Lecture 7: OFDM

Recall: **If** $x(t) = A\cos(\omega_c t) - B\sin(\omega_c t),\ 0 \le t \le T_s$

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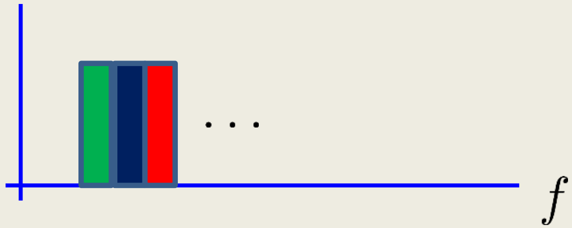
We can transmit multiple signals at different sub-carriers: This is OFDM



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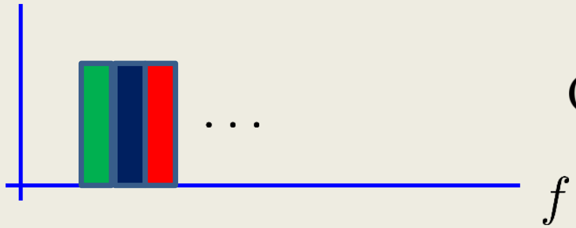

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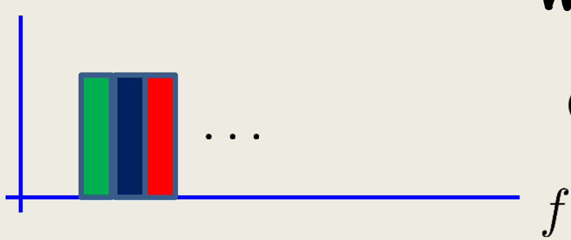

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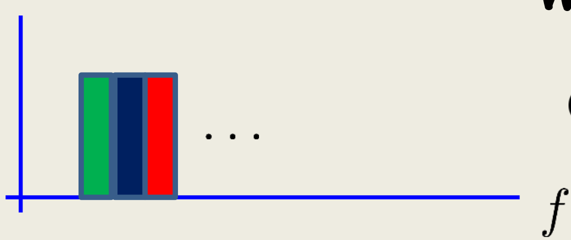

Carrier

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One signal

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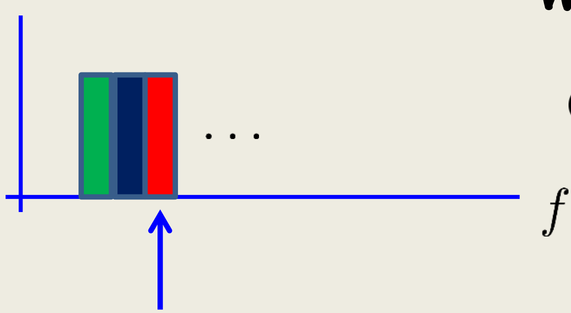

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We can transmit multiple signals at different sub-carriers:
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All signals

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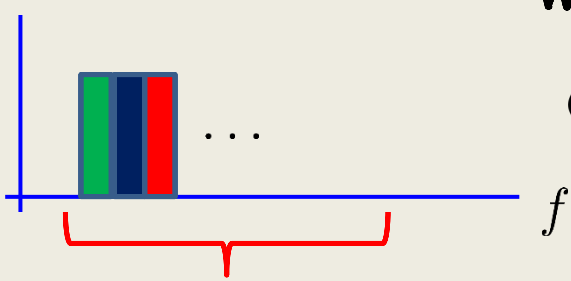

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Lecture 7: OFDM

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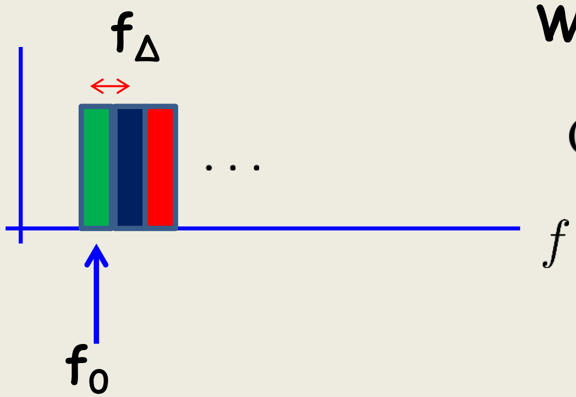

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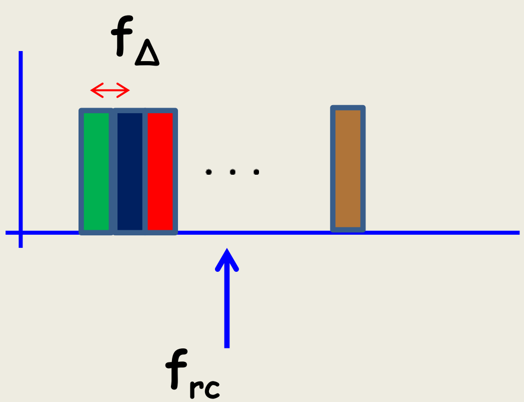

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$g_k = \ldots -2, -1, 0, 1, 2, \ldots$

Lecture 7: OFDM

$$g_k : -\frac{K-1}{2} = g_0, \dots, -1, 0, 1, \dots, \frac{K-1}{2} = g_{K-1} \quad \text{if K is odd}$$

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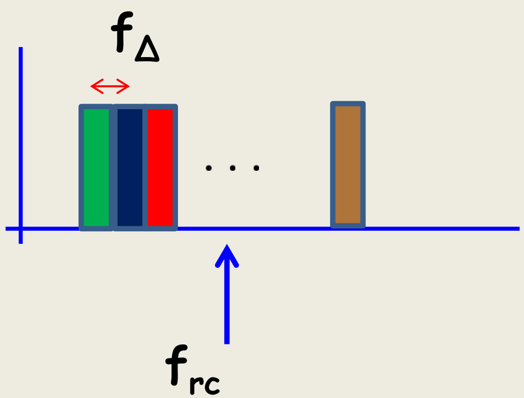

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Lecture 7: OFDM

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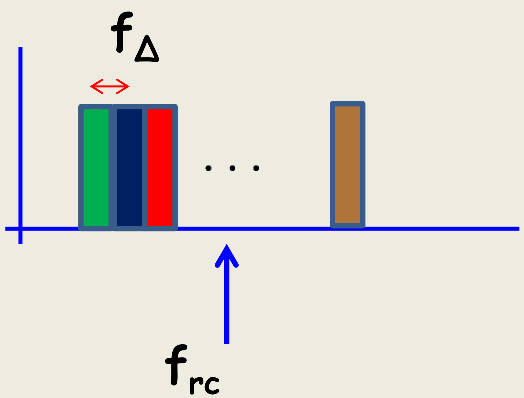

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Important: $f_\Delta T_{\text{obs}} > 1$

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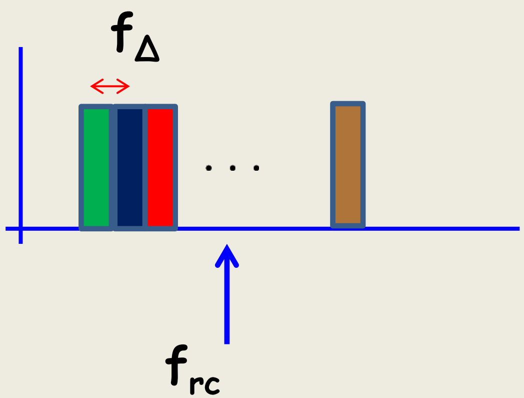

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To synthesize the signal, we

1. Sample the OFDM signal
2. Check how we can efficiently construct those samples
3. Perform D/A conversion

Lecture 7: OFDM

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Lecture 7: OFDM

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$$f_{samp} = N/T_{obs} \; = N f_\Delta$$

Lecture 7: OFDM

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The above gives a formula for how to compute the samples of the OFDM signal

Lecture 7: OFDM

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Let us now compute the Fourier transform of the samples (as of now, for no particular reason)

$$X(\nu) = \sum_{n=0}^{N-1} x_n \exp\left(-i2\pi\nu n\right)$$

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$$X_m = X(m/N) = \sum_{n=0}^{N-1} x_n \exp\left(-\frac{i2\pi mn}{N}\right)$$

Lecture 7: OFDM

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DFT

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

IDFT

Lecture 7: OFDM

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IDFT is <u>VERY</u> fast.
We can get x_n FAST if we know X_m

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(as of now, for no particular reason)

$$X_m = \sum_{n=0}^{N-1} x_n \exp\left(-\frac{i2\pi mn}{N}\right)$$

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IDFT

Lecture 7: OFDM

Logics

We have $\{a_n\}$

We need $\{x_n\}$

We know that $$x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

And that this can be computed FAST

Lecture 7: OFDM

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And that this can be computed FAST

Conclusion: We need to link $\{a_n\}$ and $\{X_m\}$

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi(g_0 + k)n}{N}\right)$$

Lecture 7: OFDM

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$$g_0 < 0$$

$$= \sum_{k=0}^{-g_0-1} a_k \exp\left(\frac{i2\pi(g_0 + k)n}{N}\right) + \sum_{k=-g_0}^{K-1} a_k \exp\left(\frac{i2\pi(g_0 + k)n}{N}\right)$$

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Variable substitutions

$$[m = g_0 + k + N] \qquad [m = g_0 + k]$$

Lecture 7: OFDM

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$$= \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

This we know from the IDFT

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi (g_0 + k) n}{N}\right)$$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Thus,

$$\boxed{X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}}$$

Lecture 7: OFDM

$$x_n = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi g_k n}{N}\right) = \sum_{k=0}^{K-1} a_k \exp\left(\frac{i2\pi (g_0 + k) n}{N}\right)$$

$$= \sum_{m=g_0+N}^{N-1} a_{m-(g_0+N)} \exp\left(\frac{i2\pi mn}{N}\right) + \sum_{m=0}^{g_{K-1}} a_{m-g_0} \exp\left(\frac{i2\pi mn}{N}\right)$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Thus,

$$\boxed{\begin{aligned} X_m &= N a_{m-g_0}, \quad 0 \le m \le g_{K-1} \\ X_m &= N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N-1 \end{aligned}}$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$

$$X_m = Na_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = Na_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$

2. Select a sampling rate, by choosing $N \geq K$

$$X_m = Na_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = Na_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$

2. Select a sampling rate, by choosing $N \geq K$

3. Find N values $\{X_m\}$ according to the box below

$$\boxed{\begin{aligned} X_m &= N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1} \\ X_m &= 0, \quad g_{K-1}+1 \leq m \leq g_0 + N - 1 \\ X_m &= N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N-1 \end{aligned}}$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IDFT

$$\boxed{x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)}$$

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IDFT

$$x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

Speed: N^2 multiplications

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Altogether, FFT = "Fast Fourier transform"

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N=2^L \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IFFT

