Lecture 8: OFDM cont.

Recall:

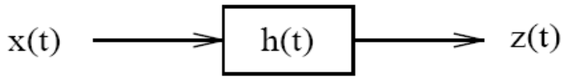


Figure 3.11: Bandpass filtering.

$$x(t) = \mathrm{Re}\left\{a \exp(i2\pi\omega_c t)\right\}, \quad 0 \leq t \leq T_s$$

$$h(t) = 0, \quad t > T_h \text{ or } t < 0$$

$$z(t) = \mathrm{Re}\left\{H(\omega_c) a \exp(i2\pi\omega_c t)\right\}, \quad T_h \leq t \leq T_s$$

(from Lecture 6)

Lecture 8: OFDM cont.

Recall:

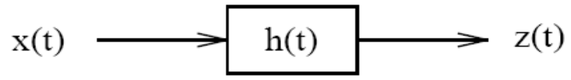


Figure 3.11: Bandpass filtering.

Consequence

$$x(t) = \sum_{k=0}^{K-1} \mathrm{Re}\left\{a_k \exp(i2\pi\omega_k t)\right\}, \quad 0 \le t \le T_s$$

$$h(t) = 0, \quad t > T_h \text{ or } t < 0$$

$$z(t) = \sum_{k=0}^{K-1} \mathrm{Re}\left\{H(\omega_k) a_k \exp(i2\pi\omega_k t)\right\}, \quad T_h \le t \le T_s$$

Lecture 8: OFDM cont.

Recall:

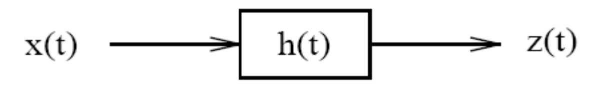


Figure 3.11: Bandpass filtering.

Consequence

$$x(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} a_k \exp(i2\pi\omega_k t) \right\}, \quad 0 \le t \le T_s$$

$$h(t) = 0, \quad t > T_h \text{ or } t < 0$$

$$z(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i2\pi\omega_k t) \right\}, \quad T_h \le t \le T_s$$

Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp}) \exp(i2\pi f_{rc} t) \right\}, \quad 0 \leq t \leq T_s$$

$$x(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} a_k \exp(i2\pi \omega_k t) \right\}, \quad 0 \leq t \leq T_s$$

$$z(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i2\pi \omega_k t) \right\}, \quad T_h \leq t \leq T_s$$

Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp}) \exp(i2\pi f_{rc} t)\right\}, \quad 0 \le t \le T_s$$

Or alternatively

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i2\pi f_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i2\pi \omega_k t)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i2\pi \omega_k t)\right\}, \quad T_h \le t \le T_s$$

Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i2\pi\omega_k t)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i2\pi\omega_k t)\right\}, \quad T_h \le t \le T_s$$

Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i\omega_k t + \theta_k)\right\}, \quad T_{cp} \le t \le T_s$$

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i2\pi\omega_k t)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i2\pi\omega_k t)\right\}, \quad T_h \le t \le T_s$$

Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k) a_k \exp(i2\pi g_k f_\Delta t)\exp(i2\pi f_{rc} t)\right\}, \quad T_{cp} \le t \le T_s$$

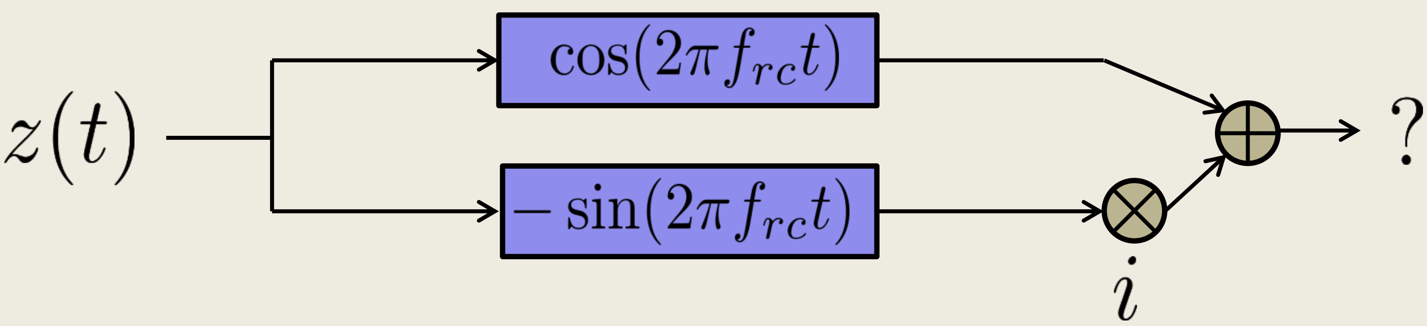
Expand using the reference-carrier

Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k) a_k \exp(i2\pi g_k f_\Delta t)\exp(i2\pi f_{rc} t)\right\}, \quad T_{cp} \le t \le T_s$$

Frequency down-conversion

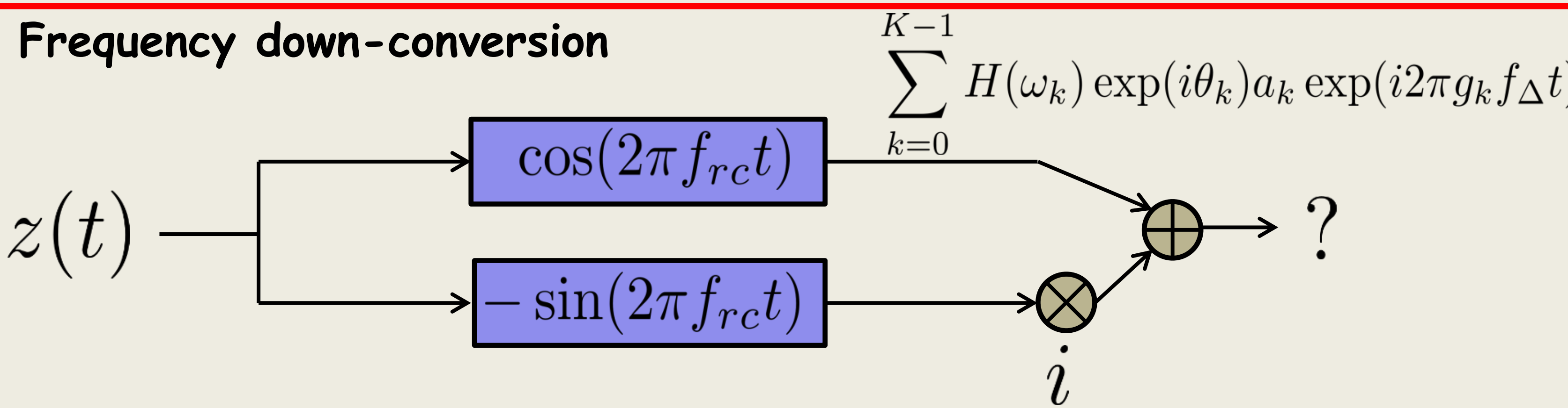


Lecture 8: OFDM cont.

$$x(t) = \text{Re}\left\{ \sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k) \right\}, \quad 0 \leq t \leq T_s$$

$$z(t) = \text{Re}\left\{ \sum_{k=0}^{K-1} H(\omega_k) \exp(i\theta_k) a_k \exp(i2\pi g_k f_\Delta t) \exp(i2\pi f_{rc} t) \right\}, \quad T_{cp} \leq t \leq T_s$$

