Lecture 4: Capacity

Project info

1. Each project group consists of two students.
2. Each project group should as soon as possible, send an email to fredrik.rusek@eit.lth.se and mg7107ma-s@student.lu.se containing Name and email address to each project member.'
3. The project group should contact Fredrik Rusek to decide about project and articles!
4. Each group should write a project report.
5. The structure of the project report should follow journal articles published by IEEE. However, two columns are not needed.
6. The project report should be written in English *with your own words, tables and figures*, and contain 4-5 pages.
7. The report should be clearly written, and written to the other students in this course!
8. Observe copyright rules: "copy & paste" is in general strictly forbidden!

Lecture 4: Capacity

Project info

9. Book a meeting with Mgeni at your earliest possible convenience in case you are interested in discussing how a good report should be written (book meetings with him via email). You can also discuss topics and articles with Mgeni
10. The project report should be sent in .pdf format to Mgeni before Wednesday 12 December, 17.00
11. Feedback on the reports will be provided in a meeting with Mgeni (book meetings with him via email)
12. Oral presentations in the week starting with Monday December 17
13. Each group should have relevant comments and questions on the project report and on the oral presentation of another group. NOTE! The project presentation should be clear and aimed to the other students in this course! After the oral presentation the project report and the presentation will be discussed (5 min).
14. Final report should be sent to Fredrik and Mgeni at latest January 11, 2019.

Lecture 4: Capacity

Power efficiency

We know from before (e.g., union bound) that $P_{\mathrm{s}} \leq cQ\left(\sqrt{d_{\min}^2 \frac{E_b}{N_0}}\right)$

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To meet a specific error probability target, this implies $\frac{E_b}{N_0} \geq \frac{\mathcal{X}}{d^2_\mathrm{min}}$

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We also know that the transmit power satisfies $\mathcal{P} = E_b R_b$

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Now, divide both sides with the bandwidth W $\frac{R_b}{W} \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

Lecture 4: Capacity

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We have seen this before, it is defined as…

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We have seen this before, it is defined as bandwidth efficiency

Now, divide both sides with the bandwidth W $\rho = \frac{R_b}{W} \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

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Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$ Power efficiency

Lecture 4: Capacity

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Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W}\frac{d_{\min}^2}{\mathcal{X}}$ Performance req

Lecture 4: Capacity

Power efficiency

We know from before (e.g., union bound) that $P_s \leq cQ\left(\sqrt{d^2_{\min}\frac{E_b}{N_0}}\right)$

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Bandwidth and power efficiencies are linked

Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W}\frac{d^2_{\min}}{\mathcal{X}}$

Power efficiency

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Unit ?

Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

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Power

Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

W

Lecture 4: Capacity

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Bandwidth

Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

$\frac{W}{\quad Hz}$

Lecture 4: Capacity

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Spectral density

Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

$$\frac{\text{W}}{\text{? Hz}}$$

Power efficiency

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Spectral density

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$\frac{\text{W}}{\text{W/Hz Hz}}$

Lecture 4: Capacity

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Has no unit (dimensionless)

Now, divide both sides with the bandwidth W $\rho \le \frac{\mathcal{P}}{N_0 W} \frac{d_{\min}^2}{\mathcal{X}}$

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Received signal-to-noise-power-ratio

Now, divide both sides with the bandwidth W $\rho \leq \frac{\mathcal{P}}{N_0 W}\frac{d^2_{\min}}{\mathcal{X}}$

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Definition

Now, divide both sides with the bandwidth W $\boxed{\rho \leq \mathcal{SNR}_r \frac{d_{\min}^2}{\mathcal{X}}}$

Lecture 4: Capacity

Power efficiency

"BW efficiency" = "Signal-to-noise-power-ratio" x "Power efficiency"

$$\boxed{\rho \leq \mathcal{SNR}_r \frac{d_{\min}^2}{\mathcal{X}}}$$

Lecture 4: Capacity

Shannon Capacity

Before going on, we go through what the term capacity means

Given a scalar channel of form $y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$

We know that the capacity is $C = \log_2\left(1 + \frac{A}{N_0}\right)$

But what does this mean?

