1/

a $$E_0 = E_1 = \int_0^{T_s/2} s_1^2(t)\,dt = \underline{\underline{s_1^2 + s_2^2}}$$

b $$z_0(t) = s_0(t) * h(t)$$
$$z_1(t) = s_1(t) * h(t) = -z_0(t)$$

$$z_0(t) = s_1 h_1 y_{11}(t) + s_1 h_2 y_{12}(t) + s_1 h_3 y_{13}(t) + s_2 h_1 y_{21}(t) + s_2 h_2 y_{22}(t) + s_2 h_3 y_{23}(t)$$

where

$$y_{mn}(t) = \phi_m(t) * \phi_n(t)$$

$$y_{mn}(t) \neq 0 \quad \text{for} \quad 0 \le t \le T_s \quad (\text{in general})$$

$\boxed{\text{orthogonality knowledge not usable}}$

$$E_{z_0} = \int_0^{T_s} z_0^2(t)\,dt = s_1^2 h_1^2 E_{1111} + 2s_1^2 h_1 h_2 E_{1112} + 2s_1^2 h_1 h_3 E_{1113}$$
$$+ 2s_1 s_2 h_1^2 E_{1121} + 2s_1 s_2 h_1 h_2 E_{1122} + 2s_1 s_2 h_1 h_3 E_{1123}$$
$$+ s_1^2 h_2^2 E_{1212} + 2s_1^2 h_2 h_3 E_{1213} + 2s_1 s_2 h_2 h_1 E_{1221}$$
$$+ 2s_1 s_2 h_2^2 E_{1222} + 2s_1 s_2 h_2 h_3 E_{1223}$$
$$+ s_1^2 h_3^2 E_{1313} + 2s_1 s_2 h_3 h_1 E_{1321} + 2s_1 s_2 h_3 h_2 E_{1322}$$
$$+ 2s_1 s_2 h_3^2 E_{1323} + s_2^2 h_1^2 E_{2121} + 2s_2^2 h_1 h_2 E_{2122} + 2s_2^2 h_1 h_3 E_{2123}$$
$$+ s_2^2 h_2^2 E_{2222} + 2s_2^2 h_2 h_3 E_{2223} + s_2^2 h_3^2 E_{2323}$$

where $$E_{mnlk} = \int_0^{T_s} y_{mn}(t)\, y_{lk}(t)\,dt$$ (which are unknown) b cont.

collect terms:

$$E_{z_0} = s_1^2 \left[h_1^2 E_{1111} + 2h_1h_2 E_{1112} + 2h_1h_3 E_{1113} + 2h_2^2 E_{1212} + 2h_2h_3 E_{1213} + h_3^2 E_{1313} \right]$$

$$+ s_2^2 \left[h_1^2 E_{2121} + 2h_1h_2 E_{2122} + 2h_1h_3 E_{2123} + h_2^2 E_{2222} + 2h_2h_3 E_{2223} + h_3^2 E_{2323} \right]$$

$$+ s_1 s_2 \left[2h_1^2 E_{1121} + 2h_1h_2 E_{1122} + 2h_1h_3 E_{1123} + 2h_2h_1 E_{1221} + 2h_2^2 E_{1222} + 2h_2h_3 E_{1223} + 2h_3h_1 E_{1321} + 2h_3h_2 E_{1322} + 2h_3^2 E_{1323} \right]$$

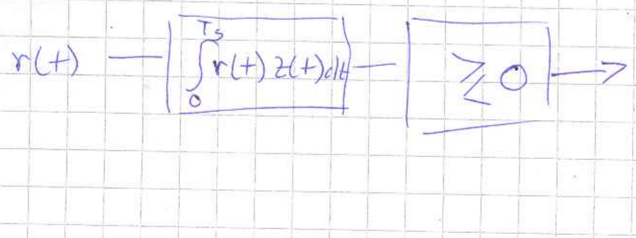
$$= s_1^2 A_{11} + s_2^2 A_{22} + 2s_1s_2 A_{12}$$ with A_{11} A_{22} A_{12} defined from above