Frequency down-conversion



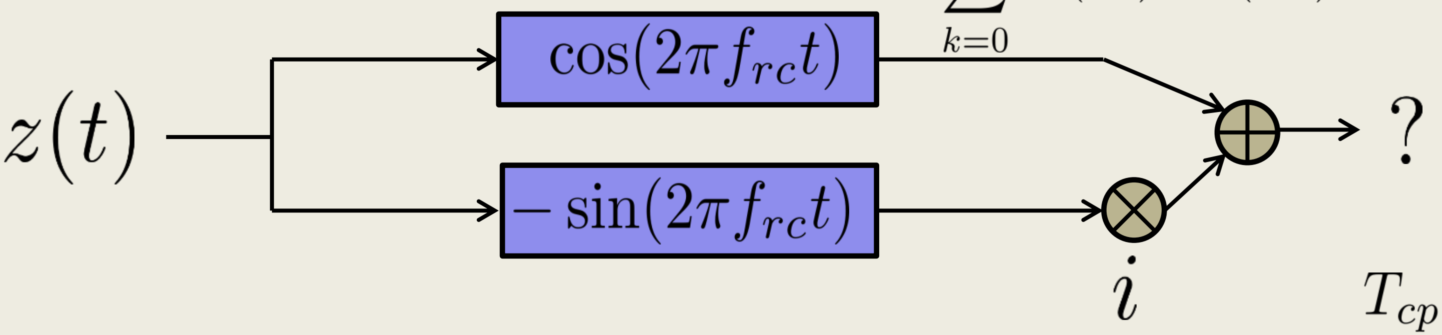
Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\boxed{\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k)a_k \exp(i2\pi g_k f_\Delta t)}\exp(i2\pi f_{rc} t)\right\}, \quad T_{cp} \le t \le T_s$$

Frequency down-conversion

$$\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k)a_k \exp(i2\pi g_k f_\Delta t)$$



$$T_{cp} \le t \le T_s$$

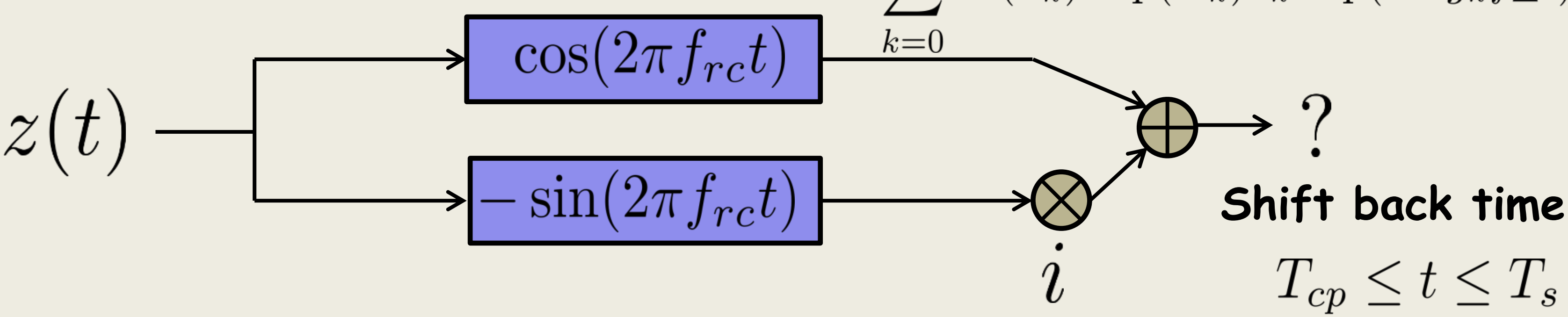
Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\boxed{\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k)a_k \exp(i2\pi g_k f_\Delta t)}\exp(i2\pi f_{rc} t)\right\}, \quad T_{cp} \le t \le T_s$$

Frequency down-conversion

$$\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k)a_k \exp(i2\pi g_k f_\Delta t)$$



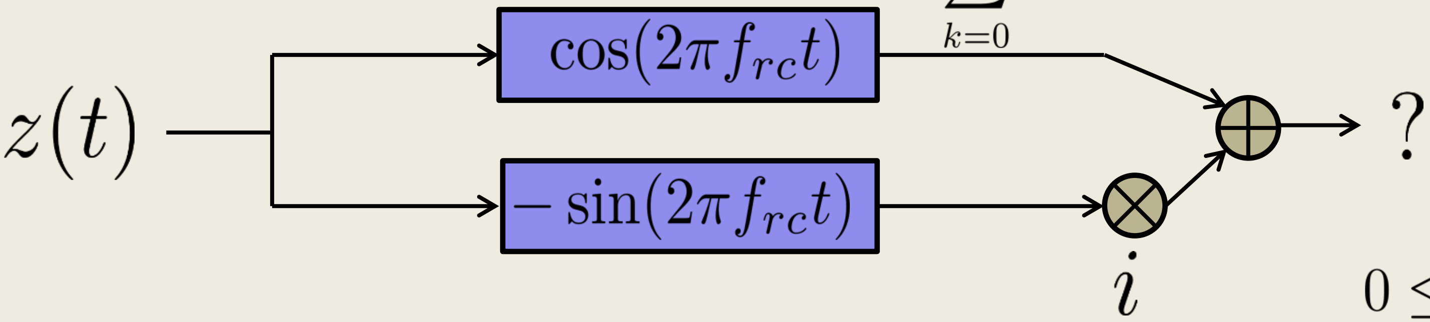
Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k)a_k \exp(i2\pi g_k f_\Delta t)\exp(i2\pi f_{rc}t)\right\}, \quad T_{cp} \le t \le T_s$$

Frequency down-conversion

$$\sum_{k=0}^{K-1} H(\omega_k)a_k \exp(i2\pi g_k f_\Delta t)$$



$$0 \le t \le T_{obs}$$

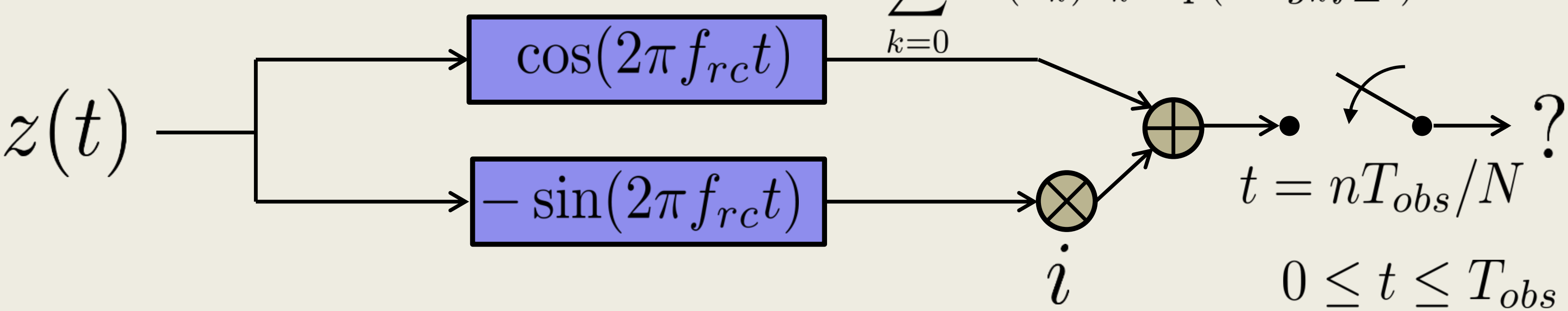
Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k) \right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{ \sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k) a_k \exp(i2\pi g_k f_\Delta t) \exp(i2\pi f_{rc} t) \right\}, \quad T_{cp} \le t \le T_s$$

