Lecture 6: Channel model (Ch.3) & coding (Ch8)

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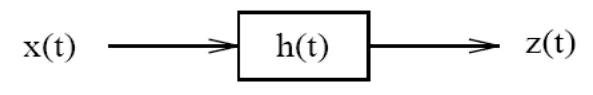

Figure 3.11: Bandpass filtering.

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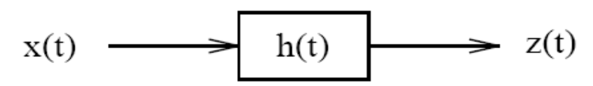

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Important

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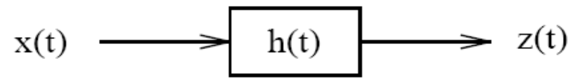

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General model for bandpass x(t):

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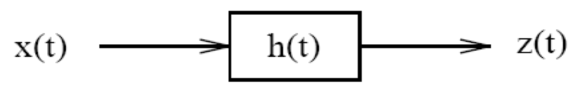

Figure 3.11: Bandpass filtering.

General model for bandpass x(t): $x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t)$

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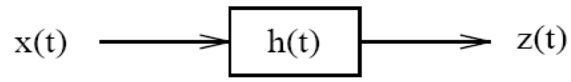

Figure 3.11: Bandpass filtering.

General model for bandpass x(t): $$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t)$$

$$= \mathrm{Re}\{\tilde{x}(t)\exp(i\omega_c t)\}$$

Where $\tilde{x}(t) = x_I(t) + i x_Q(t)$

Lecture 6: Channel model (Ch.3) & coding (Ch8)

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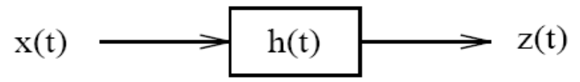

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"Complex baseband representation"

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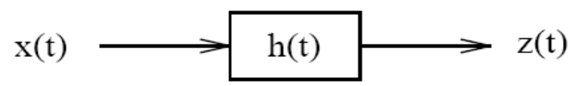

Figure 3.11: Bandpass filtering.

General model for bandpass x(t): $x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t)$

Easily verified. Do at home $= \mathrm{Re}\{\tilde{x}(t)\exp(i\omega_c t)\}$

Where $\tilde{x}(t) = x_I(t) + i x_Q(t)$

"Complex baseband representation"

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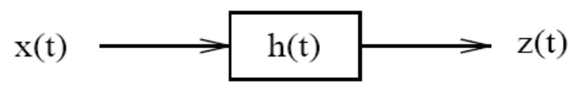

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Can be seen as "book-keeping". Allows to use 1 signal instead of 2 when doing math

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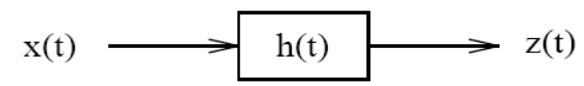

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Can be seen as "book-keeping". Allows to use 1 signal instead of 2 when doing math

…or when speaking: "… assume now that the signal $\tilde{x}(t)$ is sent…"

"… assume now that the signals $x_I(t)$ and $x_Q(t)$ are sent…"

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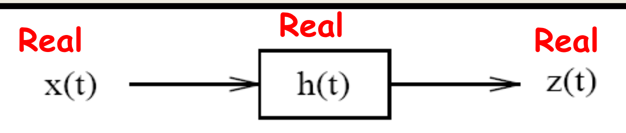

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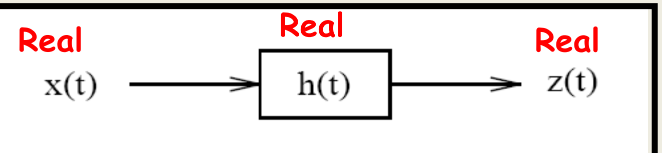

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$$z(t) = x(t) \star h(t)$$

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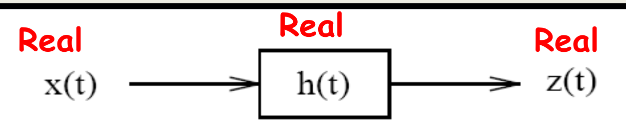

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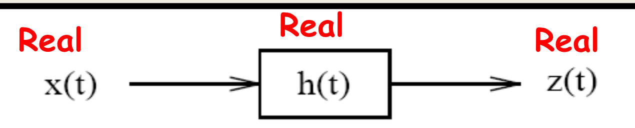

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$$\overset{h(t)\ \text{Real}}{=} \int_{-\infty}^{\infty} \mathrm{Re}\{h(\tau) \qquad \tilde{x}(t-\tau)\exp(i\omega_c(t-\tau))\}\mathrm{d}\tau$$

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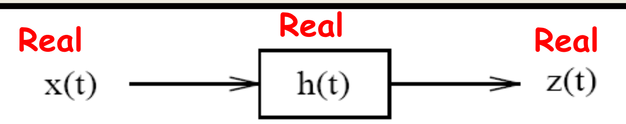

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typo

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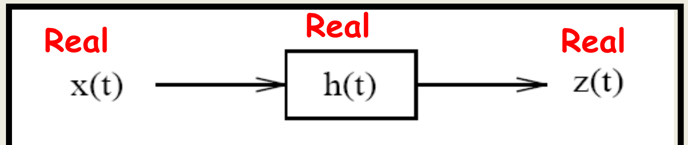
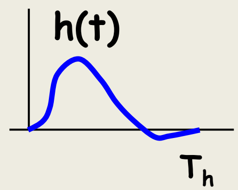


Figure 3.11: Bandpass filtering.

Assumptions:

1. h(t) of duration T_h



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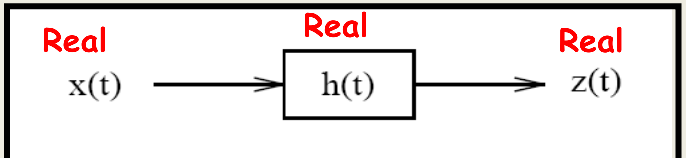

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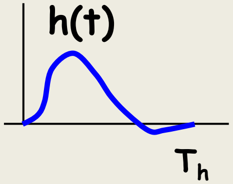

1. h(t) of duration T_h
2. x(t) is QAM of duration T_s $x(t) = A\cos(\omega_c t) - B\sin(\omega_c t),\ 0 \leq t \leq T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

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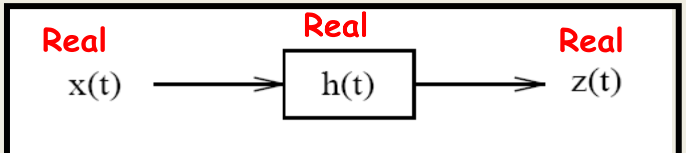

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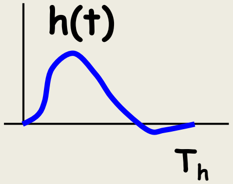

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2. x(t) is QAM of duration T_s

$$x(t) = A\cos(\omega_c t) - B\sin(\omega_c t),\ 0 \le t \le T_s$$

$$= \sqrt{A^2 + B^2}\cos(\omega_c t + \nu)$$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

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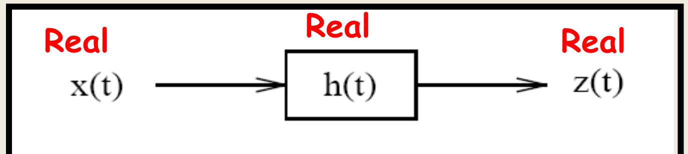

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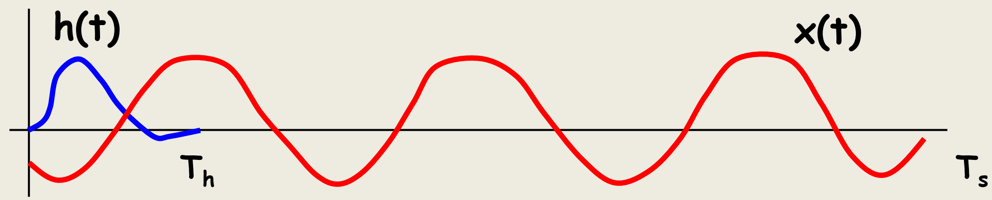

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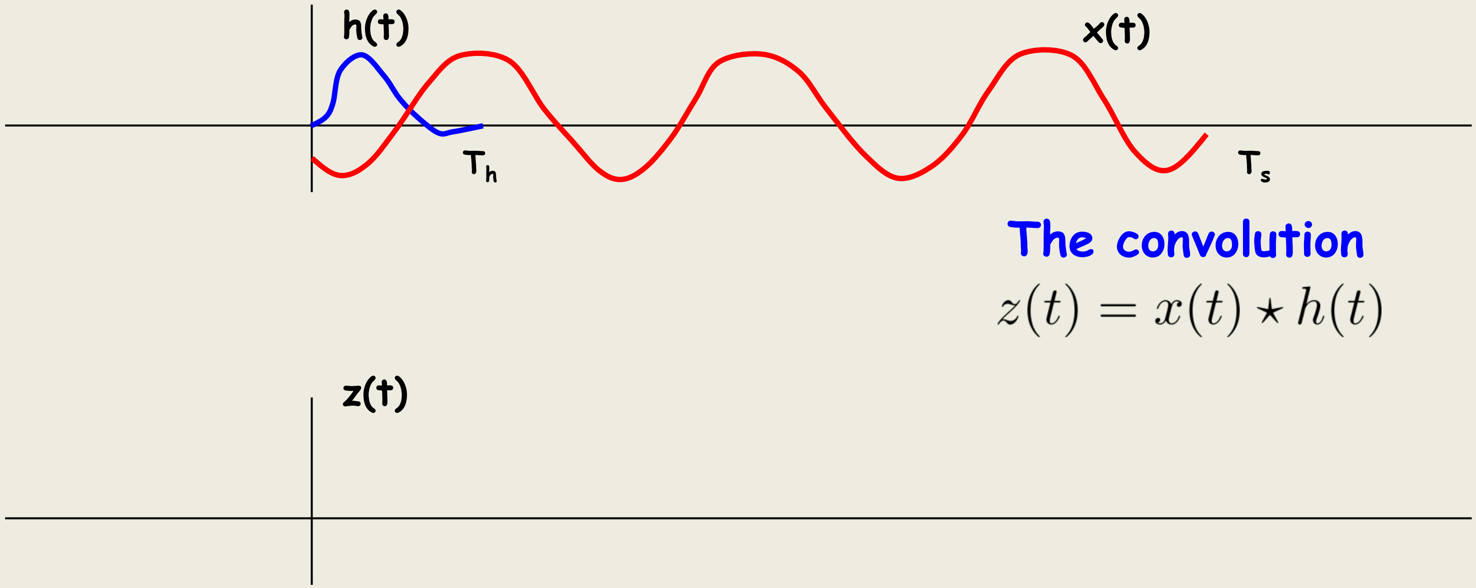
3. Low signaling rate $T_s \gg T_h$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