Thus, $\underline{E_{b,r} = s_1^2 A_{11} + s_2^2 A_{22} + 2s_1s_2 A_{12}}$

c/ received signal $z_0(t) = z(t)$

$z_1(t) = -z(t)$

<u>antipodal</u>



d/ P_b for antipodal signaling : $Q\left(\sqrt{2\frac{E_{b,r}}{N_0}}\right)$

$$\rightarrow \quad P_b = Q\left(\sqrt{2\frac{s_1^2 A_{11} + s_2^2 A_{22} + 2 s_1 s_2 A_{12}}{N_0}}\right)$$

e/ $P_b = Q\left(\sqrt{2\frac{E_{b,r}}{N_0}}\right)$ Antipodal signaling with received $E_{b,r}$

g/ optimize $E_{b,r}$

$$E_{b,t} = E \rightarrow s_1^2 + s_2^2 = E$$

Method 1 (direct): $s_2 = \pm\sqrt{E - s_1^2}$

$$\frac{\partial}{\partial s_1} \; s_1^2 A_{11} + (E - s_1^2) A_{22} \pm 2 s_1 \sqrt{E - s_1^2} A_{12} = 0$$

+ if $A_{12} > 0$

− if $A_{12} < 0$

Method 2 $\quad s_1 = \cos(x)\sqrt{E}$

$s_2 = \sin(x)\sqrt{E}$

$$f(x) = A_{11} E \cos^2(x) + A_{22} E \sin^2(x) + 2 A_{12} E \sin(x)\cos(x)$$

$$\frac{\partial}{\partial x} f(x) = -2 A_{11} E \cos(x)\sin(x) + 2 A_{22} E \cos(x)\sin(x) + 2 A_{12} E \cos(2x) = 0$$

$$\sin(2x)\left[A_{22} - A_{11}\right] + 2 A_{12} \cos(2x) = 0$$

$$\sqrt{[A_{22} - A_{11}]^2 + 4 A_{12}^2}\; \sin(2x + \varphi) \qquad \varphi = \tan^{-1}\left(\frac{A_{22} - A_{11}}{2 A_{12}}\right)$$

$$2x + \varphi = 0 \rightarrow \boxed{x = -\frac{1}{2}\tan^{-1}\left(\frac{A_{22} - A_{11}}{2 A_{12}}\right)}$$

(2)

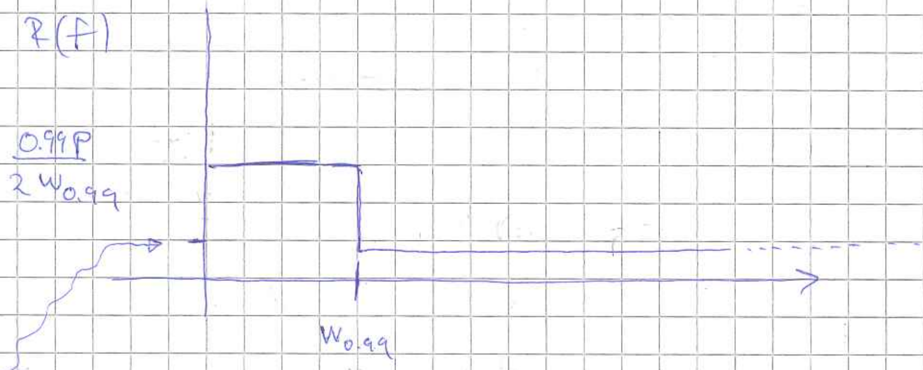
a) $W \log_2\left(1+\frac{P}{N_0 W}\right)$