Sample the signal

$$\sum_{k=0}^{K-1} H(\omega_k) a_k \exp(i2\pi g_k f_\Delta t)$$



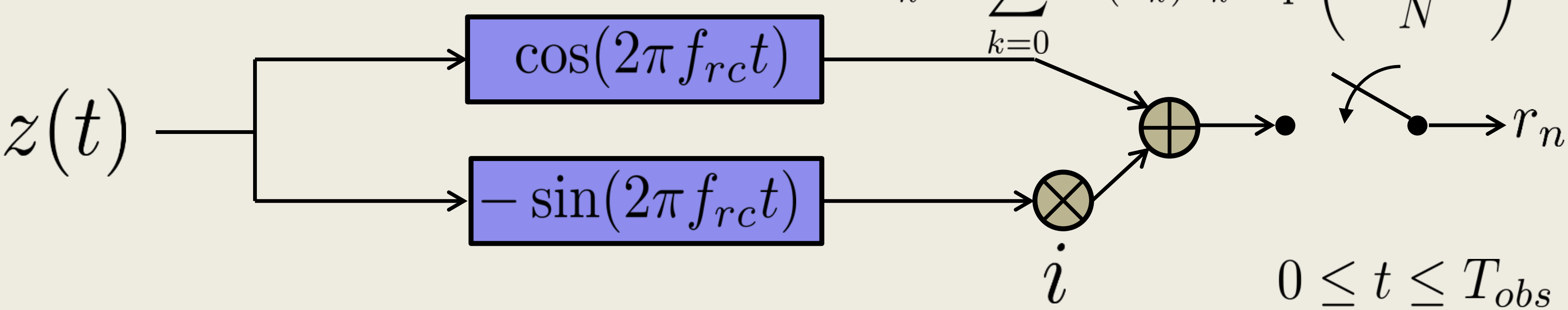
Lecture 8: OFDM cont.

$$x(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} a_k \exp(i\omega_k t + \theta_k)\right\}, \quad 0 \le t \le T_s$$

$$z(t) = \mathrm{Re}\left\{\sum_{k=0}^{K-1} H(\omega_k)\exp(i\theta_k) a_k \exp(i2\pi g_k f_\Delta t) \exp(i2\pi f_{rc} t)\right\}, \quad T_{cp} \le t \le T_s$$

Sample the signal

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

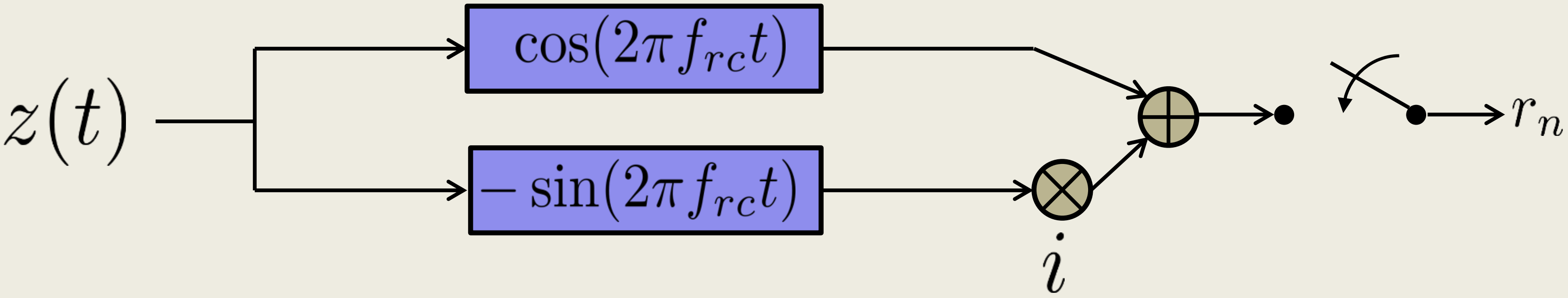


$$0 \le t \le T_{obs}$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right), \quad n = 0, 1, 2, \ldots, N-1$$

Receiver

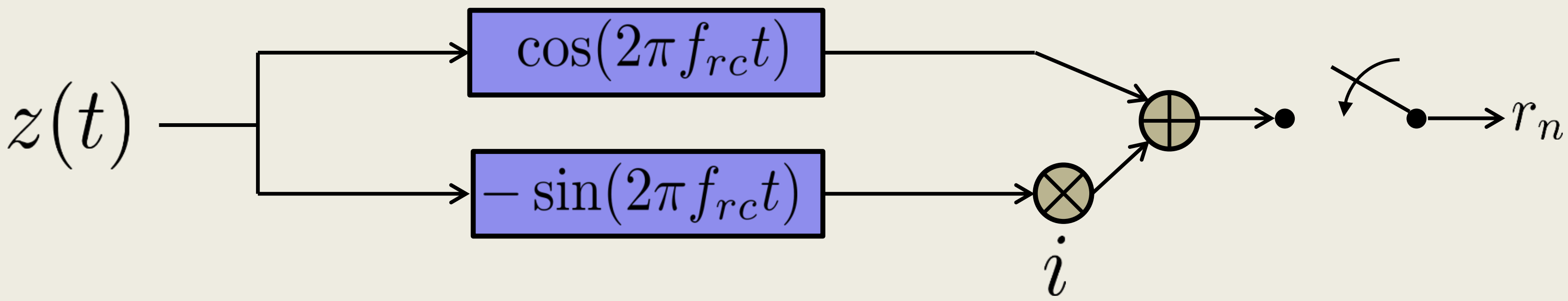


Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ **Received complex baseband signal**

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ **Transmitted complex baseband signal**

Receiver



Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X_m\}$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X_m\}$

What is so special about those ?

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X_m\}$

What is so special about those ?

They equal $\{a_n\}$ in some other order!

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X_m\}$

What is so special about those ?

They equal $\{a_n\}$ in some other order!

What order ?

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ **Received complex baseband signal**

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ **Transmitted complex baseband signal**

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X$

What is so special about

They equal $\{a_n\}$ in some

What order ? This order

$$X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H(\omega_k) a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

Define $H_k = H(\omega_k)$

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X$

What is so special about

They equal $\{a_n\}$ in some

What order ? This order

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H_k a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Received complex baseband signal

Define $H_k = H(\omega_k)$

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$ Transmitted complex baseband signal

What happens if we take the FFT of $\{x_n\}$?

We get the N values $\{X$

What is so special about

They equal $\{a_n\}$ in some

What order ? This order

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H_k a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

Received complex baseband signal

Compute FFT of $\{r_n\}$

$$R_m = \sum_{n=0}^{N-1} r_n \exp\left(\frac{-i2\pi mn}{N}\right)$$

What happens if we take the FFT of $\{r_n\}$?

We get the N values $\{X$

What is so special about

They equal $\{a_n\}$ in some

What order ? This order

$$X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H_k a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

Received complex baseband signal

Compute FFT of $\{r_n\}$

$$R_m = \sum_{n=0}^{N-1} r_n \exp\left(\frac{-i2\pi m n}{N}\right)$$

What happens if we take the FFT of $\{r_n\}$?

We get the N values $\{X$

What is so special about

They equal $\{a_n\}$ in some

What order ? This order

$$R_m = N H_{m-g_0} a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$R_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$R_m = N H_{m-(g_0+N)} a_{m-(g_0+N)}$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H_k a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

Received complex baseband signal

Get back $\{a_n\}$ from $\{R_m\}$

$$R_m = \sum_{n=0}^{N-1} r_n \exp\left(\frac{-i2\pi mn}{N}\right)$$

$$a_n = \frac{R_{\tilde{n}}}{NH_n}$$

$\tilde{n}$: The index where a_n is

What happens if we take the FFT of $\{r_n\}$?