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$

$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Build a codebook of all information sequences possible to send of length K

$$\underbrace{\begin{matrix} 000000 \;\ldots\ldots\; 00 \\ 000000 \;\ldots\ldots\; 01 \\ 000000 \;\ldots\ldots\; 10 \\ \\ \\ 1111111 \;\ldots\ldots\; 10 \\ 1111111 \;\ldots\ldots\; 11 \end{matrix}}_{K}$$

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Build a codebook of all information sequences possible to send of length K

000000 00
000000 01
000000 10

1111111 10
1111111 11

K

Sending K bits of information means:

pick one of the rows, and tell the receiver which row it is

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n,\ \ n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Build a codebook of codewords to send for each information word, length N

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

K

$x_{11}x_{12}x_{13}x_{14}$ ..... $x_{1(N-1)}x_{1N}$
$x_{21}x_{22}x_{23}x_{24}$ ..... $x_{2(N-1)}x_{2N}$

$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4}$ ..... $x_{2^K(N-1)}x_{2^KN}$

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

K

Codebook

$x_{11}x_{12}x_{13}x_{14}$ ..... $x_{1(N-1)}x_{1N}$
$x_{21}x_{22}x_{23}x_{24}$ ..... $x_{2(N-1)}x_{2N}$

$x_{2^K 1}x_{2^K 2}x_{2^K 3}x_{2^K 4}$ ..... $x_{2^K (N-1)}x_{2^K N}$

N

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$

$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Information book

000000 00

000000 01

000000 10

If this is my data

1111111 10

1111111 11

K

Codebook

$x_{11}x_{12}x_{13}x_{14} \; ..... \; x_{1(N-1)}x_{1N}$

$x_{21}x_{22}x_{23}x_{24} \; ..... \; x_{2(N-1)}x_{2N}$

$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4} \; ..... \; x_{2^K(N-1)}x_{2^KN}$

N

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n,\ n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Information book

000000 00
$\boxed{000000 \ldots\ldots 01}$
000000 10

If this is my data

1111111 10
1111111 11

$\underbrace{\hspace{6em}}_{K}$

Codebook

$x_{11}x_{12}x_{13}x_{14} \ldots\ldots x_{1(N-1)}x_{1N}$
$\boxed{x_{21}x_{22}x_{23}x_{24} \ldots\ldots x_{2(N-1)}x_{2N}}$

I send this one

$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4} \ldots\ldots x_{2^K(N-1)}x_{2^KN}$

$\underbrace{\hspace{6em}}_{N}$

Lecture 4: Capacity

Shannon Capacity

As x over this channel used N times

$$\boxed{y = \sqrt{A}x + n,} \quad n \sim CN(0, N_0)$$

$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Information book

000000 00
$\boxed{000000 \; ..... \; 01}$
000000 10

If this is my data

1111111 10
1111111 11

$\underbrace{\hspace{5em}}_{K}$

Codebook

$x_{11}x_{12}x_{13}x_{14} \;.....\; x_{1(N-1)}x_{1N}$

$\boxed{x_{21}x_{22}x_{23}x_{24} \;.....\; x_{2(N-1)}x_{2N}}$

$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4} \;.....\; x_{2^K(N-1)}x_{2^KN}$

$\underbrace{\hspace{5em}}_{N}$

Lecture 4: Capacity

Shannon Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Clearly, bit rate is K/N bits/channel use

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

K

Codebook

$x_{11}x_{12}x_{13}x_{14}$ ..... $x_{1(N-1)}x_{1N}$
$x_{21}x_{22}x_{23}x_{24}$ ..... $x_{2(N-1)}x_{2N}$

$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4}$ ..... $x_{2^K(N-1)}x_{2^KN}$

N

Lecture 4: Capacity

Receiver observes

$y_1 y_2 y_3 y_4 \cdots\cdots y_{(N-1)} y_N$

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Information book

$$\underbrace{\begin{array}{lll} 000000 & \cdots\cdots & 00 \\ 000000 & \cdots\cdots & 01 \\ 000000 & \cdots\cdots & 10 \\ & & \\ 1111111 & \cdots\cdots & 10 \\ 1111111 & \cdots\cdots & 11 \end{array}}_{K}$$

