1

(1)

a) $P_I = Q\left(\sqrt{2\frac{|s_1|^2+|s_2|^2}{N_0}}\right)$

$P_E = Q\left(\sqrt{2\alpha^2\frac{|s_1|^2+|s_2|^2}{N_0}}\right)$

b) Eaves dropper is 6dB weaker

From Fig 5.13 it is seen that

~~SN~~

$\frac{E_b}{N_0} \leq -1$ dB For the message to be lost

but at $-1+6=5$ dB $P_b > 10^{-3}$

so it is not possible.

c) $r_E = \int r_E(t)\left[\phi_1(t)+\phi_2(t)\right]dt$

$= \pm\left[\alpha s_1 + \alpha s_2\right] + n$

if $s_1 = -s_2 \Rightarrow$

$r_E = n$ only noise

d/ TYPO IN EXAM

Should be

$$r_I = \int_0^{T_s} r_E(t)\,\phi_1(t)$$

$$r_E = \pm[\alpha s_1 + \alpha s_2] + n_e$$

$$r_I = \pm s_1 + n_I$$

$$s_1 = -s_2$$

half of power is lost at Intended user

$$P_b = Q\left(\sqrt{\frac{E_{sent}}{N_0}}\right) = Q\left(\sqrt{\frac{2|s_1|^2}{N_0}}\right) \qquad E_{sent} = |s_1|^2 + |s_2|^2$$

e/

$$r_E = \pm[\alpha s_1 + \alpha s_2] + n_E$$

$$r_I = \pm[s_1 - s_2] + n_I \quad \leftarrow \quad Var = 2\,N_0/2$$

$$s_1 = -s_2 \quad \Rightarrow$$

$$r_I = \pm 2 s_1 + n_I$$

$$P_b = Q\left(\sqrt{2\,\frac{4\,\frac{E_{sent}}{2}}{2N_0}}\right) = Q\left(\sqrt{\frac{4\,E_{sent}}{2N_0}}\right)$$

$$= Q\left(\sqrt{2\,\frac{E_{sent}}{N_0}}\right)$$

f) again $s_1 = -s_2$

optimal receiver

$$\text{Var} = N_0/2$$

$$r_{I,1} = \int r_I(t)\,\phi_1(t)\,dt = \pm s_1 + n_1$$

$$r_{I,2} = \int r_I(t)\,\phi_2(t)\,dt = \mp s_1 + n_2$$

$$P_b = Q\left(\sqrt{2\,\frac{2|s_1|^2}{N_0}}\right) = Q\left(\sqrt{2\,\frac{E_{sent}}{N_0}}\right) \quad (= e)$$

(2)

a) $P = \int_{-\infty}^{\infty} R(f)\,df = \int_{-\infty}^{\infty} \beta\,|P(f)|^2\,df =$

$= \beta \int_{-4000}^{4000} \frac{1}{8000}\,df = \beta$

b) $P = E_s R_s E_p = E_s \cdot 8000 \cdot 1 = 8000 E_s$

c) $C = \frac{1}{2} \int_{-4000}^{4000} \log_2\left(1 + \frac{2\beta}{N_0}|P(f)|^2\right) df$

$= \frac{1}{2} \cdot 8000 \log_2\left(1 + \frac{2\beta}{8000 N_0}\right)$

$\overset{a)}{=} 4000 \log_2\left(1 + \frac{2P}{8000 N_0}\right) = 4000 \log_2\left(1 + \frac{1}{4000}\frac{P}{N_0}\right)$

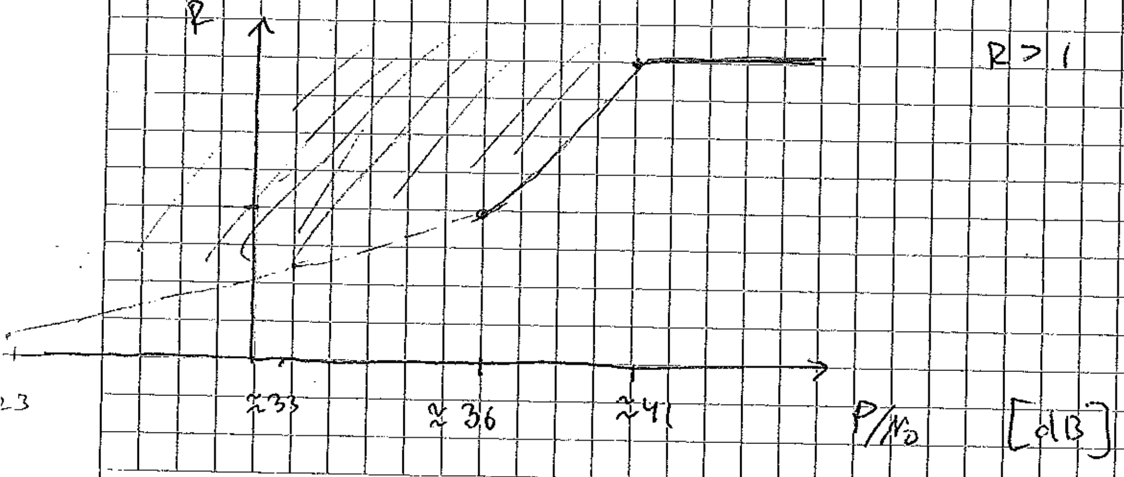
d) $4000 \log_2\left(1 + \frac{1}{4000}\frac{P}{N_0}\right) > \begin{cases} 8000 R & (M=1) \\ 8000 \cdot 2R & (M=2) \end{cases}$

