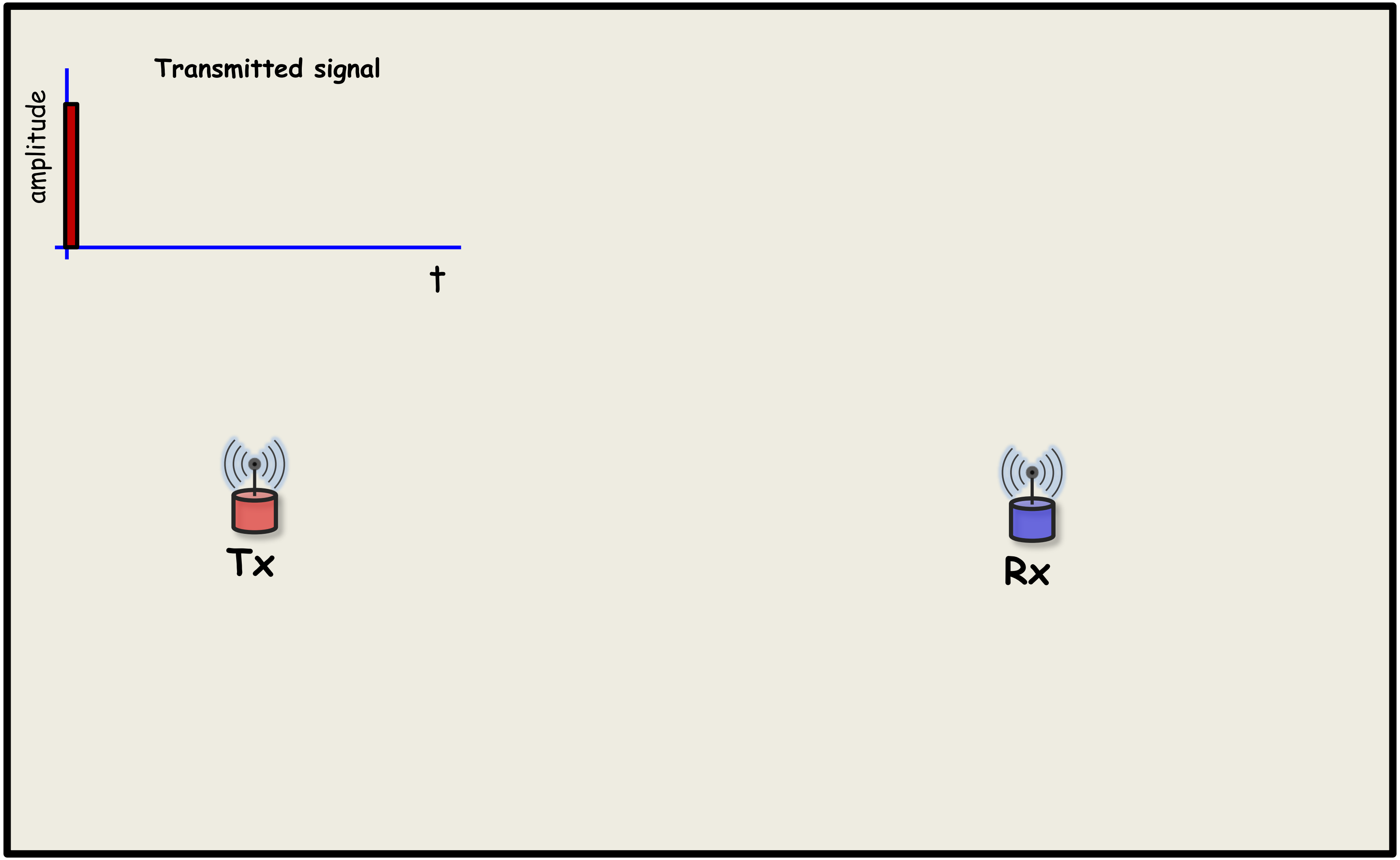
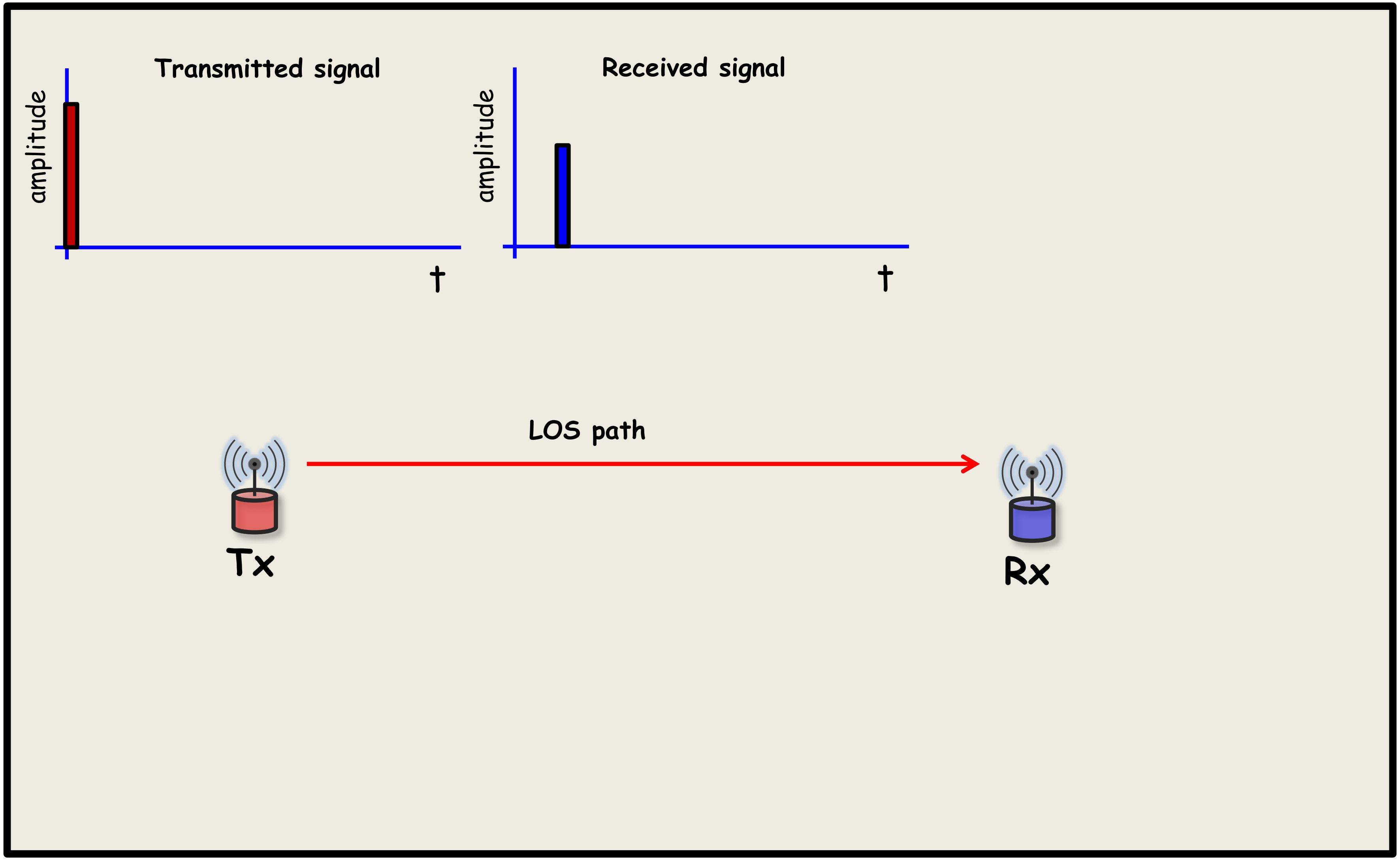
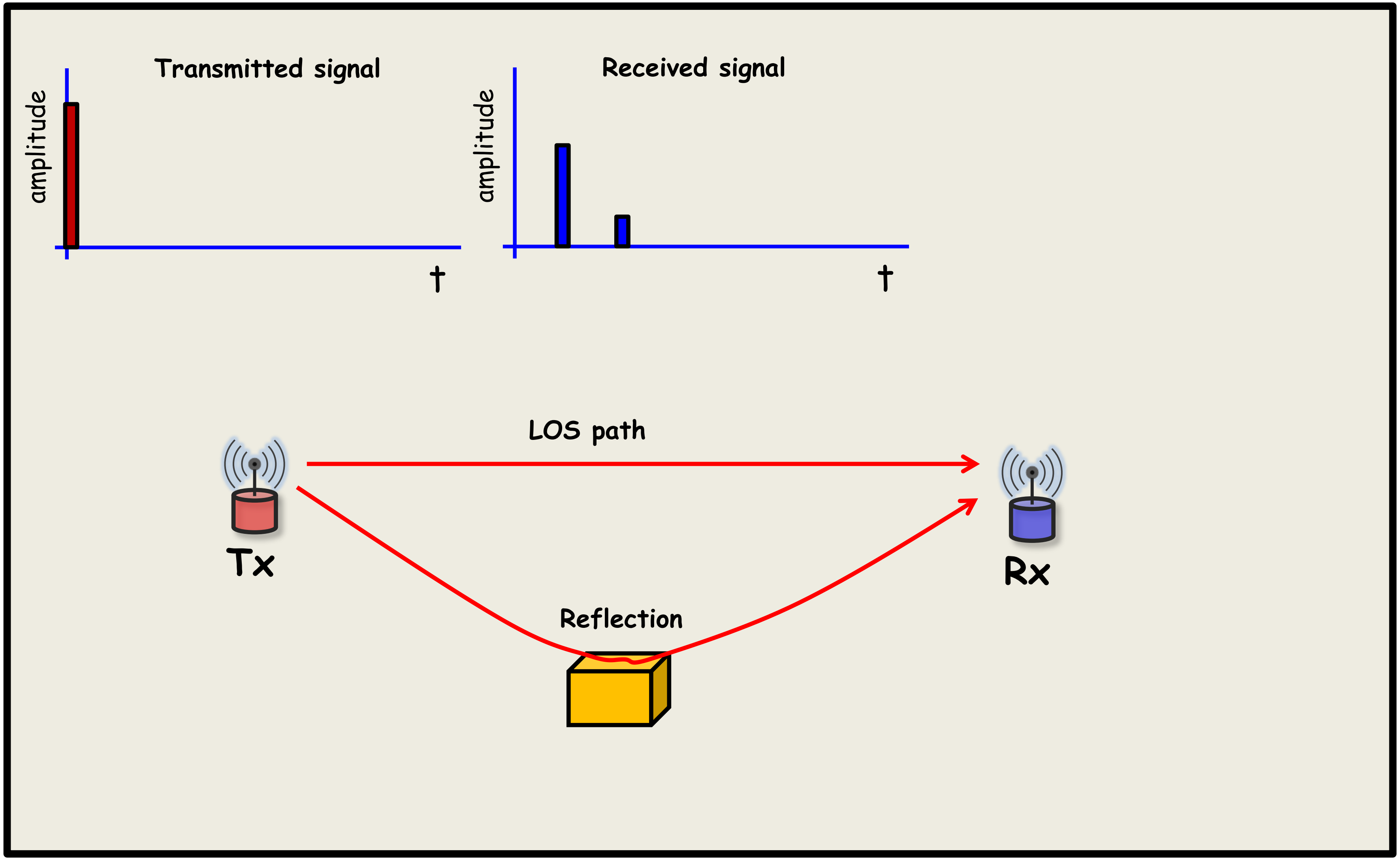
Lecture 9: Time variant channels