We get the N values $\{X$

What is so special about

They equal $\{a_n\}$ in some

What order ? This order

$$R_m = NH_{m-g_0} a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$R_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$R_m = NH_{m-(g_0+N)} a_{m-(g_0+N)}$$

Lecture 8: OFDM cont.

$$r_n = \sum_{k=0}^{K-1} H_k a_k \exp\left(\frac{i2\pi g_k n}{N}\right)$$

Received complex baseband signal

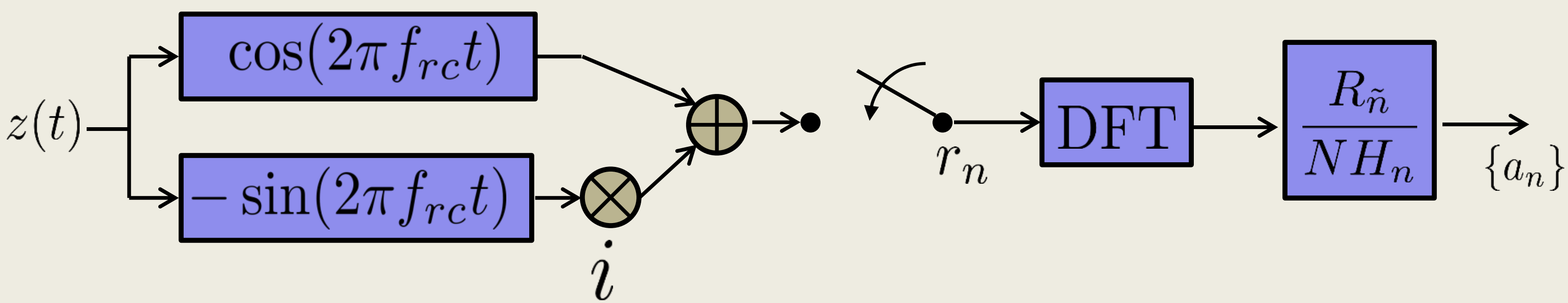
Get back $\{a_n\}$ from $\{R_m\}$

$$R_m = \sum_{n=0}^{N-1} r_n \exp\left(\frac{-i2\pi mn}{N}\right)$$

$$a_n = \frac{R_{\tilde{n}}}{NH_n}$$

$\tilde{n}$: The index where a_n is

Receiver

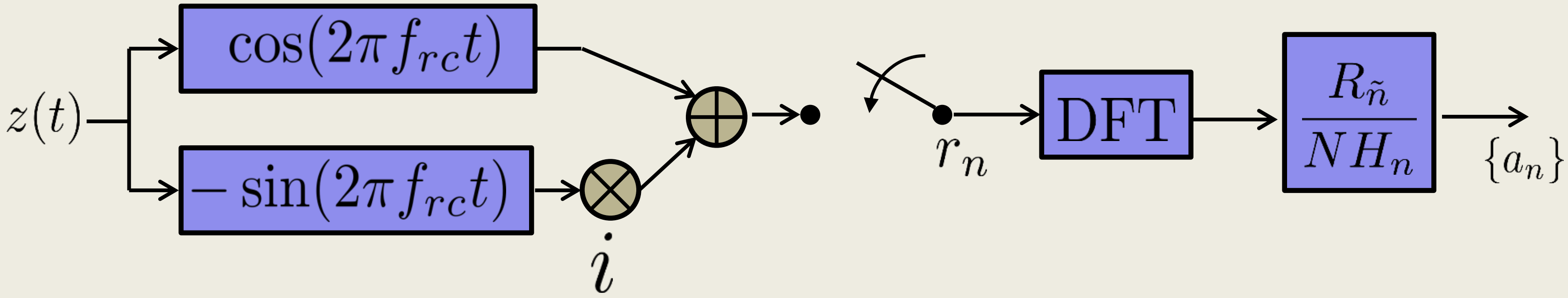


Lecture 8: OFDM cont.

With noise

$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$ **Variance N_0**

Receiver



Lecture 8: OFDM cont.

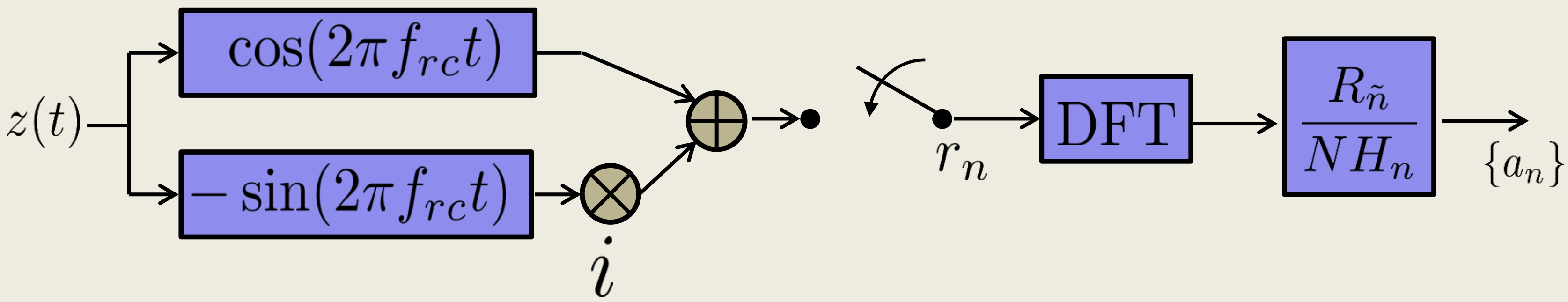
With noise

$$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$$

Variance N_0

$$\frac{R_{\tilde{n}}}{NH_n} = a_n + \frac{w_{\tilde{n}}}{NH_n}$$

Receiver



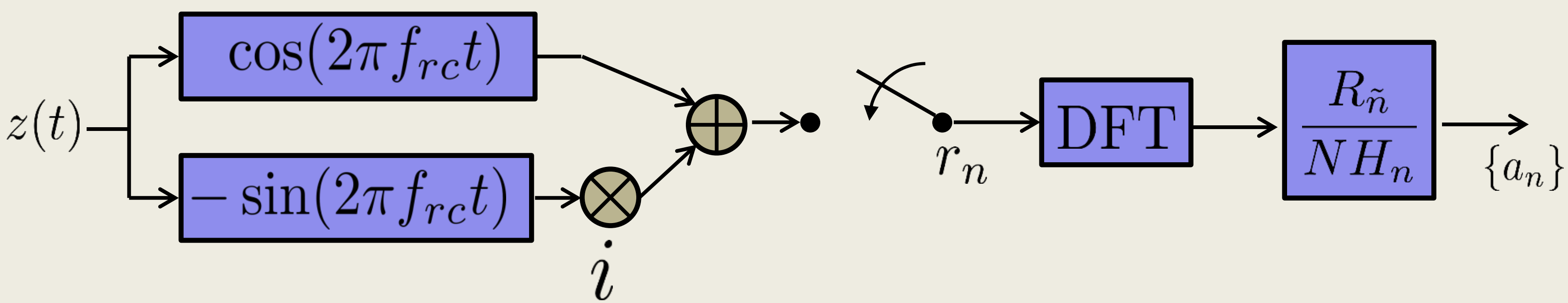
Lecture 8: OFDM cont.