Codebook

$x_{11} x_{12} x_{13} x_{14} \cdots\cdots x_{1(N-1)} x_{1N}$

$x_{21} x_{22} x_{23} x_{24} \cdots\cdots x_{2(N-1)} x_{2N}$

$$\underbrace{x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \cdots\cdots x_{2^K (N-1)} x_{2^K N}}_{N}$$

Lecture 4: Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Receiver observes

$y_1 y_2 y_3 y_4 \,\cdots\, y_{(N-1)} y_N$

Compare with this one

$$\boxed{d_1 = \sum_{n=1}^{N} |y_n - x_{1n}|^2}$$

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

K

Codebook

$x_{11} x_{12} x_{13} x_{14} \,\cdots\, x_{1(N-1)} x_{1N}$
$x_{21} x_{22} x_{23} x_{24} \,\cdots\, x_{2(N-1)} x_{2N}$

$x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \,\cdots\, x_{2^K (N-1)} x_{2^K N}$

N

Lecture 4: Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$

$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Receiver observes

$y_1 y_2 y_3 y_4 \;\cdots\; y_{(N-1)} y_N$

Compare with this one

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

$\underbrace{\hspace{6em}}_{K}$

Codebook

$x_{11} x_{12} x_{13} x_{14} \;\cdots\; x_{1(N-1)} x_{1N}$

$x_{21} x_{22} x_{23} x_{24} \;\cdots\; x_{2(N-1)} x_{2N}$

$x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \;\cdots\; x_{2^K (N-1)} x_{2^K N}$

$\underbrace{\hspace{6em}}_{N}$

Lecture 4: Capacity

$$y = \sqrt{A}x + n, \; n \sim CN(0, N_0)$$

$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Receiver observes

$y_1 y_2 y_3 y_4 \cdots\cdots y_{(N-1)} y_N$

Compare with this one

$$\boxed{d_{2^K} = \sum_{n=1}^{N} |y_n - x_{2^K n}|^2}$$

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

$\underbrace{\hphantom{1111111 \cdots\cdots 11}}_{K}$

Codebook

$x_{11}x_{12}x_{13}x_{14} \cdots\cdots x_{1(N-1)}x_{1N}$

$x_{21}x_{22}x_{23}x_{24} \cdots\cdots x_{2(N-1)}x_{2N}$

$\underbrace{x_{2^K 1}x_{2^K 2}x_{2^K 3}x_{2^K 4} \cdots\cdots x_{2^K (N-1)}x_{2^K N}}_{N}$

Lecture 4: Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$
$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Receiver observes

$y_1y_2y_3y_4$ ..... $y_{(N-1)}y_N$

Take smallest

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

Information book

000000 00
000000 01
000000 10

1111111 10
1111111 11

K

Codebook

$x_{11}x_{12}x_{13}x_{14}$ ..... $x_{1(N-1)}x_{1N}$
$\boxed{x_{21}x_{22}x_{23}x_{24} \;\ldots\ldots\; x_{2(N-1)}x_{2N}}$

$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4}$ ..... $x_{2^K(N-1)}x_{2^KN}$

N

Lecture 4: Capacity

$$y = \sqrt{A}x + n, \;\; n \sim CN(0, N_0)$$

$$C = \log_2\left(1 + \frac{A}{N_0}\right)$$

Receiver observes

$y_1 y_2 y_3 y_4 \;\cdots\; y_{(N-1)} y_N$

Take smallest

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

Information book

000000 00

$\boxed{000000 \; ..... \; 01}$

000000 10

So data is this one

1111111 10

1111111 11

$\underbrace{\hspace{6em}}_{K}$

Codebook

$x_{11} x_{12} x_{13} x_{14} \;\cdots\; x_{1(N-1)} x_{1N}$

$\boxed{x_{21} x_{22} x_{23} x_{24} \;\cdots\; x_{2(N-1)} x_{2N}}$

$x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \;\cdots\; x_{2^K (N-1)} x_{2^K N}$