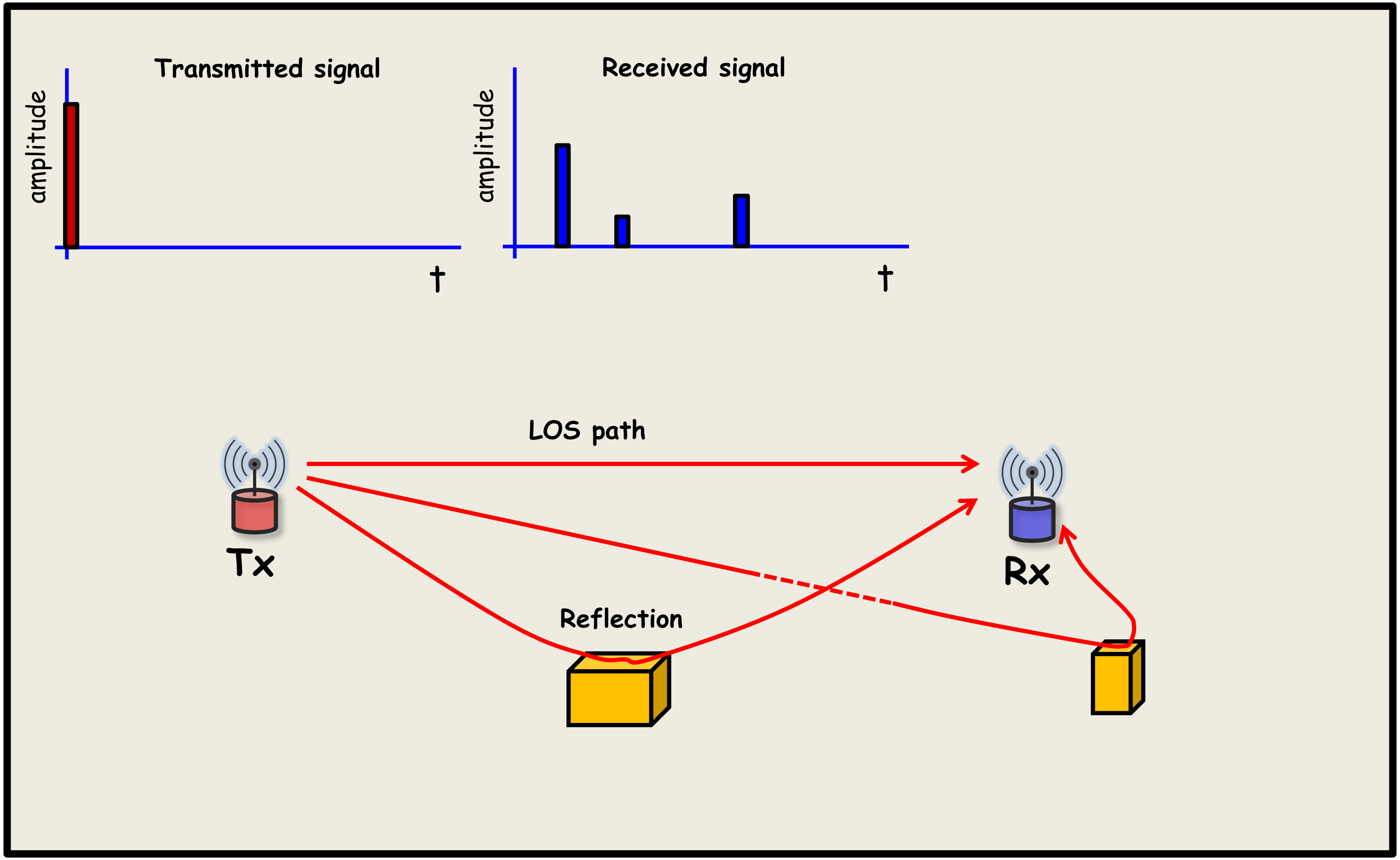
Lecture 9: Time variant channels



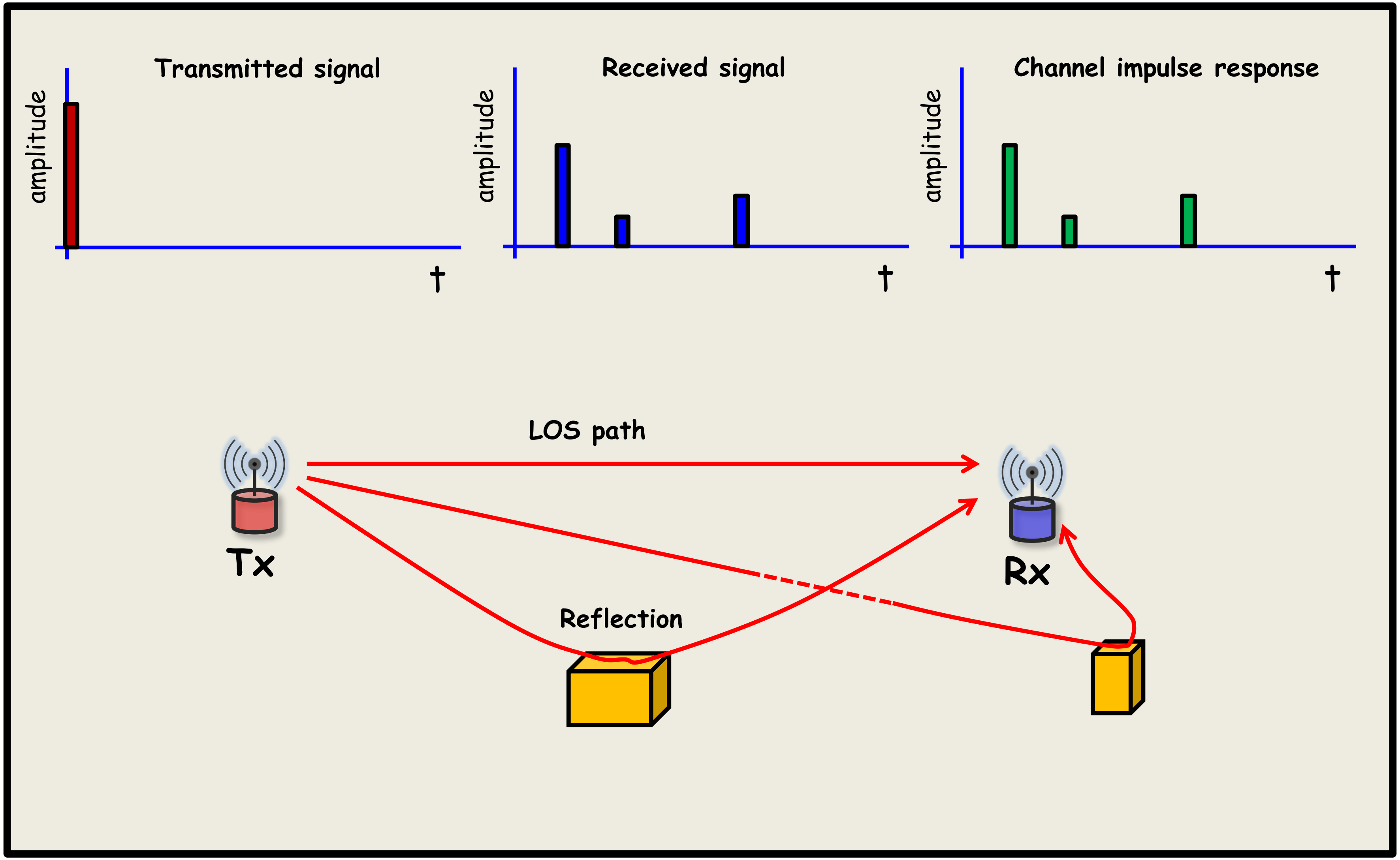
Lecture 9: Time variant channels



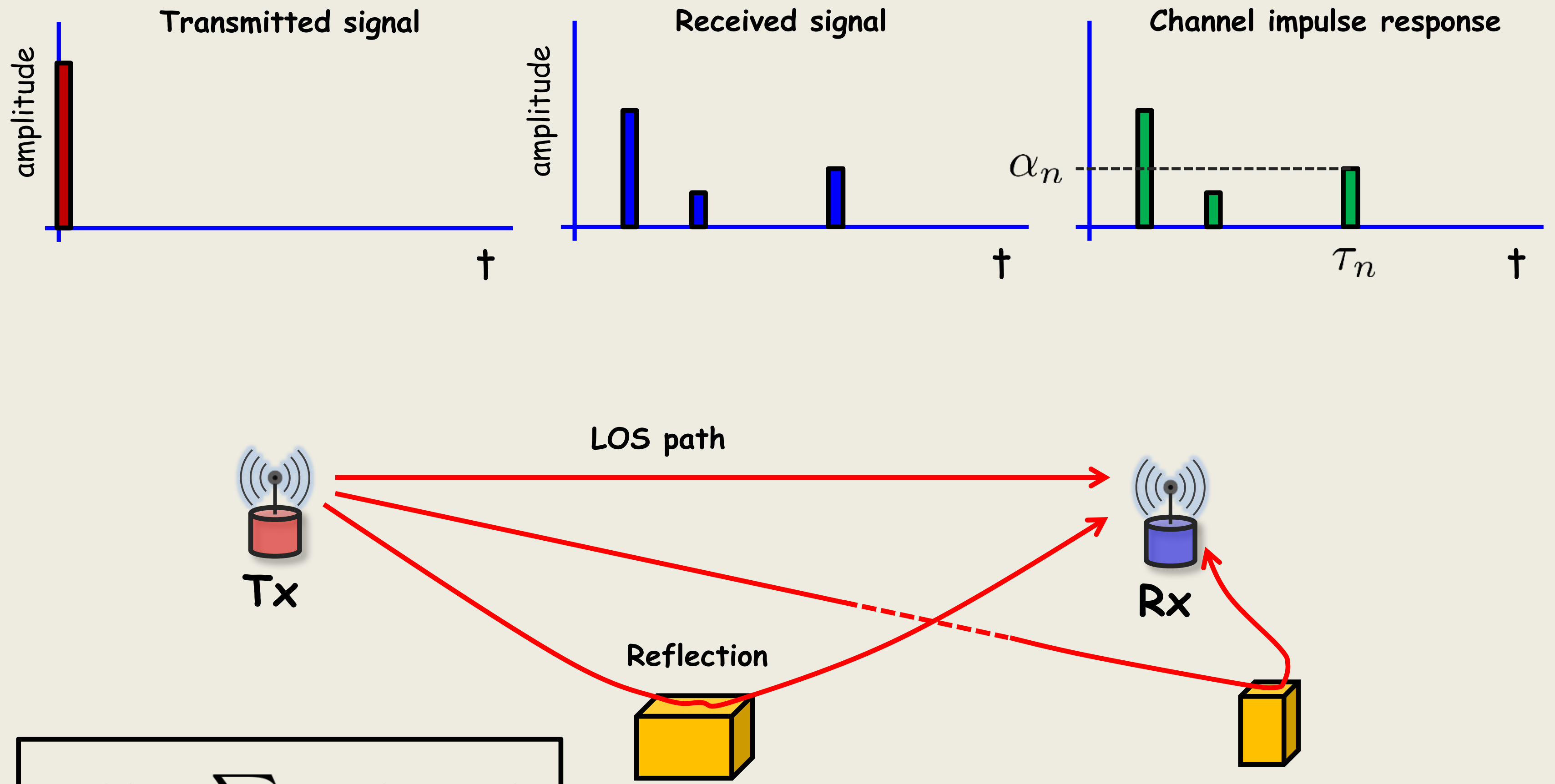
Lecture 9: Time variant channels



Lecture 9: Time variant channels

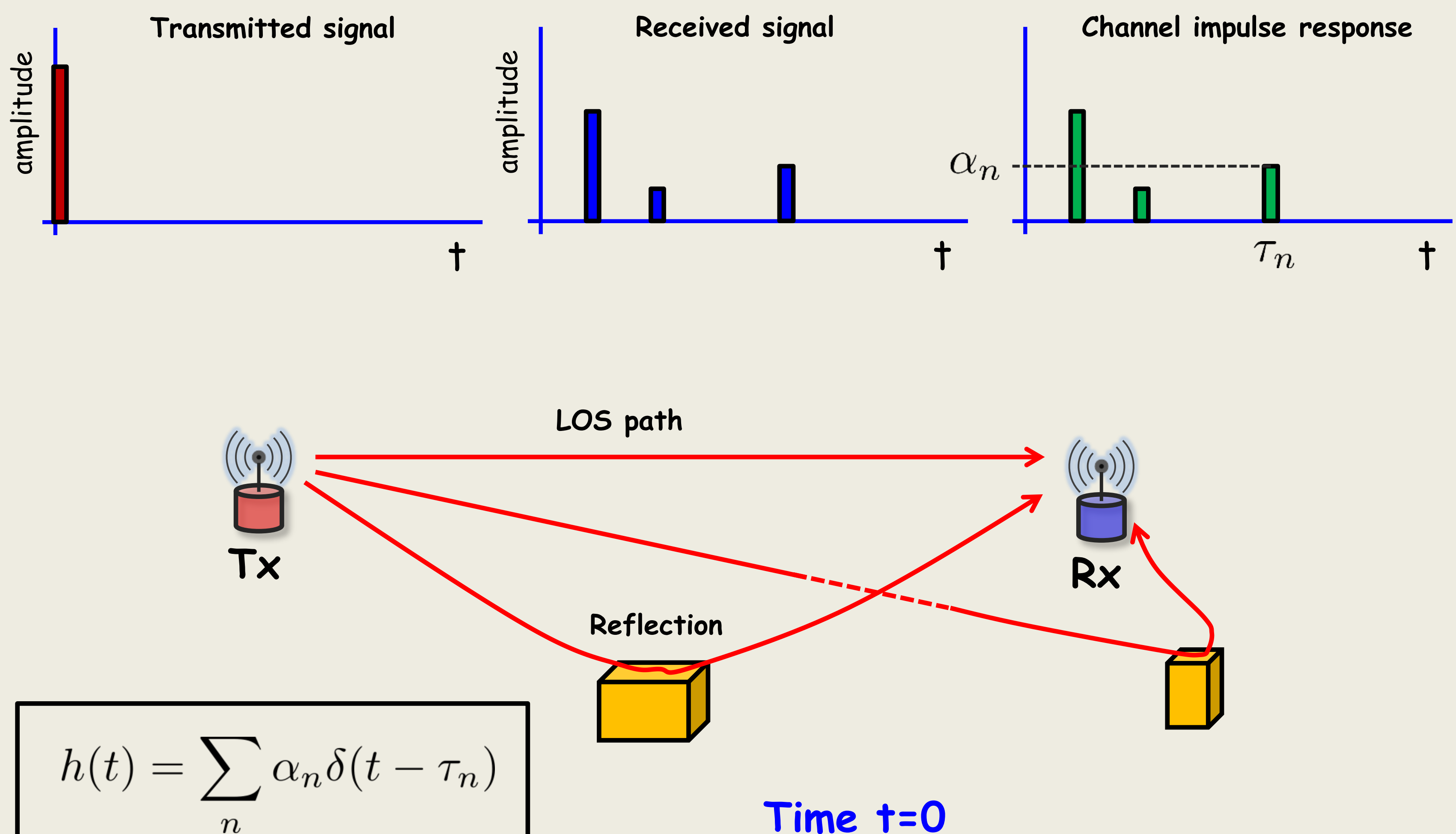


Lecture 9: Time variant channels



$$h(t) = \sum_n \alpha_n \delta(t - \tau_n)$$

Lecture 9: Time variant channels



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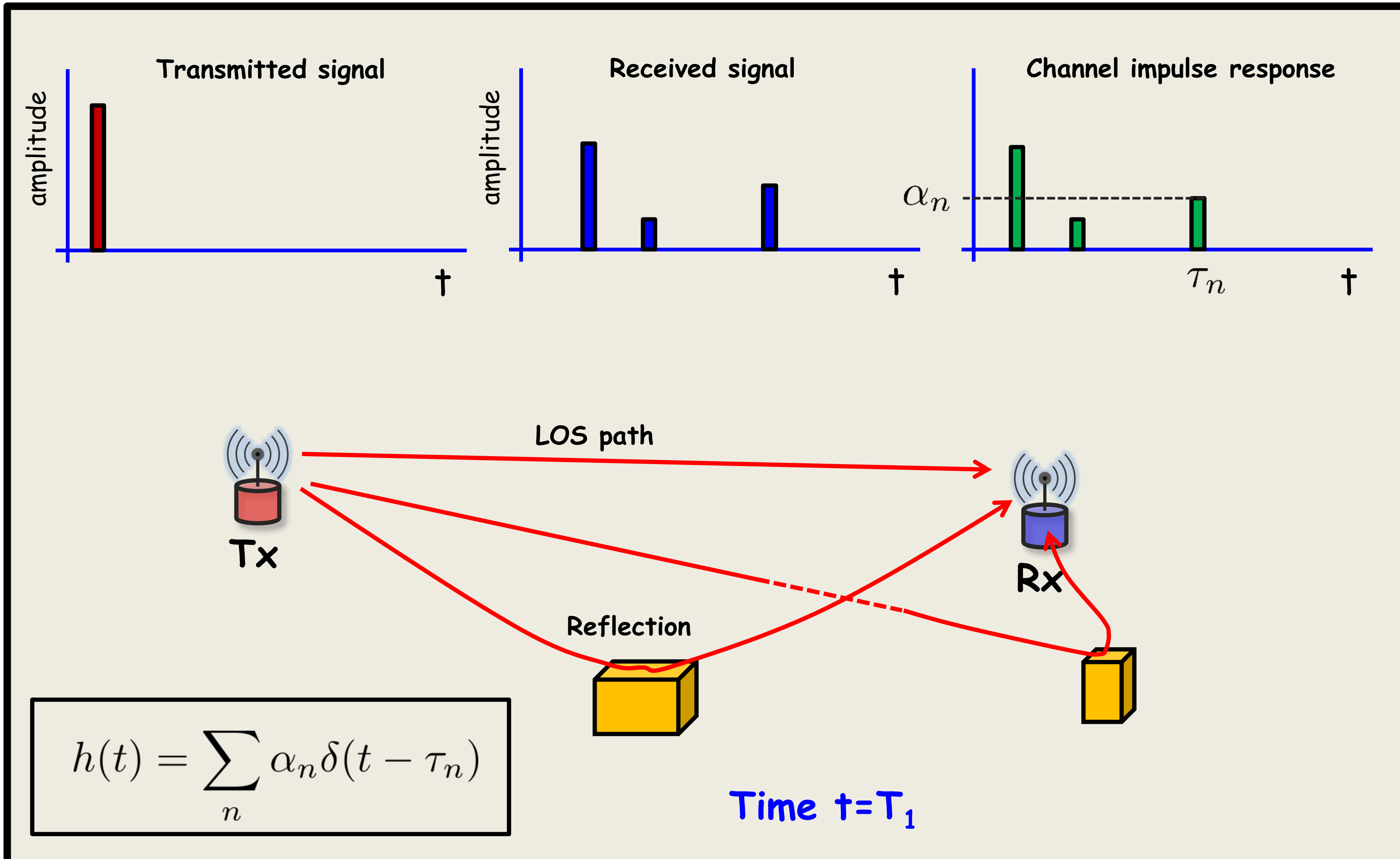
Lecture 9: Time variant channels



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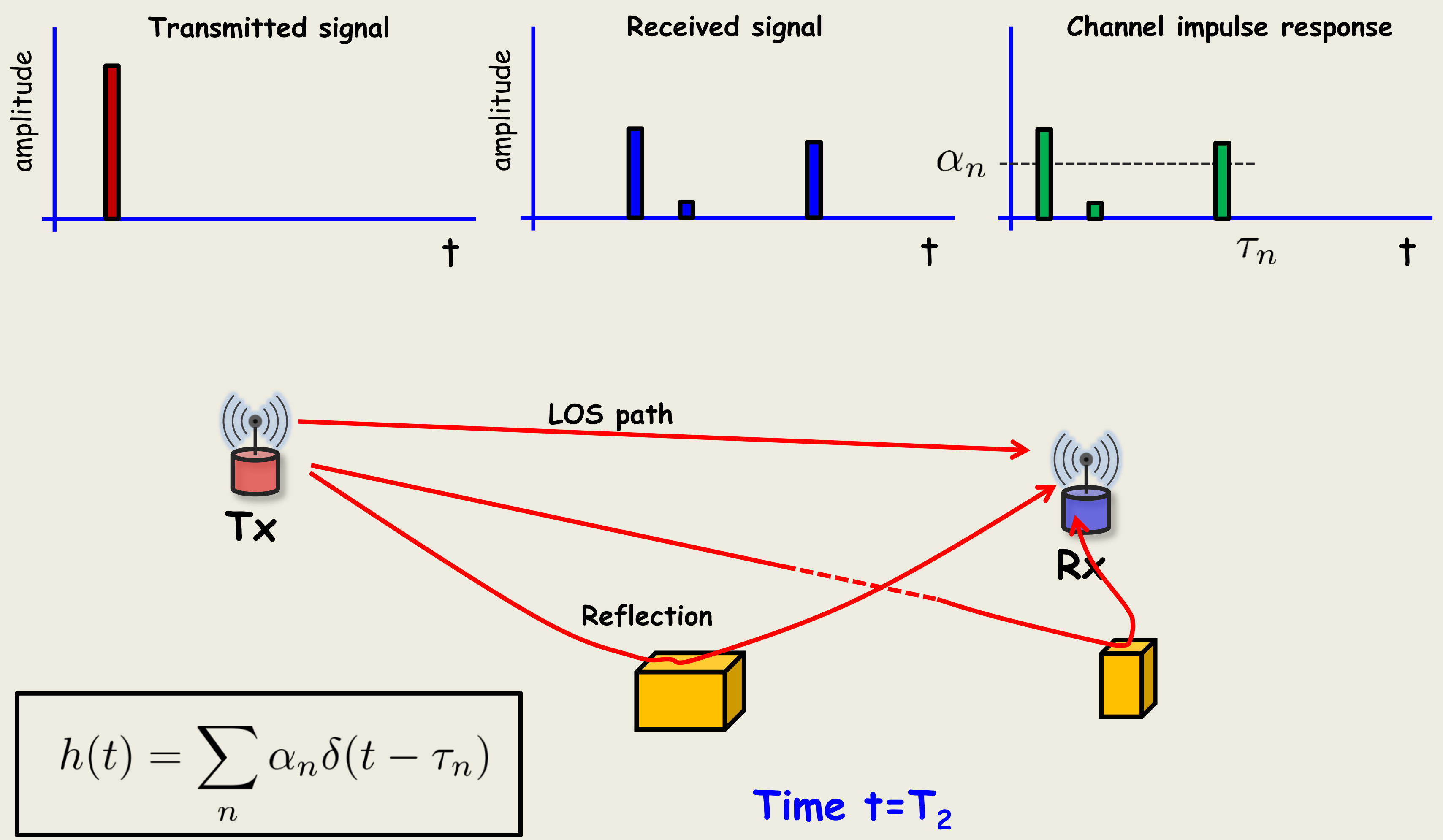
Lecture 9: Time variant channels



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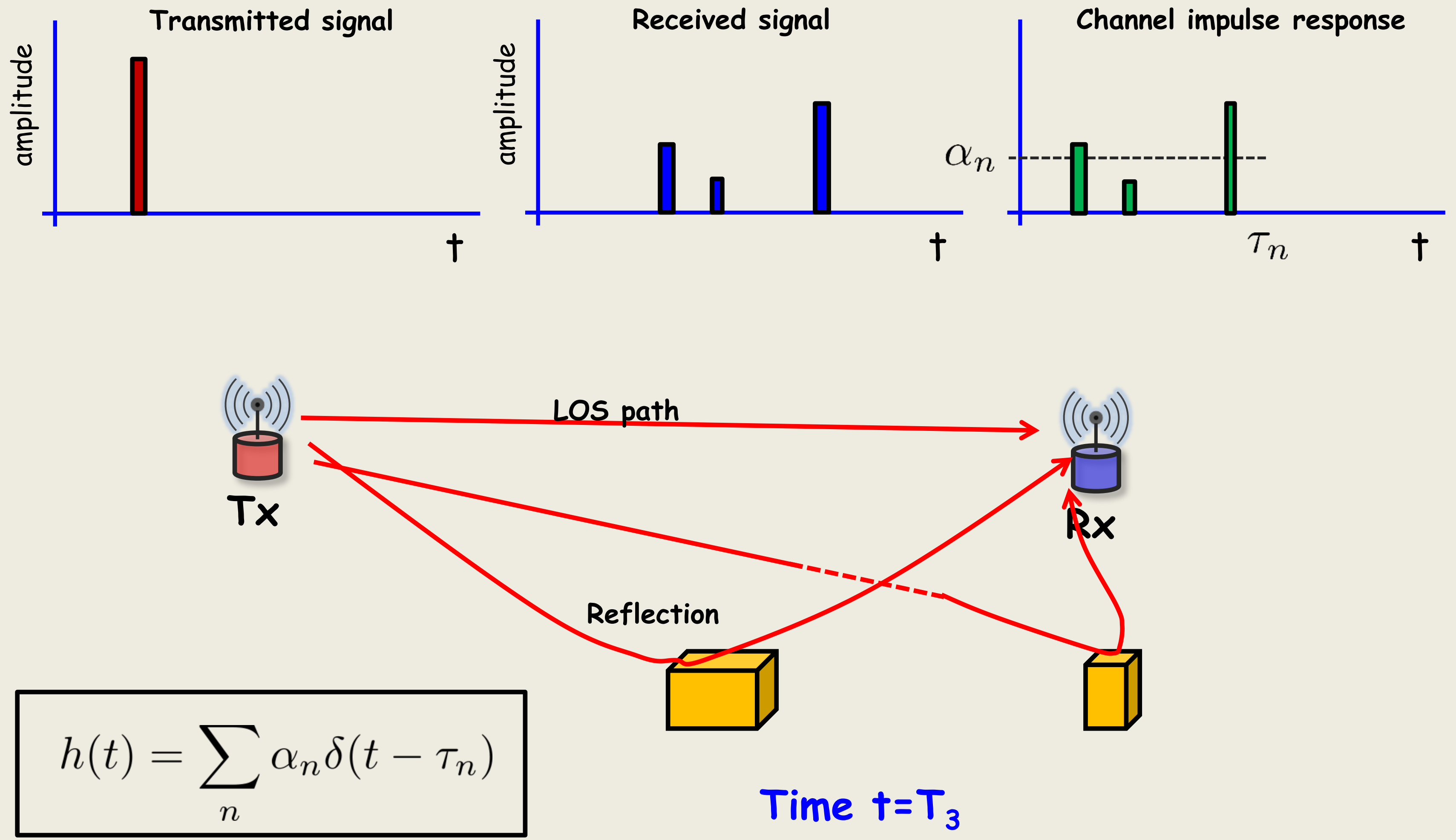
Lecture 9: Time variant channels