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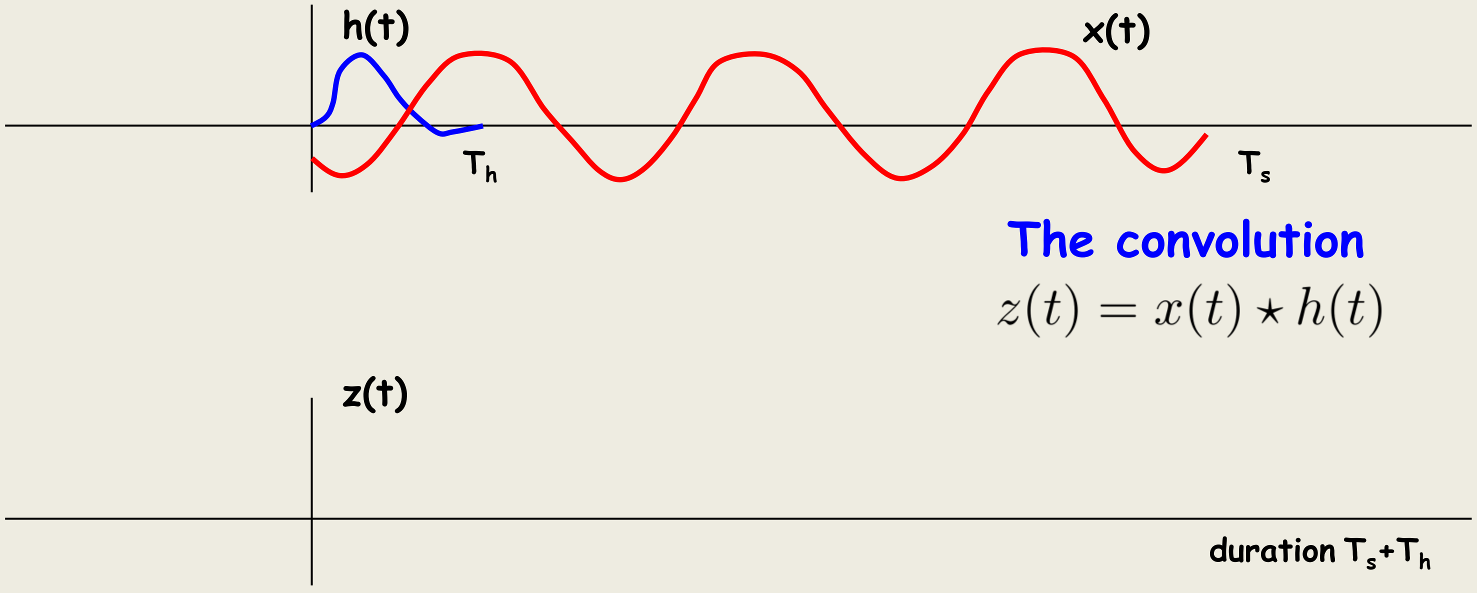

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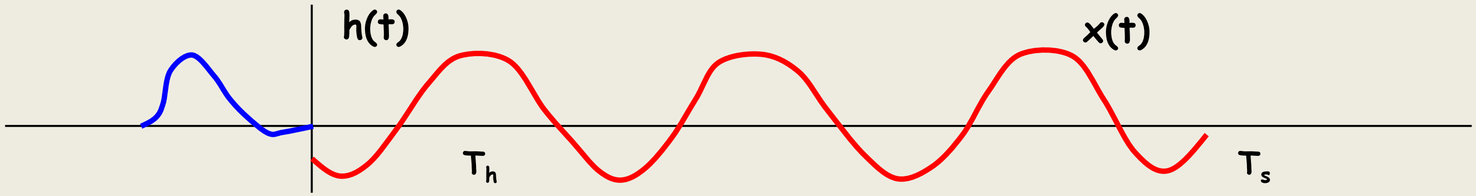

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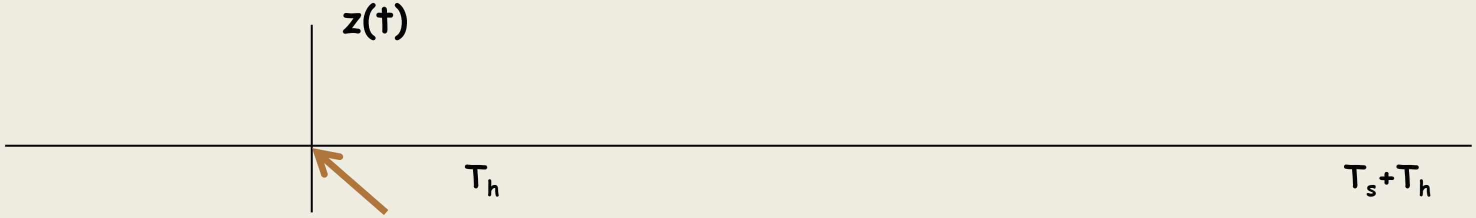

z(0) obtained by multiplying and integrating the above curves

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

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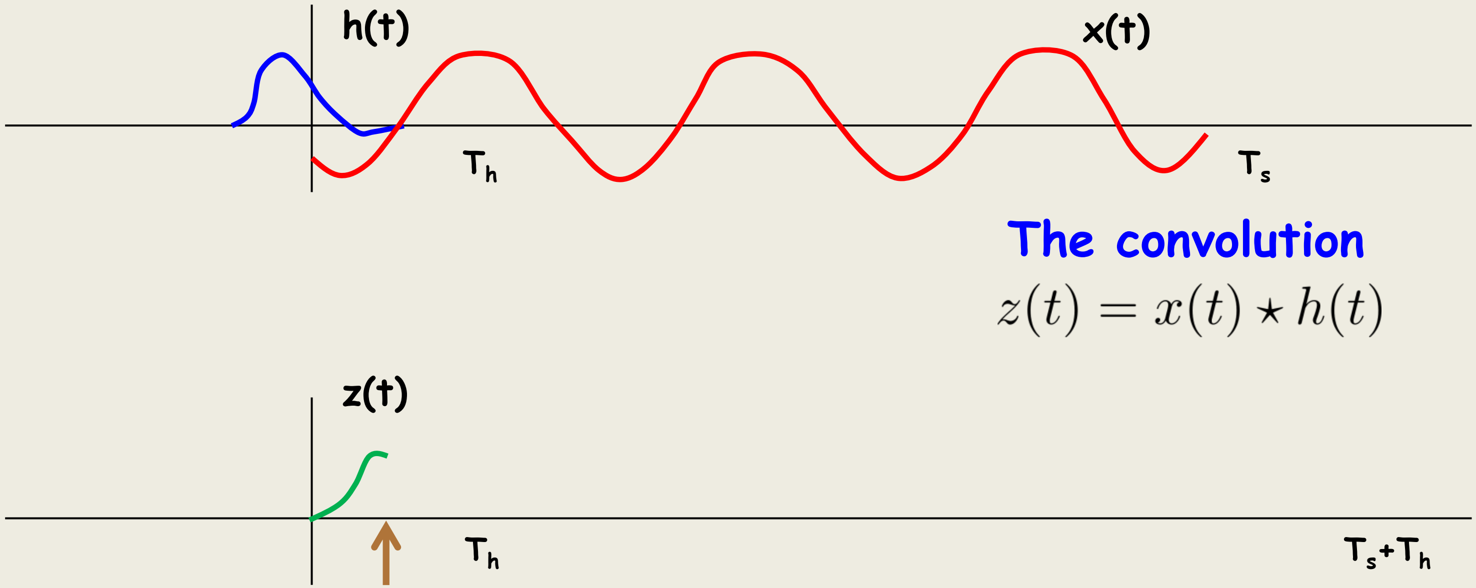

The convolution

$$z(t) = x(t) \star h(t)$$

z(T_h/2) obtained by multiplying and integrating the above curves

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

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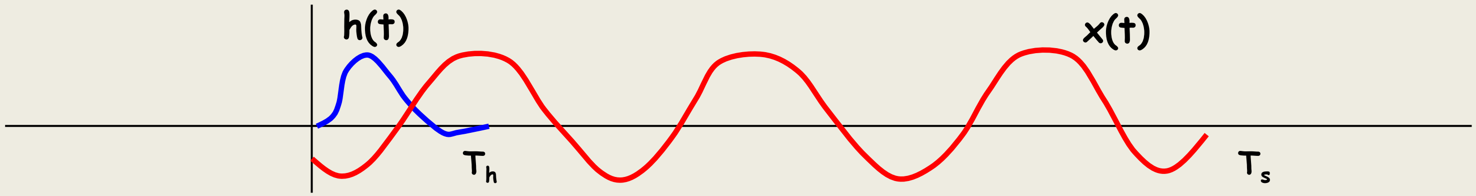

The convolution

$$z(t) = x(t) \star h(t)$$

$z(T_h)$ obtained by multiplying and integrating the above curves

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

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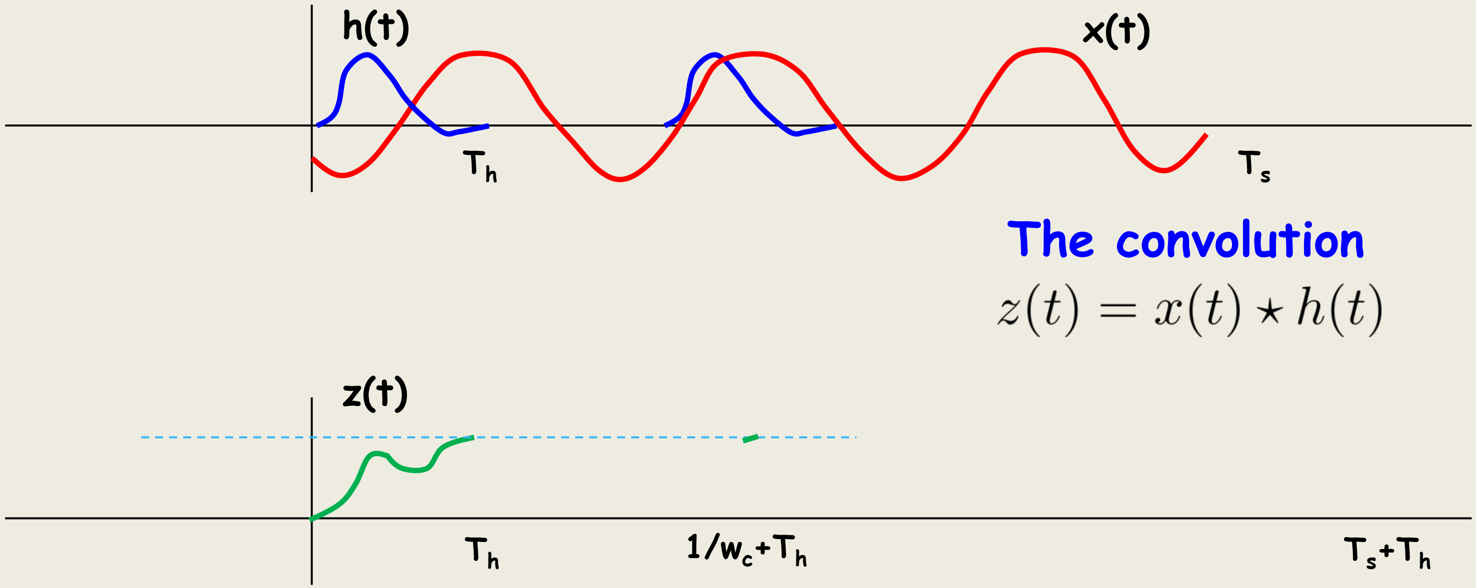

The convolution

$$z(t) = x(t) \star h(t)$$

$z(1/w_c+T_h)$ obtained by multiplying and integrating the above curves
Clearly the same!