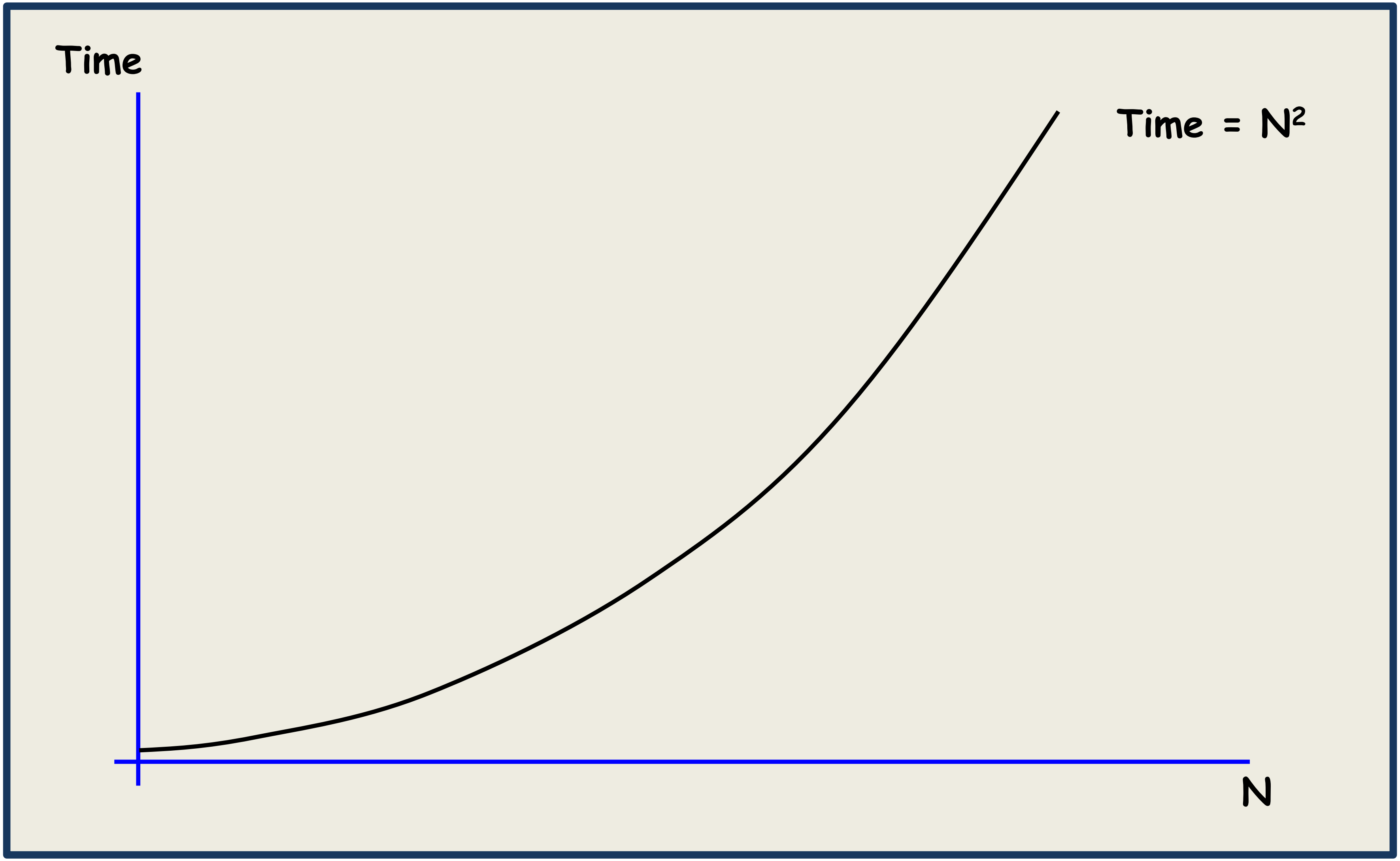
$$\boxed{x_n = \frac{1}{N}\sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)}$$

Speed: N^2 multiplications

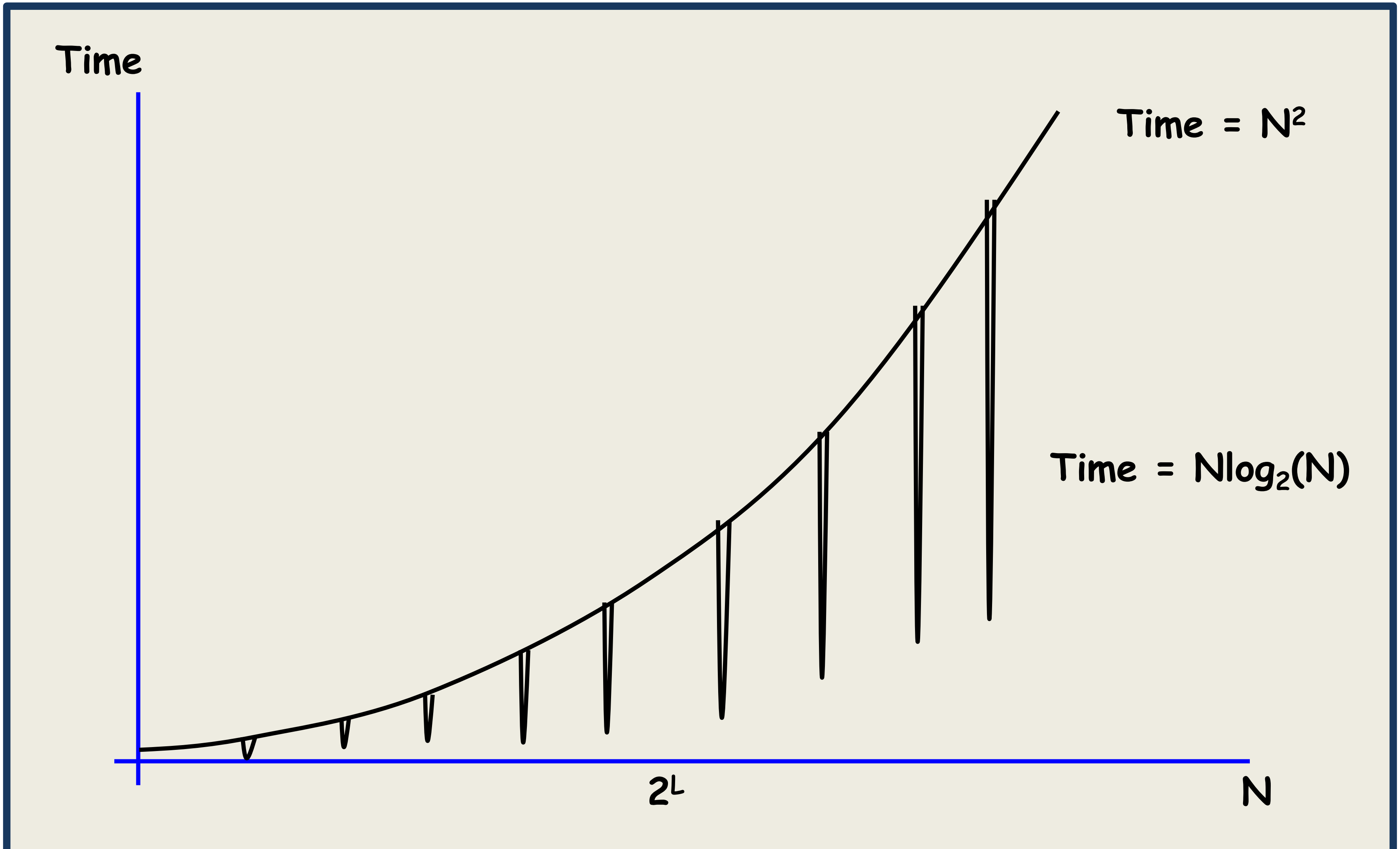
Speed: $N\log_2 N$

$$\boxed{\begin{aligned} X_m &= N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1} \\ X_m &= 0, \quad g_{K-1}+1 \leq m \leq g_0+N-1 \\ X_m &= N a_{m-(g_0+N)}, \quad g_0+N \leq m \leq N-1 \end{aligned}}$$

Lecture 7: OFDM



Lecture 7: OFDM



Lecture 7: OFDM

Left to do

1.