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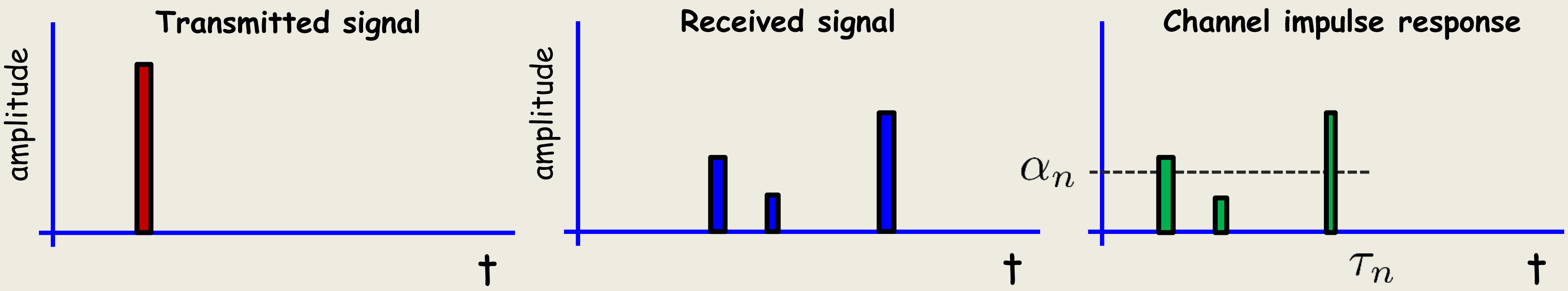
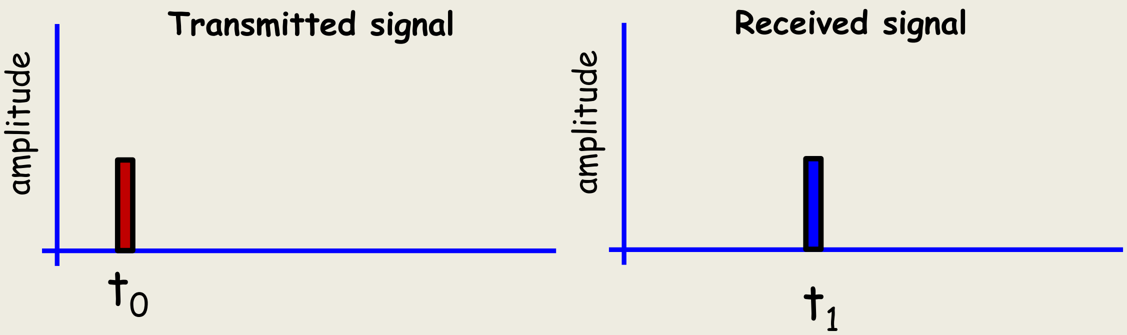
Lecture 9: Time variant channels



$$h(\tau, t) = \sum_n \alpha_n(t)\delta(t - \tau_n(t))$$

Meaning: Output at time t **for input at time** $t - \tau$

Lecture 9: Time variant channels

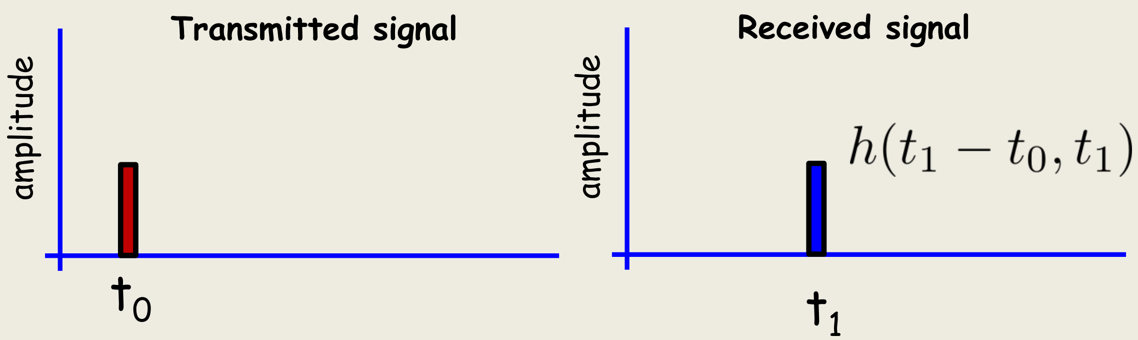


$$h(\tau, t) = \sum_n \alpha_n(t)\delta(t - \tau_n(t))$$

Meaning: Output at time t for input at time $t - \tau$

Example

Lecture 9: Time variant channels



$$\boxed{h(\tau, t) = \sum_n \alpha_n(t)\delta(t - \tau_n(t))}$$

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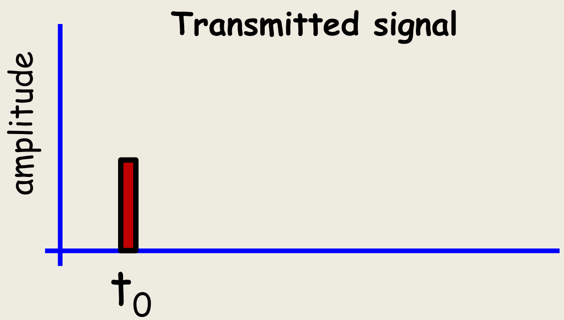
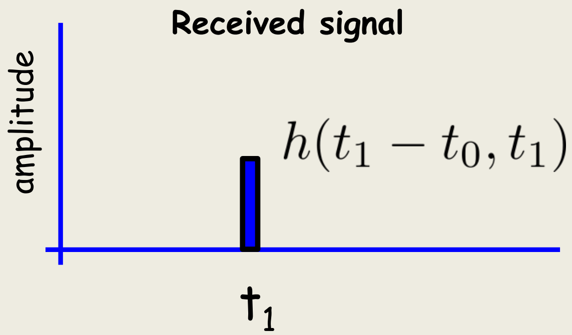
Example

Lecture 9: Time variant channels





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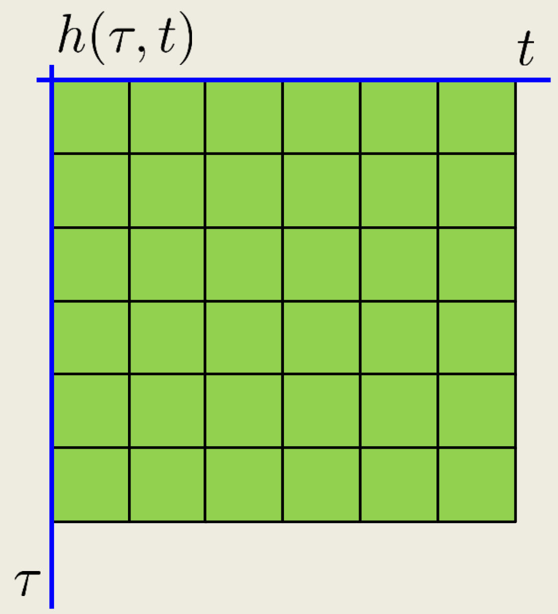
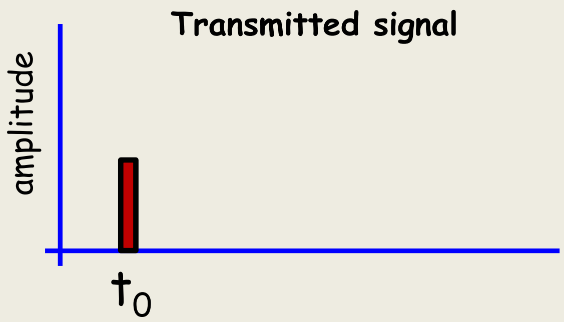
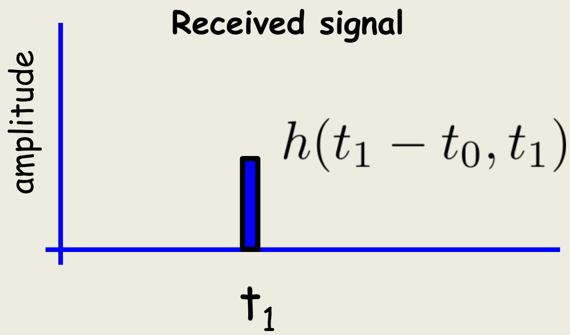


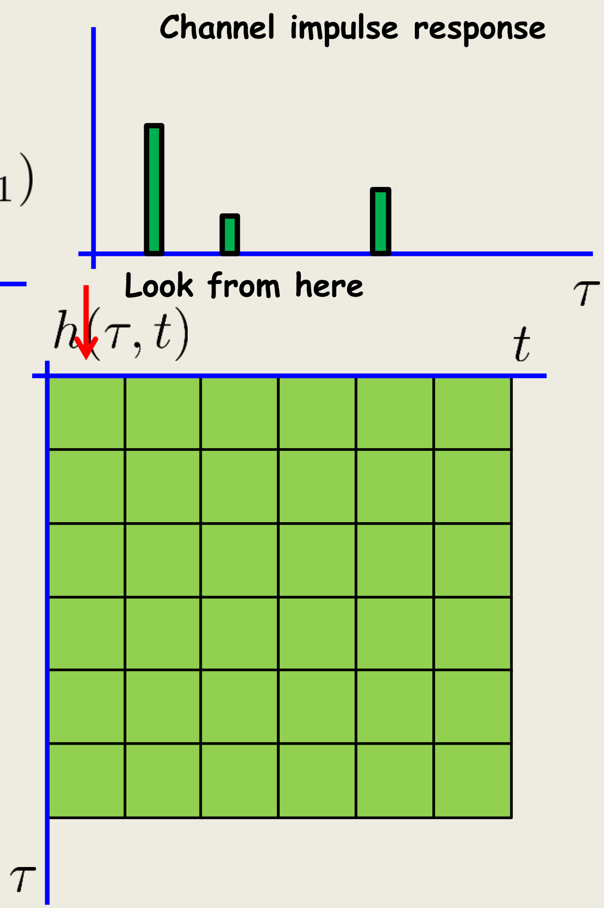
Example

Channel impulse response is 2D

Lecture 9: Time variant channels







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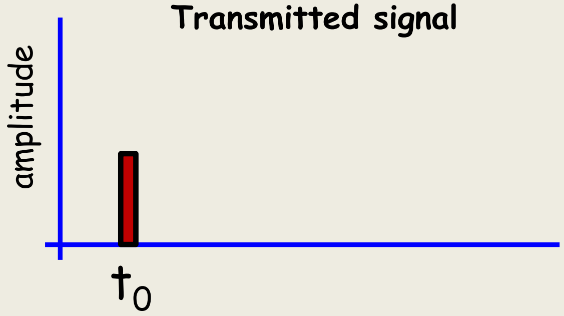
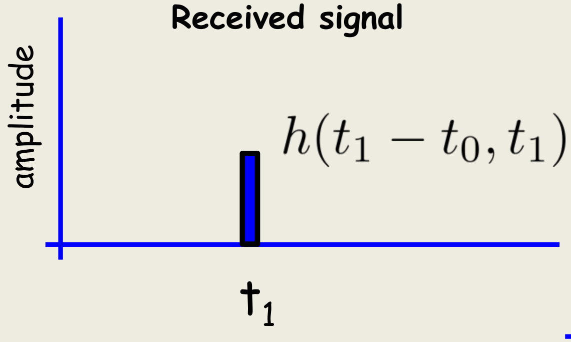
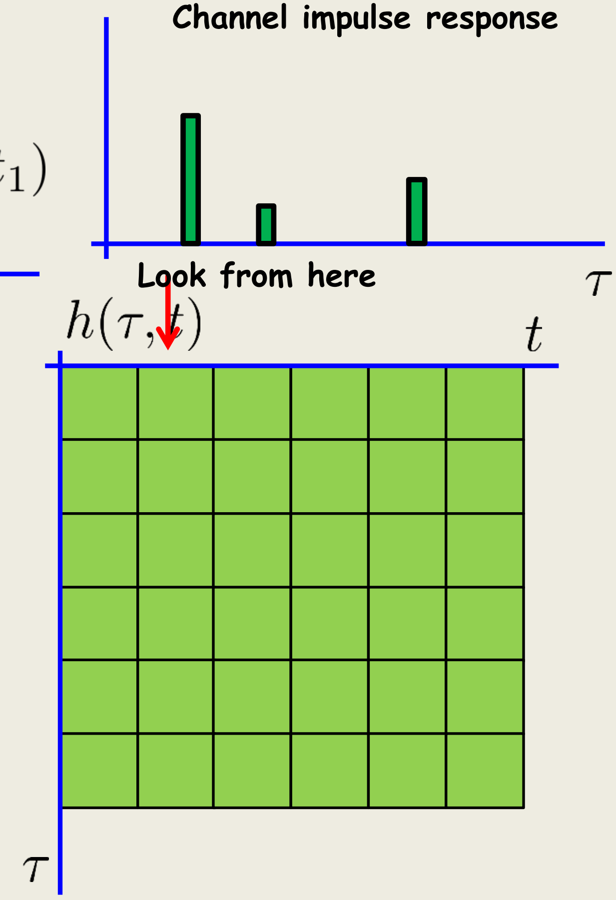
Lecture 9: Time variant channels







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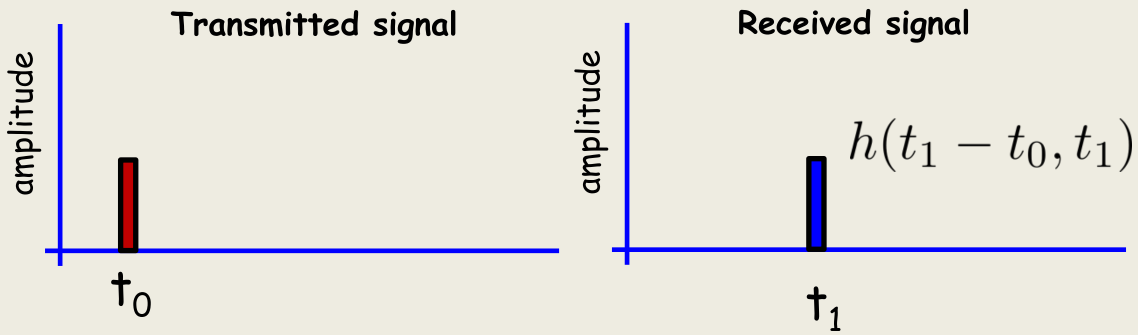
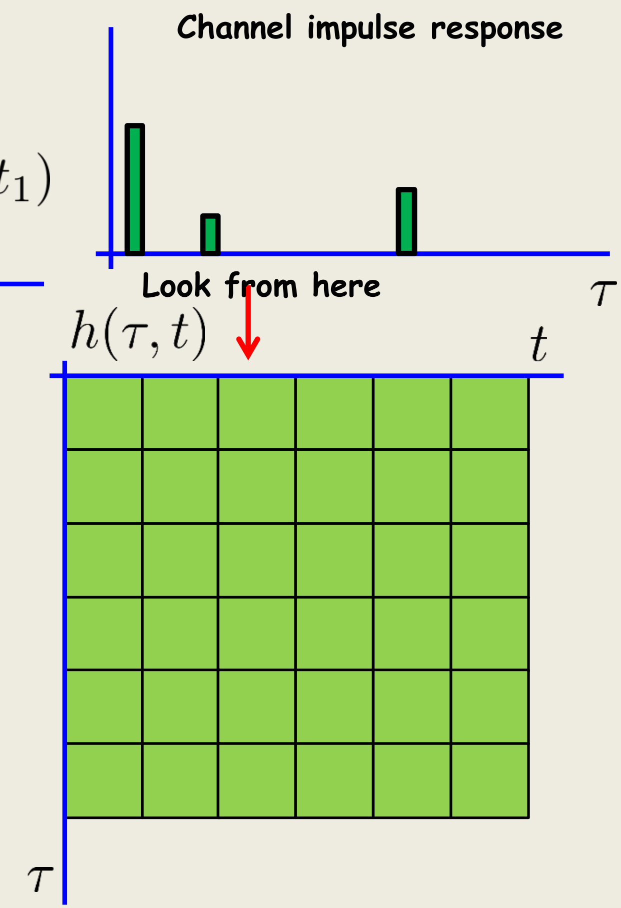
Channel impulse response is 2D

Lecture 9: Time variant channels





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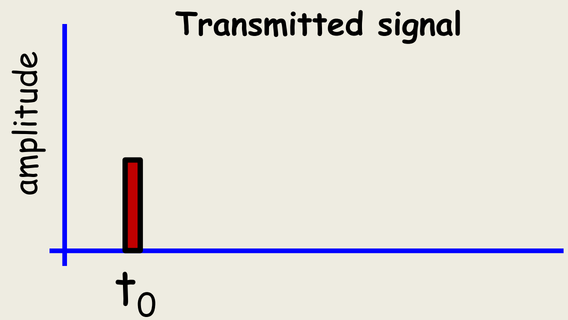
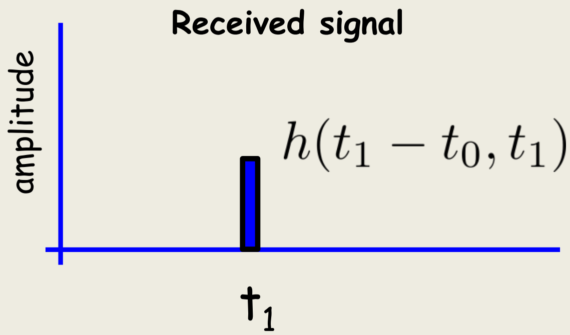
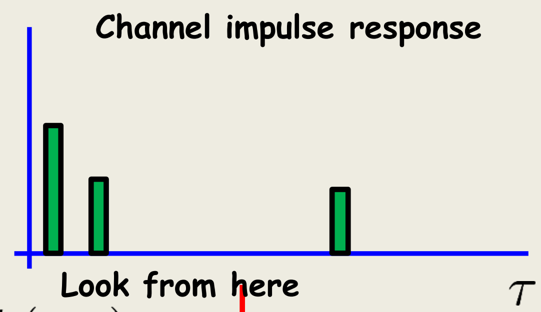
Example

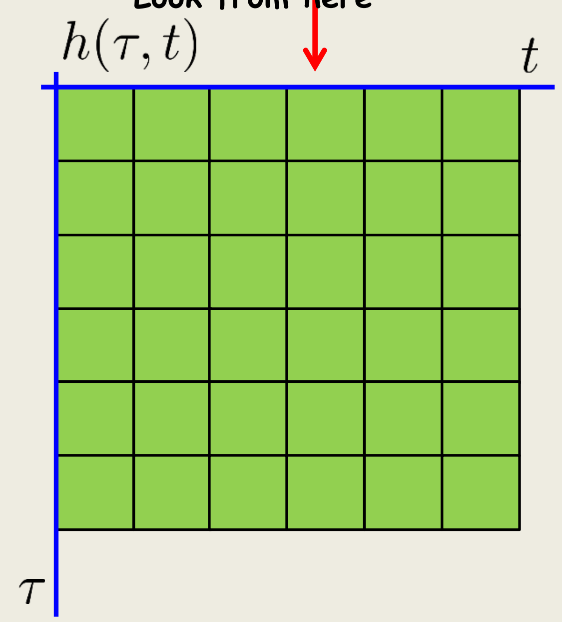
Channel impulse response is 2D

Lecture 9: Time variant channels









$$h(\tau, t) = \sum_n \alpha_n(t)\delta(t - \tau_n(t))$$

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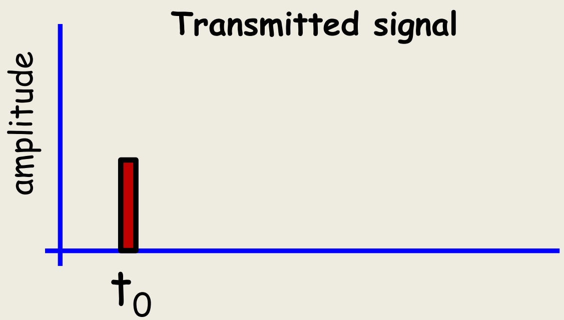
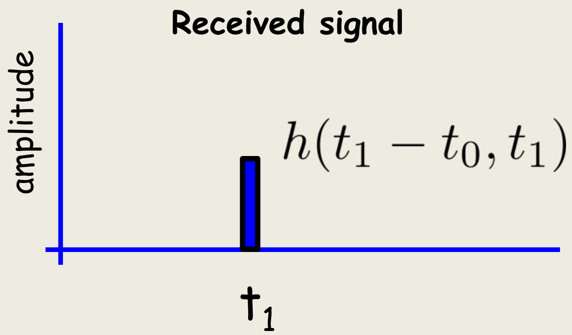
Example