$\log_2\left(1 + \frac{1}{4000}\frac{P}{N_0}\right) > 2R \quad \leftarrow M=1$

$R < \frac{1}{2} \log\left(1 + \frac{1}{4000}\frac{P}{N_0}\right)$

$R > 0$
$R < 1$

$R > 1$ Not possible



e/ $R = 1/2 \Rightarrow \frac{P}{N_0} \approx 36$ dB $(= 4000)$

Assume $N_0 = 1 \Rightarrow P = 4000$

$R_s E_s = P \Rightarrow E_s = \frac{P}{R_s} = P T_s = \frac{1}{2}$

$P_b = Q\left(\sqrt{2 \cdot \frac{1}{2}}\right) = Q(1) \approx 0.16$

3/

a/ $\int_0^4 f_k(t) f_\ell(t)\, dt = 0 \quad k \neq \ell$

$\Rightarrow \dim = 3$

b/ $\int_0^4 f_k^2(t)\, dt = 1 \quad \forall k \Rightarrow$ Orthonormal

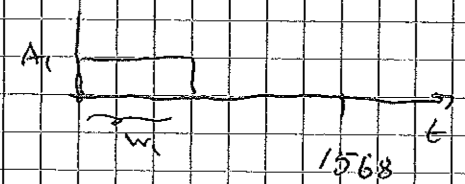
$a_1 = \int f_1(t) x(t)\, dt = 0$

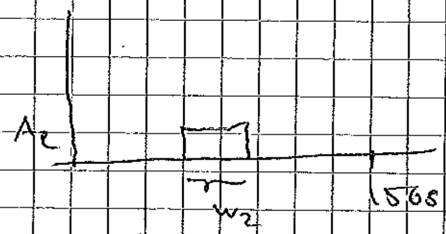
$a_2 = \int f_2(t) x(t)\, dt = 0$

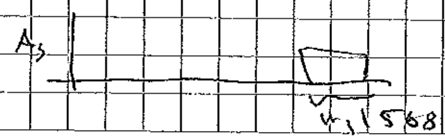
$a_3 = \int f_3(t) x(t)\, dt = 0$

$\Rightarrow$ No

c/







where $A_k^2 w_k = 1$

ORIGINAL

d) $D^2_{min} = 2$

$E_b = \frac{E_s}{\log_2(3)} = \frac{1}{\log_2(3)}$

$d^2_{min} = \frac{D^2_{min}}{2E_b} = \log_2(3) = 1.585$

ADD $f_4(t)$

$D^2_{min} = 1$

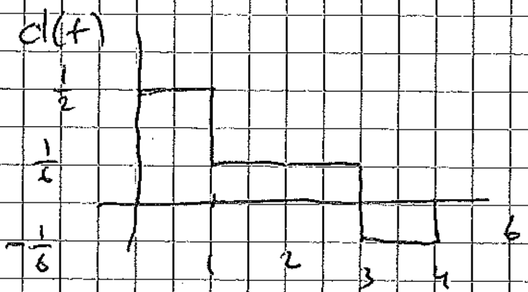
$E_b = \frac{\frac{1}{4}\left[1+1+1+0\right]}{2} = \frac{\frac{3}{4}}{2} = \frac{3}{8}$

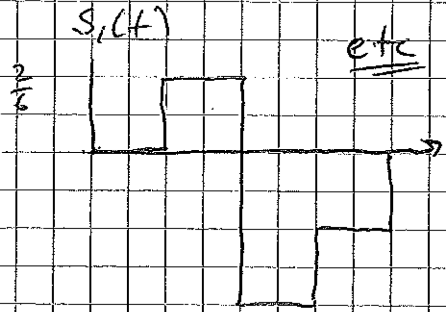
$d^2_{min} = \frac{1}{2\frac{3}{8}} = \frac{8}{6} = \frac{4}{3} = 1.33$

d^2_{min} Decrases

e) A DC-level carries no information but consumes energy. energy eff decreased due to DC-level

f)





etc

f) cont.