b) Power inside $W_{0.99} = 0.99P \rightarrow$

$$C = W_{0.99} \log_2\left(1 + \frac{0.99P}{N_0 W_{0.99}}\right)$$

c) ONLY constraint: $\underline{0.99\,P \text{ inside } W_{0.99}}$

$\rightarrow$ unlimited bandwidth $\rightarrow$



$\frac{0.01\,P}{2\,\tilde{W}}$ with $\tilde{W} \rightarrow \infty$

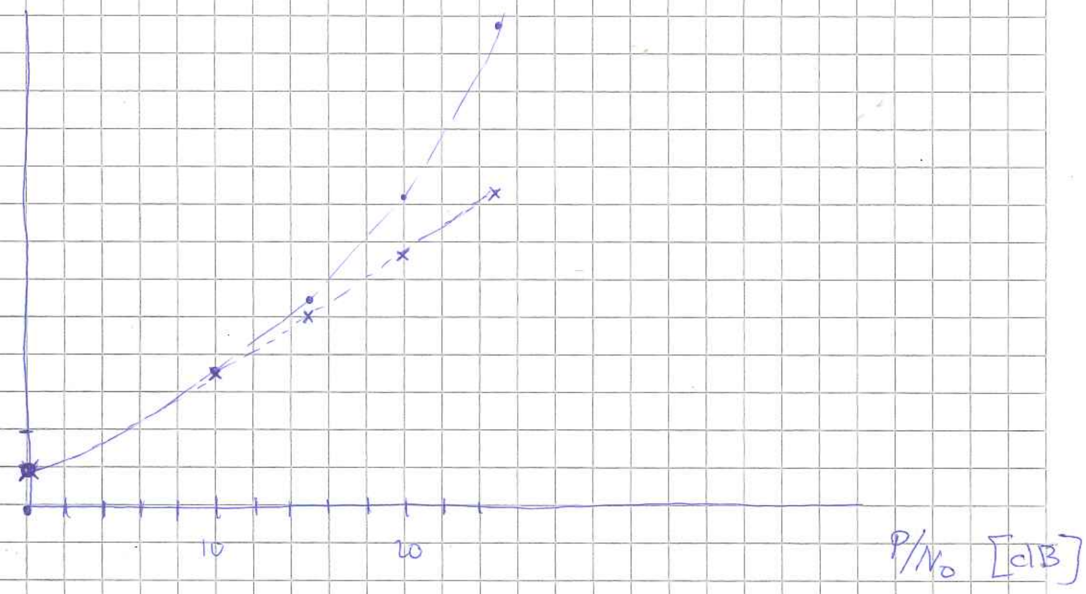
d) $C_{inside} = W_{0.99} \log_2\left(1 + \frac{0.99P}{N_0 W_{0.99}}\right)$

$$C_{outside} = \lim_{\tilde{W} \to \infty} \tilde{W} \log_2\left(1 + \frac{0.01P}{N_0 \tilde{W}}\right) = \frac{0.01P}{N_0 \ln(2)}$$

(eq. 5.63)

$$C = W_{0.99} \log_2\left(1 + \frac{0.99P}{N_0 W_{0.99}}\right) + \frac{0.01P}{N_0 \ln(2)}$$

e/



set $W_{0.99} = 1$ Hz (More rigorously: plot $C/W_{.99}$) against E_b/N_0

$\frac{P}{N_0}$ dB	C
0	1
10	3.6
15	5.47
20	8.09
25	12.86

shown with •

f/

$\frac{P}{N_0}$ dB	C
0	1
10	3.46
15	5.03
20	6.66
25	8.31

shown with x

99% BW makes sense up to about 15-20 dB $\frac{P}{N_0}$

g/ by repeating for other ε, we can see a large gap around $\frac{P}{N_0} \approx \frac{1}{1-\varepsilon}$ (linear scale)

(3)

a) 2 since $\int_0^T s_1(t)\, s_2(t)\, dt = 0$

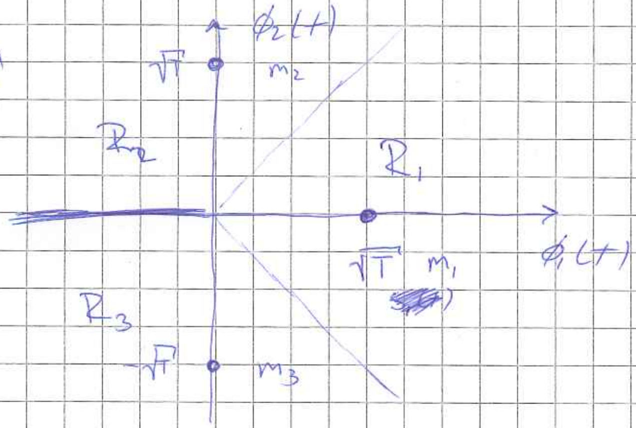
$s_2(t)$ and $s_3(t)$ clearly live in the same dimension

b) since $\int s_1(t)\, s_2(t)\, dt = 0 \rightarrow$

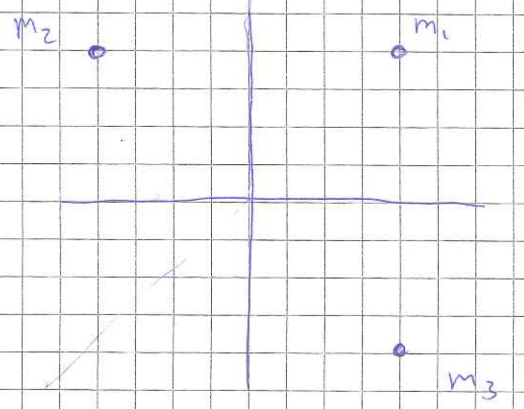
$\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}}$ $\phi_2(t) = \dfrac{s_2(t)}{\sqrt{E_2}}$