$\underbrace{\hspace{6em}}_{N}$

Lecture 4: Capacity

Receiver observes

$y_1 y_2 y_3 y_4 \ldots\ldots y_{(N-1)} y_N$

Take smallest

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

This is ML decoding and is optimal

Capacity means the following

Information book

000000 00

$\boxed{000000 \ldots\ldots 01}$

000000 10

So data is this one

1111111 10

1111111 11

K

Codebook

$x_{11} x_{12} x_{13} x_{14} \ldots\ldots x_{1(N-1)} x_{1N}$

$\boxed{x_{21} x_{22} x_{23} x_{24} \ldots\ldots x_{2(N-1)} x_{2N}}$

$x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \ldots\ldots x_{2^K (N-1)} x_{2^K N}$

N

Lecture 4: Capacity

Receiver observes

$y_1 y_2 y_3 y_4 \ldots\ldots y_{(N-1)} y_N$

Take smallest

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

Information book

000000 00

$\boxed{000000 \ \ldots\ldots \ 01}$

000000 10

So data is this one

1111111 10

1111111 11

$\underbrace{\qquad\qquad\qquad}_{K}$

This is ML decoding and is optimal

Capacity means the following

1. If K/N ≤ C, and K->∞ then
   Prob(Correct detection)=1

Codebook

$x_{11} x_{12} x_{13} x_{14} \ldots\ldots x_{1(N-1)} x_{1N}$

$\boxed{x_{21} x_{22} x_{23} x_{24} \ldots\ldots x_{2(N-1)} x_{2N}}$

$\underbrace{x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \ldots\ldots x_{2^K (N-1)} x_{2^K N}}_{N}$

Lecture 4: Capacity

Receiver observes

$y_1 y_2 y_3 y_4 \; \cdots\cdots \; y_{(N-1)} y_N$

Take smallest

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

Information book

$000000 \; \cdots\cdots \; 00$

$\boxed{000000 \; \cdots\cdots \; 01}$

$000000 \; \cdots\cdots \; 10$

So data is this one

$1111111 \; \cdots\cdots \; 10$

$\underbrace{1111111 \; \cdots\cdots \; 11}_{K}$

This is ML decoding and is optimal

Capacity means the following

1. If $K/N \leq C$, and $K \to \infty$ then
   Prob(Correct detection)=1
2. If $K/N > C$, then
   Prob(Incorrect detection)=1

Codebook

$x_{11} x_{12} x_{13} x_{14} \; \cdots\cdots \; x_{1(N-1)} x_{1N}$

$\boxed{x_{21} x_{22} x_{23} x_{24} \; \cdots\cdots \; x_{2(N-1)} x_{2N}}$

$\underbrace{x_{2^K 1} x_{2^K 2} x_{2^K 3} x_{2^K 4} \; \cdots\cdots \; x_{2^K (N-1)} x_{2^K N}}_{N}$

Lecture 4: Capacity

To reach C, code-symbols must be
Random complex Gaussian variables
That is, generate codebook randomly