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

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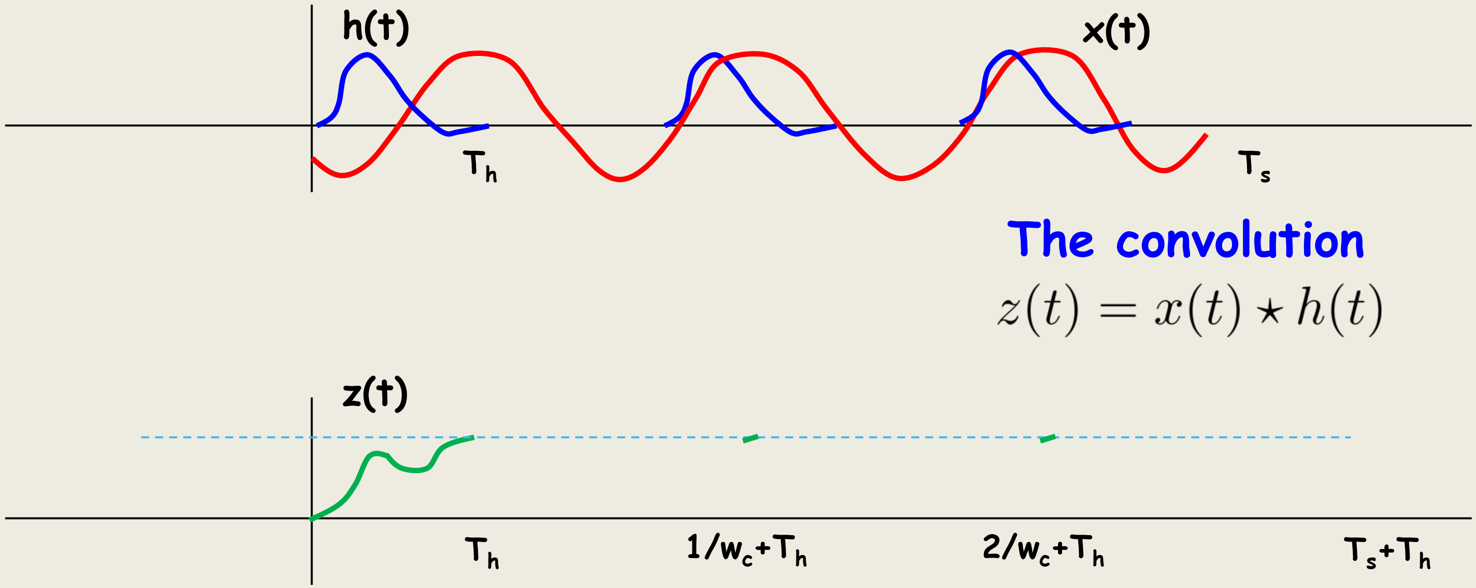

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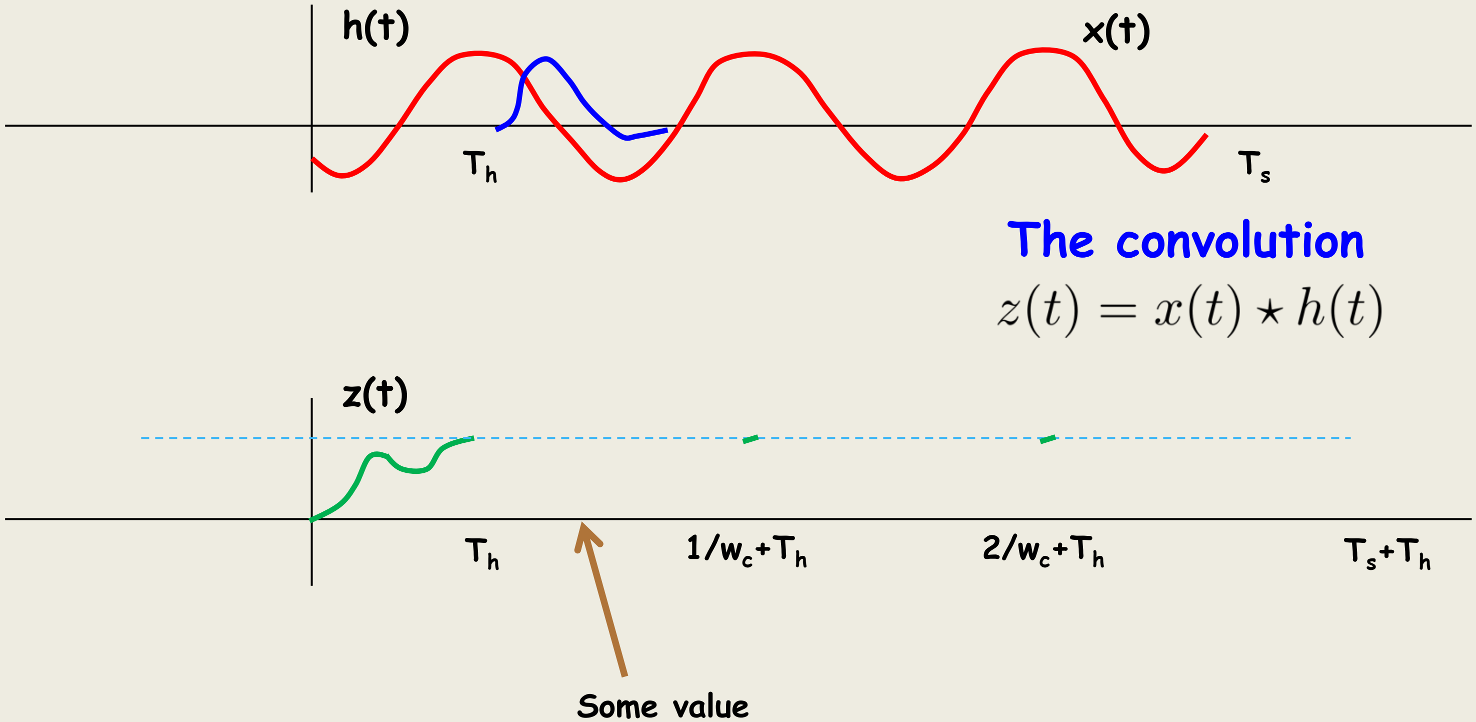

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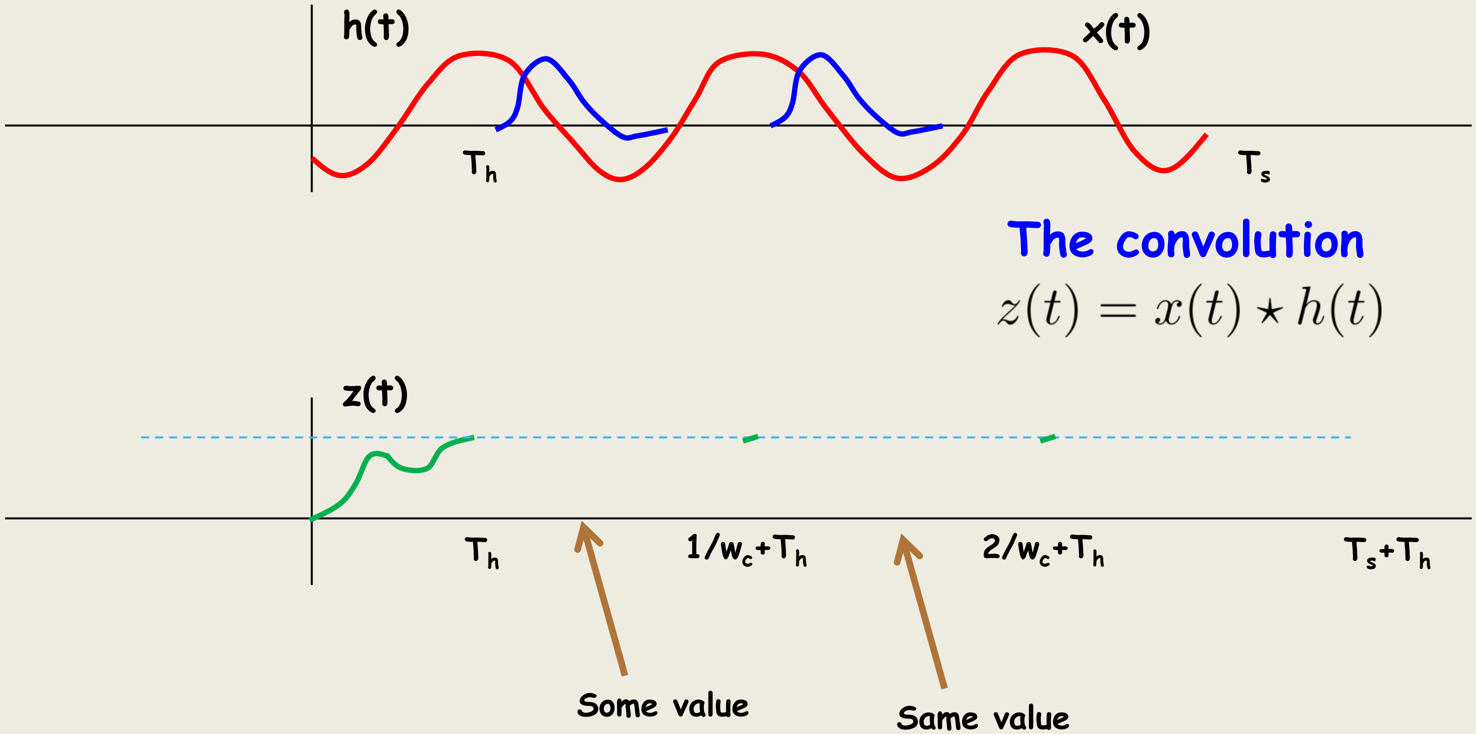