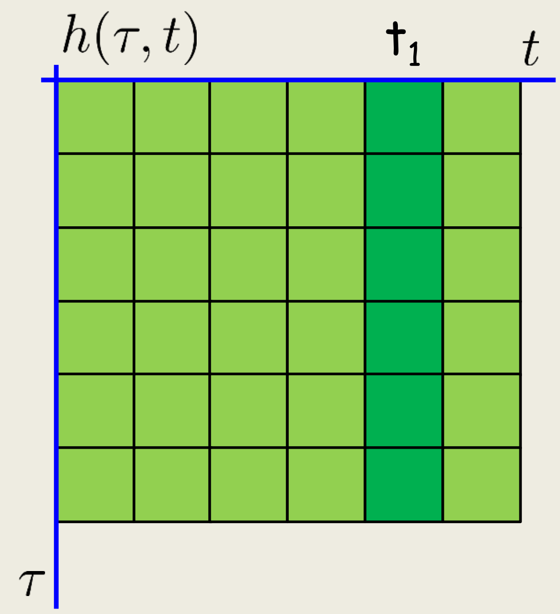
Channel impulse response is 2D

Lecture 9: Time variant channels





Interested in output at t_0

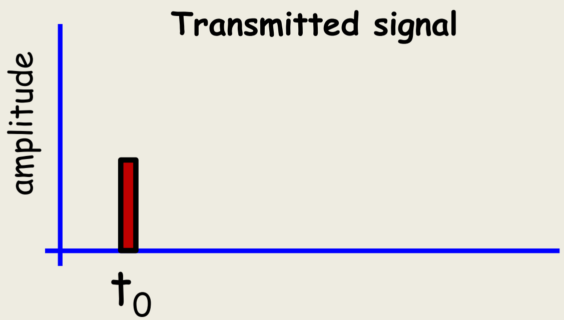


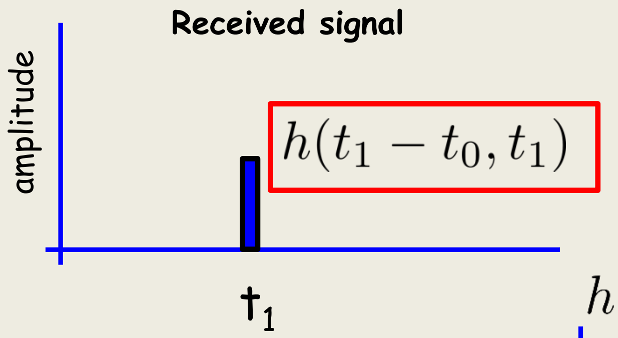
$$\boxed{h(\tau, t) = \sum_n \alpha_n(t)\delta(t - \tau_n(t))}$$

Meaning: Output at time t for input at time $t - \tau$

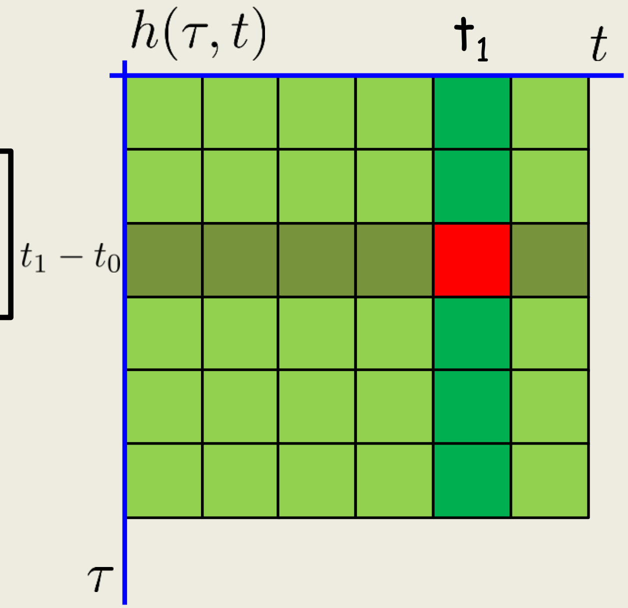
Example

Lecture 9: Time variant channels





Interested in output at t_0
What is time delay?

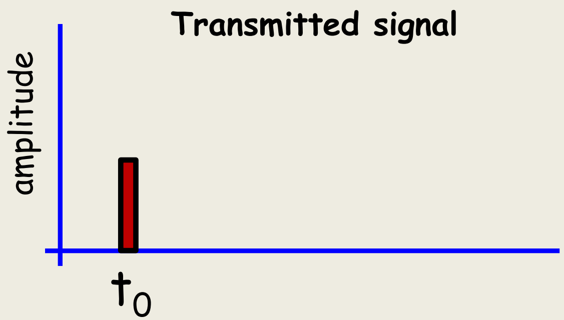


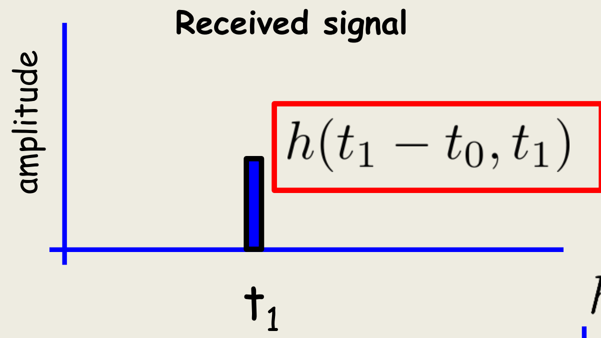
$$h(\tau, t) = \sum_n \alpha_n(t)\delta(t - \tau_n(t))$$

Meaning: Output at time t for input at time $t - \tau$

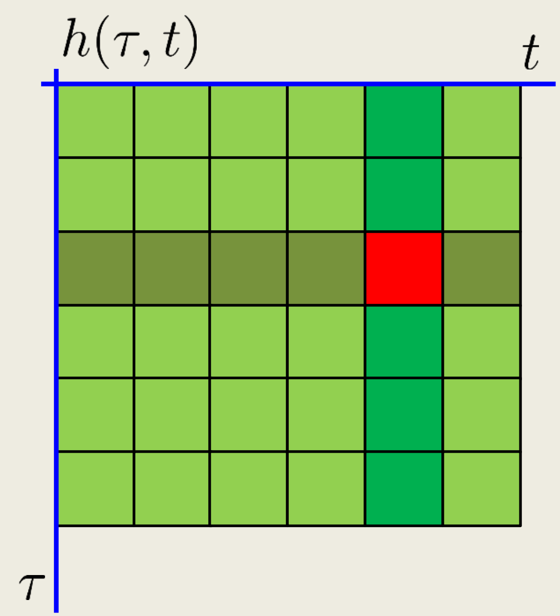
Example

Lecture 9: Time variant channels





Interested in output at t_0
What is time delay?



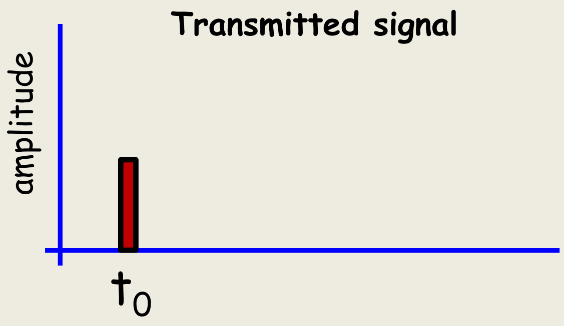
Output signal

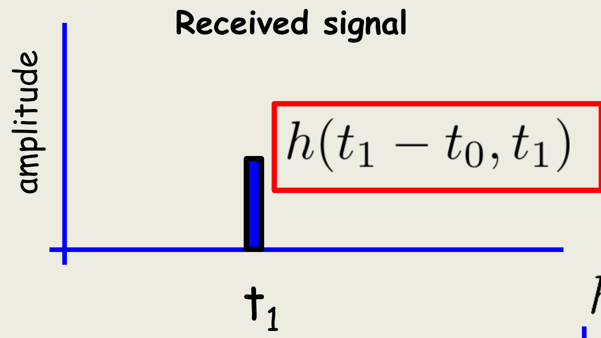
$$\boxed{z(t) =}$$

$$h(\tau, t) \qquad \mathrm{d}\tau \qquad \int_{-\infty}^{\infty} \qquad s(t) \qquad \mathrm{d}t$$

Must be a combination of these things, (possibly time-shifted)

Lecture 9: Time variant channels





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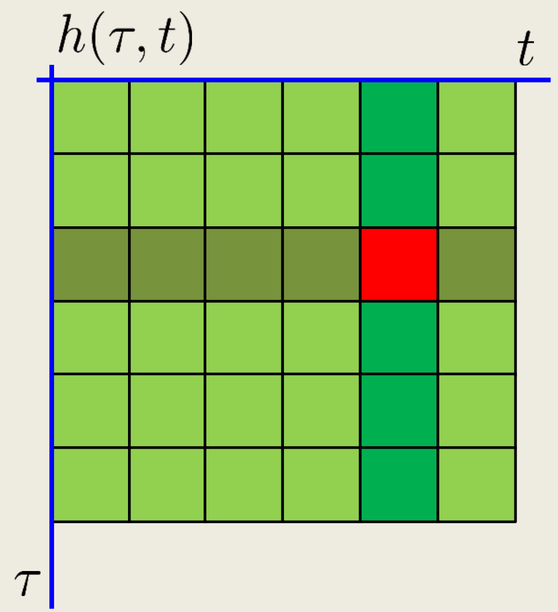


Output signal

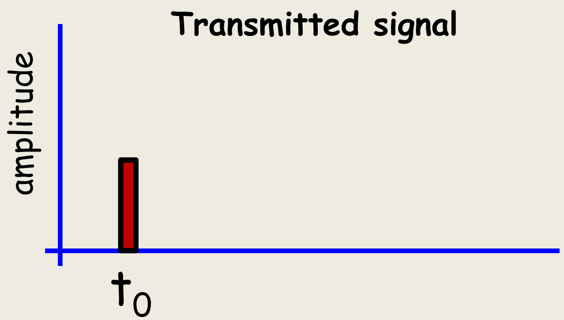
$$\boxed{z(t) = \int_{-\infty}^{\infty}}$$

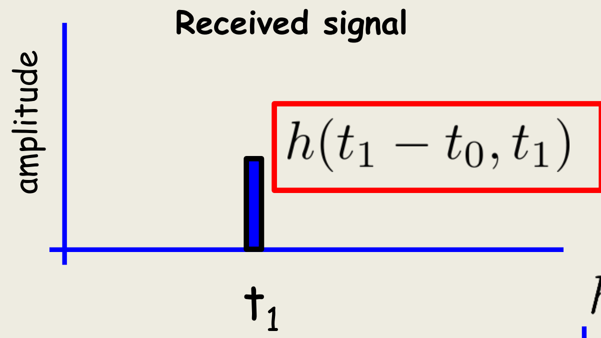
$h(\tau, t)$ $\quad$ $\mathrm{d}\tau$ $\quad$ $s(t)$ $\quad$ $\mathrm{d}t$

Must be a combination of these things, (possibly time-shifted)

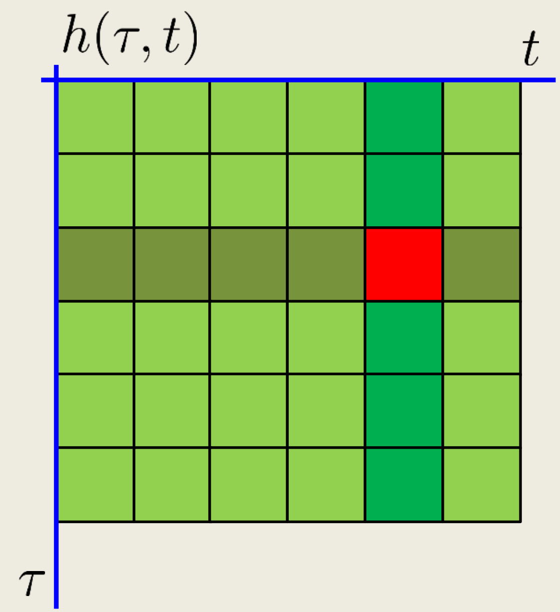
We need to integrate something
(because we do that for time-invariant channels)

Lecture 9: Time variant channels





Interested in output at t_0
What is time delay?



Output signal

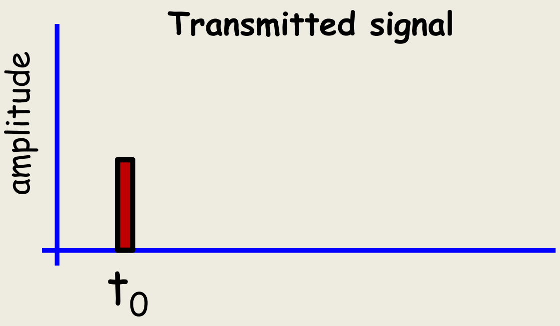
$$z(t) = \int_{-\infty}^{\infty}$$

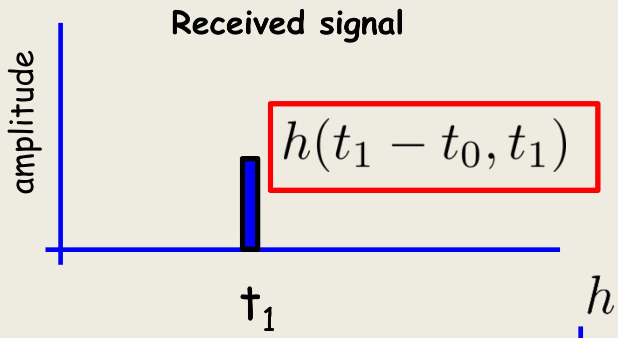
$h(\tau, t)$ $\quad \mathrm{d}\tau$ $\quad s(t)$ $\quad \mathrm{d}t$

Must be a combination of these things, (possibly time-shifted)

If we integrate f(x) over x, is result dependent on x ?

Lecture 9: Time variant channels





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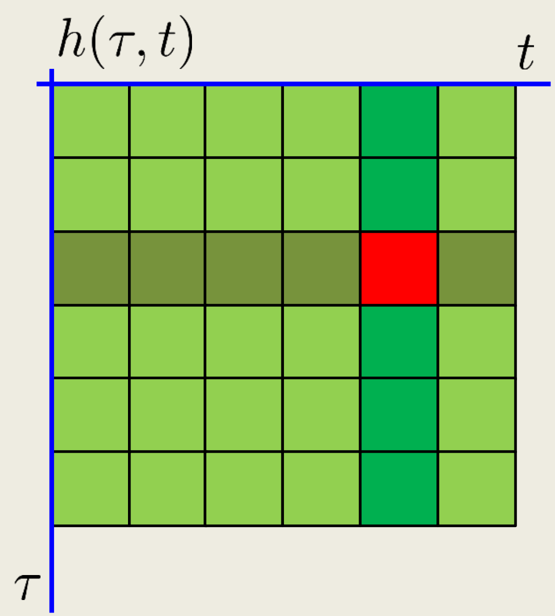


Output signal

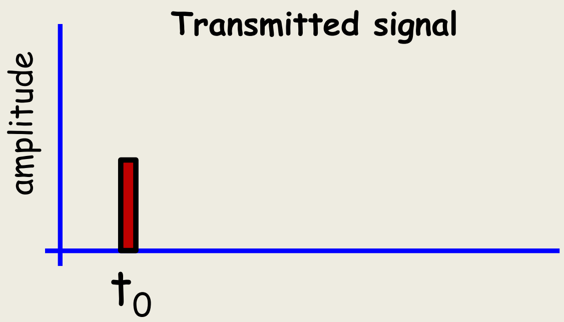
$$\boxed{z(t) = \int_{-\infty}^{\infty} \qquad \mathrm{d}\tau}$$

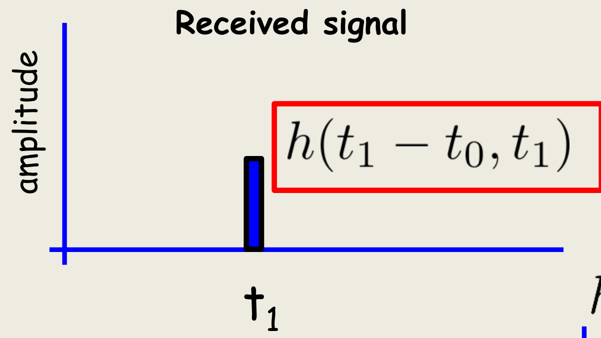
$h(\tau, t)$ $\qquad$ $s(t)$ $\qquad$ $\cancel{\mathrm{d}t}$

Must be a combination of these things, (possibly time-shifted)

If we integrate f(x) over x, is result dependent on x ?
NO! So we cannot integrate over t since z(t) depends on it

Lecture 9: Time variant channels





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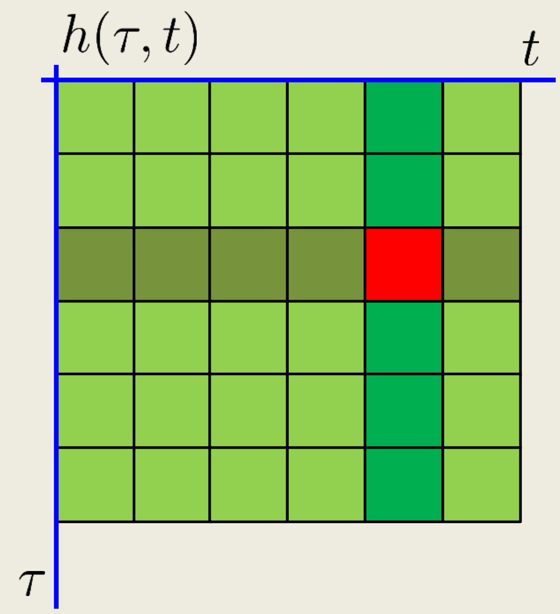


Output signal

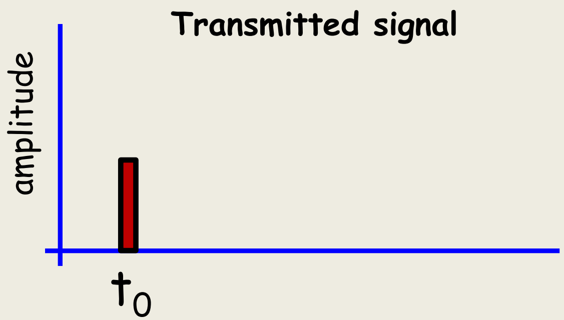
$$z(t) = \int_{-\infty}^{\infty} \qquad \mathrm{d}\tau$$

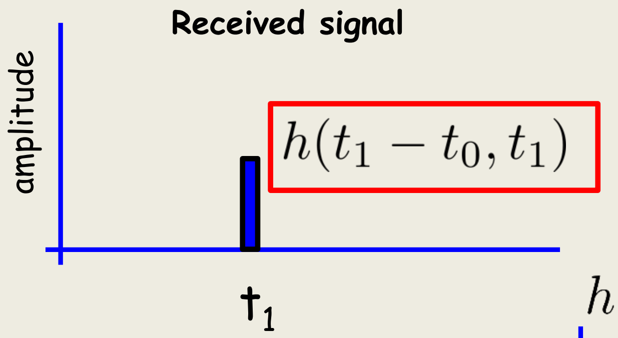
$h(\tau, t)$ $s(t)$

Must be a combination of these things, (possibly time-shifted)

At time t and a given τ for what x does $s(x)$ affect the result ?