If it is generated with, say, 16QAM
C cannot be reached

Receiver observes

$y_1y_2y_3y_4 \cdots y_{(N-1)}y_N$

Take smallest

$$\boxed{d_2 = \sum_{n=1}^{N} |y_n - x_{2n}|^2}$$

Information book

000000 00
$\boxed{\text{000000 ..... 01}}$
000000 10

So data is this one

1111111 10
1111111 11

$\underbrace{\hspace{6em}}_{K}$

Codebook

$x_{11}x_{12}x_{13}x_{14} \cdots x_{1(N-1)}x_{1N}$

$\boxed{x_{21}x_{22}x_{23}x_{24} \cdots x_{2(N-1)}x_{2N}}$

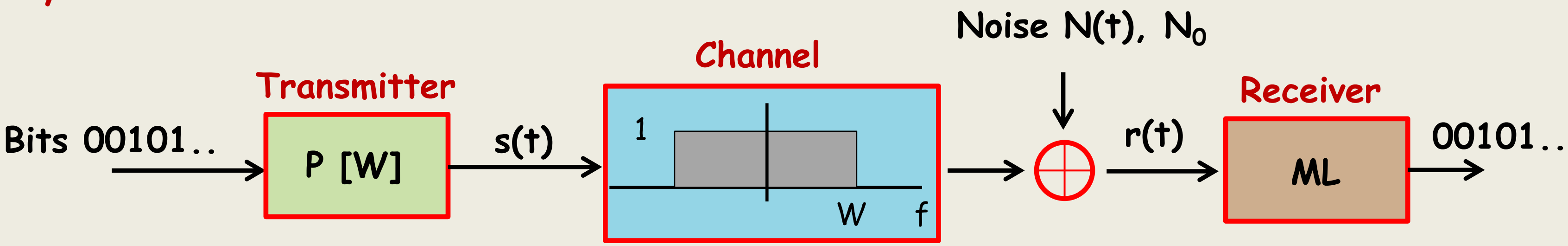
$x_{2^K1}x_{2^K2}x_{2^K3}x_{2^K4} \cdots x_{2^K(N-1)}x_{2^KN}$

$\underbrace{\hspace{10em}}_{N}$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

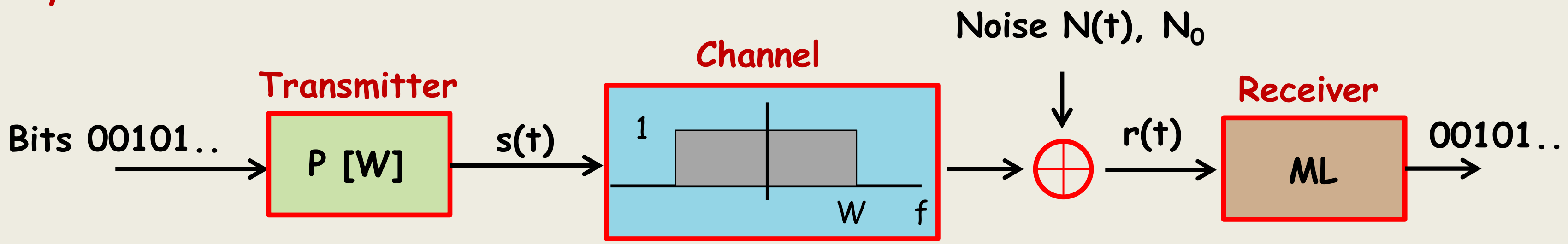
System model:



Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



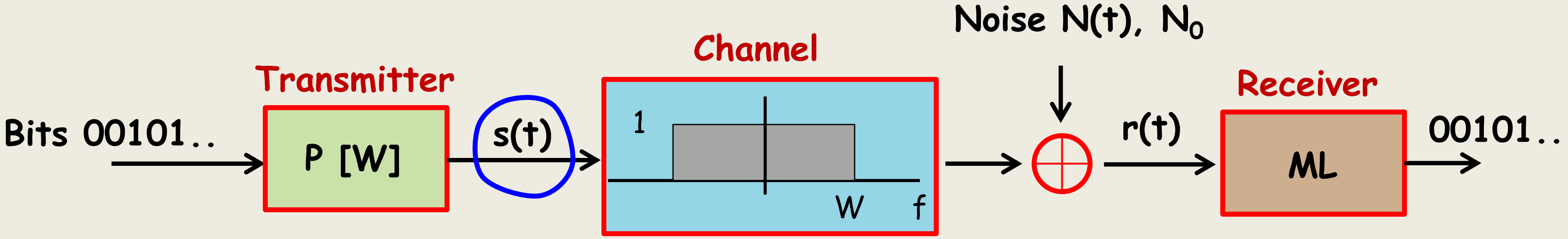
Interpretation of capacity:

Given a transmission of length T (seconds)