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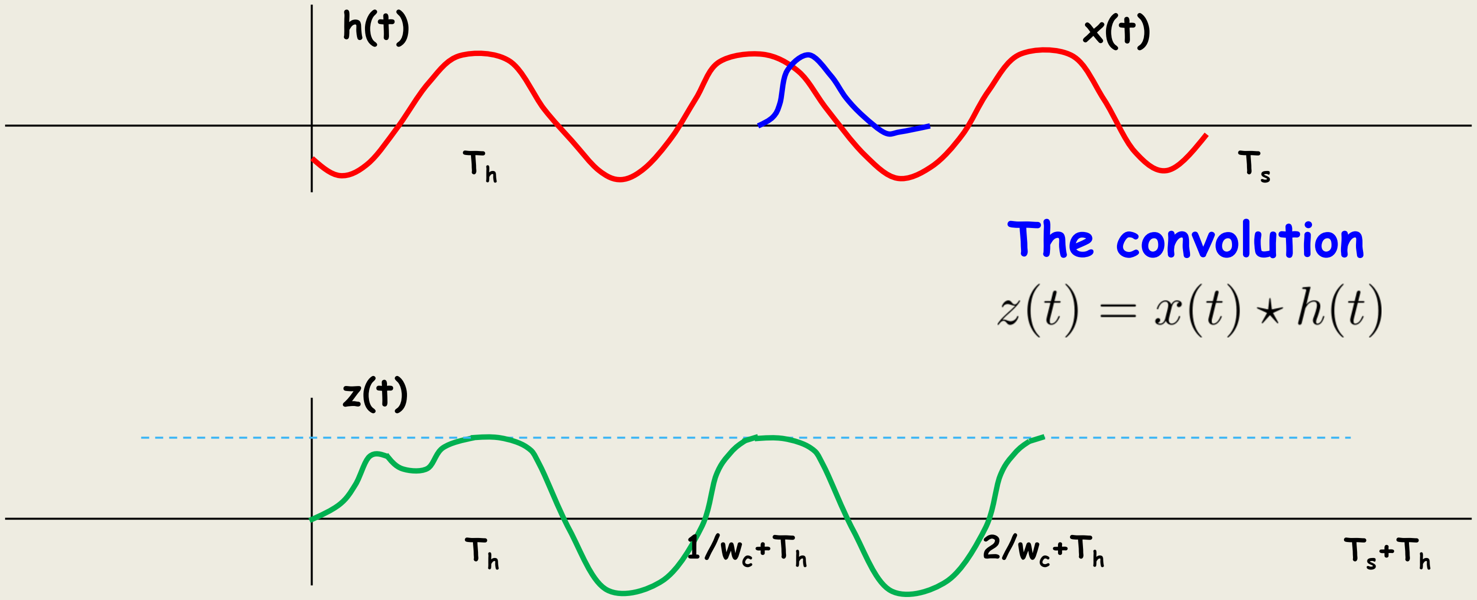

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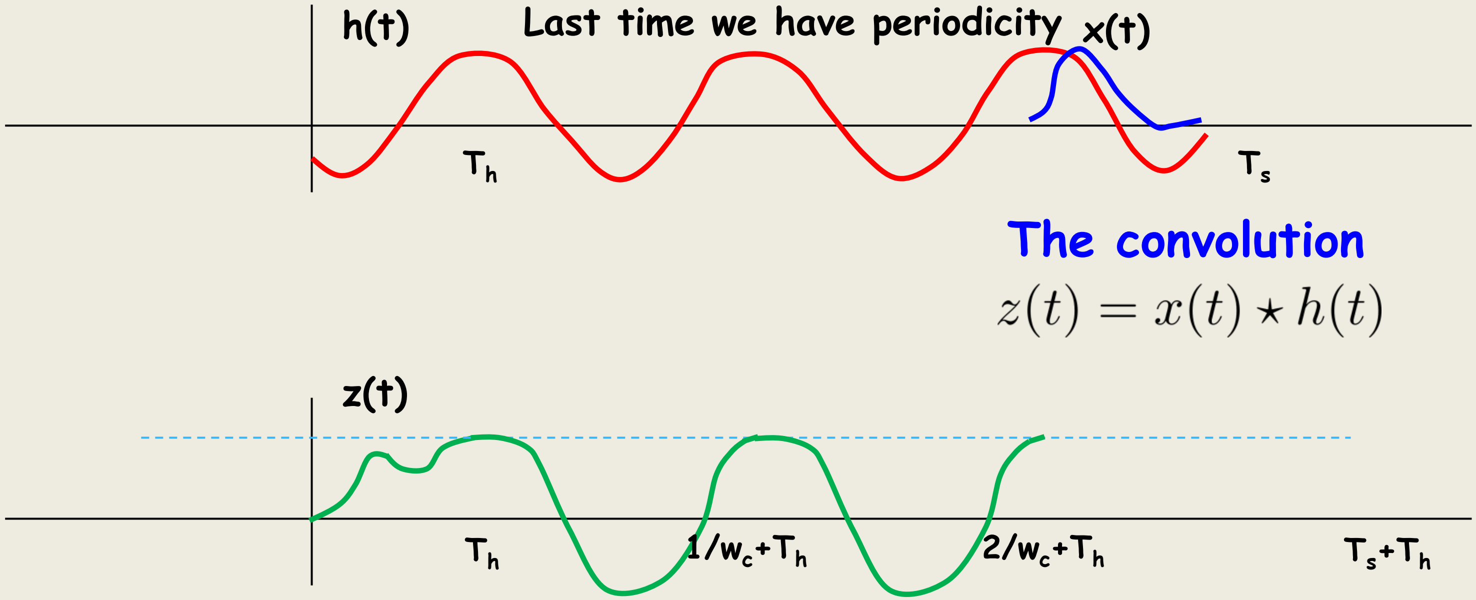

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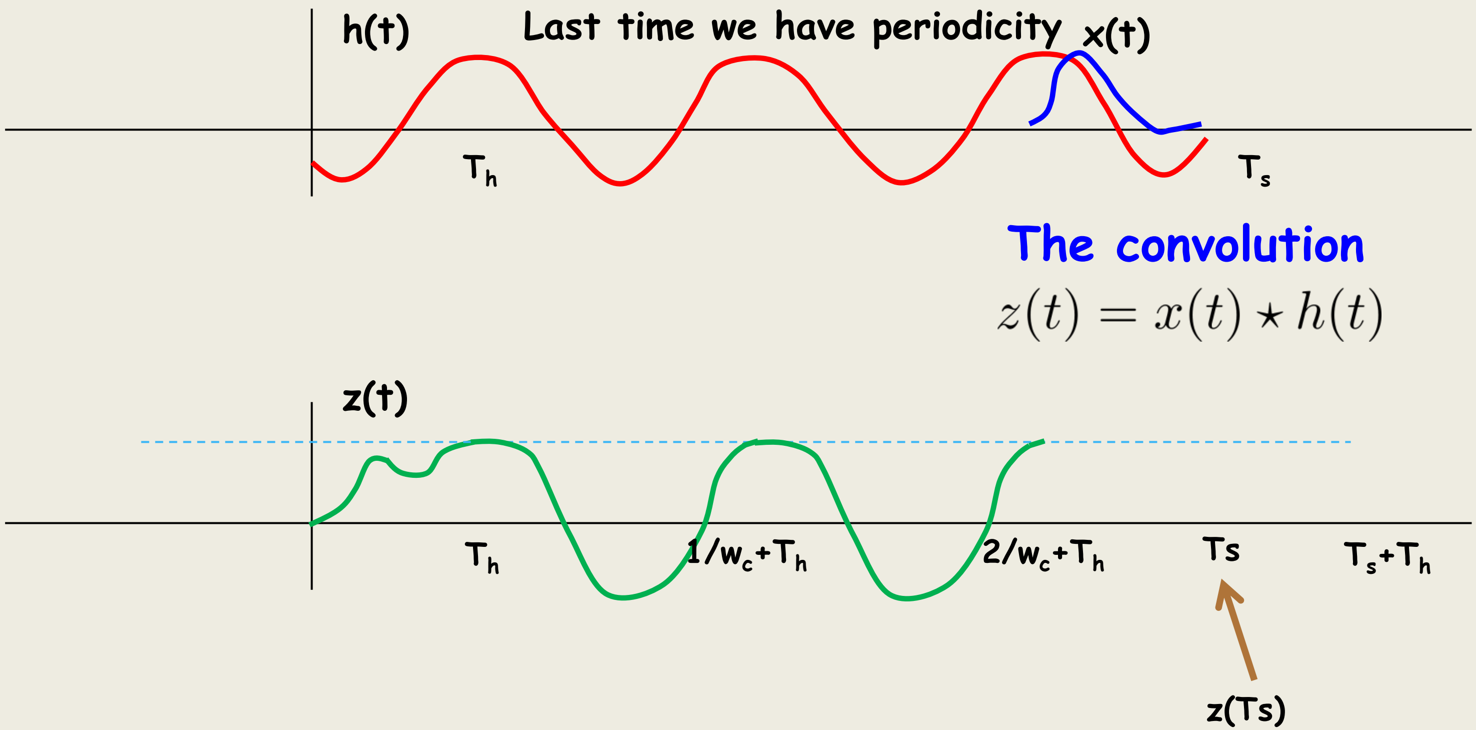