2. D/A conversion

3. Modulation to band-pass

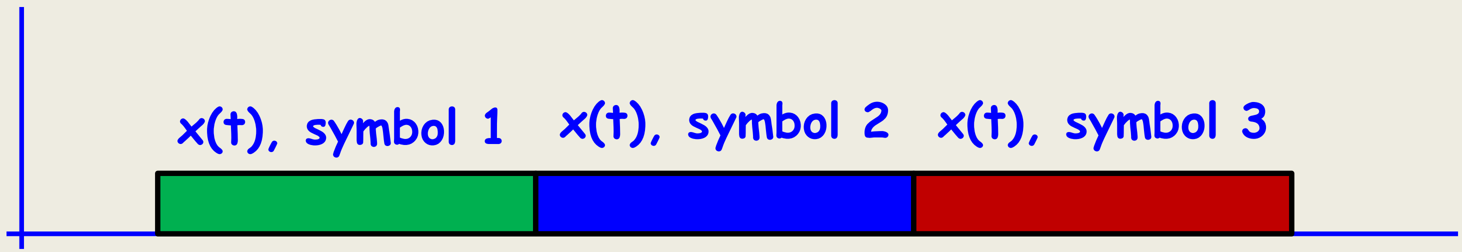
4. Recevier

Lecture 7: OFDM

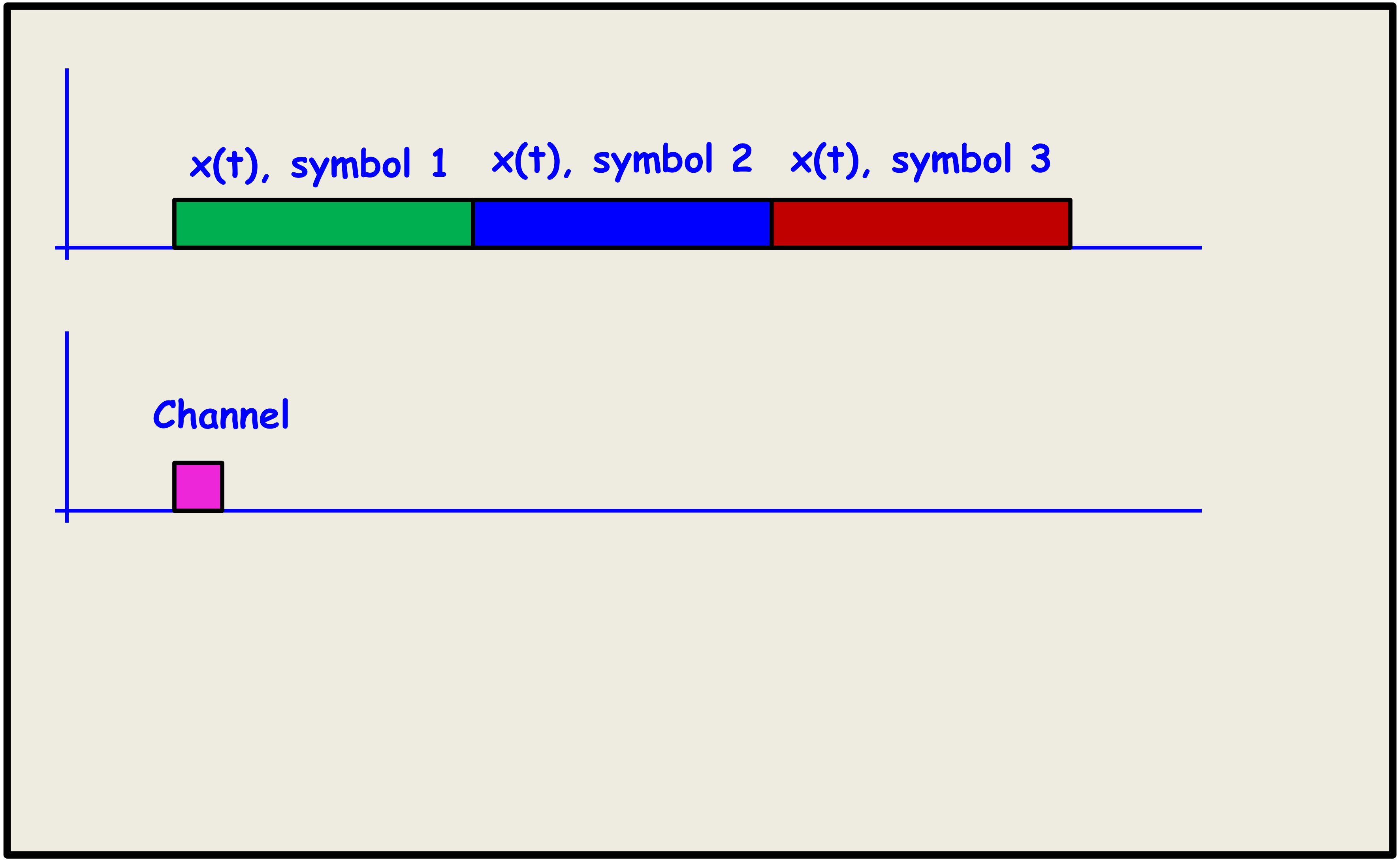
Left to do

1. Add a cyclic prefix to deal with channel effects
2. D/A conversion
3. Modulation to band-pass
4. Recevier

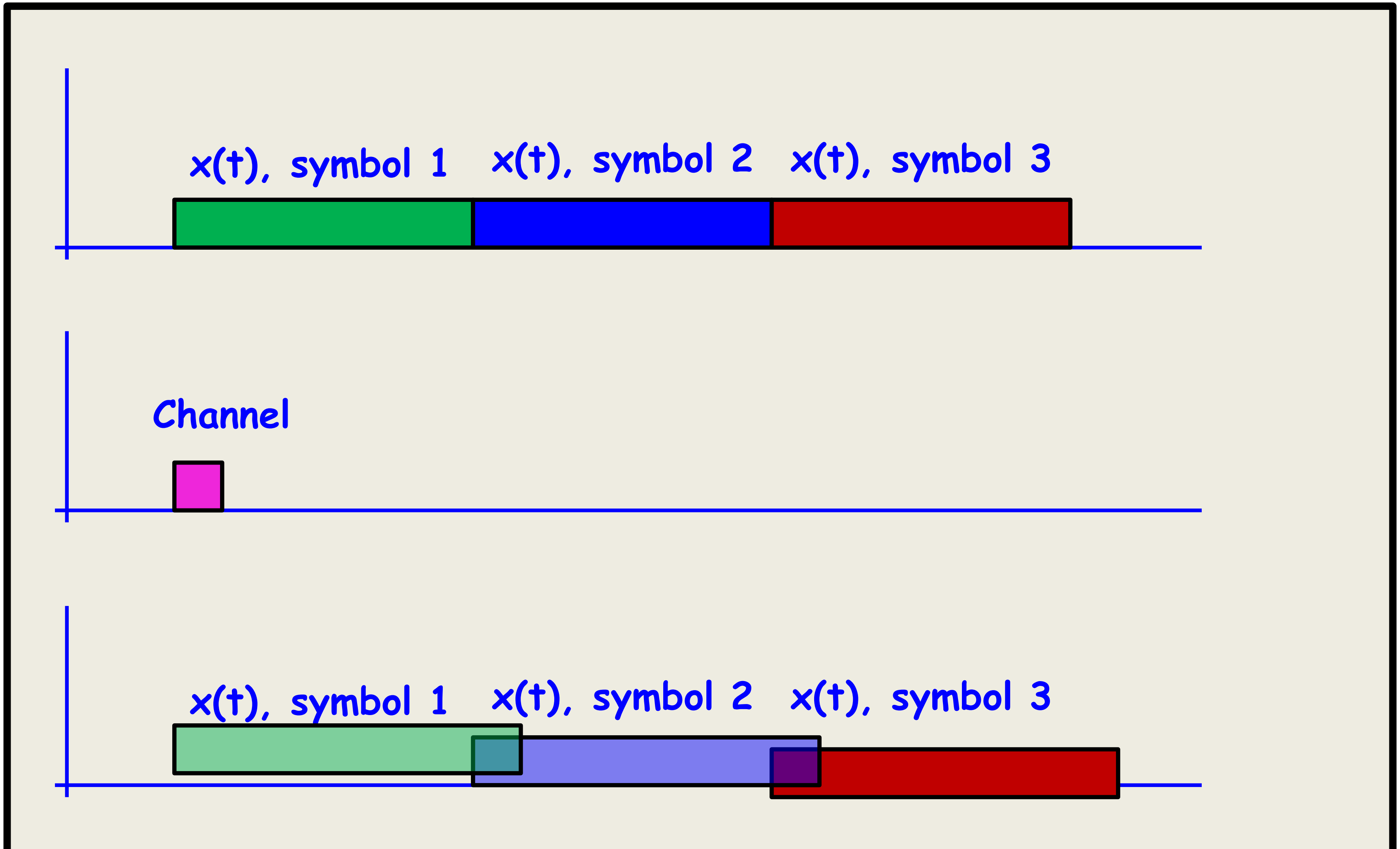