With noise

$$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$$ **Variance N_0**

$$\frac{R_{\tilde{n}}}{NH_n} = a_n + \frac{w_{\tilde{n}}}{NH_n} = a_n + \eta_{\tilde{n}}$$

Receiver



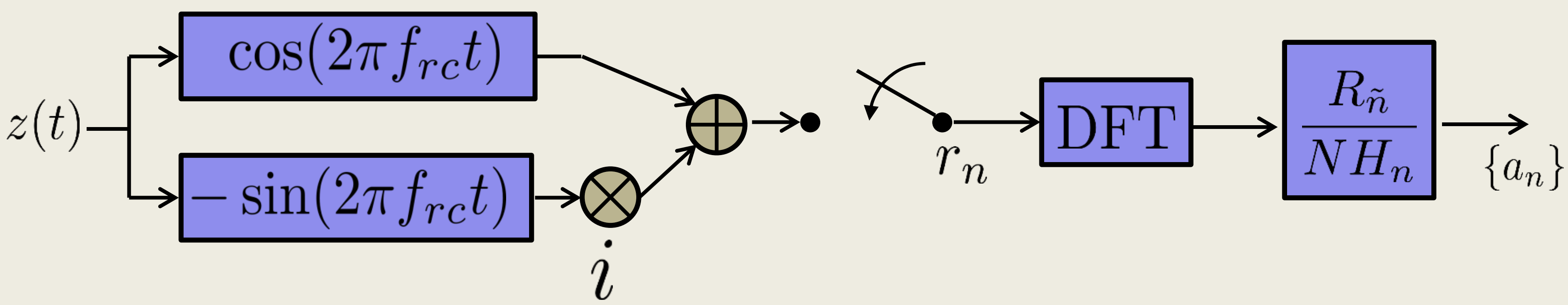
Lecture 8: OFDM cont.

With noise

$$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$$ **Variance N_0**

$$\frac{R_{\tilde{n}}}{NH_n} = a_n + \frac{w_{\tilde{n}}}{NH_n} = a_n + \eta_{\tilde{n}}$$ **Variance** $\frac{N_0}{N^2|H_n|^2}$

Receiver



Lecture 8: OFDM cont.

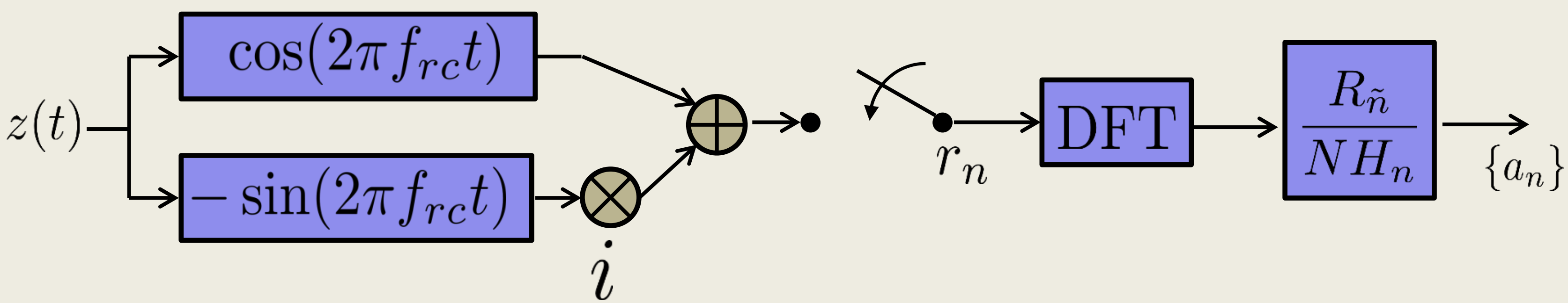
With noise

$$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$$ **Variance N_0**

$$\frac{R_{\tilde{n}}}{NH_n} = a_n + \frac{w_{\tilde{n}}}{NH_n} = a_n + \eta_{\tilde{n}}$$ **Variance** $\frac{N_0}{N^2|H_n|^2}$

$$\mathrm{SNR}_n = \frac{N^2|H_n|^2}{N_0}$$

Receiver



Lecture 8: OFDM cont.

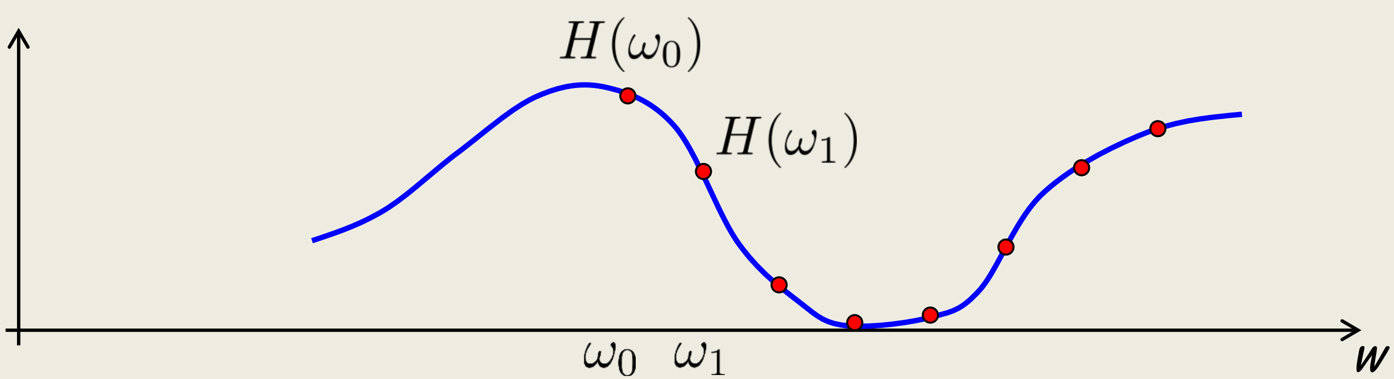
With noise

$$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$$ **Variance N_0**

$$\frac{R_{\tilde{n}}}{NH_n} = a_n + \frac{w_{\tilde{n}}}{NH_n} = a_n + \eta_{\tilde{n}}$$ **Variance** $\frac{N_0}{N^2|H_n|^2}$

$$\mathrm{SNR}_n = \frac{N^2|H_n|^2}{N_0}$$

Channel



Lecture 8: OFDM cont.

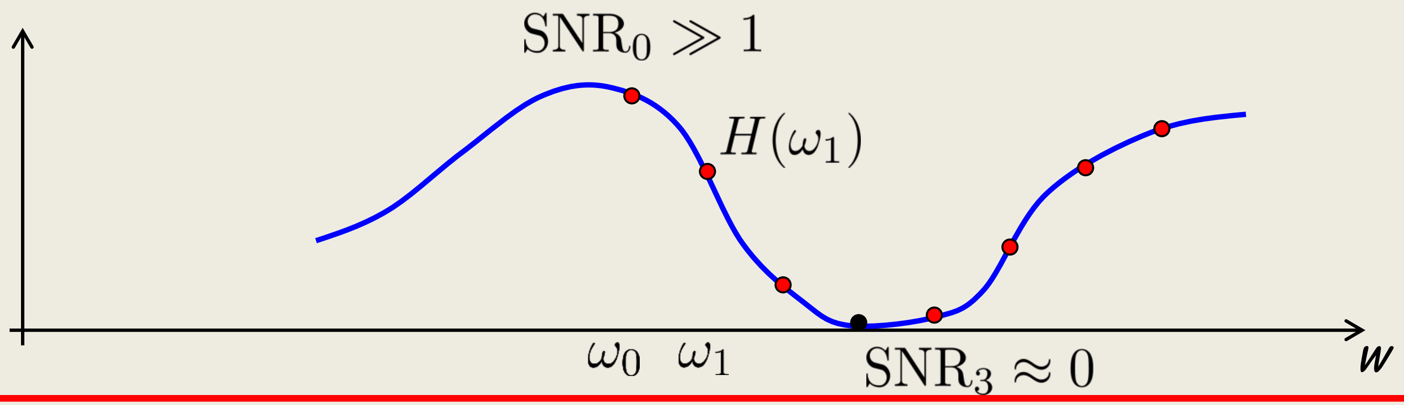
With noise

$$R_{\tilde{n}} = NH_n a_n + w_{\tilde{n}}$$ **Variance N_0**

$$\frac{R_{\tilde{n}}}{NH_n} = a_n + \frac{w_{\tilde{n}}}{NH_n} = a_n + \eta_{\tilde{n}}$$ **Variance** $\frac{N_0}{N^2|H_n|^2}$