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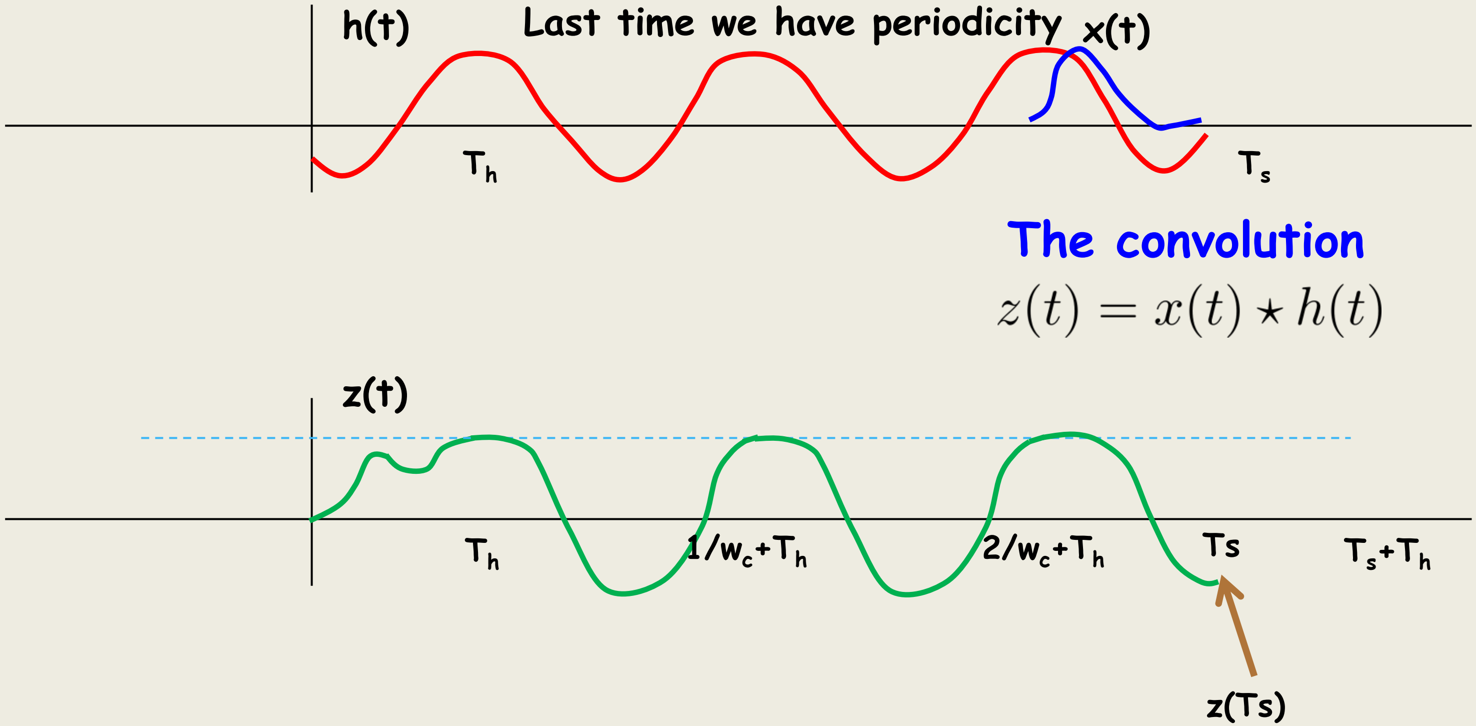
Lecture 6: Channel model (Ch.3) & coding (Ch8)

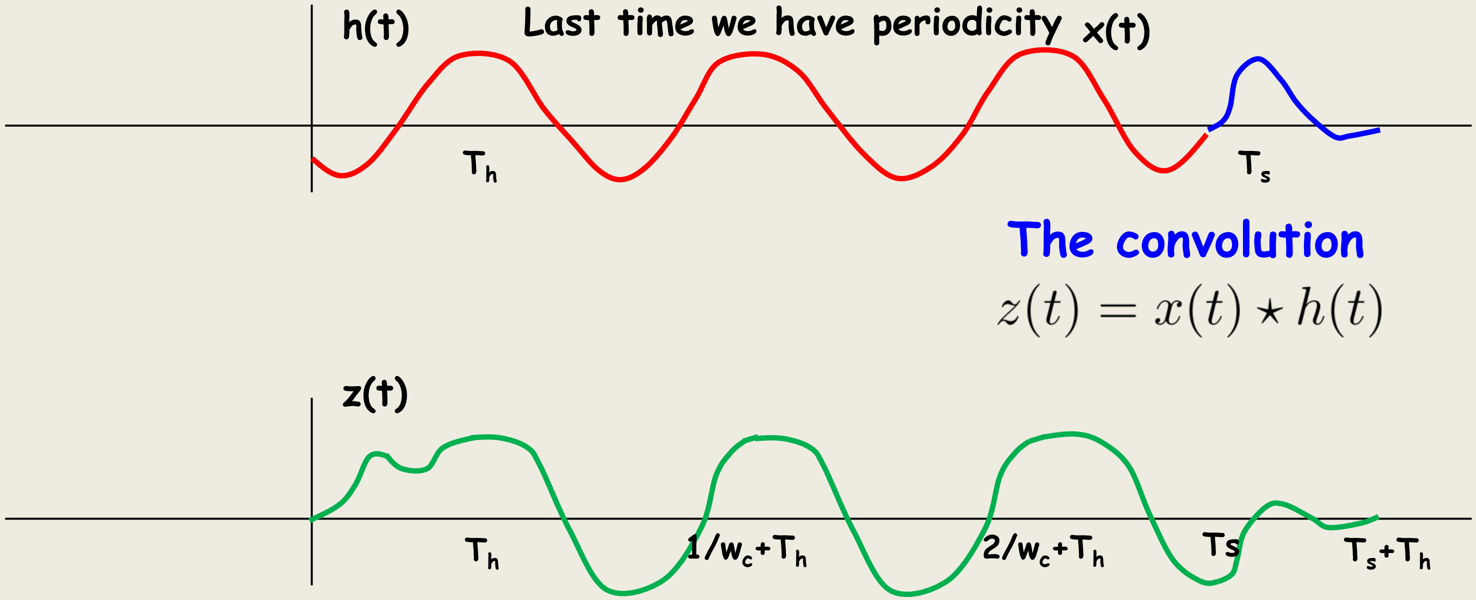


The convolution

$$z(t) = x(t) \star h(t)$$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

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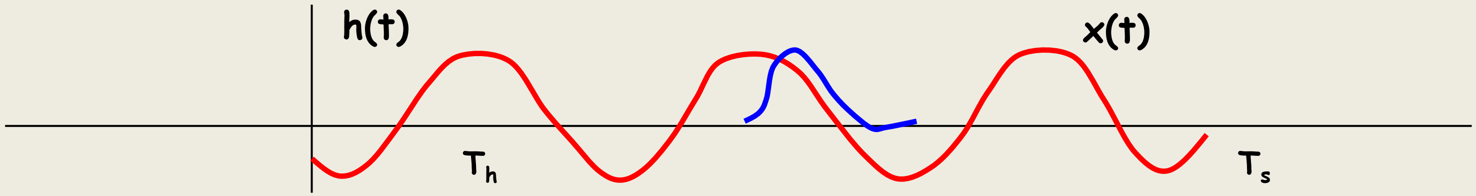
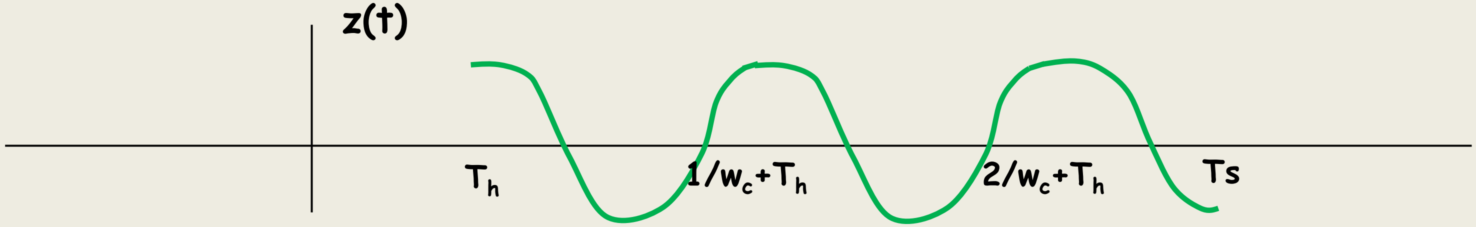
Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

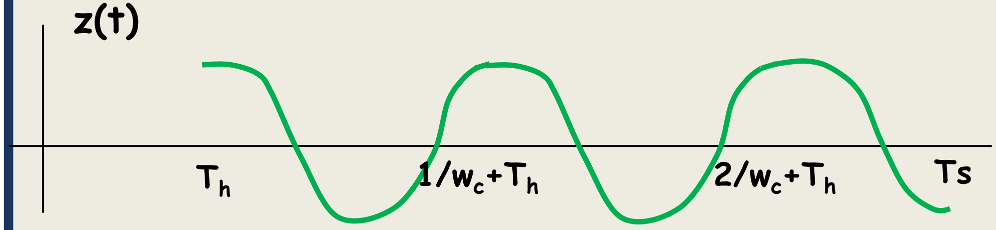
$$z(t) = x(t) \star h(t)$$



Let us now focus on $T_h < t < T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

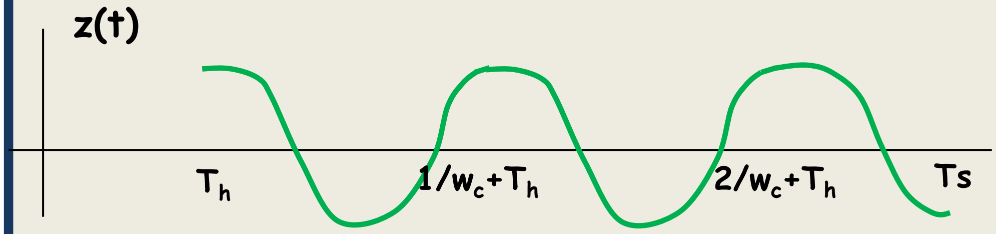
$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB$$

By definition

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

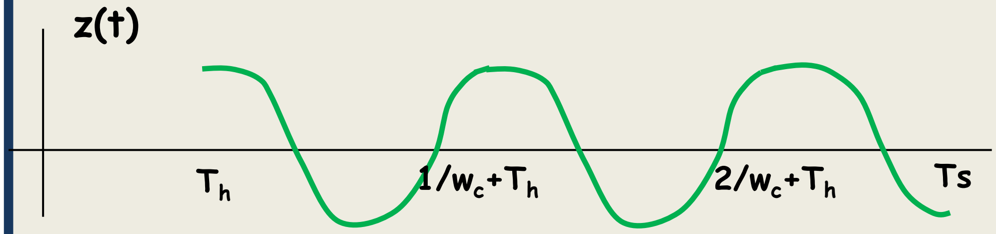
$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

By definition **By manipulation**

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

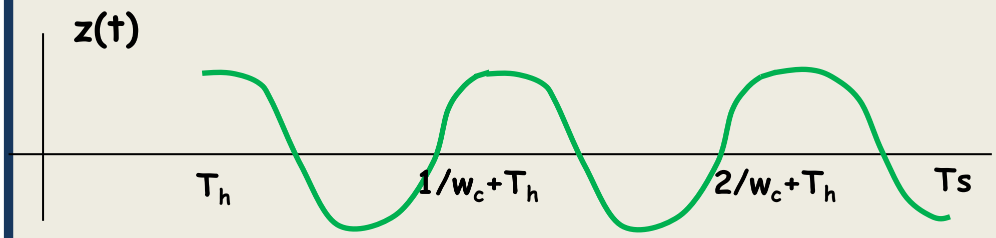
By definition

By manipulation

Alive in $0<t<T_h$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_{-\infty}^{\infty} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

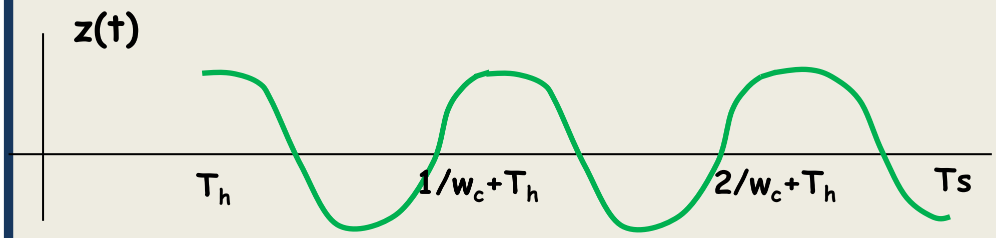
$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