Lecture 7: OFDM



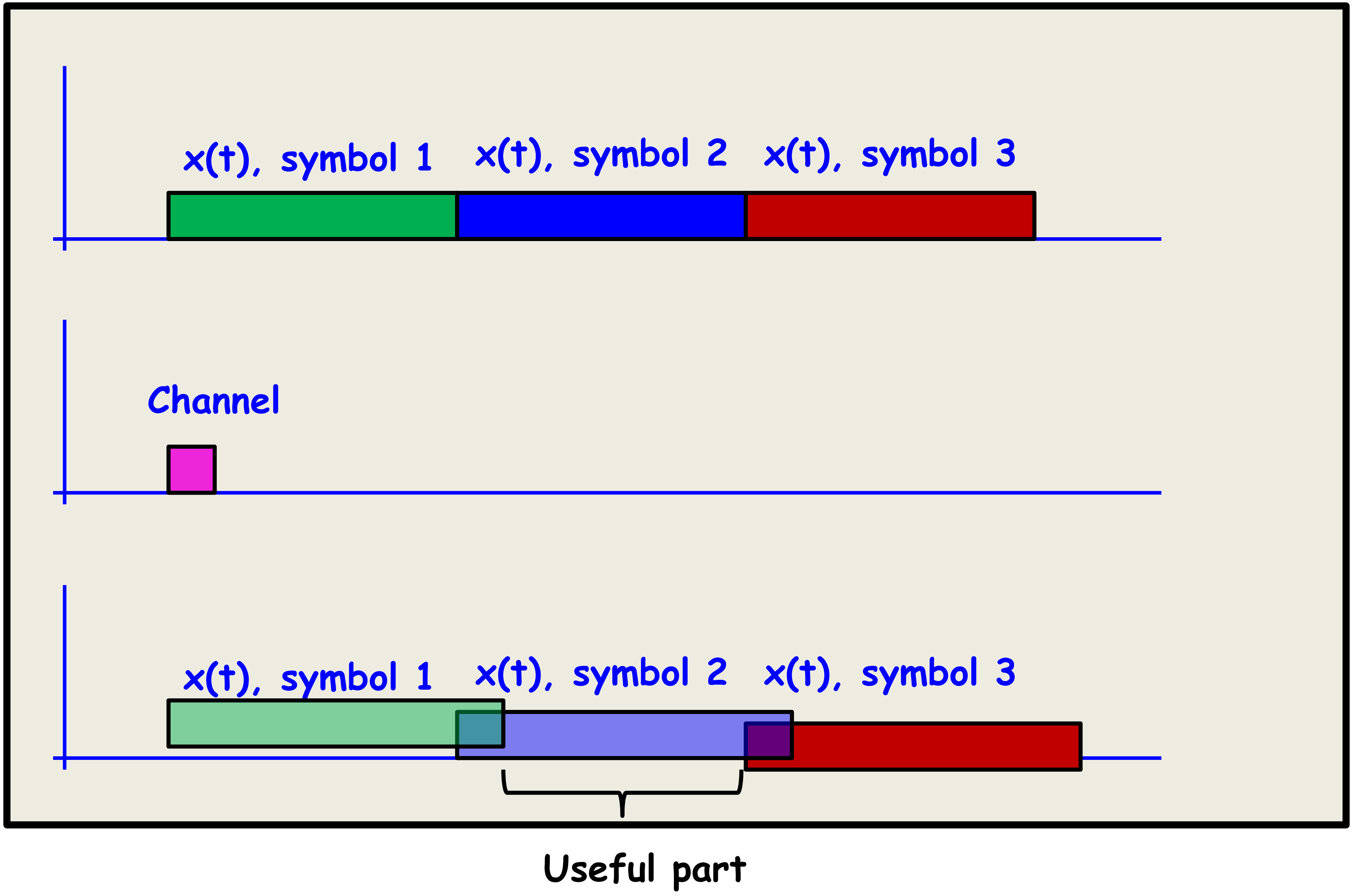
Lecture 7: OFDM



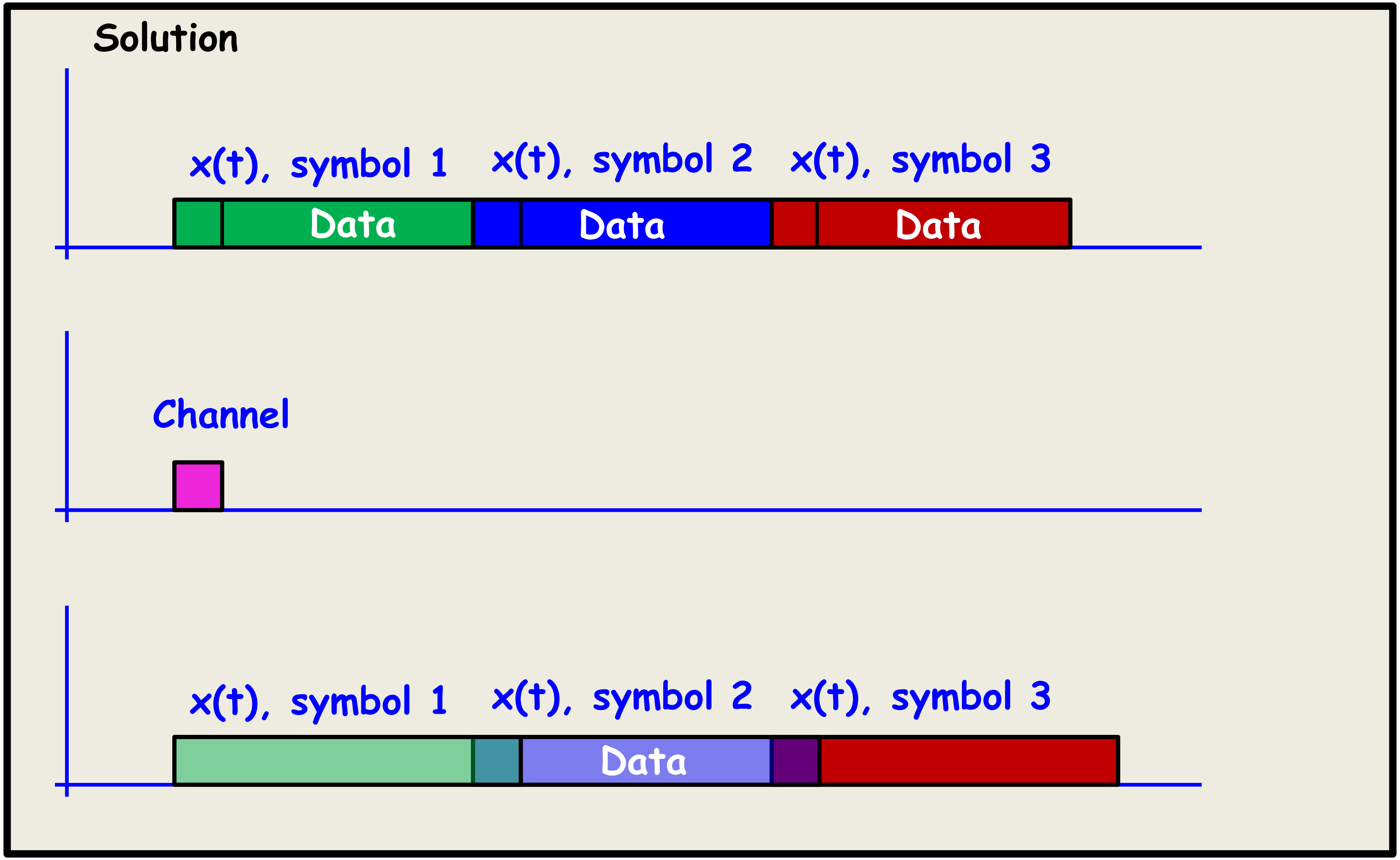
Lecture 7: OFDM



Lecture 7: OFDM

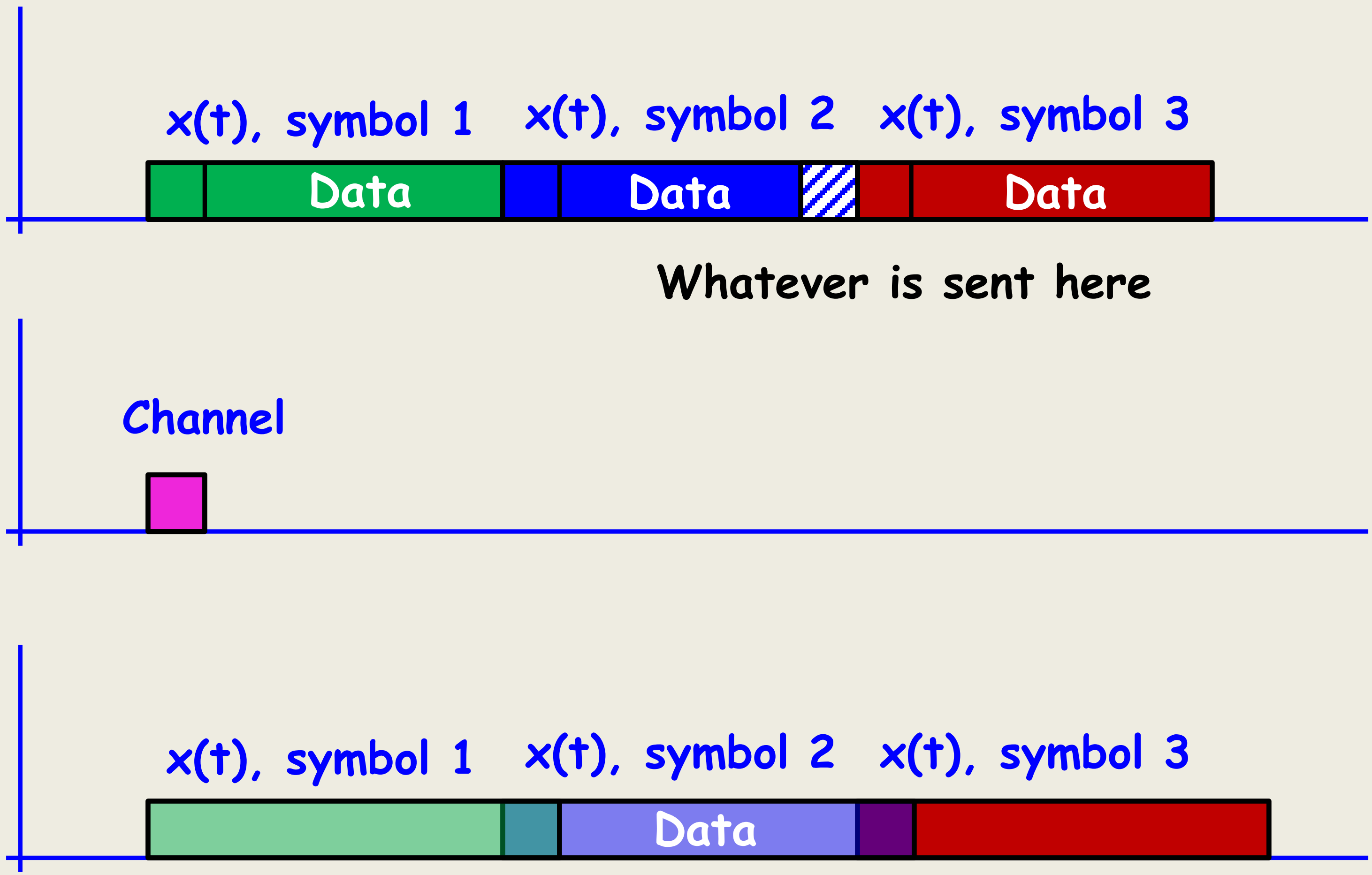


Lecture 7: OFDM

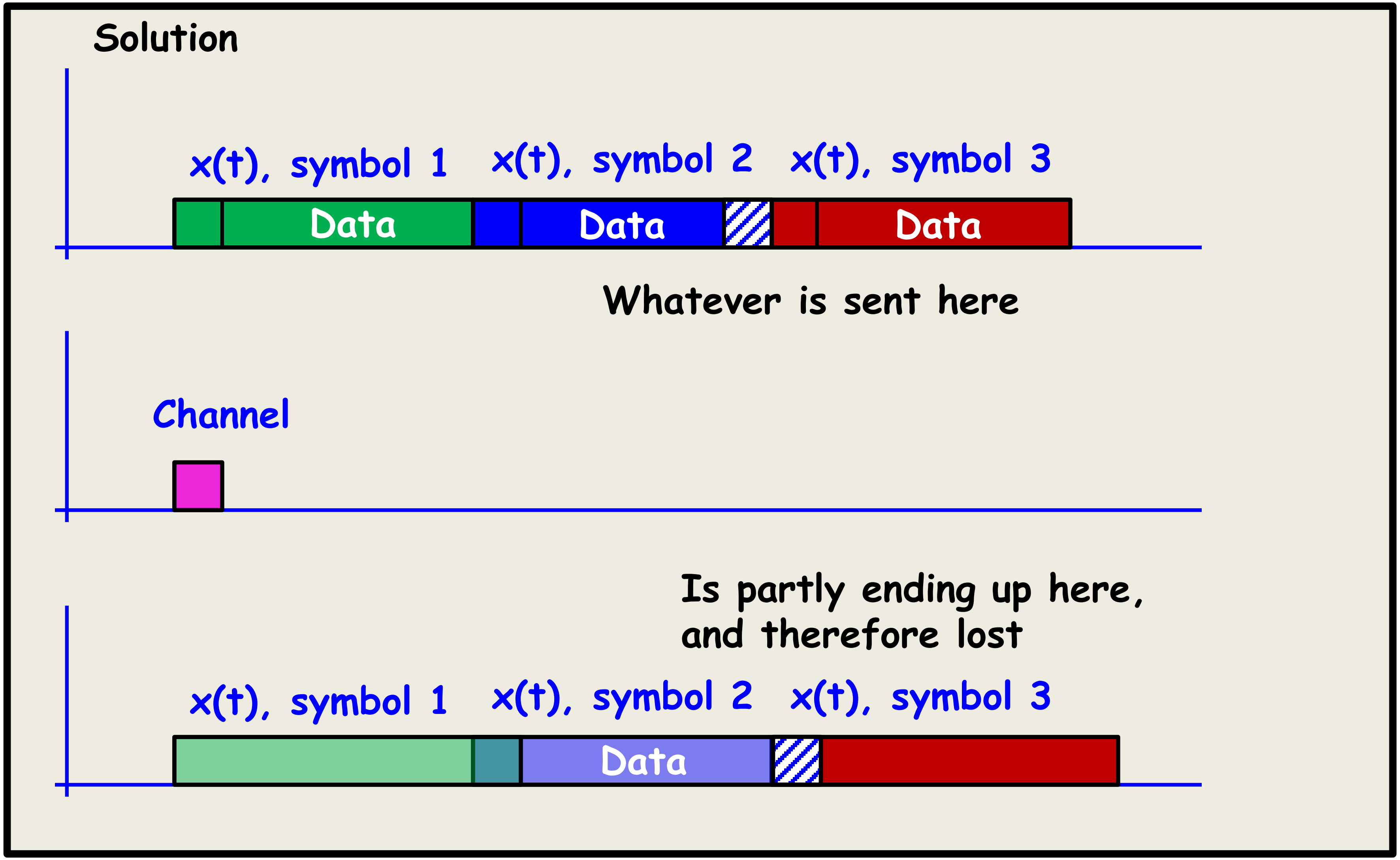


Lecture 7: OFDM

Solution

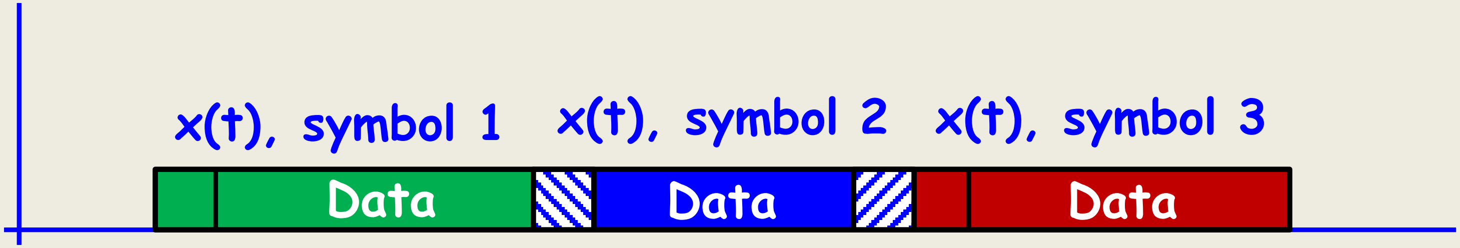


