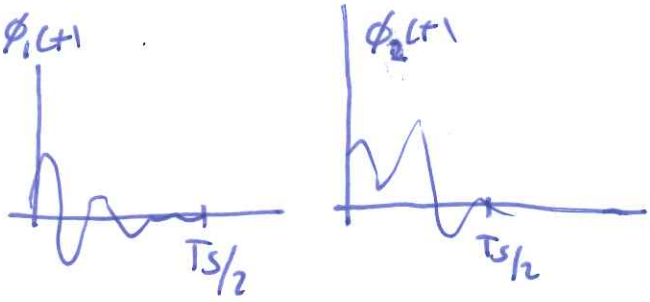
$$\frac{\partial f}{\partial x_n} = 2x_n - 2y_n = 0 \Rightarrow \boxed{x_n = y_n, \quad 1 \leq n \leq N}$$

e/ The amount of energy in $s_4(t)$ that cannot be represented in the signal space generated from $s_0(t), \ldots, s_3(t)$

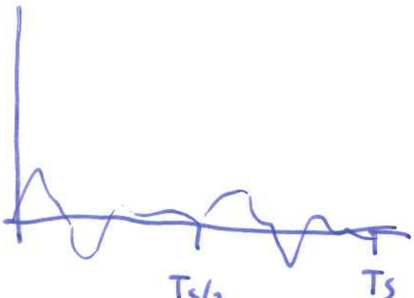
f/ (1) can happen if $h(t) = \delta(t)$

(2) can happen if $h(t)$





received signal is



$\phi_1(t)$ and $\phi_2(t)$ cannot represent this in $T_s/2 \le t \le T_s$

f cont./

(3) can happen, as we have seen, in non coherent FSK

if $\nu_j = \frac{\pi}{2} \quad \forall j$

(Note that with Noncoherent FSK, (2) is the normal case)

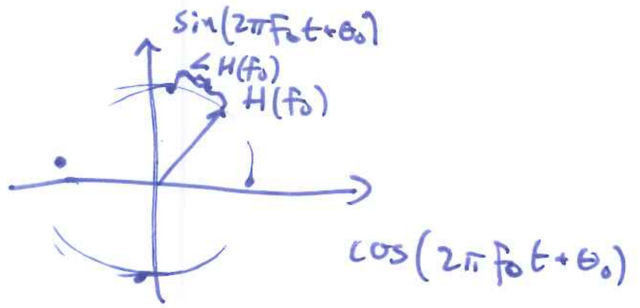
③ a/ OFDM-comp.

From (5.11) $\left\{ \cos(2\pi f_k t + \theta_k), \sin(2\pi f_k t + \theta_k) \right\}_{k=0}^{K-1}$

b/ From (5.13) assume QPSK inputs 

Signal space is $2K$ dimensional, but is appearing in mutually orthogonal pairs.

Pair 1 (symbol a_0)



b/ Because of the cyclic prefix.

The OFDM signal is designed to be orthogonal only after removal of the CP.

The transmitted signal includes the CP, so it is not orthogonal.

4 coherence time t_{coh} and coherence bandwidth f_{coh}

Assume a T_s and f_s

we need a pilot every t_{coh} seconds and one every f_{coh} Hz.

Thus $X = \frac{f_{coh}}{f_s}$ $Y = \frac{t_{coh}}{T_s}$

Loss : 1 pilot in XY symbols

$$\frac{\frac{f_{coh}\,t_{coh}}{f_s T_s} - 1}{\frac{f_{coh}\,t_{coh}}{f_s T_s}} = \frac{f_{coh}\,t_{coh} - 1}{f_{coh}\,t_{coh}}$$

(5)

a/ engineer B forgets that by reducing T, then P needs to be increased to maintain the same E_b (since $E_b = TP$).

Thus, to compare fairly, Engineer A should not only let W grow, but also P.

Then $\lim_{W,P \to} W \log\left(1 + \frac{P}{W N_0}\right) = \infty$

b/ $C = 1000\alpha \log_2\left(1 + \frac{A}{1000\alpha N_0}\right) + 1000\alpha \log_2\left(1 + \frac{4A}{1000\alpha N_0}\right)$

$+ 1000\alpha \log_2\left(1 + \frac{16A}{1000\alpha N_0}\right)$

c/ High SNR:

$C \approx 1000\alpha \log_2\left(\frac{A}{1000\alpha N_0}\right) + 1000\alpha \log_2\left(1 + \frac{4A}{1000\alpha N_0}\right) + \ldots$

Now $\dfrac{\log_2\left(\frac{A}{1000\alpha N_0}\right)}{\log_2\left(\frac{4A}{1000\alpha N_0}\right)} \to 1, \quad N_0 \to 0$

So, yes. All bands contribute roughly the same c cont./

Low SNR

$$C \approx \frac{A}{N_0 \ln(2)} + \frac{4A}{N_0 \ln(2)} + \frac{16A}{N_0 \ln(2)}$$

No. All bands do not contribute equally.
(band "3" is giving 16x band "1")

d/ No, since WF takes N_0 into account.

thus, a fixed $R(f)$ cannot be optimal