By definition **By manipulation**

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

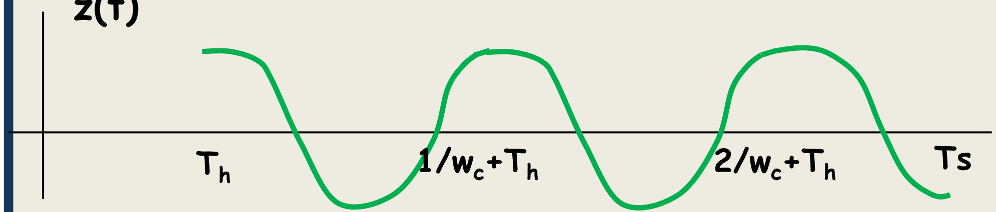
By definition

By manipulation

Of interest: $z(t),\ T_h \leq t \leq T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

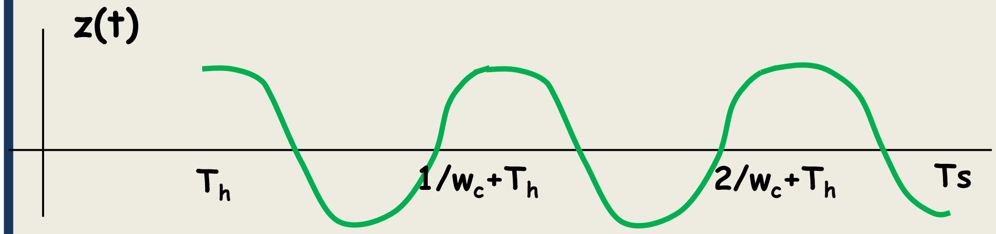
$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

By definition **By manipulation**

Of interest: $z(t),\ T_h \le t \le T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

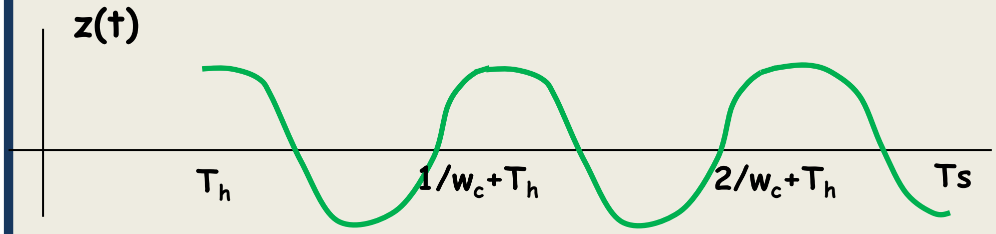
$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

By definition **By manipulation**

Of interest: $z(t),\ T_h \le t \le T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

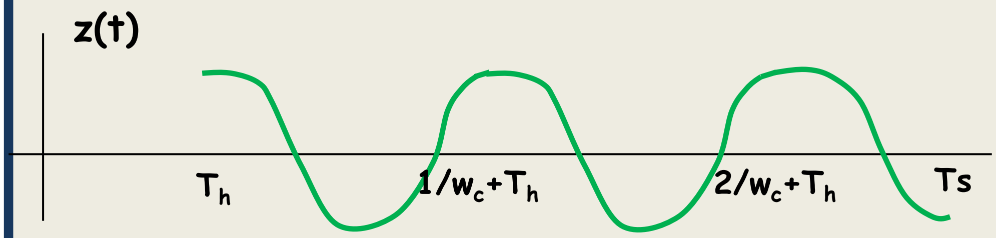
$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

By definition **By manipulation**

Of interest: $z(t),\ T_h \le t \le T_s$ $\qquad 0 \le t - \tau \le T_s - T_h$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\mathrm{Re}\{\tilde{x}(t-\tau)\exp(i\omega_c(-\tau))\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

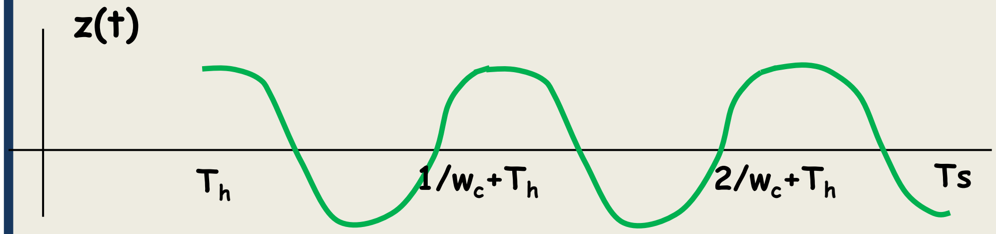
By definition By manipulation

Of interest: $z(t),\ T_h \le t \le T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\sqrt{A^2+B^2}\exp(i\omega_c(\nu-\tau))\mathrm{d}\tau\right\}$$

Typo fixed

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

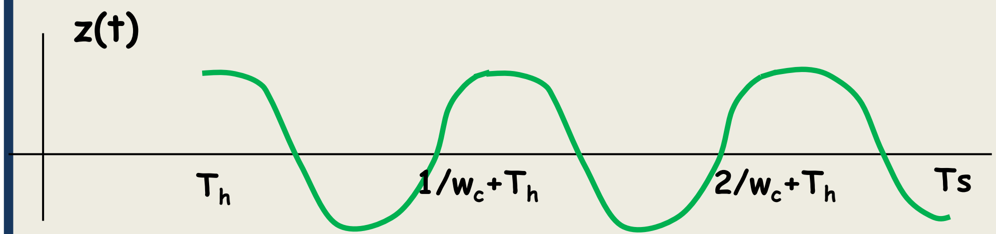
By definition **By manipulation**

Of interest: $z(t),\ T_h \leq t \leq T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i\omega_c t)\int_0^{T_h} h(\tau)\underline{\sqrt{A^2+B^2}}\exp(i\omega_c(\underline{\nu} - \tau))\mathrm{d}\tau\right\}$$

Constants wrt integration

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

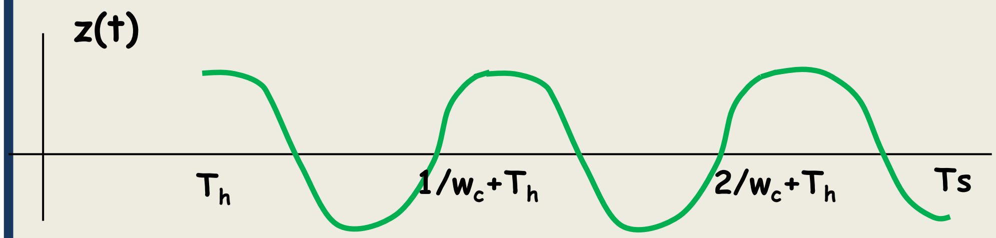
By definition

By manipulation

Of interest: $z(t), \; T_h \leq t \leq T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i(\omega_c t + \nu))\sqrt{A^2 + B^2}\int_0^{T_h} h(\tau)\exp(-i\omega_c\tau)\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

By definition

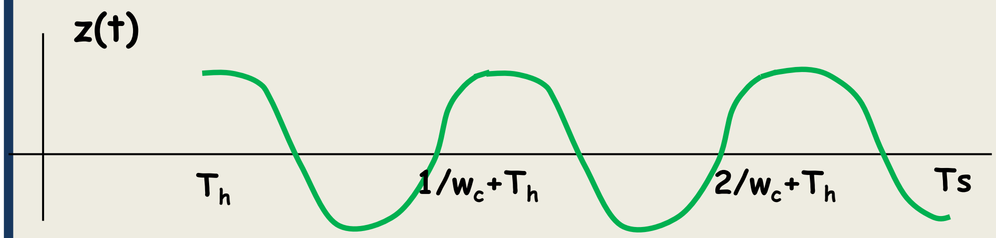
By manipulation

Of interest: $z(t),\ T_h \leq t \leq T_s$

Definition of…

$$z(t) = \mathrm{Re}\left\{\exp(i(\omega_c t + \nu))\sqrt{A^2 + B^2}\int_0^{T_h} h(\tau)\exp(-i\omega_c\tau)\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

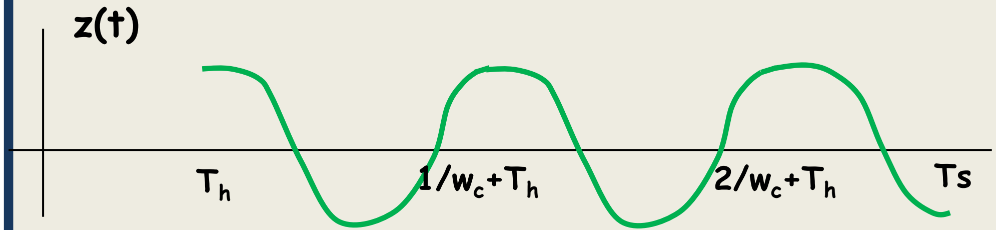
By definition   By manipulation

Of interest: $z(t),\ T_h \le t \le T_s$

Definition of Fourier transform

$$z(t) = \mathrm{Re}\left\{\exp(i(\omega_c t + \nu))\sqrt{A^2 + B^2}\int_0^{T_h} h(\tau)\exp(-i\omega_c \tau)\mathrm{d}\tau\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

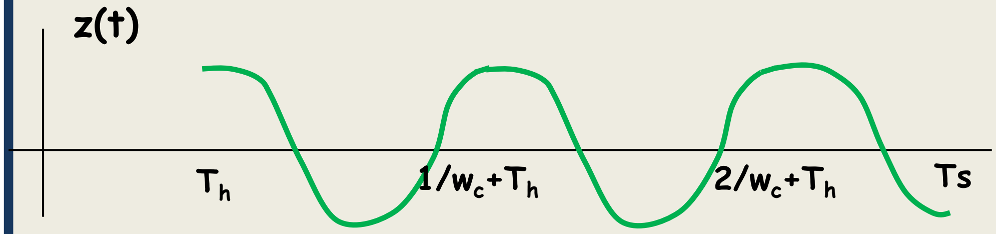
By definition

By manipulation

Of interest: $z(t),\ T_h \leq t \leq T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i(\omega_c t + \nu))\sqrt{A^2 + B^2}H(\omega_c)\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