Lecture 7: OFDM

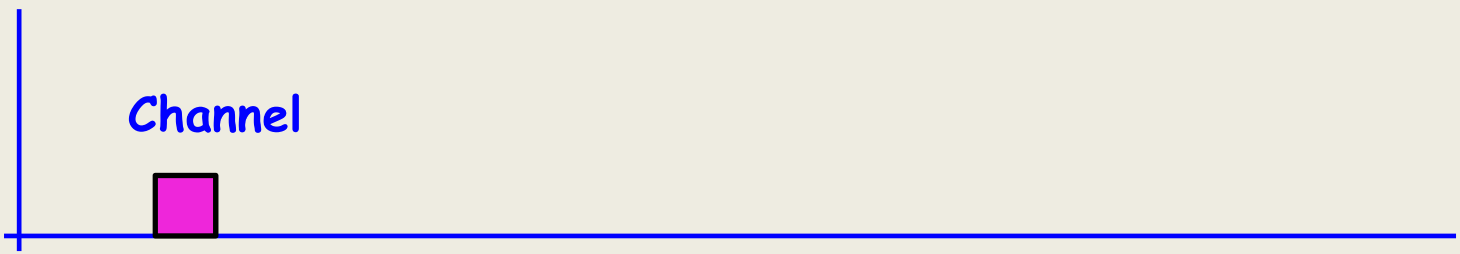


Lecture 7: OFDM

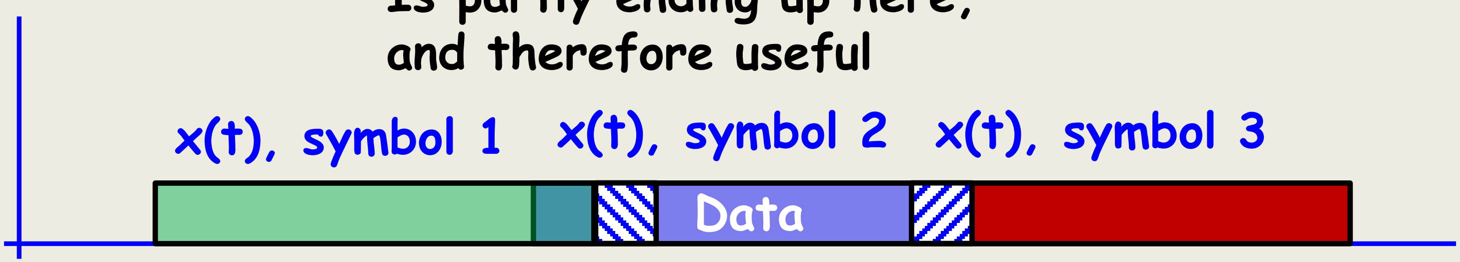
Solution



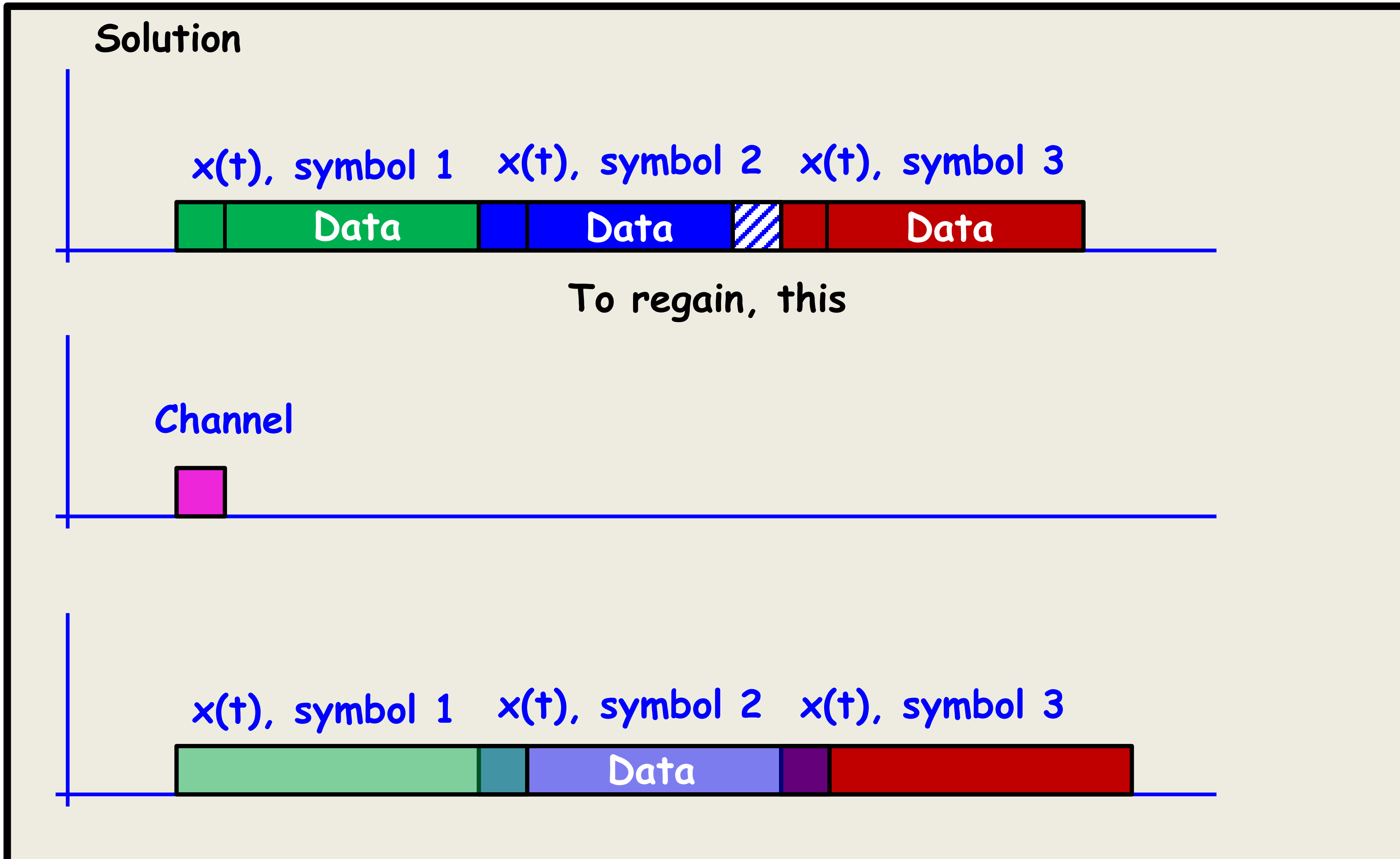
However, whatever is sent here



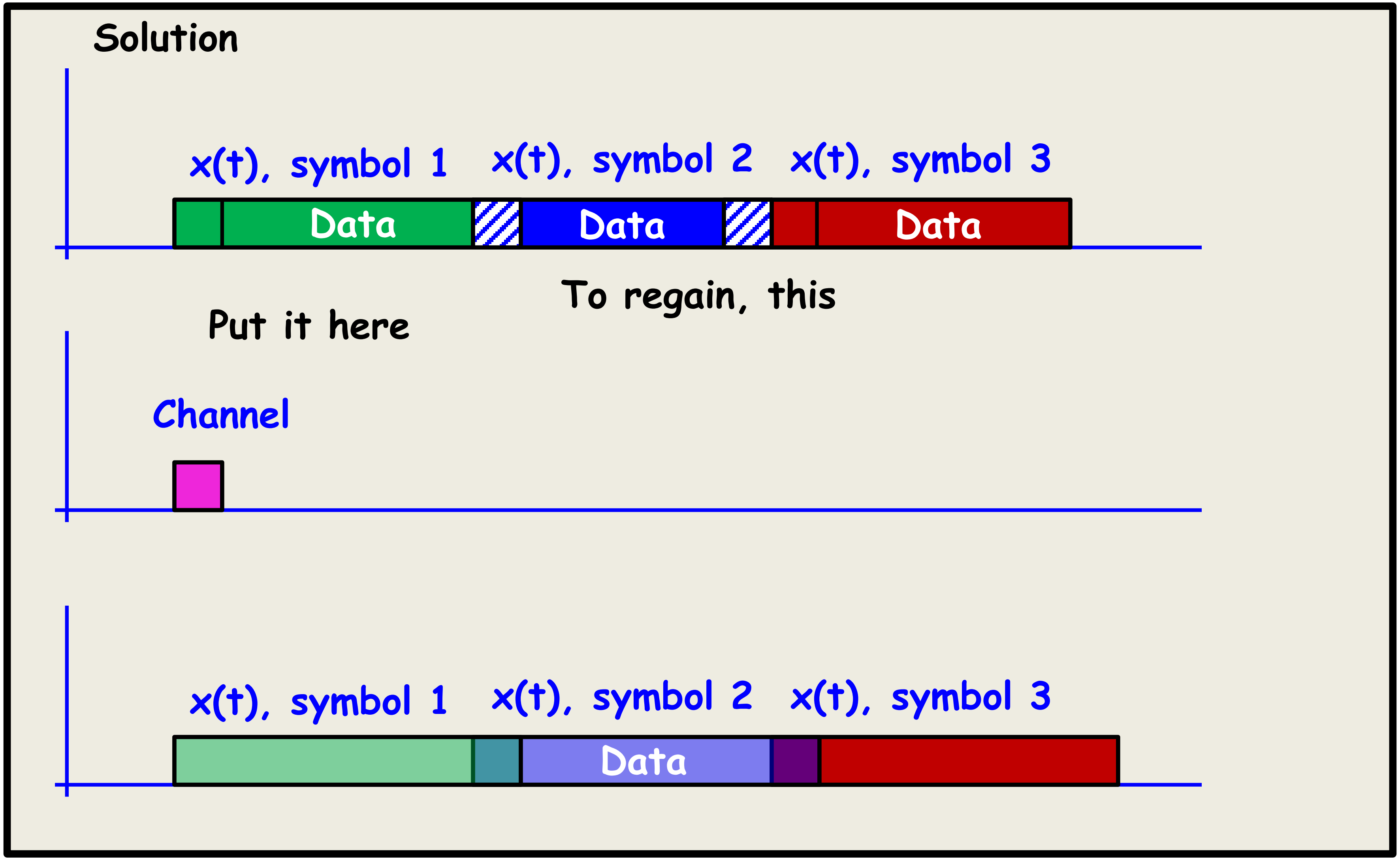
Is partly ending up here,
and therefore useful