$$\mathrm{SNR}_n = \frac{N^2|H_n|^2}{N_0}$$

Channel



Lecture 8: OFDM cont.

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

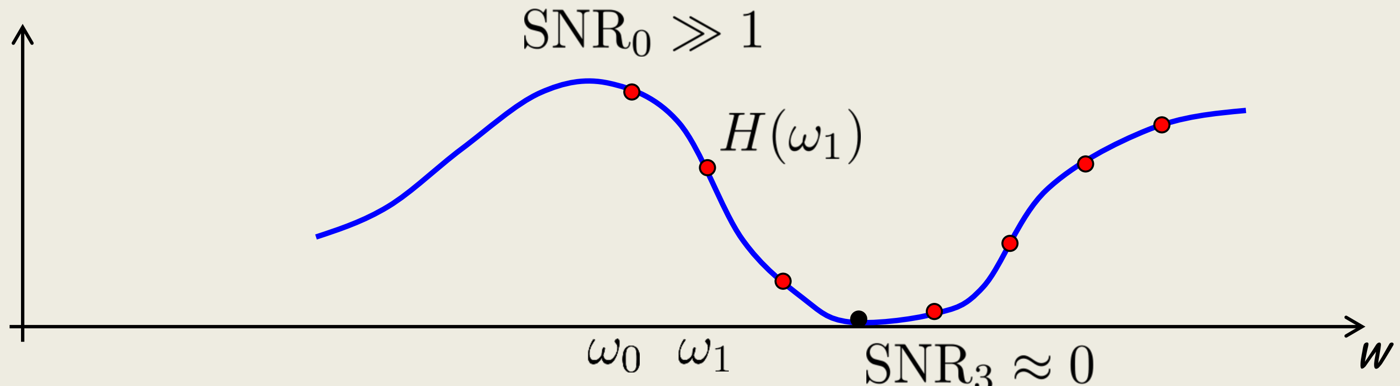
$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

$$\text{SNR}_n = \frac{N^2 |H_n|^2}{N_0}$$

Power allocation: use different powers at different sub-carriers (Waterfilling).

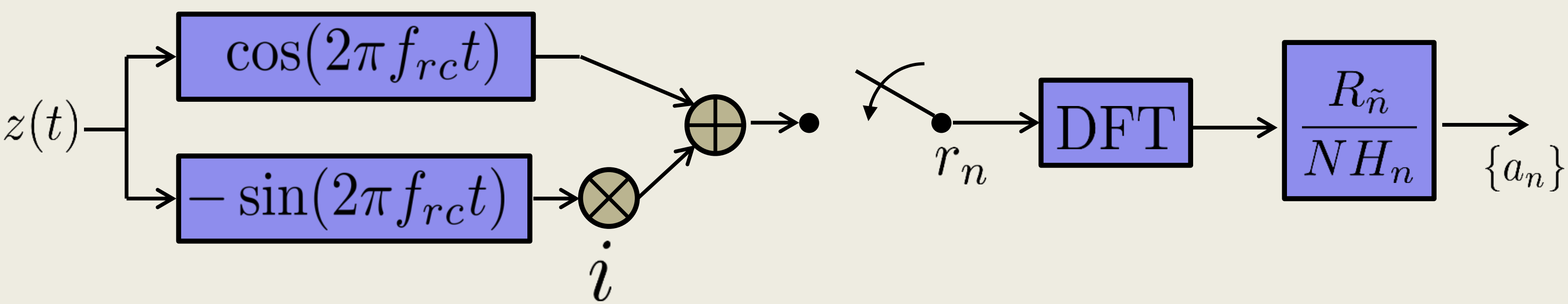
Channel



Lecture 8: OFDM cont.

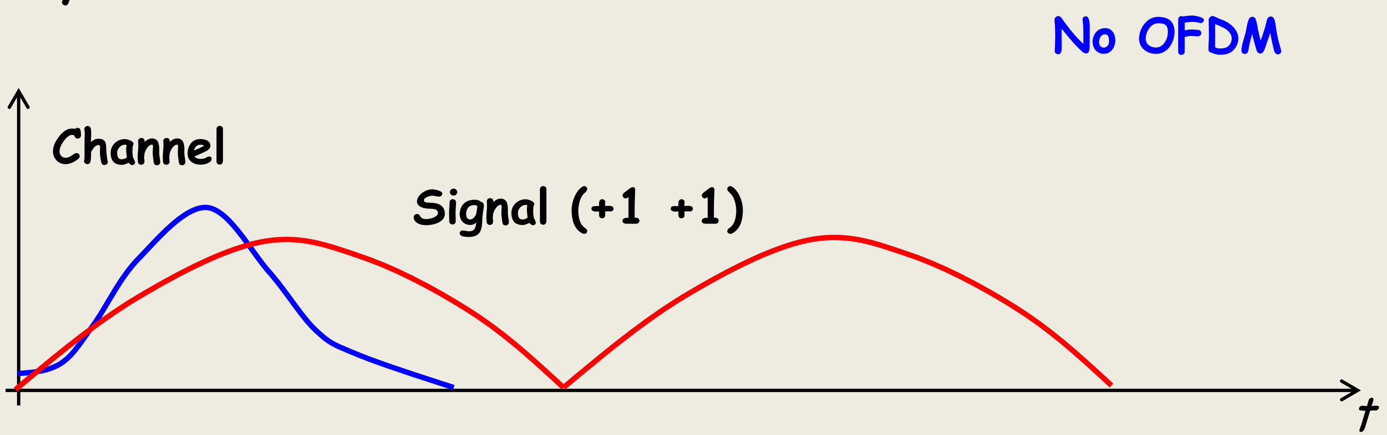
Why OFDM

Receiver

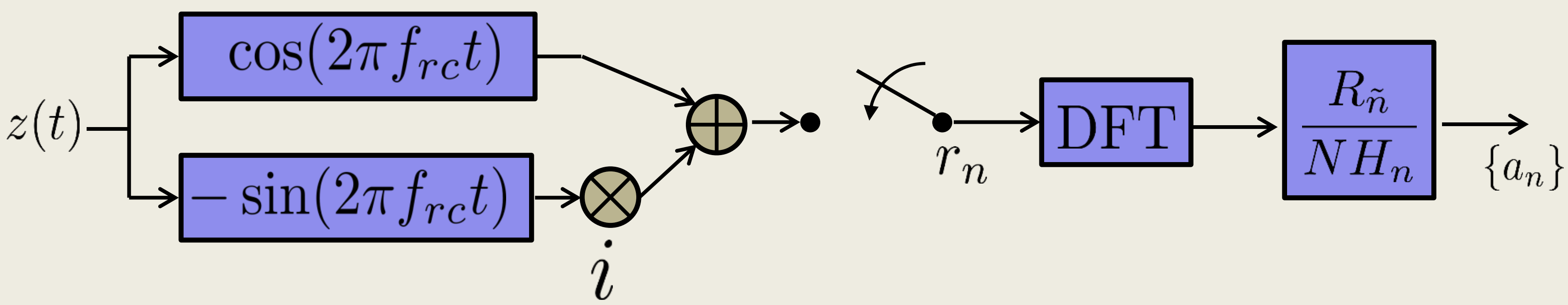


Lecture 8: OFDM cont.