Lecture 9: Time variant channels





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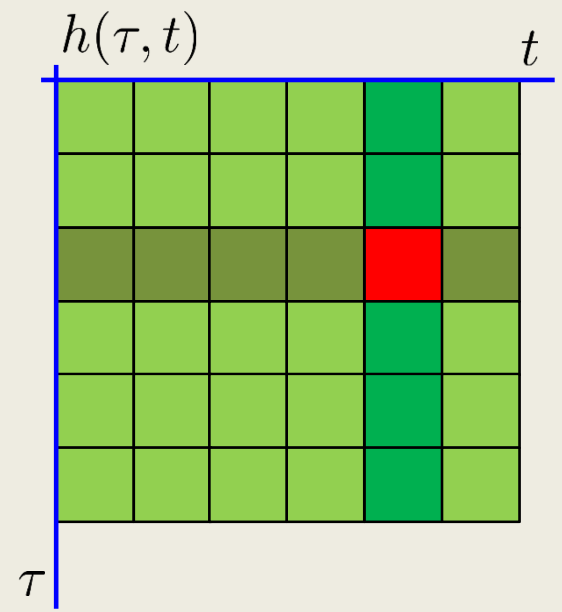


Output signal

$$z(t) = \int_{-\infty}^{\infty} \qquad \mathrm{d}\tau$$

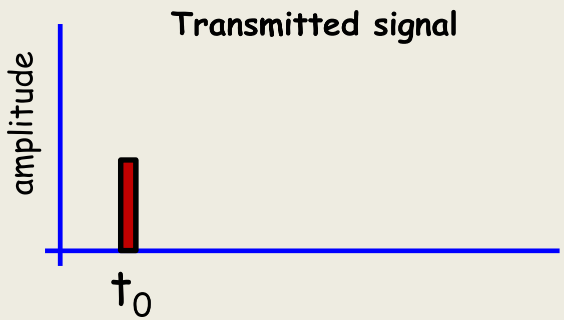
$h(\tau, t)$ $s(t)$

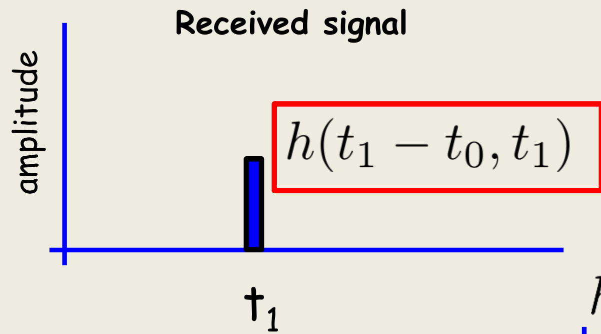
Must be a combination of these things, (possibly time-shifted)

At time t and a given τ for what x does $s(x)$ affect the result ?

$$x = t - \tau$$

Lecture 9: Time variant channels





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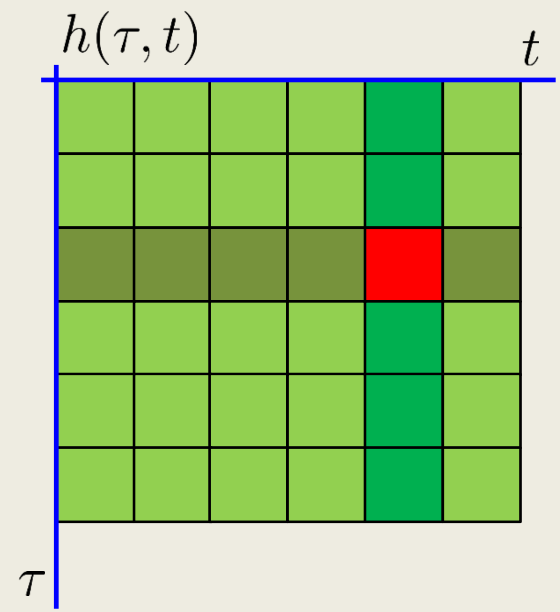
What is time delay?



Output signal

$$z(t) = \int_{-\infty}^{\infty} \qquad s(t - \tau)\, \mathrm{d}\tau$$

$h(\tau, t)$

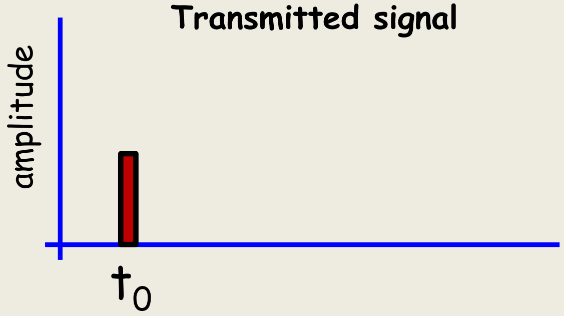
Must be a combination of these things, (possibly time-shifted)

At time t and a given τ for what x does $s(x)$ affect the result ?

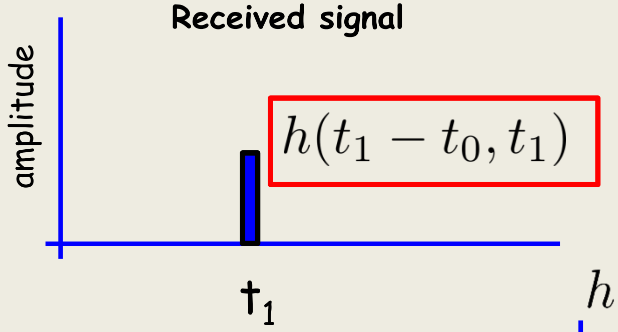
$$x = t - \tau$$

Lecture 9: Time variant channels

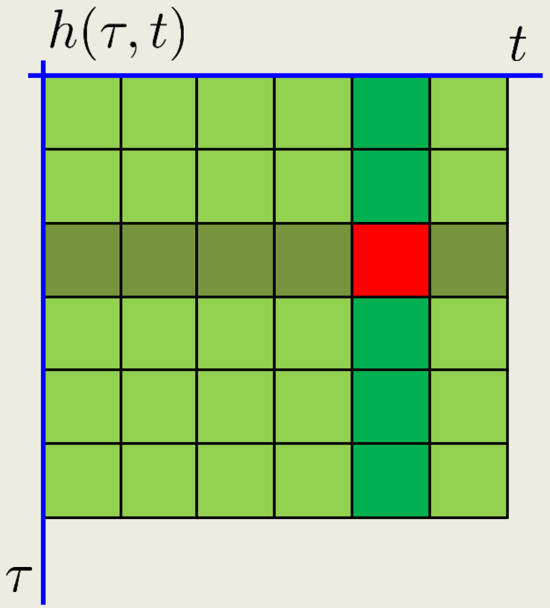
Transmitted signal



Received signal



Interested in output at t_0
What is time delay?



Output signal

$$z(t) = \int_{-\infty}^{\infty} \qquad s(t-\tau)\,\mathrm{d}\tau$$

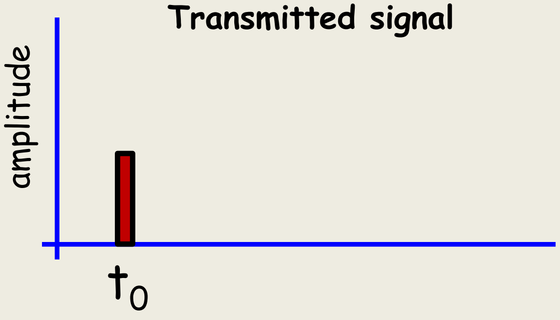
$h(\tau, t)$

Must be a combination of these things, (possibly time-shifted)

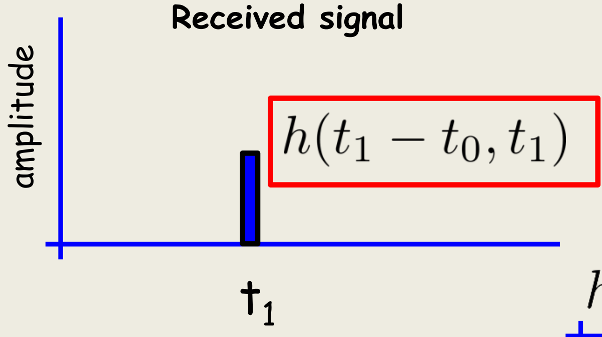
At time t and a given τ for what x does $s(x)$ affect the result ?
How "much" does it impact ?

Lecture 9: Time variant channels

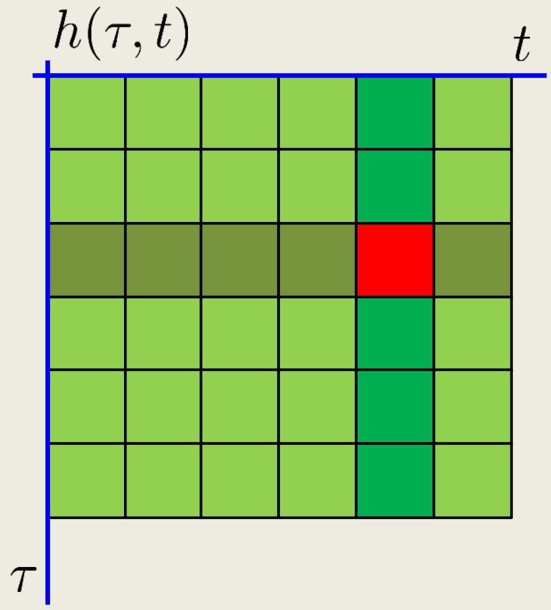
Transmitted signal



Received signal



Interested in output at t_0
What is time delay?



Output signal

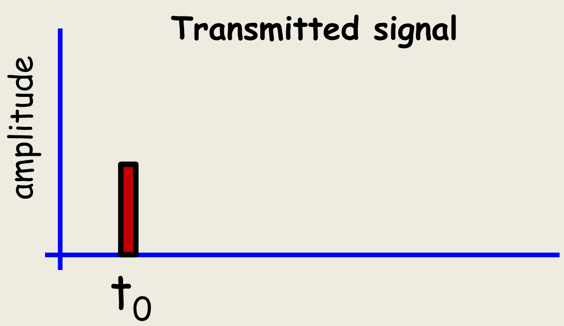
$$z(t) = \int_{-\infty}^{\infty} h(\tau, t)\, s(t-\tau)\,\mathrm{d}\tau$$

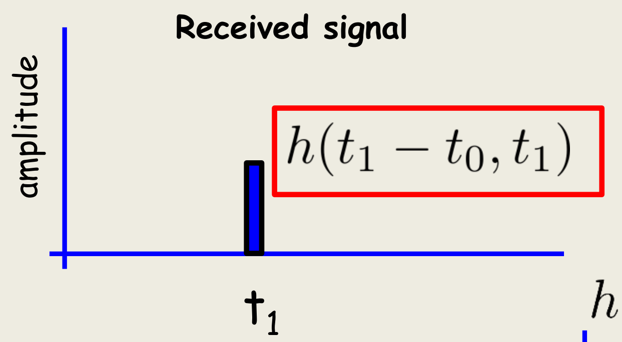
Must be a combination of these things, (possibly time-shifted)

At time t and a given τ for what x does $s(x)$ affect the result ?
How "much" does it impact ?

Exactly $h(\tau, t)$

Lecture 9: Time variant channels





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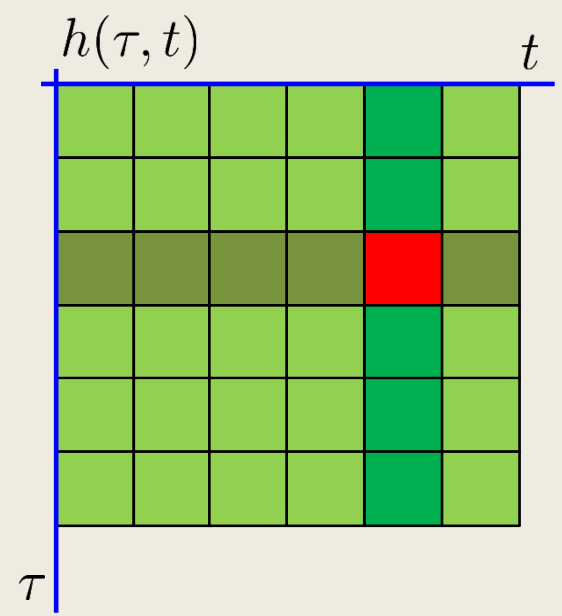

Output signal

$$z(t) = \int_{-\infty}^{\infty} h(\tau, t)\, s(t - \tau)\, \mathrm{d}\tau$$

Channel impulse response

$$h(\tau, t) = \sum_n \alpha_n(t) \delta(t - \tau_n(t))$$

$$z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))$$

Lecture 9: Time variant channels

Assume pure cosine input

$$s(t) = \cos((\omega_c + \omega_1)t), \quad -\infty < t < \infty$$

$$\boxed{z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))}$$

Lecture 9: Time variant channels

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$$s(t) = \cos((\omega_c + \omega_1)t), \quad -\infty < t < \infty$$

If time-invariant channel,
we get cosine out at same frequency

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Lecture 9: Time variant channels

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$$z(t) = \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t)))$$

$$\boxed{z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))}$$

Lecture 9: Time variant channels

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$$\begin{aligned} z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) \\ &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)t - (\omega_c + \omega_1)\tau_n(t)) \end{aligned}$$

$$\boxed{z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))}$$

Lecture 9: Time variant channels

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$$\begin{aligned} z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) \\ &= \sum_n \alpha_n(t) \cos(\underbrace{\boxed{(\omega_c + \omega_1)t}}_{x} - \underbrace{\boxed{(\omega_c + \omega_1)\tau_n(t))}}_{y} \end{aligned}$$

$$\boxed{\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)}$$

Lecture 9: Time variant channels

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z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) \\
&= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)t - (\omega_c + \omega_1)\tau_n(t)) \\
&= \left[\sum_n \alpha_n(t) \cos((\omega_c + \omega_1)\tau_n(t)) \right] \cos((\omega_c + \omega_1)t) \\
&+ \left[\sum_n \alpha_n(t) \sin((\omega_c + \omega_1)\tau_n(t)) \right] \sin((\omega_c + \omega_1)t)
\end{aligned}$$

$$\boxed{\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)}$$

Lecture 9: Time variant channels

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&= \underbrace{\left[\boxed{\sum_n \alpha_n(t)\cos((\omega_c + \omega_1)\tau_n(t))}\right]}_{z_I(t)} \cos((\omega_c + \omega_1)t) \\
&\quad + \underbrace{\left[\boxed{\sum_n \alpha_n(t)\sin((\omega_c + \omega_1)\tau_n(t))}\right]}_{-z_Q(t)} \sin((\omega_c + \omega_1)t)
\end{aligned}$$

Lecture 9: Time variant channels

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If time-invariant channel,
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$$s(t) = \cos((\omega_c + \omega_1)t), \quad -\infty < t < \infty$$

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z(t) &= \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)(t-\tau_n(t))) \\
&= \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)t-(\omega_c+\omega_1)\tau_n(t)) \\
&= \underbrace{\boxed{\sum_n \alpha_n(t)\cos((\omega_c+\omega_1)\tau_n(t))}}_{z_I(t)}\cos((\omega_c+\omega_1)t) \\
&\quad + \underbrace{\boxed{\sum_n \alpha_n(t)\sin((\omega_c+\omega_1)\tau_n(t))}}_{-z_Q(t)}\sin((\omega_c+\omega_1)t) \\
&= z_I(t)\cos((\omega_c+\omega_1)t) - z_Q(t)\sin((\omega_c+\omega_1)t) \\
&= e_z(t)\cos((\omega_c+\omega_1)t+\theta_z(t))
\end{aligned}$$

Lecture 9: Time variant channels

Assume pure cosine input

If time-invariant channel,
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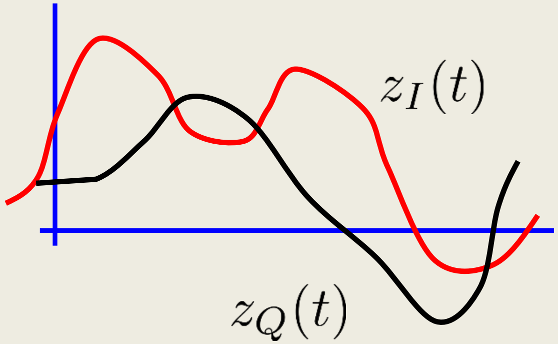

Baseband signals are time-variant

Lecture 9: Time variant channels

Assume pure cosine input

If time-invariant channel,
we get cosine out at same frequency

$$s(t) = \cos((\omega_c + \omega_1)t), \quad -\infty < t < \infty$$

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$$= e_z(t)\cos((\omega_c + \omega_1)t + \theta_z(t))$$

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c + \omega_1)\tau_n(t))$$

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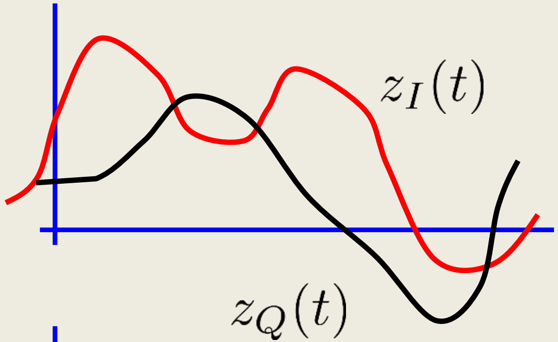
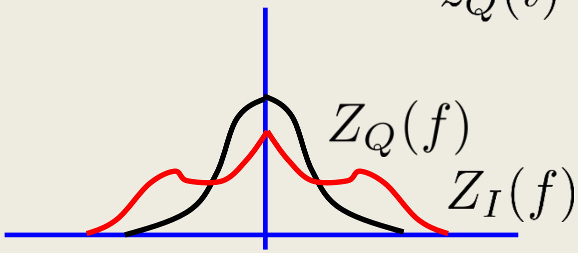


Baseband signals are time-variant

Fourier transforms are NOT have spread



Lecture 9: Time variant channels

Assume pure cosine input

If time-invariant channel,
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$$s(t) = \cos((\omega_c + \omega_1)t), \quad -\infty < t < \infty$$

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$$= e_z(t)\cos((\omega_c + \omega_1)t + \theta_z(t))$$

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c + \omega_1)\tau_n(t))$$

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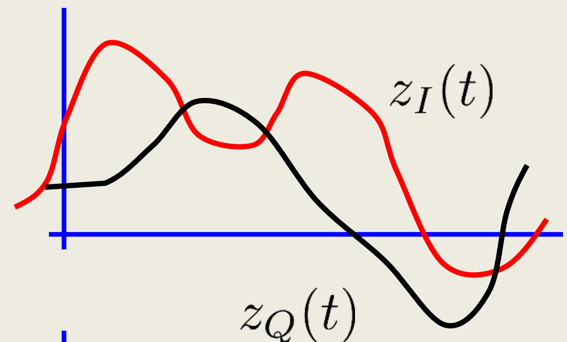

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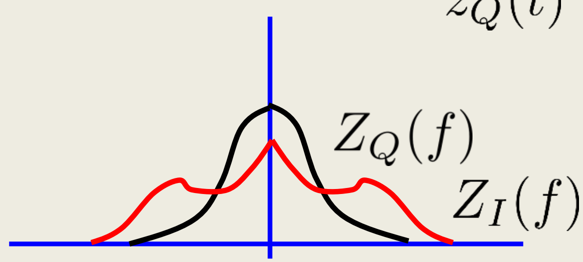

A pure cosine has spread to other frequencies

Lecture 9: Time variant channels

Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c + \omega_1)\tau_n(t))$$

$$z_Q(t) = -\sum_n \alpha_n(t)\sin((\omega_c + \omega_1)\tau_n(t))$$

We assume that both $z_I(t)$ and $z_Q(t)$ are Gaussian distributed with mean 0 and variance σ^2

Envelope is Rayleigh distributed $e_z(t) = \sqrt{z_I^2(t) + z_Q^2(t)}$

Lecture 9: Time variant channels

Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)\tau_n(t))$$

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We would like to understand how severe the spectral broadening is