Lecture 7: OFDM

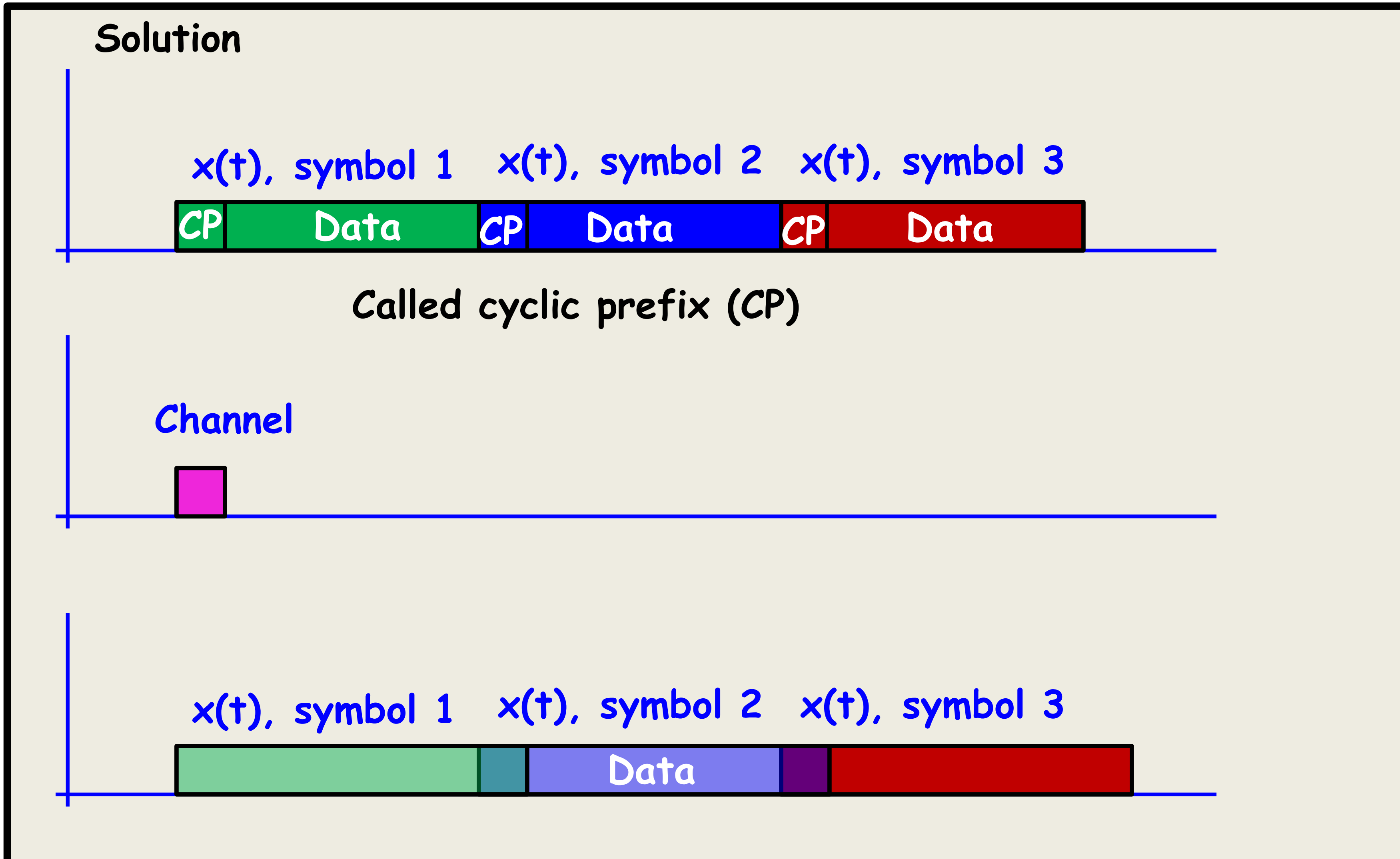


Lecture 7: OFDM



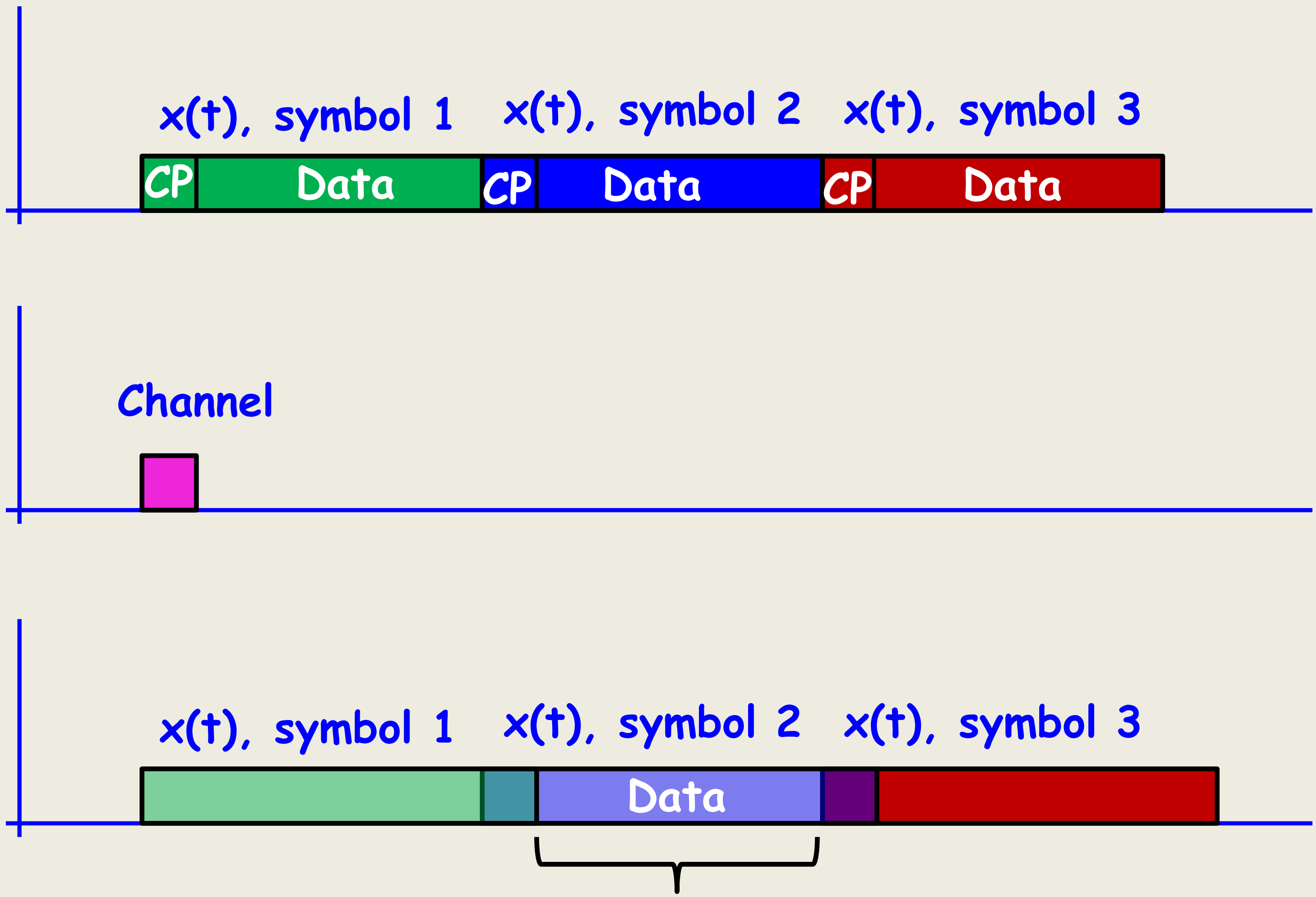
Useful part

Lecture 7: OFDM



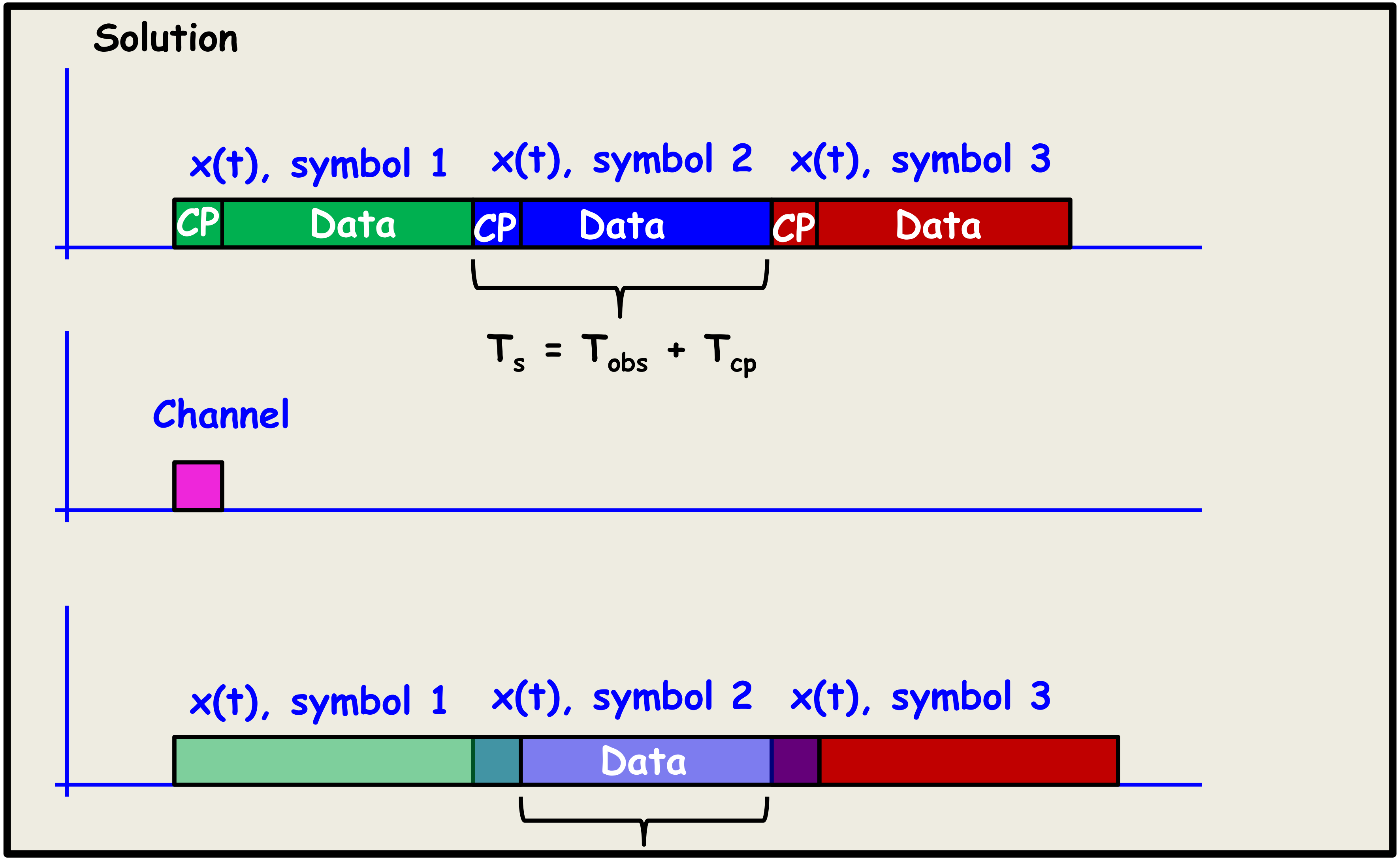
Lecture 7: OFDM

Solution



Useful part, should be length T_{obs}

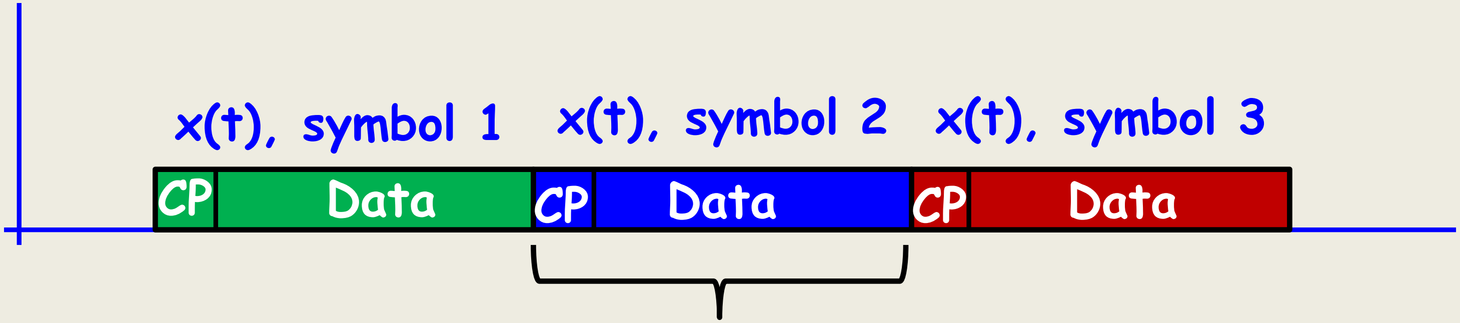
Lecture 7: OFDM



Useful part, should be length T_{obs}

Lecture 7: OFDM

Solution

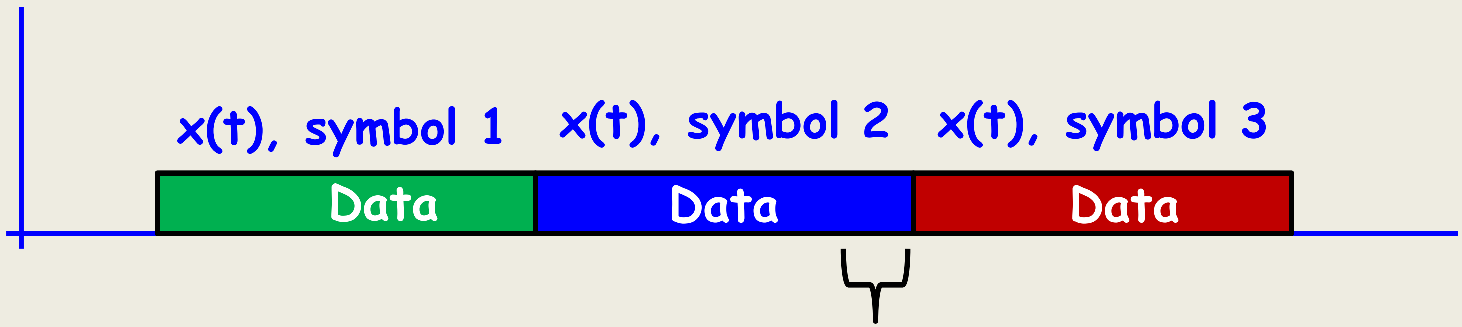


$$T_s = T_{obs} + T_{cp}$$

Summary:

Lecture 7: OFDM

Solution

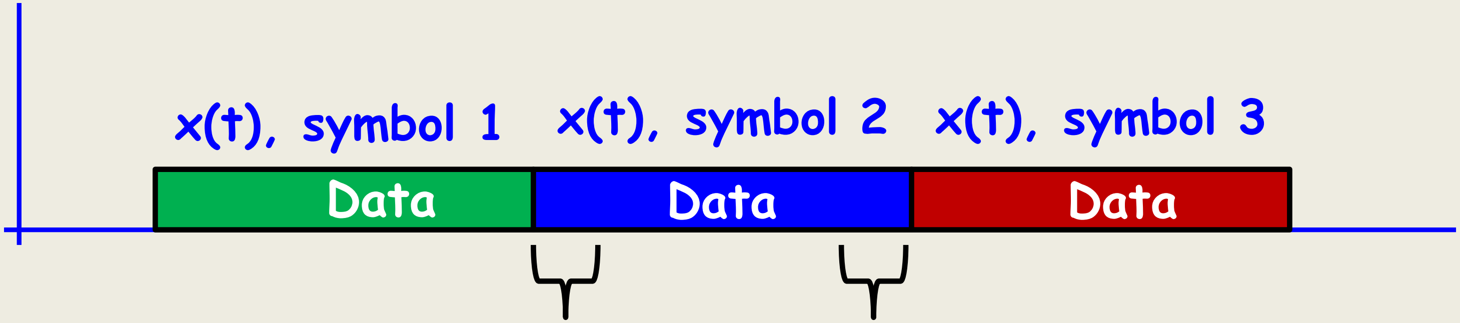


Summary:

1. Marked region is partly lost since it interferes with next block

Lecture 7: OFDM

Solution

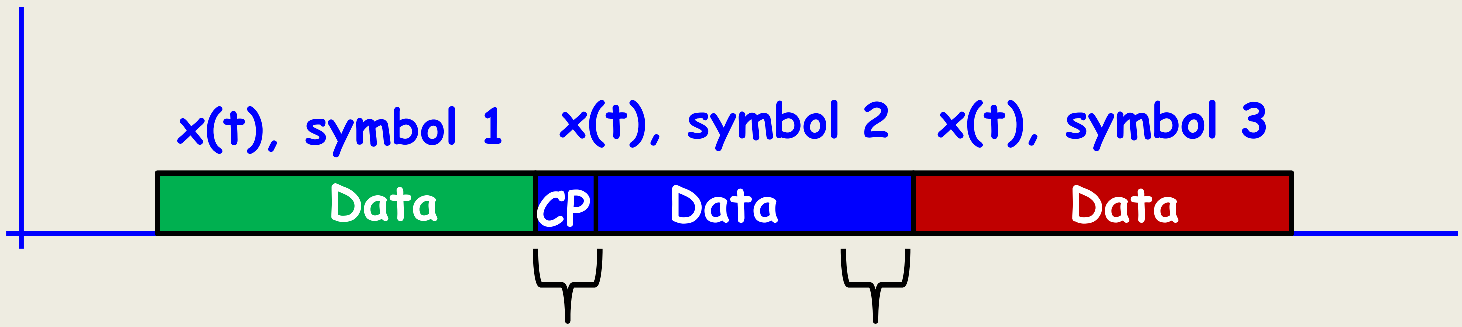


Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it

Lecture 7: OFDM

Solution

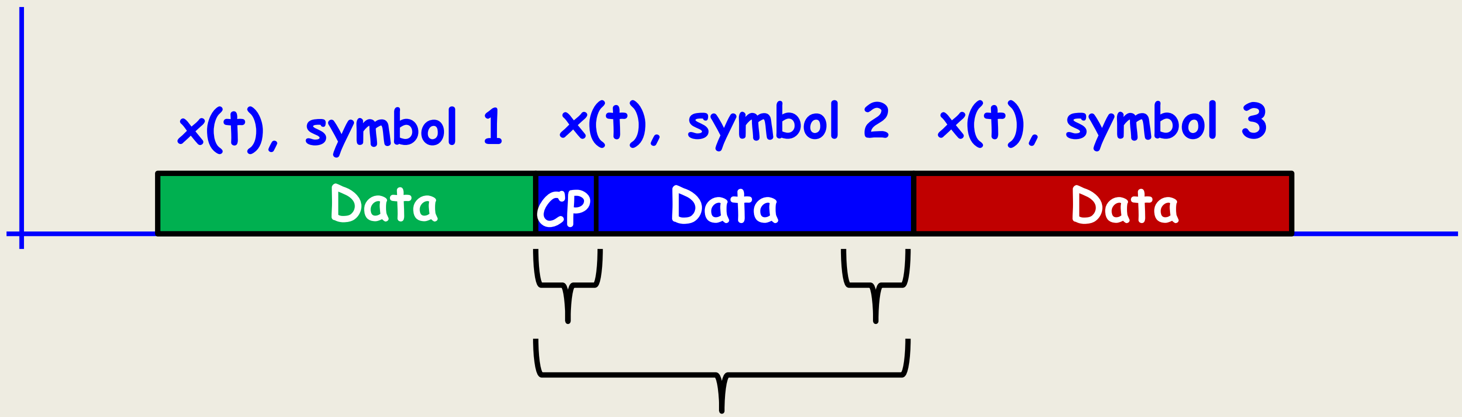


Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP

Lecture 7: OFDM

Solution



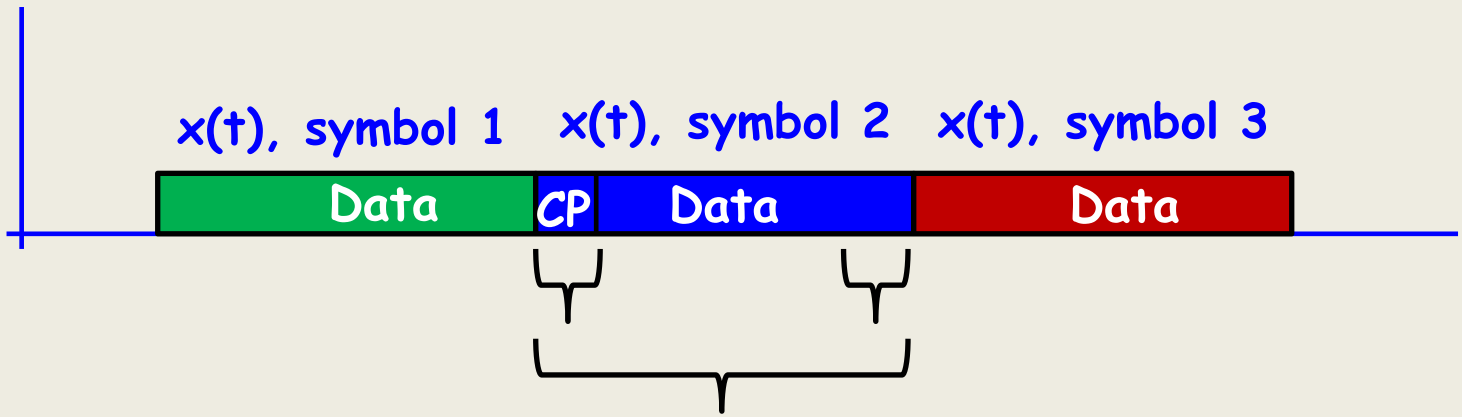
$T_s = T_{obs} + T_{cp}$

Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP
4. Length of data block should be $T_{obs}=1/f_\Delta$

Lecture 7: OFDM

Solution



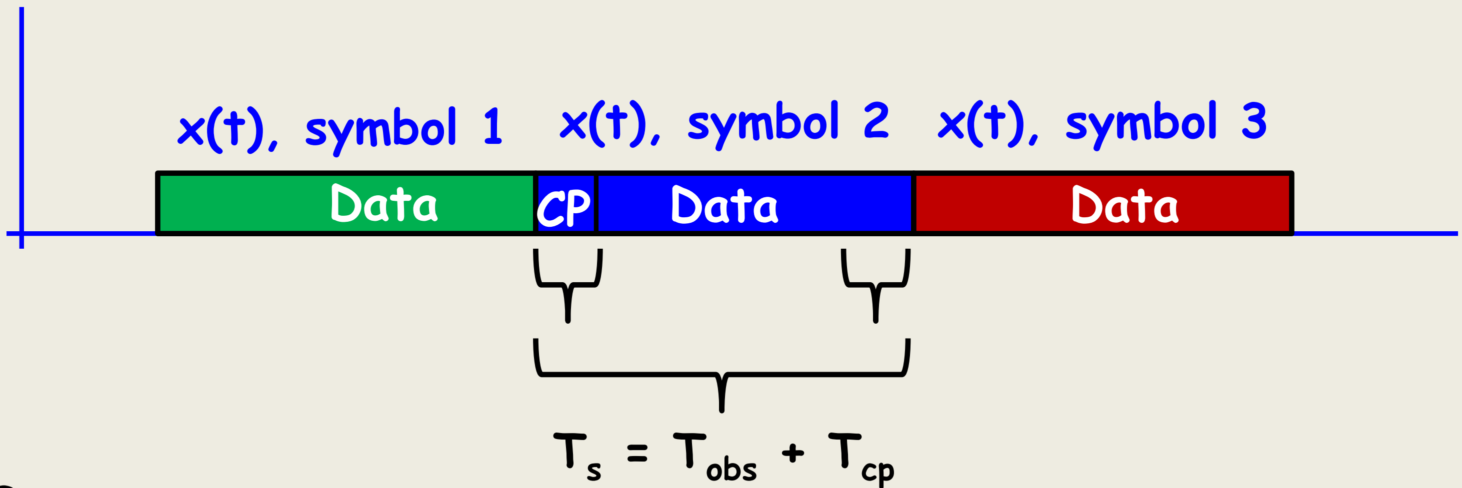
$T_s = T_{obs} + T_{cp}$

Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP
4. Length of data block should be $T_{obs}=1/f_\Delta$
5. No proof, yet, why this should be good, but it is plausible, since the lost part is present in the beginning

Lecture 7: OFDM

Solution

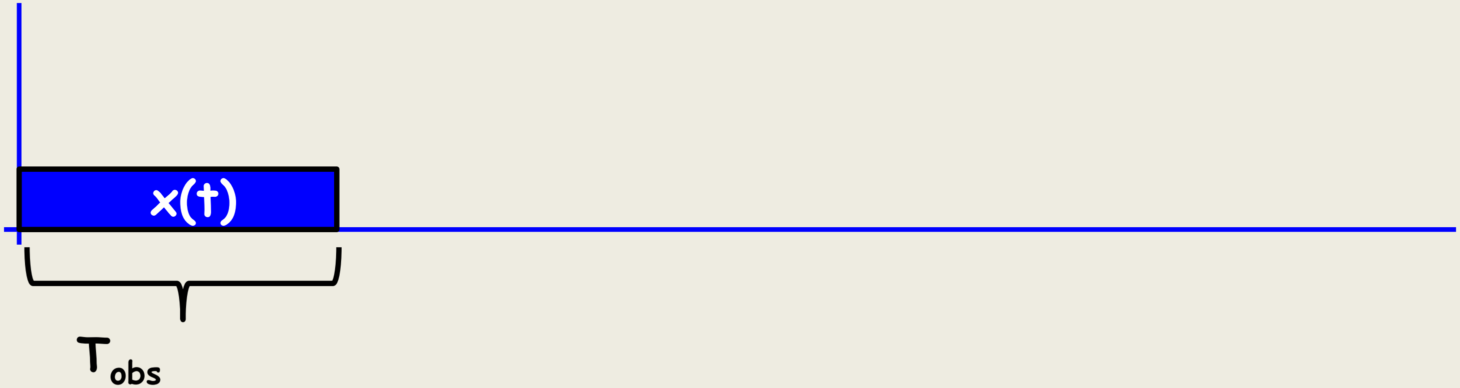


Summary:

1. Marked region is partly lost since it interferes with next block
2. First part is partly lost, since previous block interferes with it
3. Put the last part in the first part, call it CP
4. Length of data block should be $T_{obs}=1/f_\Delta$
5. No proof, yet, why this should be good, but it is plausible, since the lost part is present in the beginning
6. Spectral efficiency loss

Lecture 7: OFDM

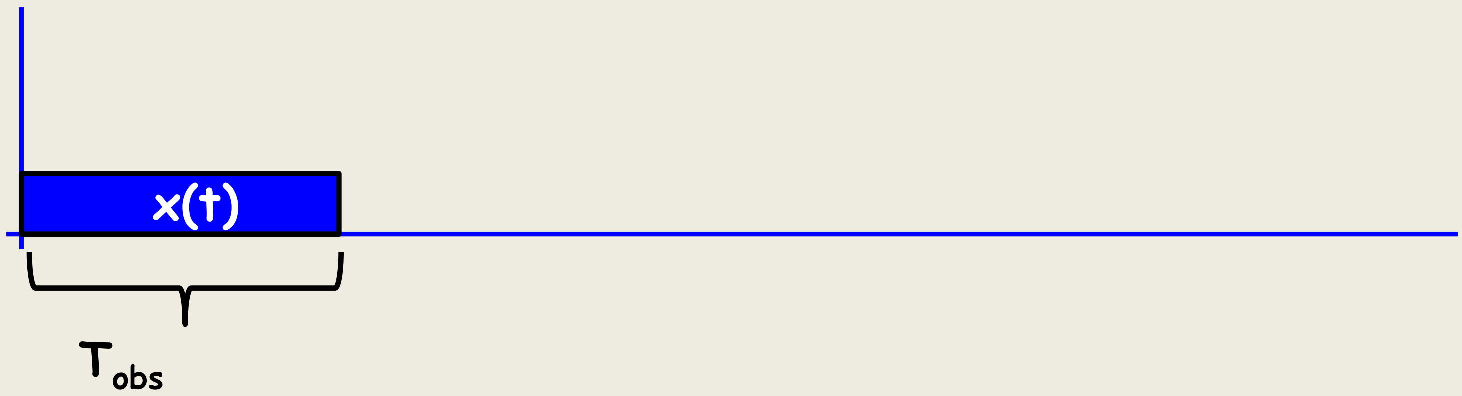
Formulas



$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t), \quad 0 \le t \le T_{obs}$$

Lecture 7: OFDM

Formulas

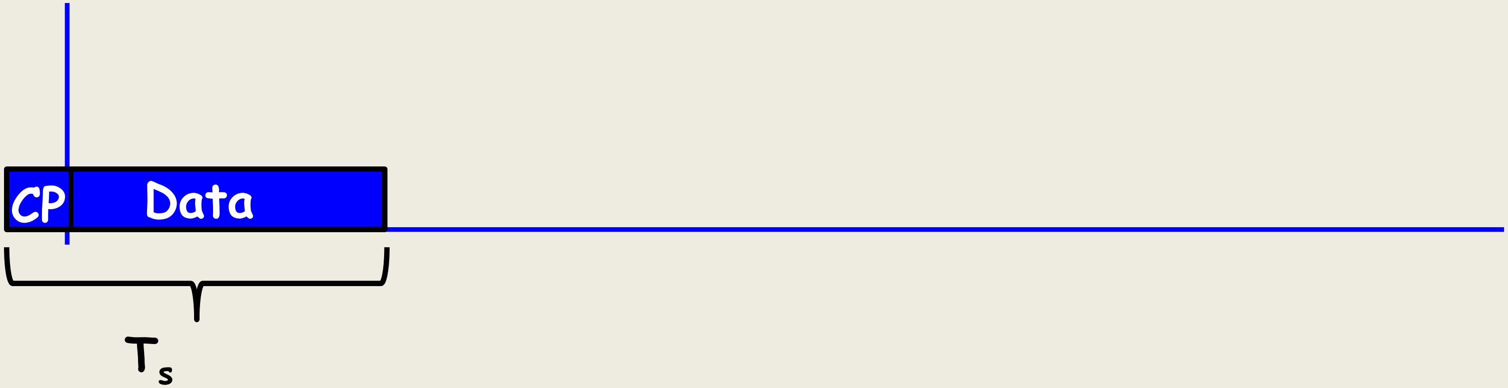


$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t)$$

Periodic with period $T_{obs} = 1/f_\Delta$

Lecture 7: OFDM

Formulas

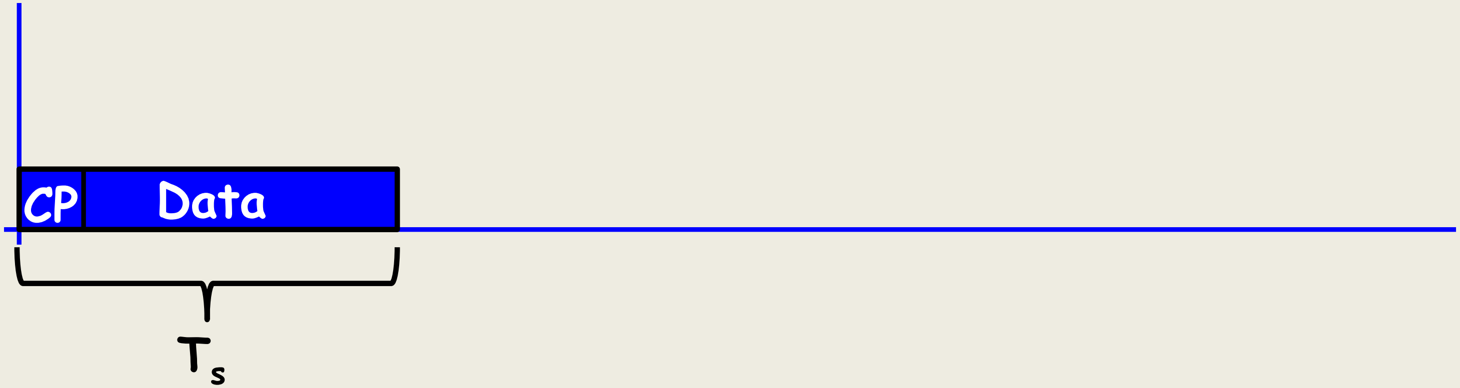


$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta t), \quad -T_{cp} \leq t \leq T_{obs}$$