ORIGINAL

$d^2_{mn} = \log_2(3)$

(see p. 279) for energy efficiency

New

$\int s_k(t)\, s_e(t)\, dt \neq 0$ in general

$\int s_k^2(t)\, dt = \frac{2}{3} \quad \forall k$

$E_s = \frac{2}{3}$

$E_b = \frac{2}{3} \frac{1}{\log_2(3)}$

$D_{1,2}^2 = \int |s_1(t) - s_2(t)|^2 dt = D_{13}^2 = D_{23}^2 = 2$

$\Rightarrow$

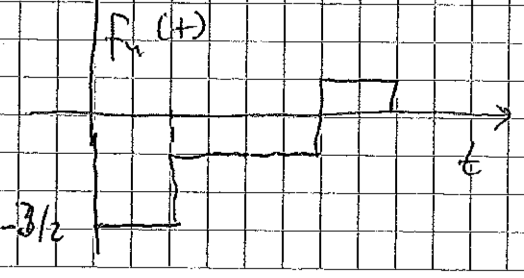
$d^2_{mn} = \frac{2}{2 \cdot \frac{2}{3} \frac{1}{\log_2(3)}} = \frac{3 \log_2(3)}{2}$

Increases by a factor $\frac{3}{2}$

g)

$\frac{1}{4} f_1(t) + \frac{1}{4} f_2(t) + \frac{1}{4} f_3(t) + \frac{1}{4} f_4(t) = 0$

$f_4 = -f_1(t) - f_2(t) - f_3(t)$



g/ cont $D^2_{k4} = 6 \quad k = 1,2,3$

$D^2_{k\ell} = 2 \quad k \neq \ell \quad 1 \leq k,\ell \leq 3$

$\Rightarrow D^2_{mn} = 2$

$$\int F_k^2(t)\,dt = \begin{cases} 1 & k = 1,2,3 \\ 3 & k = 4 \end{cases}$$

$E_s = \frac{1+1+1+3}{4} = \frac{6}{4} = \frac{3}{2} \qquad E_b = \frac{3}{2}\,\frac{1}{2} = \frac{3}{4}$

$d^2_{mn} = \frac{2}{2 \cdot \frac{3}{4}} = \frac{4}{3} = 1.33$

~~Worse~~ worse than original

4)

a) $$P_b = p(a=0)\cdot\frac{1}{2} + p(a=\cancel{0}2)\, P_{b|a=2}$$

$$= 0.1\cdot\frac{1}{2} + 0.9\, Q\left(\sqrt{2\frac{4E_s}{N_0}}\right)$$

b) $$\frac{E_s}{N_0} \to \infty \;;\quad P_b = 0.1\frac{1}{2} = \frac{1}{20}$$

c) $$P_b = p(a_1=0, a_2=0)\frac{1}{2} + p(a_1=2\, a_2=0)\, Q\left(\sqrt{2\frac{4E_s}{N_0}}\right)$$

$$+ p(a_1=0, a_2=2)\, Q\left(\sqrt{2\frac{4E_s}{N_0}}\right)$$

$$+ P(a_1=a_2=2)\, Q\left(\sqrt{2\frac{8E_s}{N_0}}\right)$$

d) $$\frac{E_s}{N_0} \to \infty \;;\quad P_b = \frac{1}{2}\cdot\frac{1}{10}\frac{1}{10} = \frac{1}{200}$$

5

a) $E = \mathbb{E} \int_0^{T_{obs}} |x(t)|^2 dt \qquad P T_{obs} = E$

$$\mathbb{E}|x(t)|^2 = \mathbb{E}\left(\sum_{k=0}^{K-1} \sum_{\ell=0}^{K-1} a_k a_\ell^* e^{j2\pi(g_k - g_\ell) f_\Delta t} \right)$$

$$= [k=\ell]$$

$$= \sum_{k=0}^{K-1} E_a = K E_a$$

$$E = T_{obs} K E_a = P T_{obs}$$

$$E_a = \frac{P}{K}$$

b) SINCE $N = K$ ~~$a_k \geq X$~~

a_n is $\{a_k\}$ permuted version of $\{X_k\}$

and there is no zero-padding.

$\{x_n\}$ are time signals and are not 16 QAM

See 2.19 - 2.20

c) From 2.19 - 2.21 X_k is some a_k multiplied with N.

Thus, from a), $E_X = N^2 E_a = \frac{N^2 P}{K} = KP$

d) No see page 41 of lecture notes

e) we obtain orthogonality among carriers. No ISI is present

f) None, since there is no ISI that we have studied