Lecture 9: Time variant channels

Gaussian assumption

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Envelope is Rayleigh distributed $e_z(t) = \sqrt{z_I^2(t) + z_Q^2(t)}$

We would like to understand how severe the spectral broadening is

Intuatively, if the channel changes fast, there is a lot of broadening

How to measure "how fast something changes"

Lecture 9: Time variant channels

Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c + \omega_1)\tau_n(t))$$

$$z_Q(t) = -\sum_n \alpha_n(t)\sin((\omega_c + \omega_1)\tau_n(t))$$

Covariance function

$$\boxed{c_z(\tau) = \frac{1}{4} E\left\{[z_I(t+\tau) + \jmath z_Q(t+\tau)][z_I(t) - \jmath z_Q(t)]\right\}}$$

Lecture 9: Time variant channels

Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)\tau_n(t))$$

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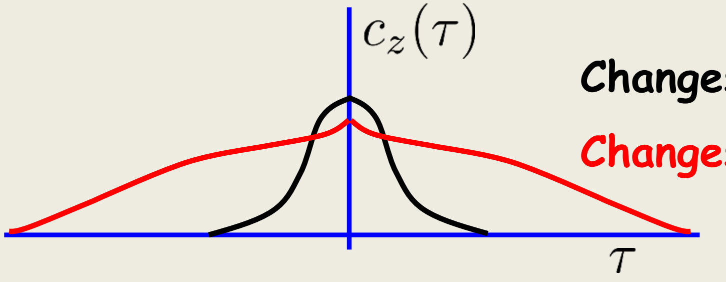


Changes rapidly

Changes slowly

Lecture 9: Time variant channels

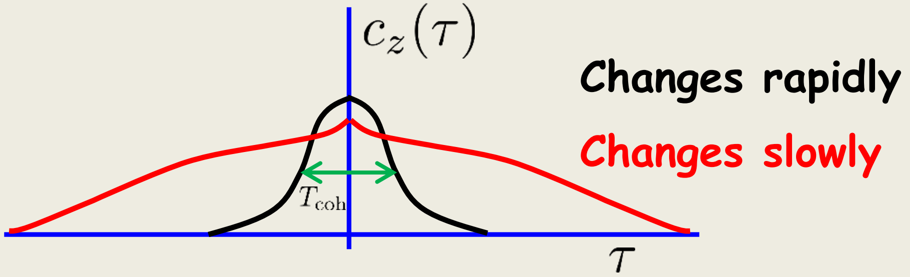
Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)\tau_n(t))$$

$$z_Q(t) = -\sum_n \alpha_n(t)\sin((\omega_c+\omega_1)\tau_n(t))$$

Covariance function

$$\boxed{c_z(\tau) = \frac{1}{4}E\left\{[z_I(t+\tau)+\jmath z_Q(t+\tau)][z_I(t)-\jmath z_Q(t)]\right\}}$$



Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)\tau_n(t))$$

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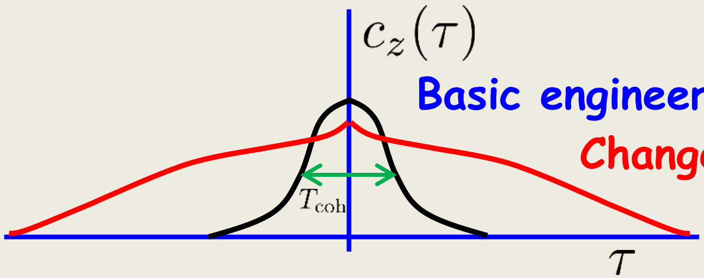

Basic engineering: inverse relationship btw time and frequency

Changes slowly

Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

Gaussian assumption

$$z_I(t) = \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)\tau_n(t))$$

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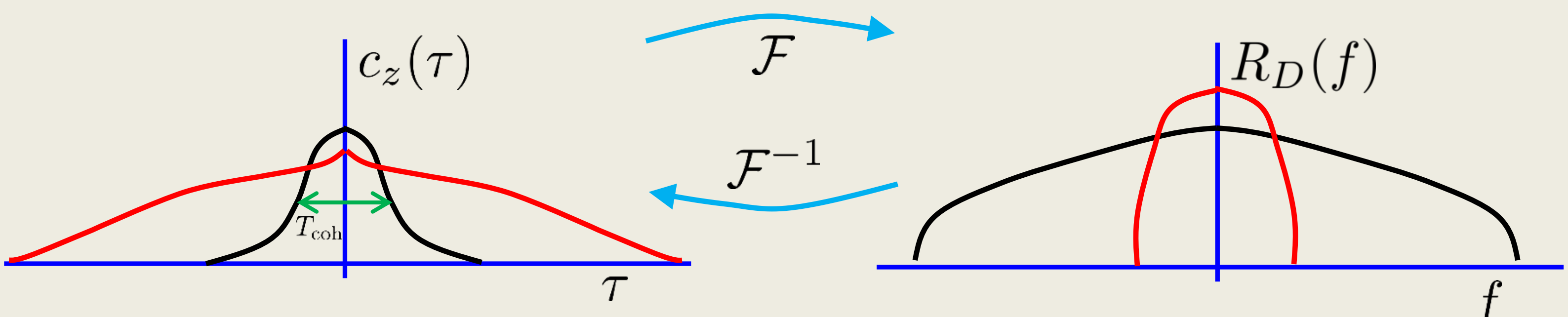

Define coherence time $T_{\rm coh}$ as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

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From Dig.com 1:
Fourier transform of covariance function is
Power Spectral Density (PSD)

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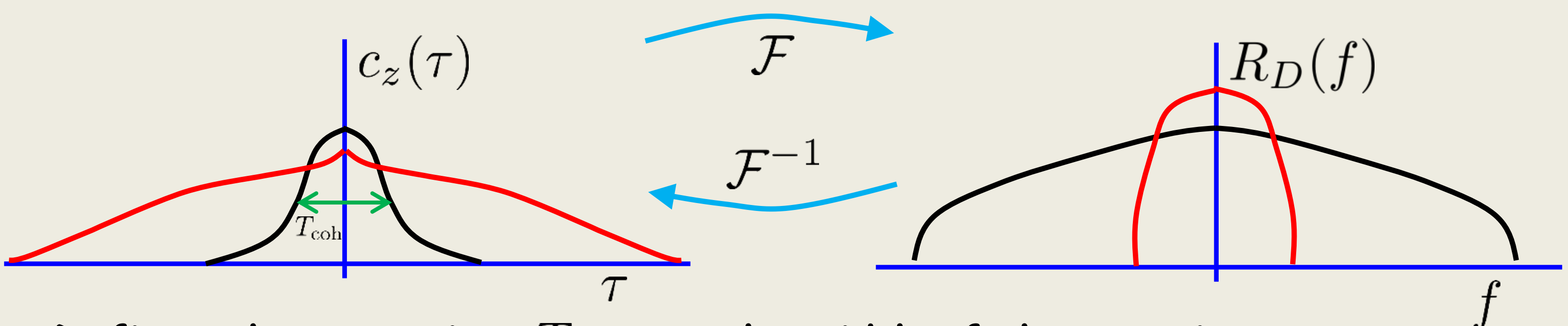

Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

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Right plot tell us how power is being spread due to time-variance

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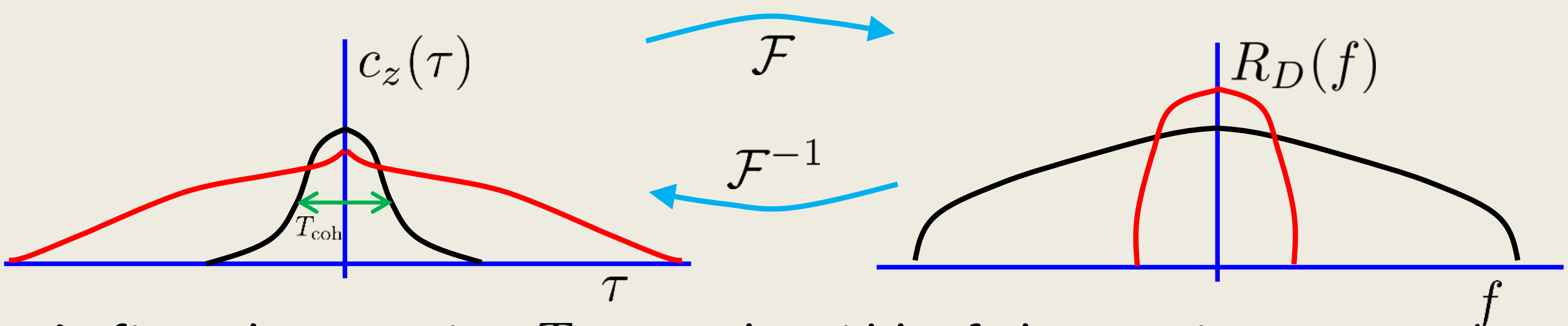

Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

What causes time-variance: Doppler

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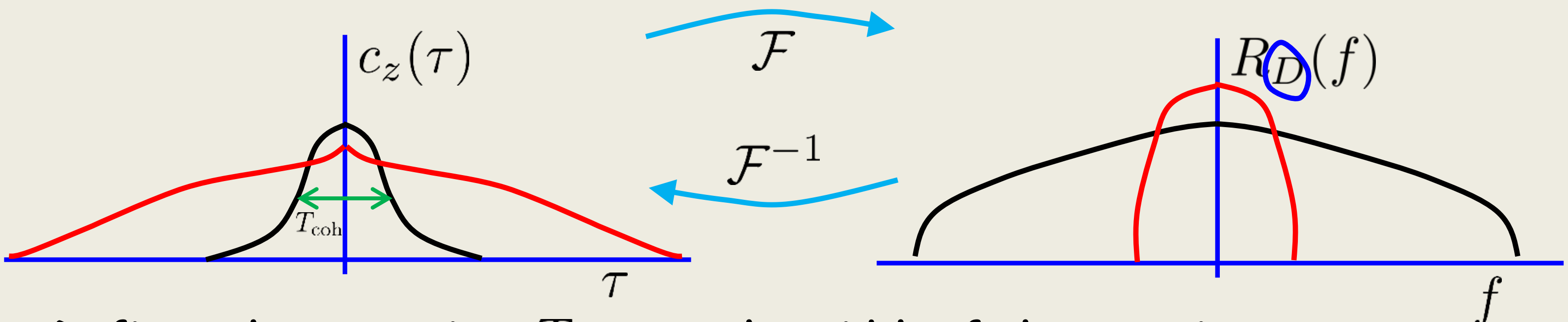

Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

What causes time-variance: Doppler

Width is called Doppler spread $B_{\mathcal{D}}$

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Fourier transform of covariance function is
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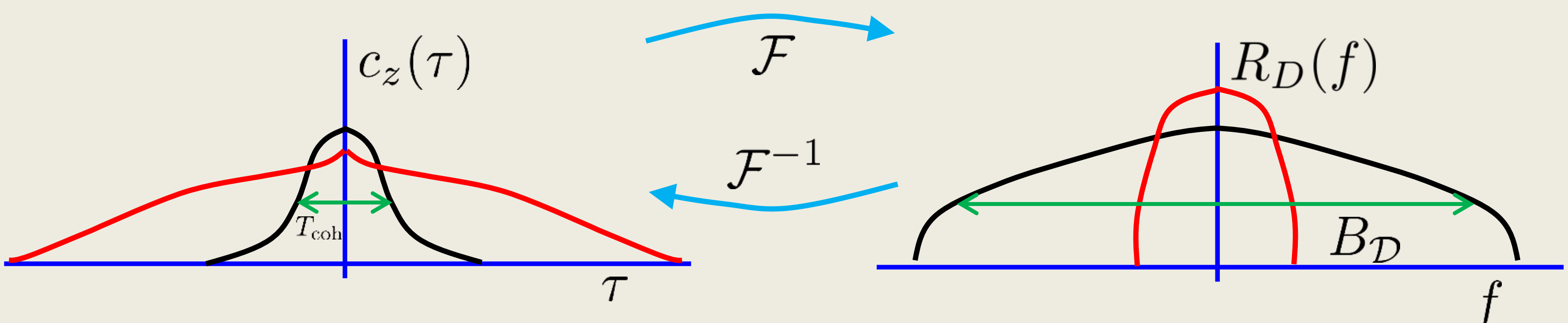


Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

What causes time-variance: Doppler

Width is called Doppler spread $B_{\mathcal{D}}$

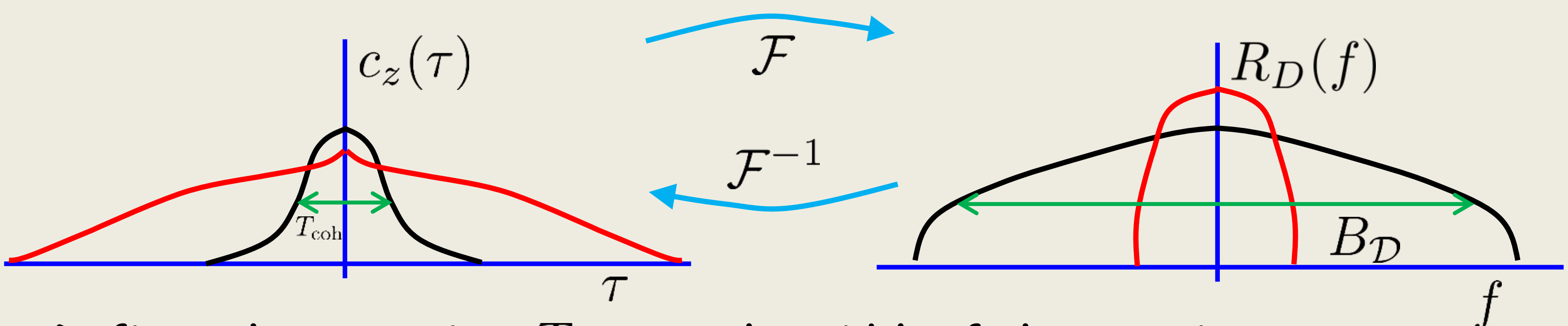
We have, roughly, $t_{\text{coh}} \approx \dfrac{1}{B_{\mathcal{D}}}$

From Dig.com 1:
Fourier transform of covariance function is
Power Spectral Density (PSD)

Right plot tell us how power is being spread due to time-variance

Covariance function

$$\boxed{c_z(\tau) = \frac{1}{4} E\left\{[z_I(t+\tau) + \jmath z_Q(t+\tau)][z_I(t) - \jmath z_Q(t)]\right\}}$$



Define coherence time T_{coh} as the width of the covariance (according to some measure)

Lecture 9: Time variant channels

Summary so far

- Wireless channels are time-variant
- Input-output relation is $$z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))$$

Lecture 9: Time variant channels

Summary so far

- **Wireless channels are time-variant**
- **Input-output relation is** $z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))$
- **A pure cosine at frequency f_0 will**
  1. For time-invariant channels produce a pure cosine at f_0
  2. For time-variant channels produce a signal around f_0

Lecture 9: Time variant channels

Summary so far

- **Wireless channels are time-variant**
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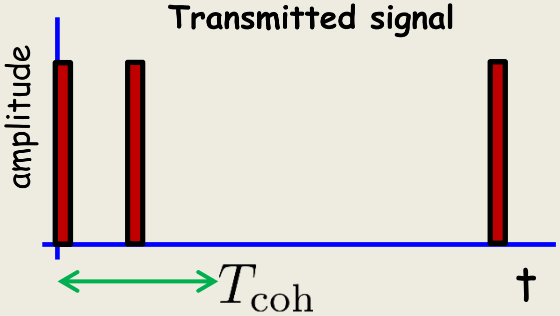
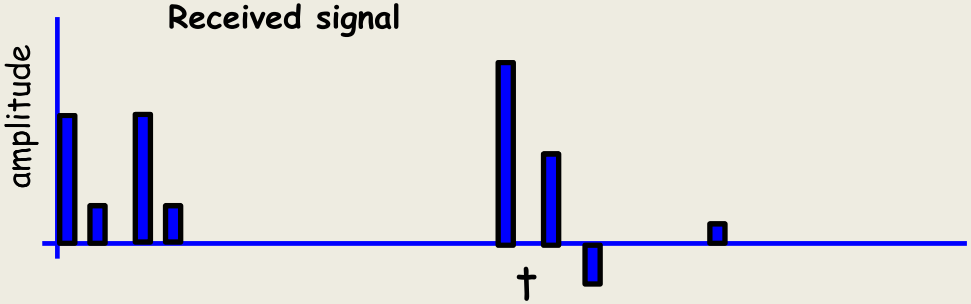
- **A pure cosine at frequency f_0 will**
  i. For time-invariant channels produce a pure cosine at f_0
  ii. For time-variant channels produce a signal around f_0
- **We measure how fast the channel changes by coherence time** t_{coh}





Lecture 9: Time variant channels

Summary so far

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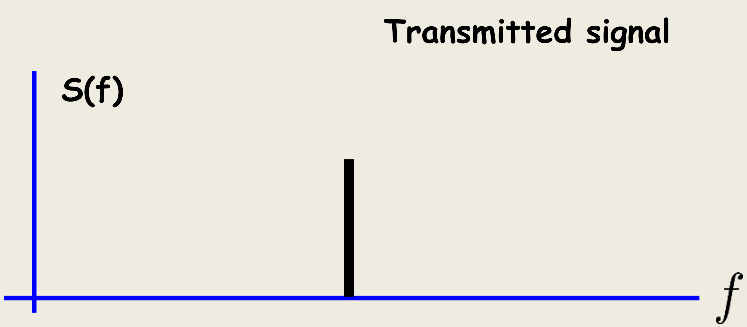
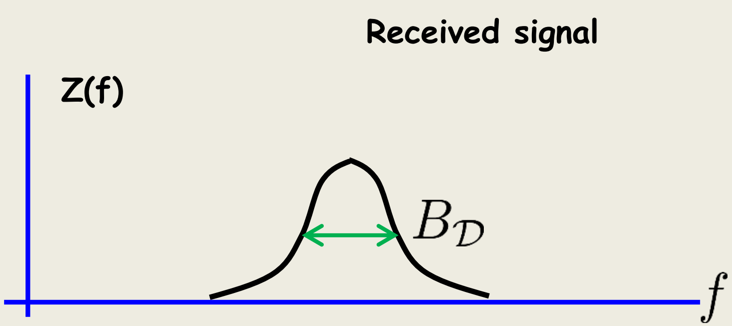
- **We measure spectral broadening with Doppler spread** $B_{\mathcal{D}}$





Lecture 9: Time variant channels

Summary so far

- Wireless channels are time-variant
- Input-output relation is $z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))$
- A pure cosine at frequency f_0 will
  i. For time-invariant channels produce a pure cosine at f_0
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- We measure spectral broadening with Doppler spread $B_{\mathcal{D}}$
- We have $t_{\text{coh}} \approx \dfrac{1}{B_{\mathcal{D}}}$

Lecture 9: Time variant channels

Summary so far

- Wireless channels are time-variant
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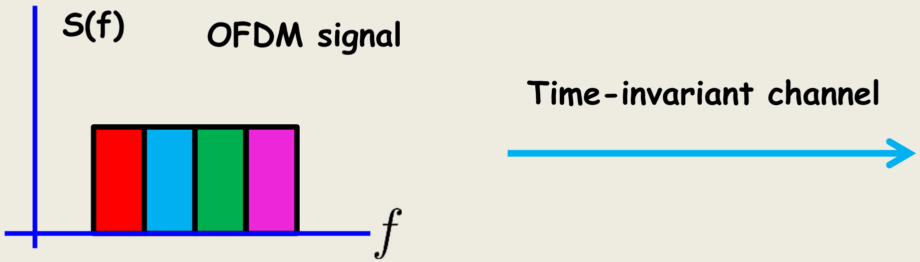
- A pure cosine at frequency f_0 will
  i. For time-invariant channels produce a pure cosine at f_0
  ii. For time-variant channels produce a signal around f_0
- We measure spectral broadening with Doppler spread $B_{\mathcal{D}}$
- We have $t_{\text{coh}} \approx \dfrac{1}{B_{\mathcal{D}}}$
- In industrial simulations, $B_{\mathcal{D}}$ is varied from low to high, thus it is an input parameter to a system