Has properties that we want: first part (left of 0) equals last part

Lecture 7: OFDM

Formulas



$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp}), \quad 0 \le t \le T_s$$

Make causal

Lecture 7: OFDM

Altogether,

1. Take a block of K data symbols $\{a_k\}$
2. Select a sampling rate, by choosing $N \geq K$
3. Find N values $\{X_m\}$ according to the box below
4. Compute $\{x_n\}$ using an IDFT
5. Add last L samples to the beginning

$$x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left(\frac{i2\pi mn}{N}\right)$$

$$X_m = N a_{m-g_0}, \quad 0 \leq m \leq g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \leq m \leq g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \leq m \leq N - 1$$

Lecture 7: OFDM

Left to do

1. Add a cyclic prefix to deal with channel effects
2. D/A conversion
3. Modulation to band-pass
4. Receiver

Explanation of 1
comes in 4

Warning: numbering of steps is not the same as in the compendium

Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp})), \quad 0 \le t \le T_s$$

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp})), \quad 0 \le t \le T_s$$

$$u_n = \mathrm{IFFT}(\mathbf{X})$$

$$X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

$$\mathbf{x} = [\underbrace{u_{N-L}\ u_{N-L+1}\ \ldots u_{N-1}}_{CP}\ u_0\ \ldots u_{N-1}]$$

Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{k=0}^{K-1} a_k \exp(i2\pi g_k f_\Delta (t - T_{cp})), \quad 0 \le t \le T_s$$

$$u_n = \mathrm{IFFT}(\mathbf{X})$$

D/A

$$X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

$$\mathbf{x} = [\underbrace{u_{N-L}\; u_{N-L+1}\; \ldots u_{N-1}}_{\text{CP}}\; u_0\; \ldots u_{N-1}]$$

Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$u_n = \text{IFFT}(\mathbf{X})$$

D/A

$$X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

$$\mathbf{x} = [\underbrace{u_{N-L}\ u_{N-L+1}\ \ldots u_{N-1}}_{\text{CP}}\ u_0\ \ldots u_{N-1}]$$

Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$\boxed{g_{ideal}(t) = \frac{\sin(\pi f_{samp}t)}{\pi f_{samp}t}}$$

$$u_n = \text{IFFT}(\mathbf{X})$$

$$X_m = N a_{m-g_0}, \quad 0 \le m \le g_{K-1}$$

$$X_m = 0, \quad g_{K-1} + 1 \le m \le g_0 + N - 1$$

$$X_m = N a_{m-(g_0+N)}, \quad g_0 + N \le m \le N - 1$$

$$\mathbf{x} = [\underbrace{u_{N-L}\ u_{N-L+1}\ \ldots u_{N-1}}_{CP}\ u_0\ \ldots u_{N-1}]$$

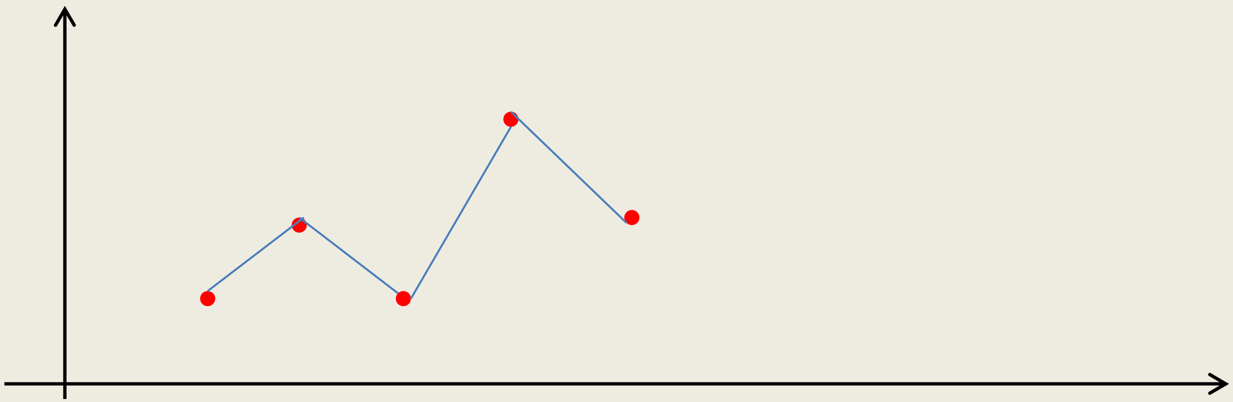
Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})$$

$$g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}$$

Alternative (suboptimal)



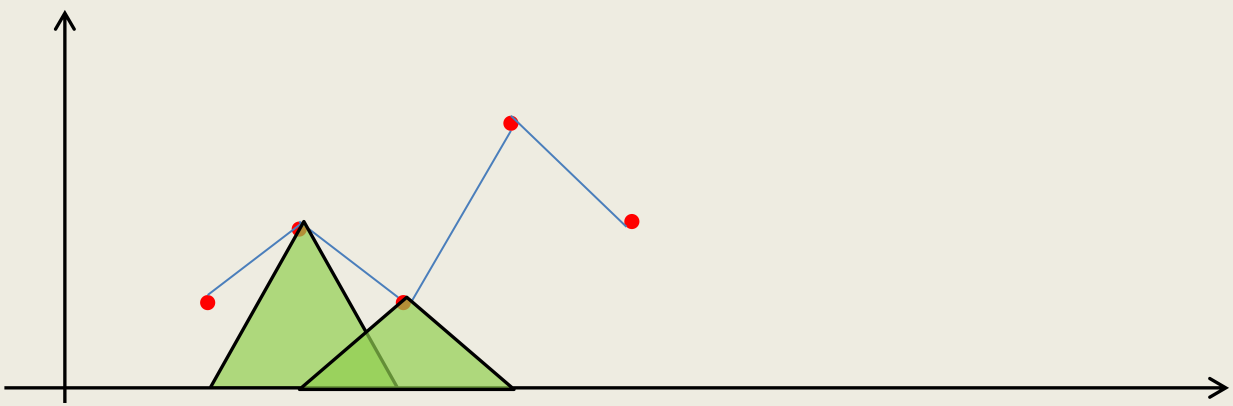
Lecture 7: OFDM

$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$\boxed{g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}}$$

Alternative (suboptimal)



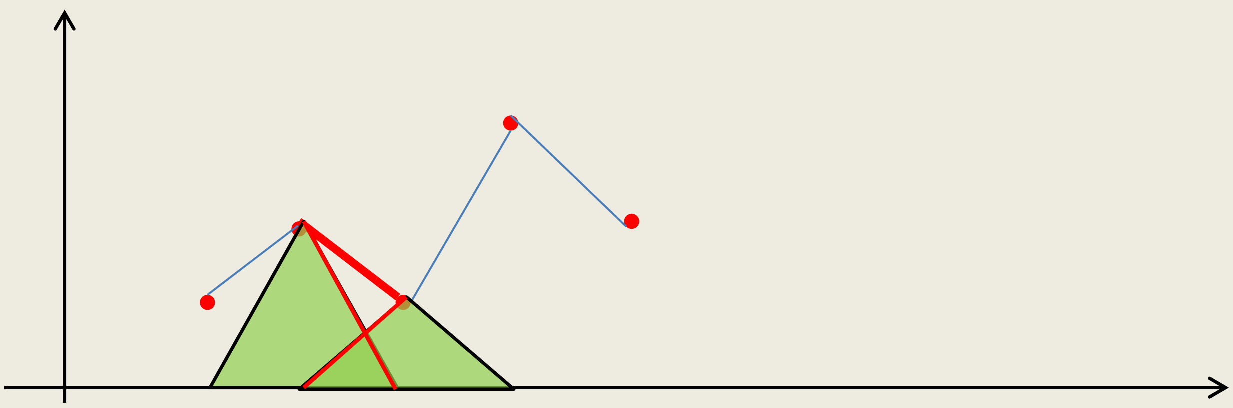
Lecture 7: OFDM

$$\text{OFDM signal} = \mathrm{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$$

$$\boxed{x(t) = \sum_{\ell} x_\ell g(t - \ell/f_{samp})}$$

$$\boxed{g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}}$$

Alternative (suboptimal)



Lecture 7: OFDM

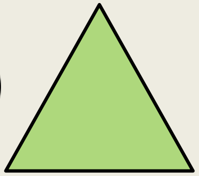
$$\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc} t)\}$$

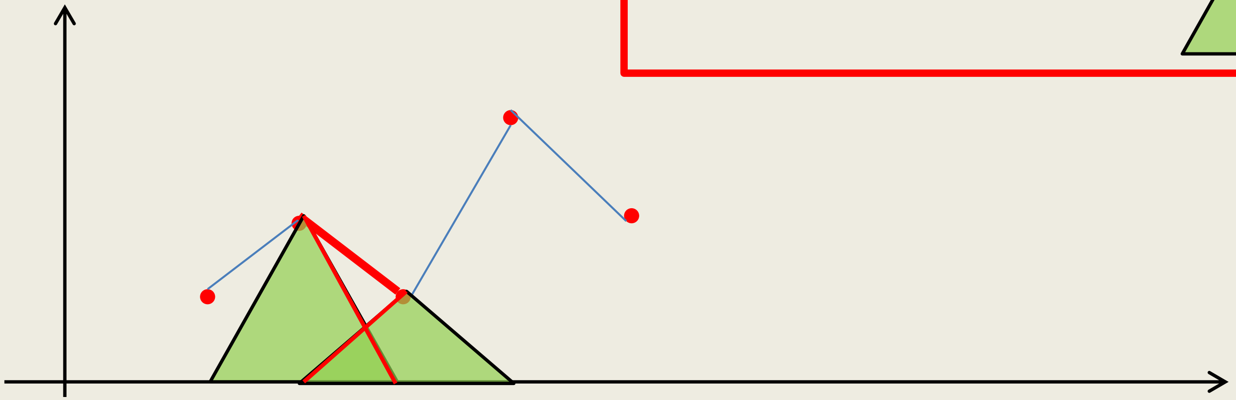
$$x(t) = \sum_{\ell} x_{\ell} g(t - \ell / f_{samp})$$

$$g_{ideal}(t) = \frac{\sin(\pi f_{samp} t)}{\pi f_{samp} t}$$

Alternative (suboptimal)

$$g_{simple}(t) = \text{tri}(t)$$





Lecture 7: OFDM

Left to do

1. Add a cyclic prefix to deal with channel effects
2. D/A conversion
3. Modulation to band-pass
4. Receiver

Explanation of 1
comes in 4

Warning: numbering of steps is not the same as in the compendium

Lecture 7: OFDM

Left to do

1. Add a cyclic prefix to deal with channel effects

2. D/A conversion

3. Modulation to band-pass $\quad \text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$

4. Receiver

Very simple

Warning: numbering of steps is not the same as in the compendium

Lecture 7: OFDM

Left to do

1. Add a cyclic prefix to deal with channel effects

2. D/A conversion

3. Modulation to band-pass $\text{OFDM signal} = \text{Re}\{x(t)\exp(i2\pi f_{rc}t)\}$

4. Receiver Next lecture

Warning: numbering of steps is not the same as in the compendium