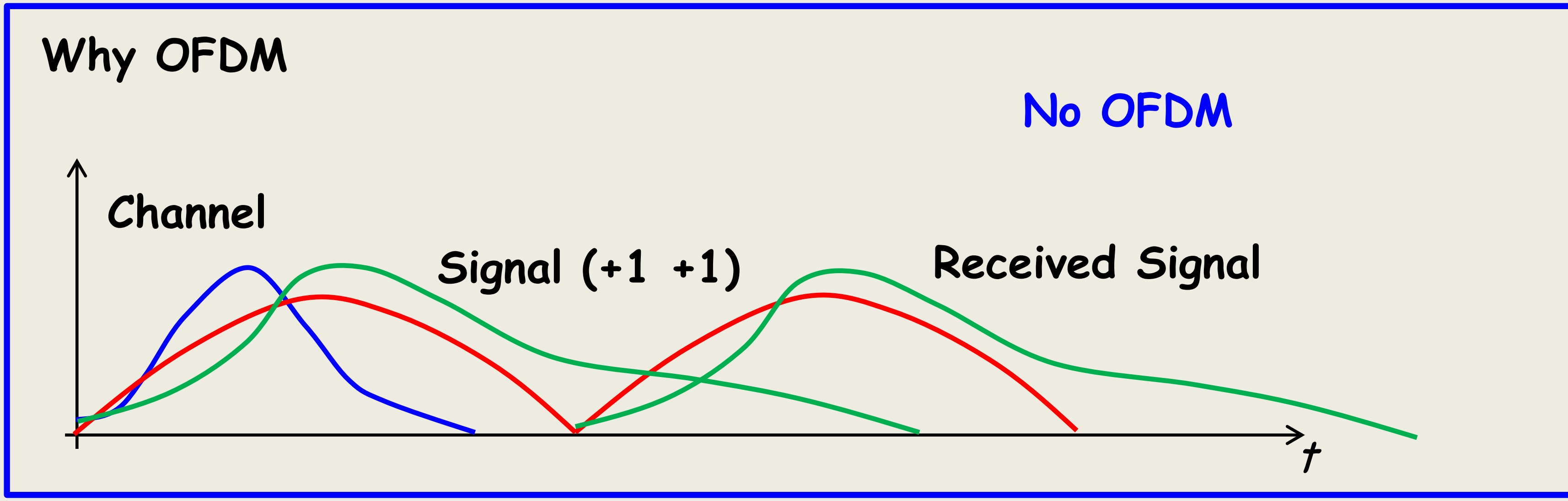
Why OFDM

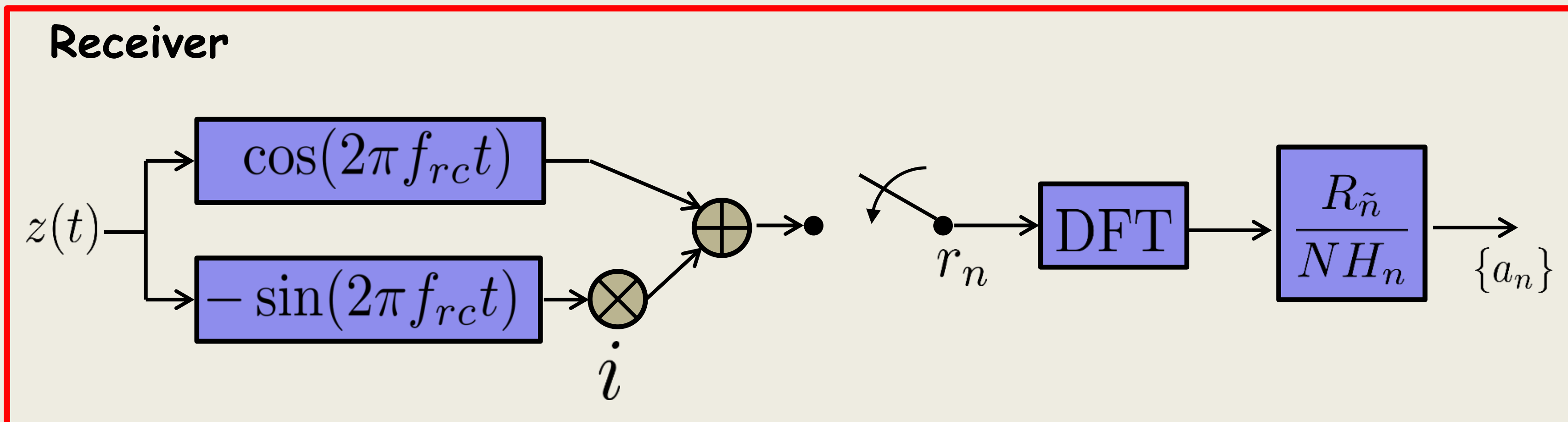


Receiver



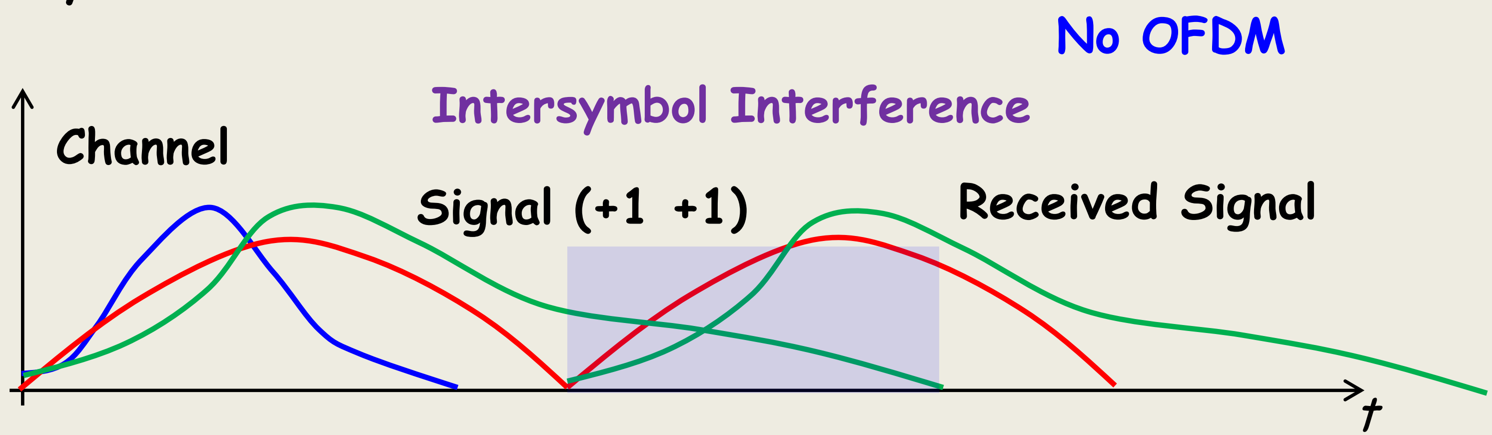
Lecture 8: OFDM cont.