Lecture 9: Time variant channels

Consequences

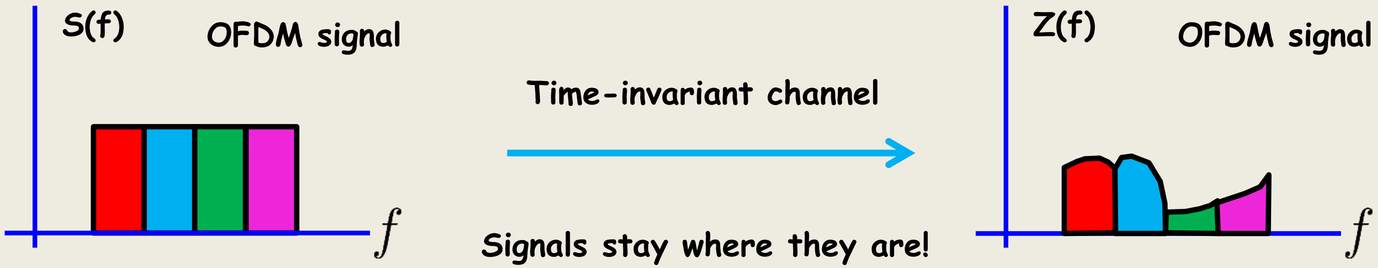
Frequency division multiplexing



Lecture 9: Time variant channels

Consequences

Frequency division multiplexing



Lecture 9: Time variant channels

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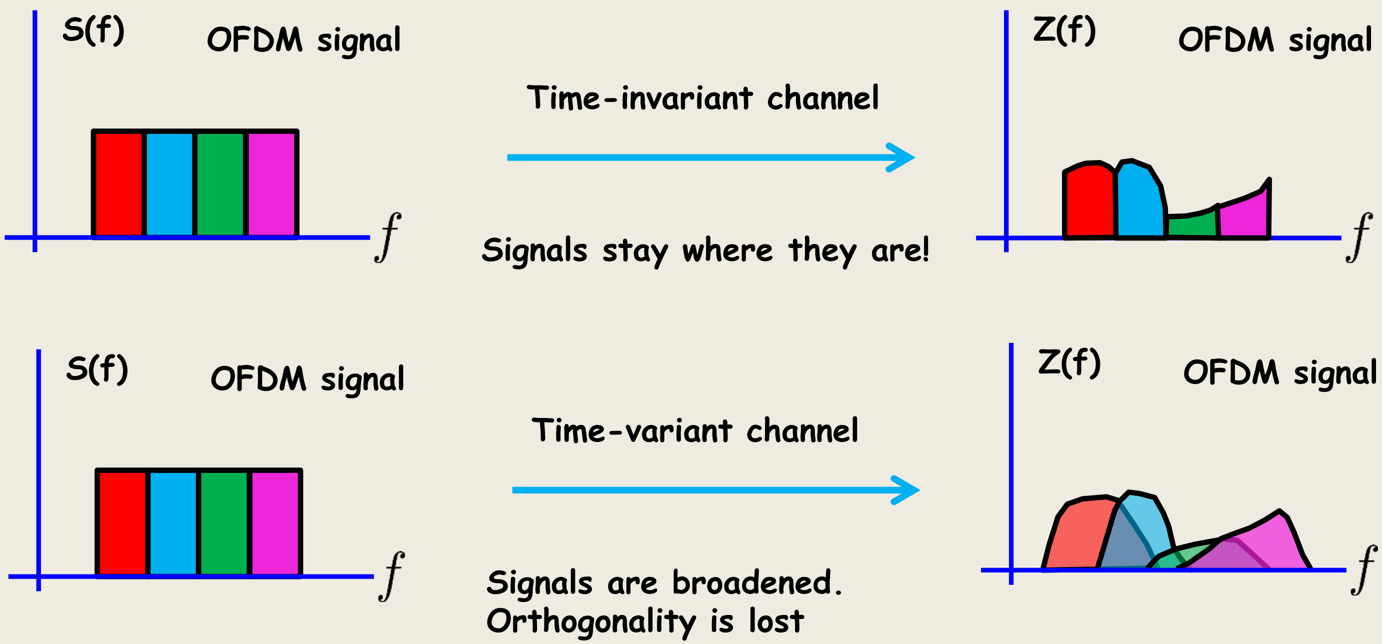

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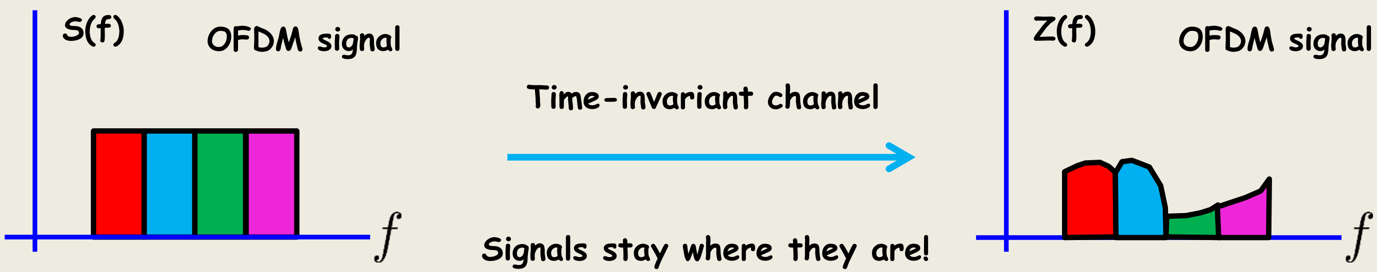
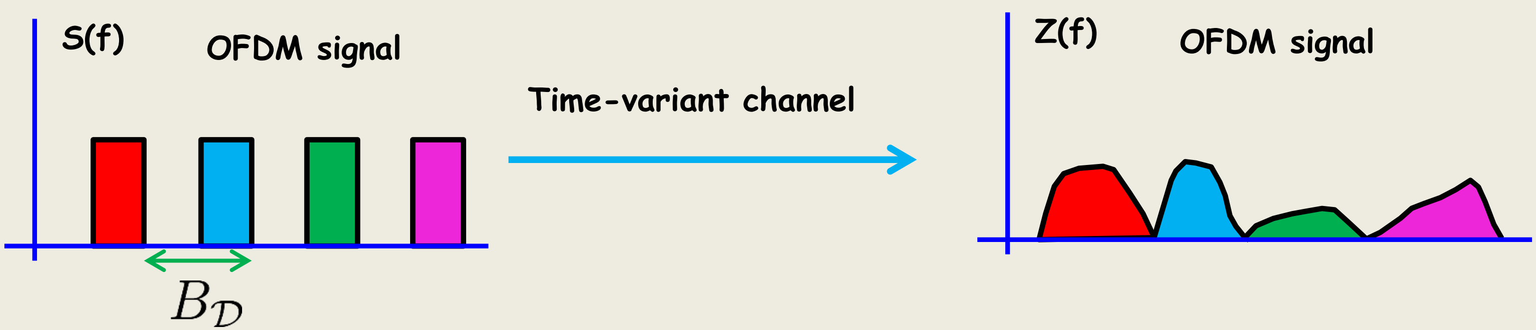


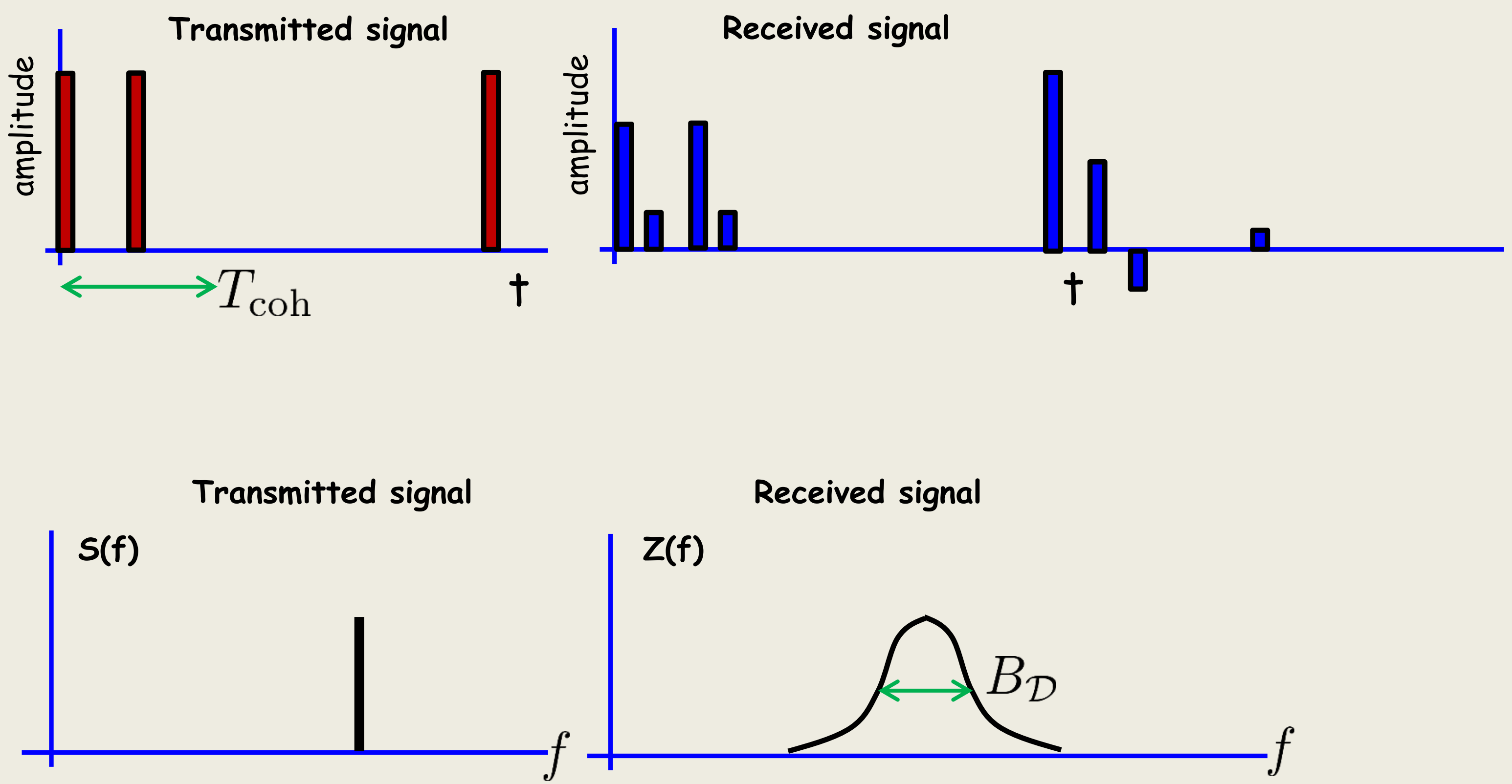


Guard space needed. Spectral efficiency loss

Lecture 9: Time variant channels

Natural question

We know:



Lecture 9: Time variant channels

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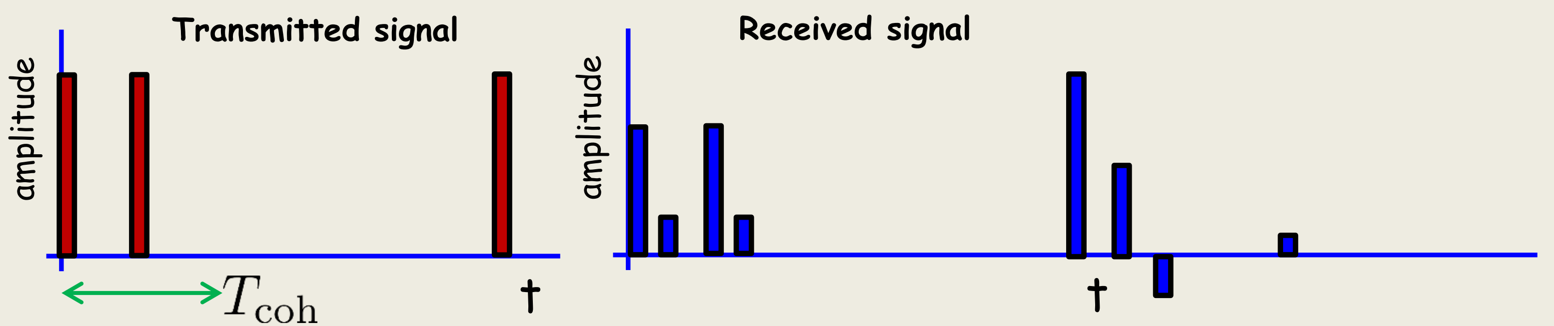
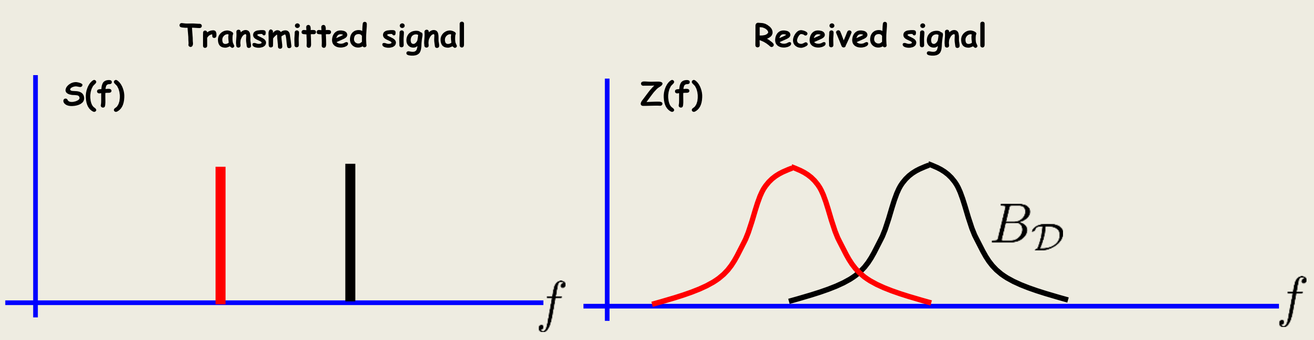


Will we get the same signal, but shifted in frequency?



Lecture 9: Time variant channels

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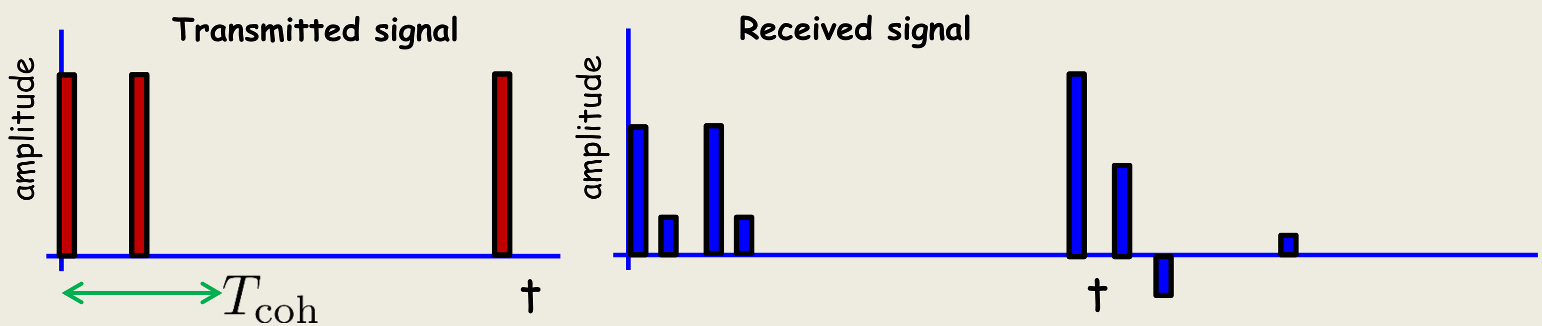
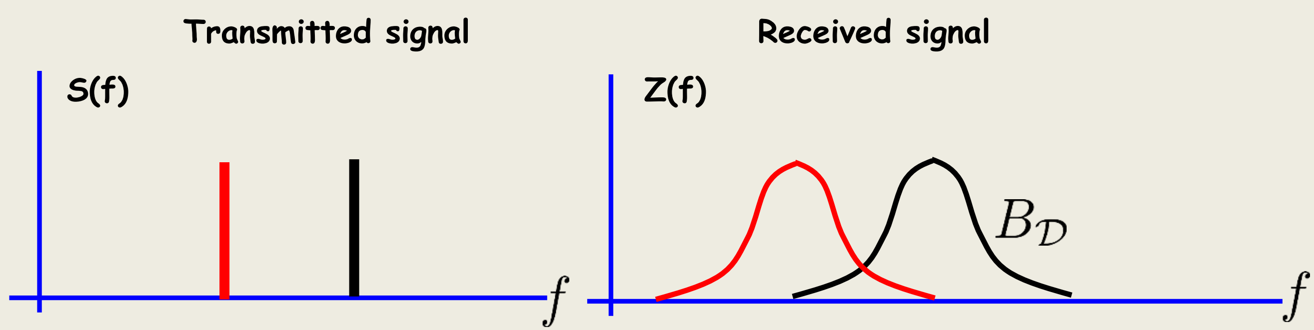


The Doppler spread does not answer this question

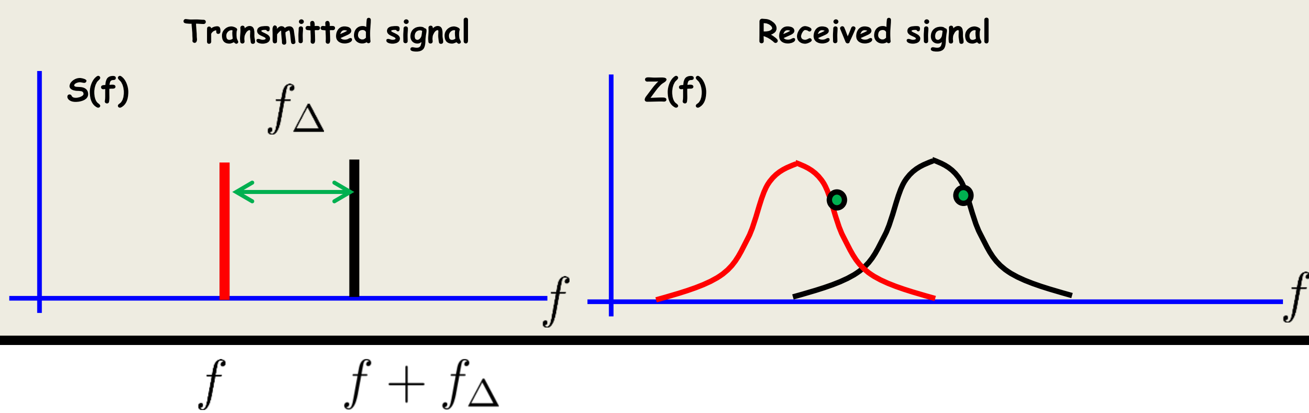


Lecture 9: Time variant channels

New concept: Coherence bandwidth

Compute covariance between the points generated from 2 signals at distance f_Δ

$$\tilde{c}_z(f_\Delta) = E\left\{z(f,t)z^*(f+f_\Delta,t)\right\}$$



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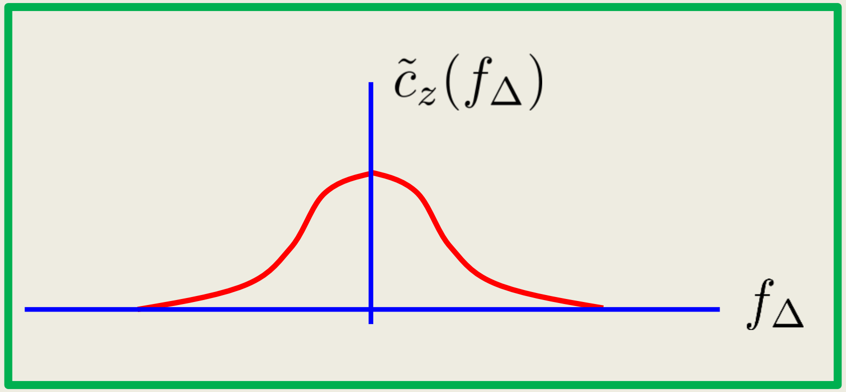