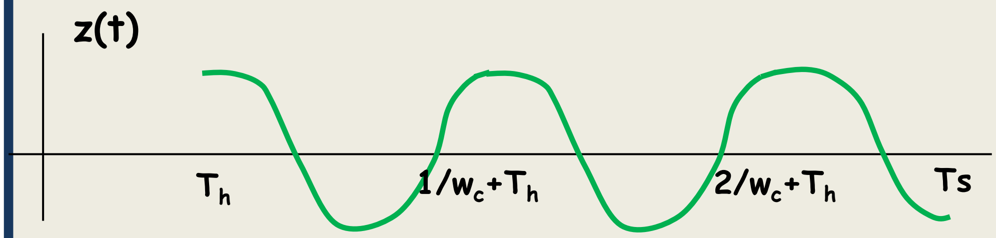
$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

By definition **By manipulation**

Of interest: $z(t),\ T_h \le t \le T_s$

$$z(t) = \mathrm{Re}\left\{\exp(i(\omega_c t + \nu + \phi(\omega_c)))\sqrt{A^2 + B^2}|H(\omega_c)|\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

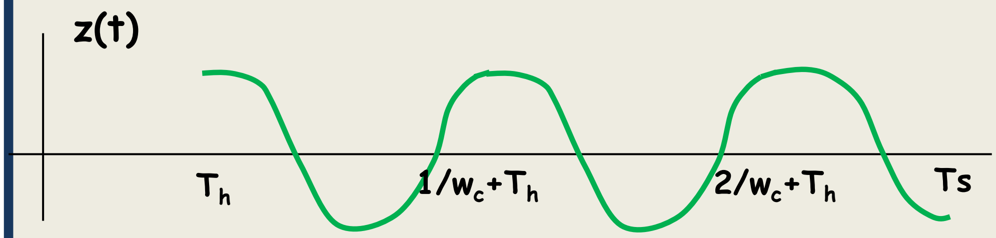
$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

Recall $x(t) = A\cos(\omega_c t) - B\sin(\omega_c t), \ 0 \leq t \leq T_s$

$$= \sqrt{A^2 + B^2}\cos(\omega_c t + \nu)$$

$$z(t) = \mathrm{Re}\left\{\exp(i(\omega_c t + \nu + \phi(\omega_c)))\sqrt{A^2 + B^2}|H(\omega_c)|\right\}$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

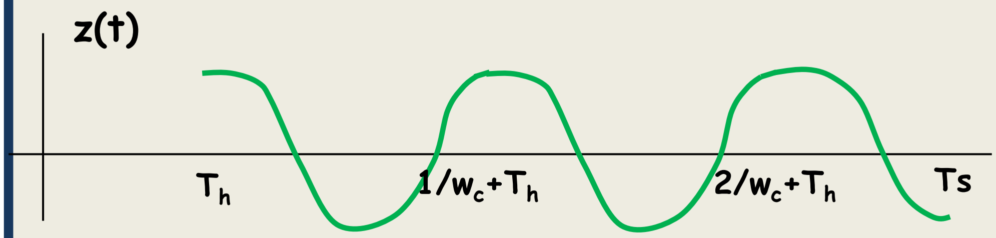
$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

Recall $x(t) = A\cos(\omega_c t) - B\sin(\omega_c t), \; 0 \leq t \leq T_s$

$$= \sqrt{A^2 + B^2}\cos(\omega_c t + \nu)$$

$$z(t) = A_z\cos(\omega_c t) - B_z\sin(\omega_c t), \; T_h \leq t \leq T_s$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

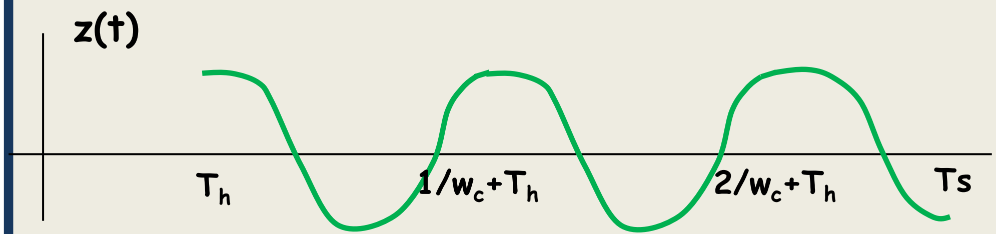
$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \le t \le T_s$$

Altogether: $A_z + iB_z = (A + iB)H(\omega_c)$

$$z(t) = A_z \cos(\omega_c t) - B_z \sin(\omega_c t), \; T_h \le t \le T_s$$

Lecture 6: Channel model (Ch.3) & coding (Ch8)



The convolution

$$z(t) = x(t) \star h(t)$$

$$\tilde{x}(t) = A + iB = \sqrt{A^2 + B^2}\exp(i\nu) \qquad 0 \leq t \leq T_s$$

Altogether: $A_z + iB_z = (A + iB)H(\omega_c)$

Lesson learned (important): For low-rate inputs, A QAM signal is changed into a new QAM signal, but coordinates are changed in signal by a multiplication with $H(w_c)$, w_c being the carrier-frequency

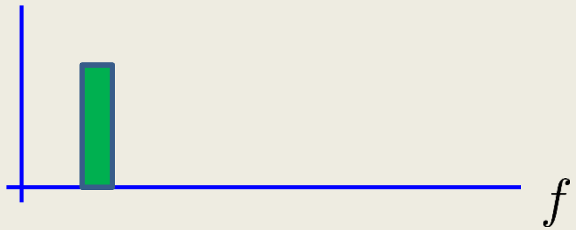
Lecture 6: Channel model (Ch.3) & coding (Ch8)

What does a "low-rate" signal look like in the Frequency domain?

Lecture 6: Channel model (Ch.3) & coding (Ch8)

What does a "low-rate" signal look like in the Frequency domain?

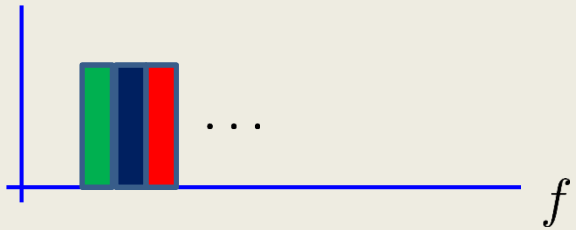
very narrow



Lecture 6: Channel model (Ch.3) & coding (Ch8)

What does a "low-rate" signal look like in the Frequency domain?

very narrow



Putting many next to eachother would result in:

1. Non-interfering transmissions
2. Simple equalization (input-output relation is just a scaling)

This is the basis of OFDM

Lecture 6: Channel model (Ch.3) & coding (Ch8)

What does a normal channel look like in the frequency domain?

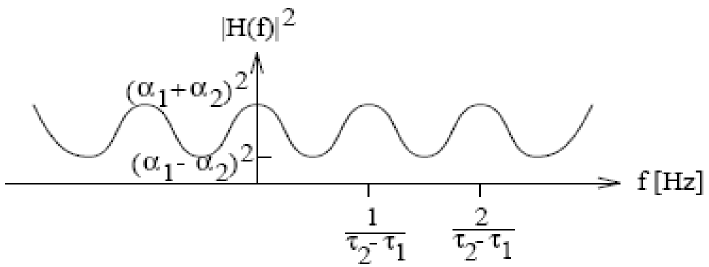
$$z(t) = x(t) * \underbrace{\left(\sum_{i=1}^{N} \alpha_i \delta(t-\tau_i)\right)}_{\text{Impulse response } h(t)} = \sum_{i=1}^{N} \alpha_i x(t-\tau_i) \qquad (3.126)$$

Channel compises N paths between tx and rx

$$H(f) = \mathcal{F}\{h(t)\} = \sum_{i=1}^{N} \alpha_i e^{-j2\pi f \tau_i} \qquad (3.128)$$

Rough sketch:

EXAMPLE 3.20



It is seen in this figure that the two signal paths add constructively or destructively (fading) depending on the frequency. Furthermore, if $\alpha_1 \approx \alpha_2$ then $|H(f)|$ is very close to zero at certain frequencies (so-called deep fades)!

Chapter 8

Trellis-coded Signals

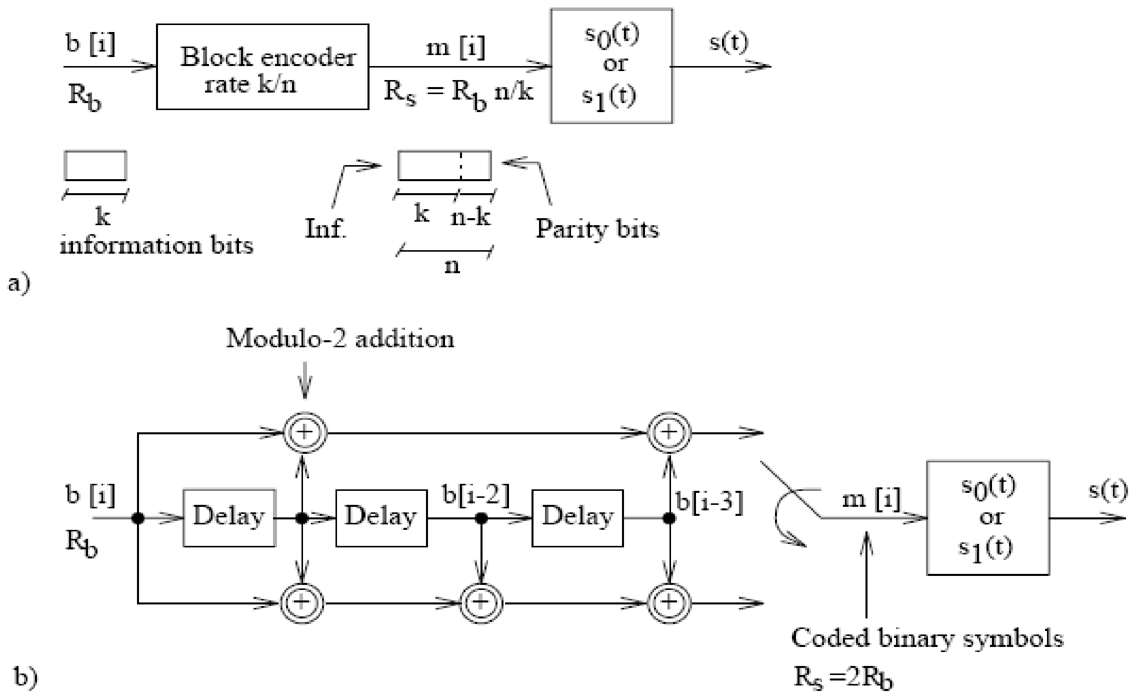


Figure 8.1: a) Block coding, $r_c = k/n$. b) Convolutional coding, $r_c = 1/2$.