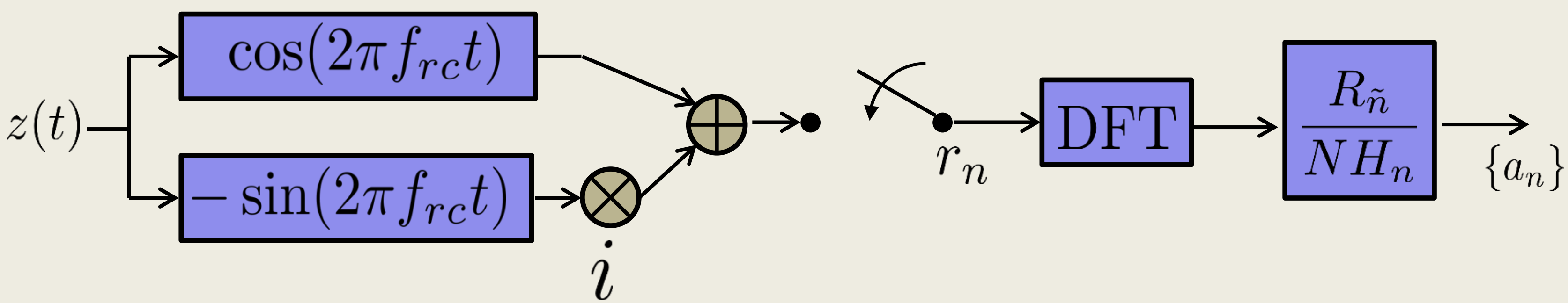


Lecture 8: OFDM cont.

Why OFDM

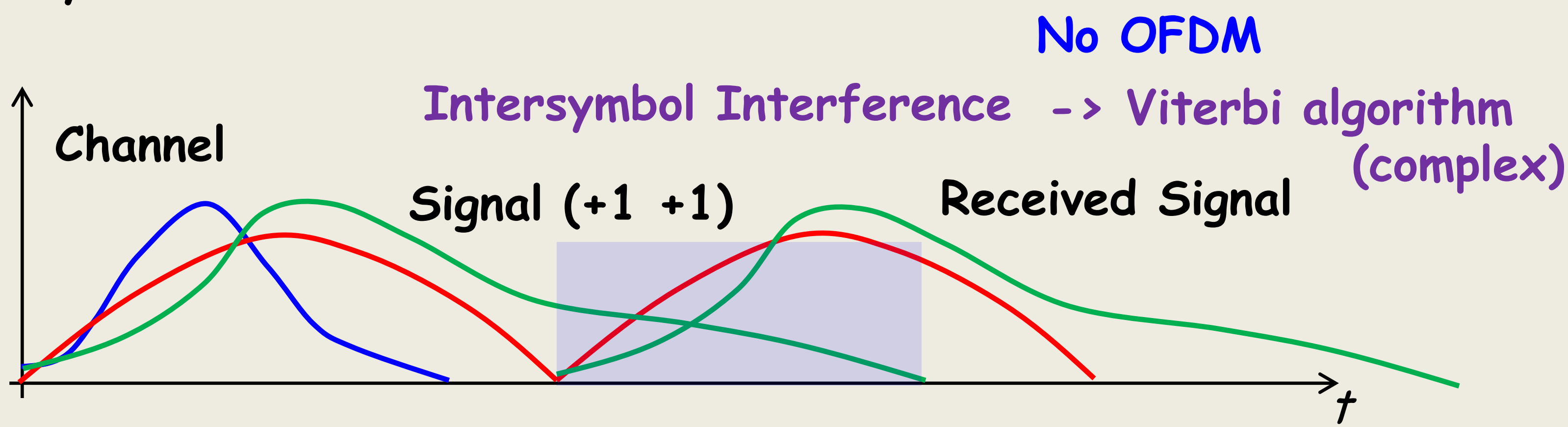


Receiver

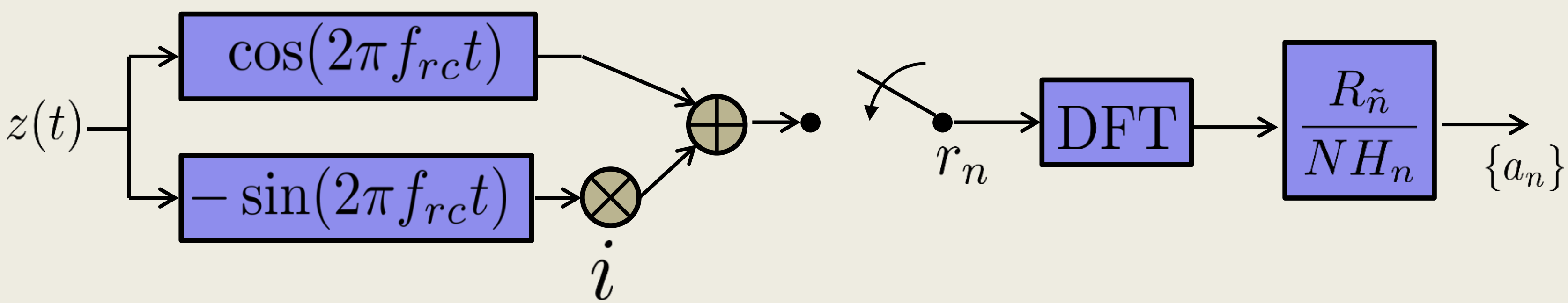


Lecture 8: OFDM cont.

Why OFDM



Receiver

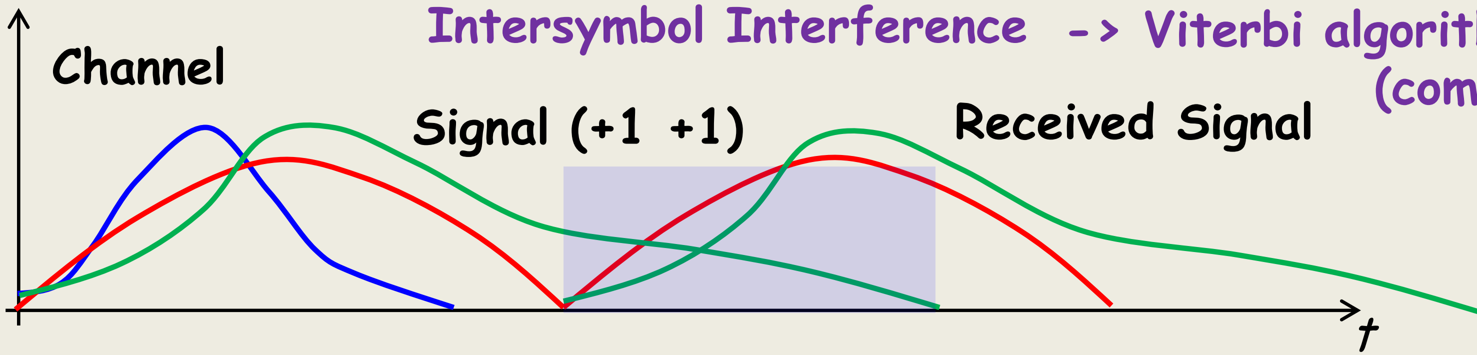


Lecture 8: OFDM cont.

Why OFDM

No OFDM

Intersymbol Interference -> Viterbi algorithm (complex)



Receiver

OFDM - very simple