Frequency autocorrelation function

Lecture 9: Time variant channels

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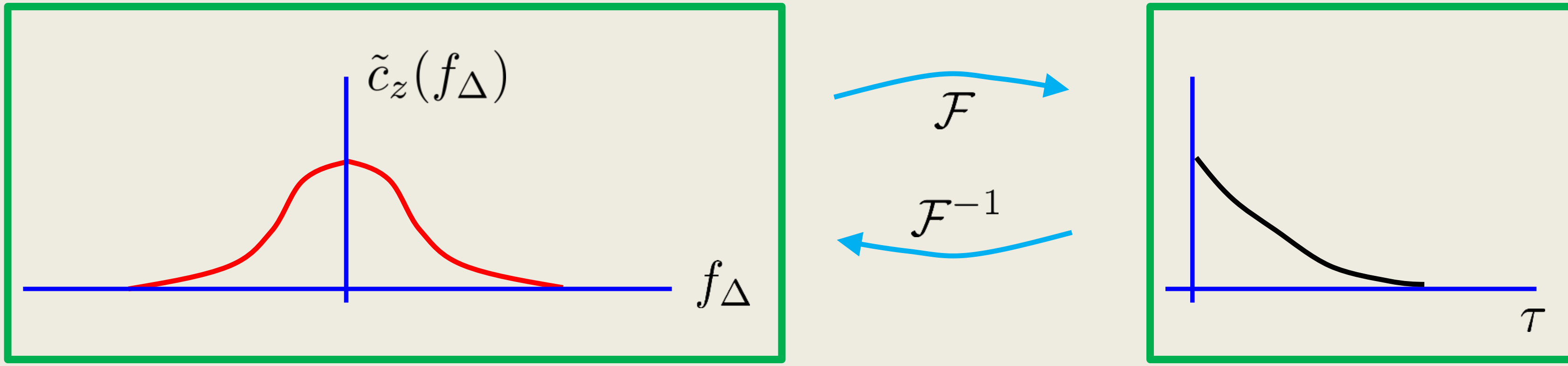

Frequency autocorrelation function

Some function

Lecture 9: Time variant channels

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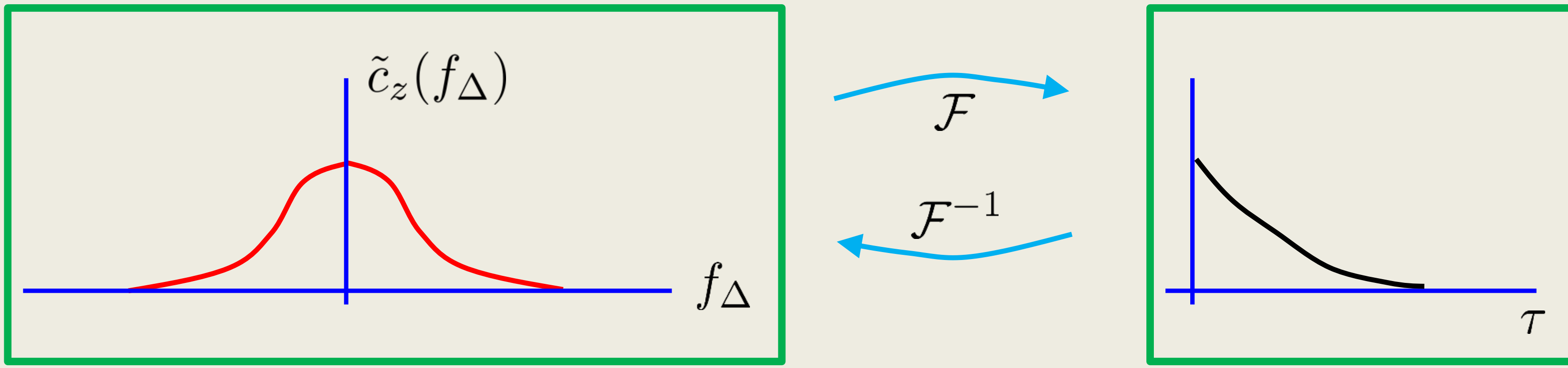

Frequency autocorrelation function

delay power spectrum

Can be shown to be the delay power spectrum $c_h(\tau) = E\left\{h^2(\tau,t)\right\}$

Lecture 9: Time variant channels

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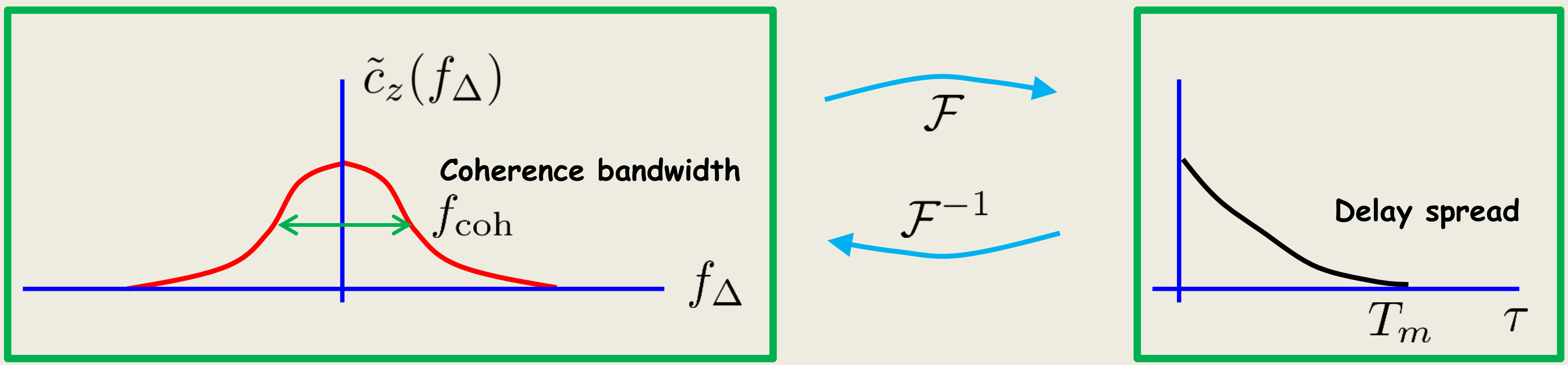

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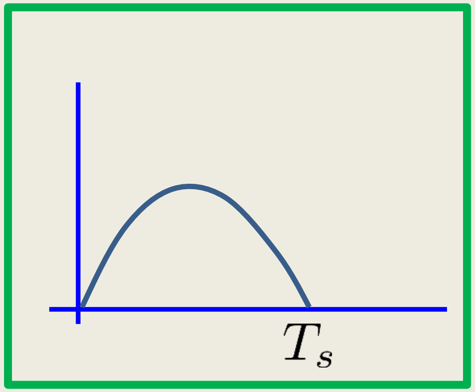
Lecture 9: Time variant channels

Consequence

Assume single carrier transmission with pulse shape p(t)



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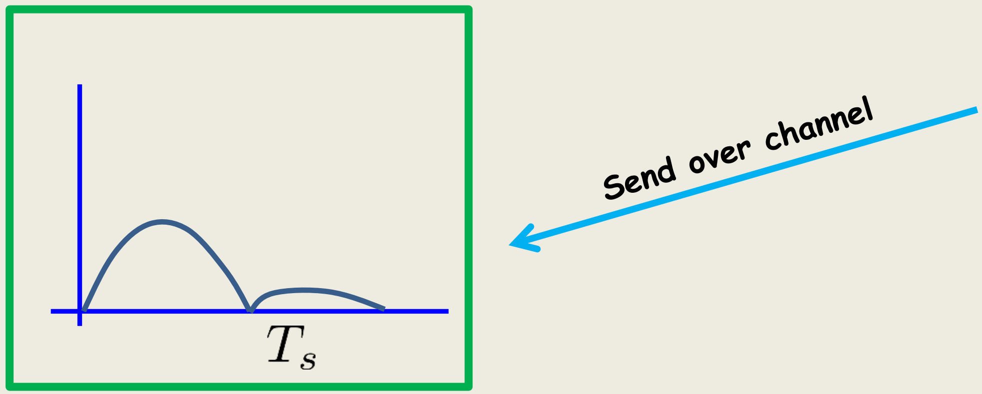

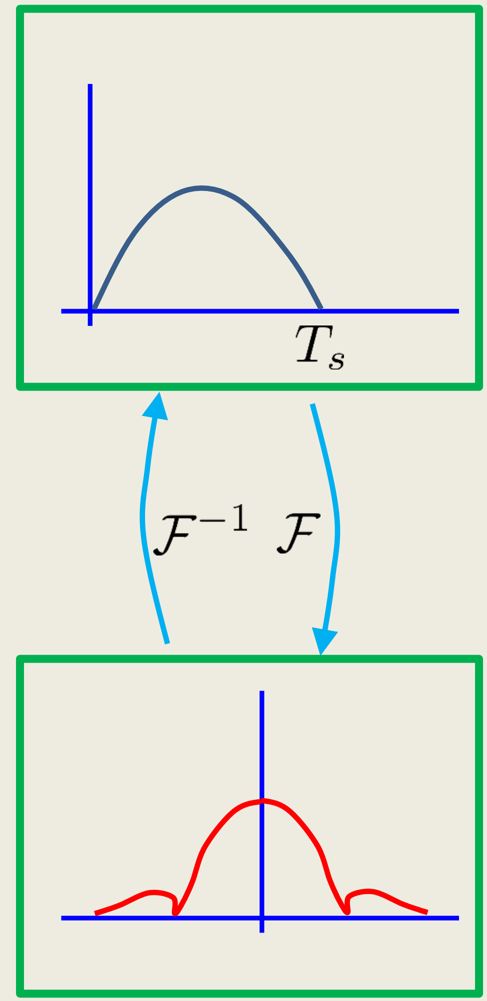

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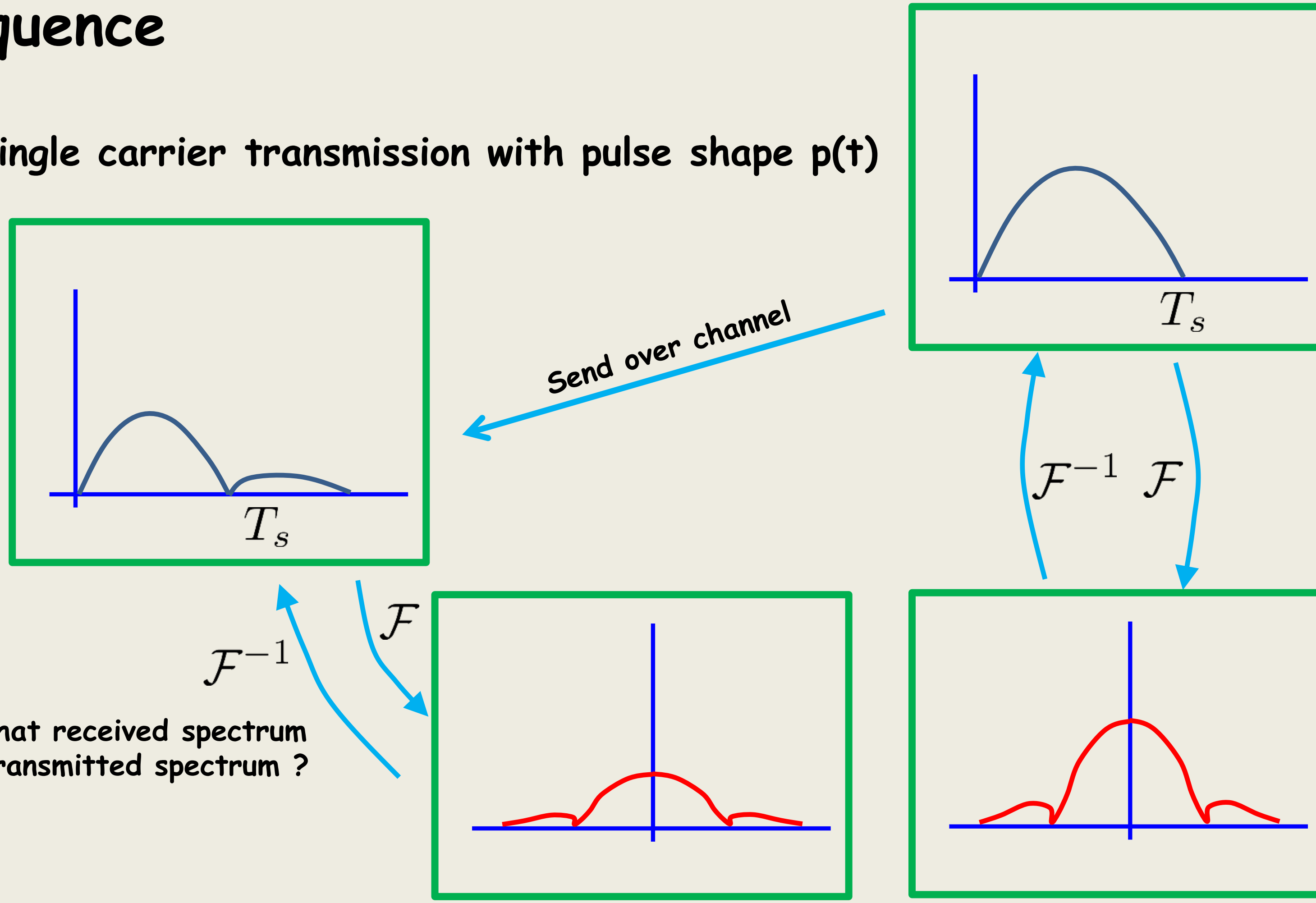

Requirement that received spectrum
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Lecture 9: Time variant channels

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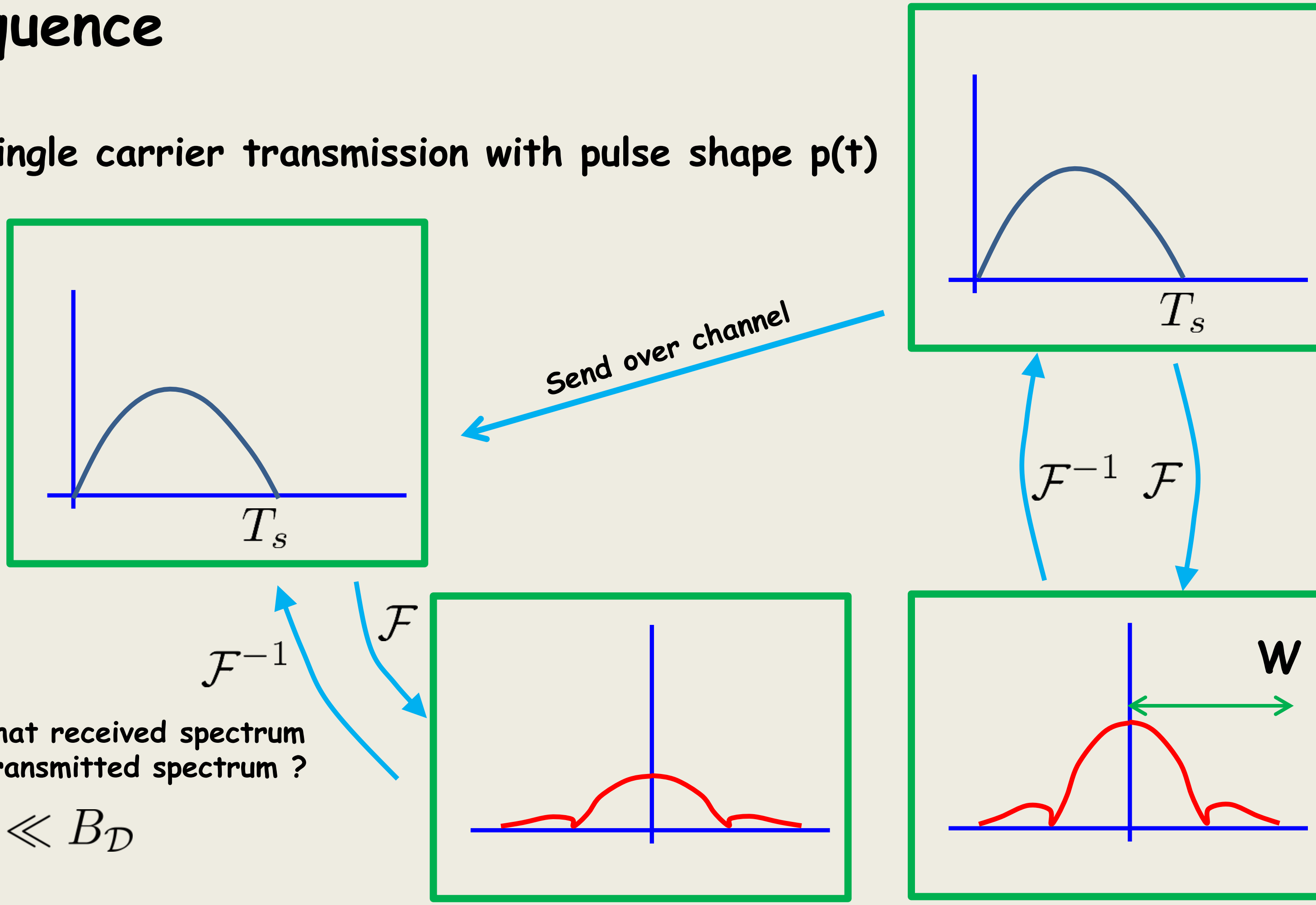

Requirement that received spectrum is scaling of transmitted spectrum ?

$$W \ll B_{\mathcal{D}}$$

Lecture 9: Time variant channels

Frequency-non-selective, slowly fading channel

Symbol rate $R_s = \frac{1}{T_s}$

Slow fading $T_s \ll t_{\mathrm{coh}}$

Lecture 9: Time variant channels

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Lecture 9: Time variant channels

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We know from dig com 1, that $W = k_w R_s$

Lecture 9: Time variant channels

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Conditions for Frequency-non-selective, slowly fading channel

$$\boxed{k_w T_m \approx \frac{k_w}{f_{\mathrm{coh}}} \ll T_s \ll t_{\mathrm{coh}} \approx \frac{1}{B_{\mathrm{D}}}}$$

$$\boxed{\frac{t}{t_{\mathrm{coh}}} \approx B_{\mathcal{D}} \ll R_s \ll \frac{f_{\mathrm{coh}}}{k_w} \approx \frac{1}{k_w T_m}}$$

Lecture 9: Time variant channels

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Significantly simplifed modelling. For complex basesband, signals are mutltiplied with a complex constant

Underspread channel: $B_{\mathcal{D}} T_m \ll 1$

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NO. Note that $B_{\mathcal{D}} T_m$ is a channel parameter, out of our control

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$$\boxed{z(t) = a e_s(t) \cos(\omega_c t + \theta_s(t) + \phi)}$$

$e_s(t)$ and $\theta_s(t)$ describe signal

a and ϕ describe channel

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a Rayleigh ϕ Uniform

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