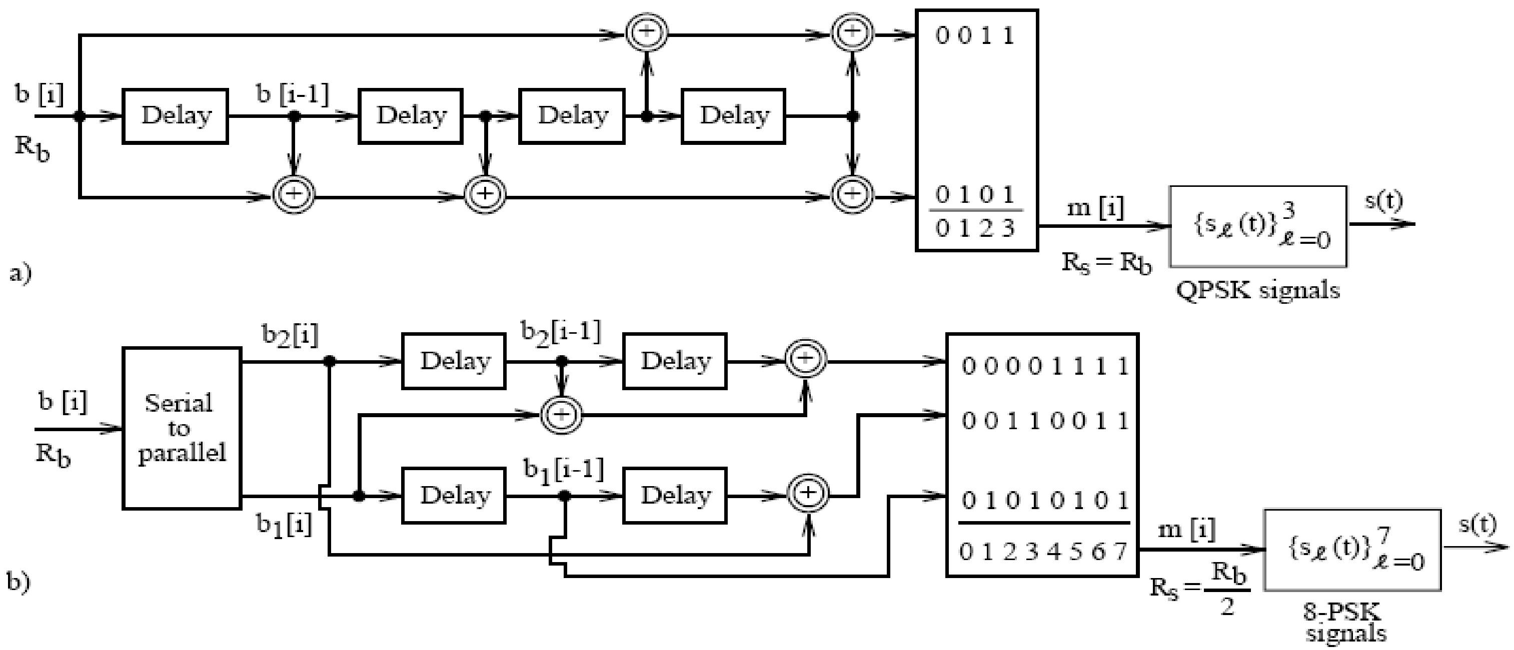


Figure 8.2: a) Rate $r_c = 1/2$ convolutional encoder combined with QPSK; b) Rate $r_c = 2/3$ convolutional encoder combined with 8-PSK, from [63], [64].

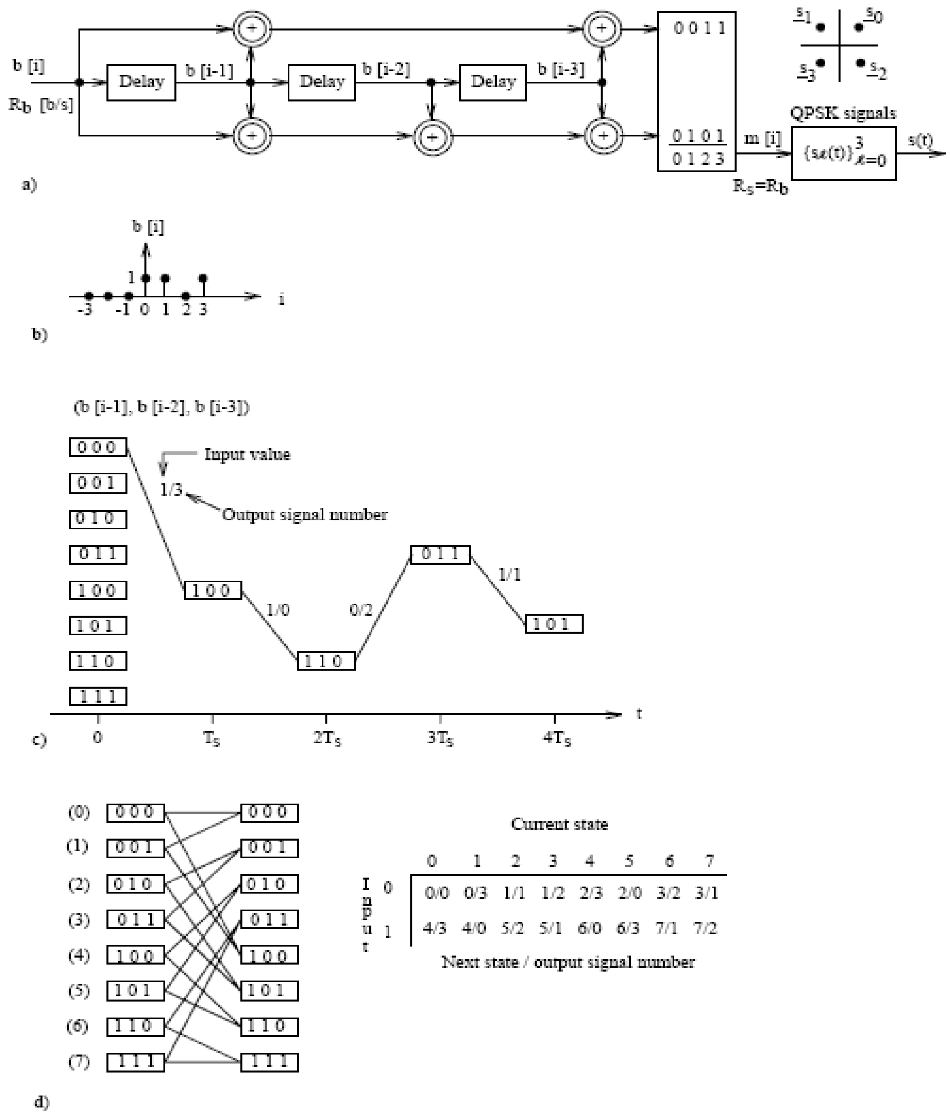
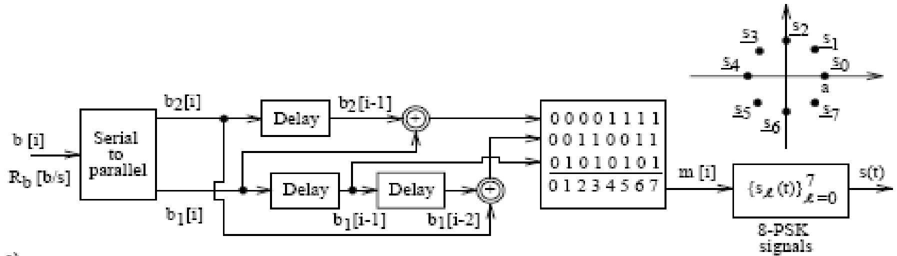


Figure 8.4: a) A rate 1/2 convolutional encoder combined with QPSK signal alternatives; b) A specific input sequence $b[i]$; c) The corresponding path in the trellis; d) A trellis section, and a table containing all relevant parameters.



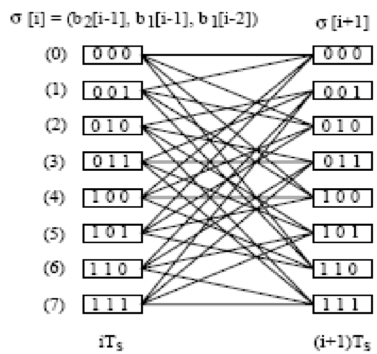
a)

Current state σ [i]

INPUT $\binom{b_2[i]}{b_1[i]}$	(000) 0	(001) 1	(010) 2	(011) 3	(100) 4	(101) 5	(110) 6	(111) 7
$\binom{0}{0}$	0/0	0/2	1/1	1/3	0/4	0/6	1/5	1/7
$\binom{0}{1}$	2/4	2/6	3/5	3/7	2/0	2/2	3/1	3/3
$\binom{1}{0}$	4/2	4/0	5/3	5/1	4/6	4/4	5/7	5/5
$\binom{1}{1}$	6/6	6/4	7/7	7/5	6/2	6/0	7/3	7/1

σ [i+1] / m [i]

b)



c)

Figure 8.6: a) An example of TCM, from [63]–[64]; b) The mappings $F(\cdot,\cdot)$ and $G(\cdot,\cdot)$; c) A trellis section.

<u>Memory (redundancy, dependancy) is introduced among the sent signal alternatives!</u>

This gives us some new properties like, e.g.,:

Which of the following signal sequences are impossible?

1. $s_3(t), s_2(t-T_b), s_1(t-2T_b), s_1(t-3T_b)$
2. $s_3(t), s_2(t-T_b), s_2(t-2T_b), s_1(t-3T_b)$
3. $s_3(t), s_1(t-T_b), s_0(t-2T_b), s_2(t-3T_b)$
4. $s_3(t), s_1(t-T_b), s_3(t-2T_b), s_1(t-3T_b)$

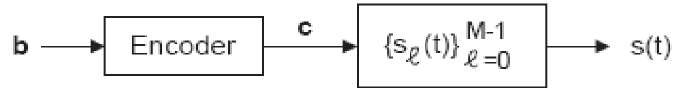
Note: In the uncoded case all signal sequences are possible.

Find the "missing" signal, in the sequence below,

$$s_1(t), s_3(t-T_b), \ ? \ , s_2(t-3T_b), s_3(t-4T_b), s_0(t-5T_b)$$

Note: This is not possible to do in the uncoded case!

2.32 Let us here study adaptive coding and modulation according to the block diagram below.



$$\bar{E}_{sent} = r_c \log_2(M) E_{b,sent} = \frac{k}{n} \log_2(M) E_{b,sent} \qquad (8.4)$$

$$R_s = 1/T_s = \frac{1}{r_c} \cdot \frac{1}{\log_2(M)} \cdot R_b = \frac{1}{k/n} \cdot \frac{1}{\log_2(M)} \cdot R_b \qquad (8.5)$$

$$W = c \cdot R_s \qquad (8.6)$$

Typically, the bandwidth W is fixed and given but:

the rate of the encoder

the number of signal alternatives

and the bit rate can be **ADAPTIVE**, see (8.5)-(8.6)!

We have memory in the sequence of sent signal alternatives!

Some sequences are impossible, see problem!

Only ”good” sequences are sent!