$E_1 = E_2 = T$

c/ and d/



e/ rotate signal space 45°



$$P(\text{error} \mid m_2) = P(\text{error} \mid m_3) = 1 - \iint_{(n_1, n_2) \in S'} p(n_1, n_2)\, dn_1 dn_2$$

$$P(\text{error} \mid m_1) = 1 - \iint_{\tilde{S}} p(n_1, n_2)\, dn_1 dn_2 \qquad \tilde{S} \subset S' \rightarrow P(\text{error} \mid m_1) > P(\text{error} \mid m_2)$$

f) Let $E_4 = \int_0^T s_4^2(t)\,dt$

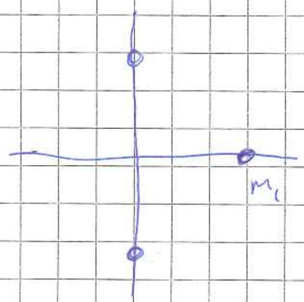
Let $a_1 = \int_0^T s_4(t)\,\phi_1(t)\,dt$

$a_2 = \int_0^T s_4(t)\,\phi_2(t)\,dt$

if $a_1^2 + a_2^2 = E \longrightarrow s_4(t)$ is in the same space

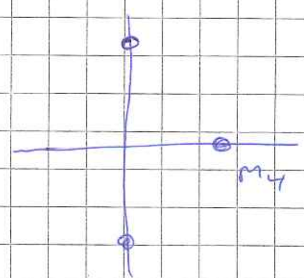
g) exactly the same situation as the original problem:

original

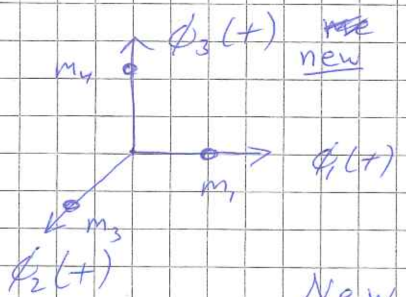


original remove s_1

add $s_4(t)$



h)



New situation! eq. to 3-FSK

all signals orthogonal, P(error | m_i) equal for all i

4

a)

P_b for Rayleigh fading $\approx \frac{1}{2d^2 \frac{E_b}{N_0}}$ (eq. 9.46)

d^2 for M-PAM : $\frac{6 \log_2(M)}{M^2-1}$

$P = R_b E_b \Rightarrow$

$$P_b \approx \frac{(M^2-1) \quad R_b \cdot N_0}{12 \log_2(M) \cdot P} = \frac{(M^2-1)}{12 \log_2(M)} \frac{1}{P} = 10^{-5}$$

$\Rightarrow$

$$P = 10^5 \cdot \frac{(M^2-1)}{12 \log_2(M)}$$

function decays with $M \Rightarrow$ choose ~~for~~ smallest.

But slow fading, freq-nonselective $\Rightarrow$

$R_s \gg \frac{1}{t_{coh}}$ and $R_s \ll \frac{1}{2T_m}$ (eq. 9.32 with $k_w=2$)

$R_s \gg 2000$ $\quad$ $R_s \ll 10000$

$\Rightarrow 2000 \ll R_s \ll 10000$

R_s as a function of M ?

$R_s = \frac{R_b}{\log_2(M)}$ $\quad$ M=2 does not work

$\Rightarrow$ choose M=4

M	R_s	ok ?
2	10000	×
4	5000	✓
8	3333	✓ ?
16	2500	✓ ? ?
32	2000	×

b/ Bandwidth $= 2R_s$

spectral broadening $= B_D = \frac{1}{t_{coh}} = 2000\ Hz$

total need $2R_s + 2000$

$R_s = \frac{R_b}{\log_2(M)} \rightarrow$

$$\boxed{BW = \frac{20000}{\log_2(M)} + 2000}$$

d/ $$P_b = \frac{1}{2p(R)\,\frac{6\log_2(M)}{M^2-1}\,\frac{E_b}{N_0}}$$

$P = E_b R_b$ even with coding

$$\Rightarrow \boxed{P = 10^5 \frac{(M^2-1)}{12\,p(R)\log_2(M)}}$$

$$R_s = \frac{R_b}{R\log_2(M)}$$

$2000 \ll R_s \leq 10000$

M	Range of R to satisfy	
2	$1 \ll R \ll 5$	Not possible
4	$\frac{1}{2} \ll R \ll 2.5$	
8	$\frac{1}{3} \ll R \ll 1.67$	wide range of possibilities.
16	$\frac{1}{4} \ll R \leq 1.25$	optimal choice depends on $p(R)$
32	$\frac{1}{8} \ll R \ll 1$	if $p(\frac{1}{3}) \gg p(\frac{1}{2})$ $M=8$ may be better than $M=4$

etc

5

a) since $T_{obs} f_\Delta = 1$

$$P = \frac{\log_2(M)}{\frac{N_{FFT} + N_{CP}}{N_{FFT}}}$$