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



Interpretation of capacity:
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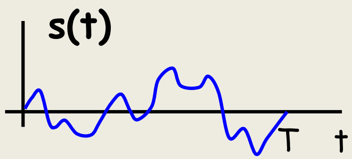
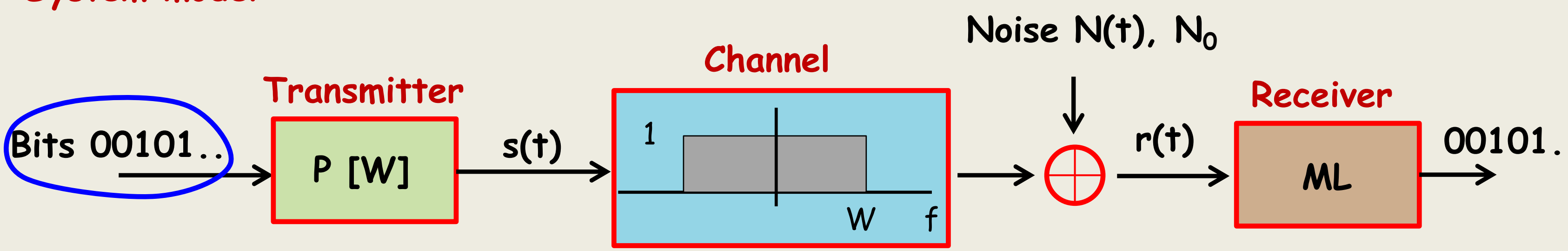


Lecture 4: Capacity

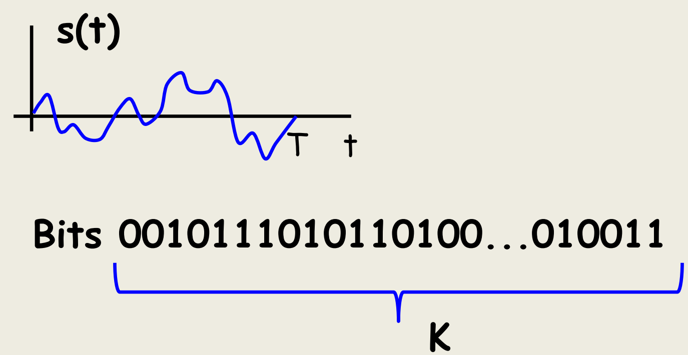
Extension to continuous channel (Shannon '48)

System model:



Interpretation of capacity:
Given a transmission of length T (seconds)

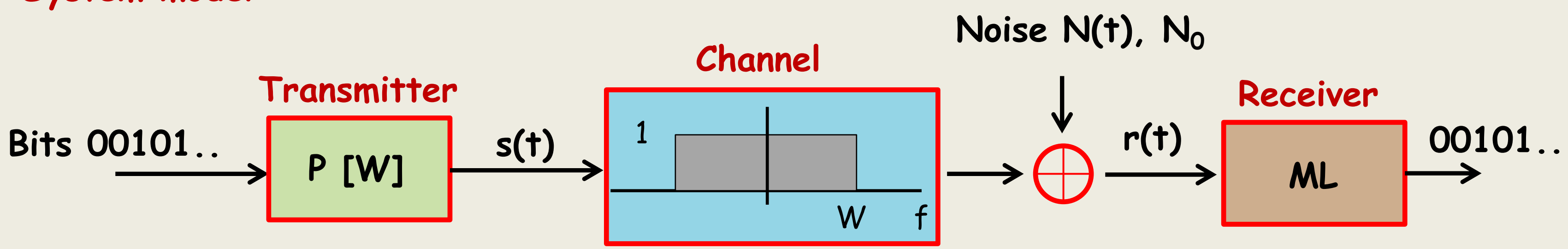
And a number of bits K



Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

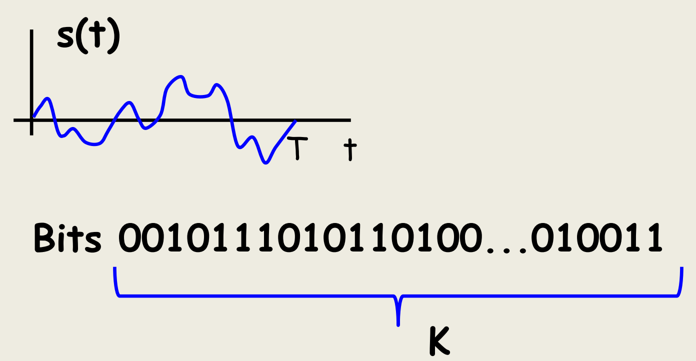
System model:



Interpretation of capacity:
Given a transmission of length T (seconds)

And a number of bits K

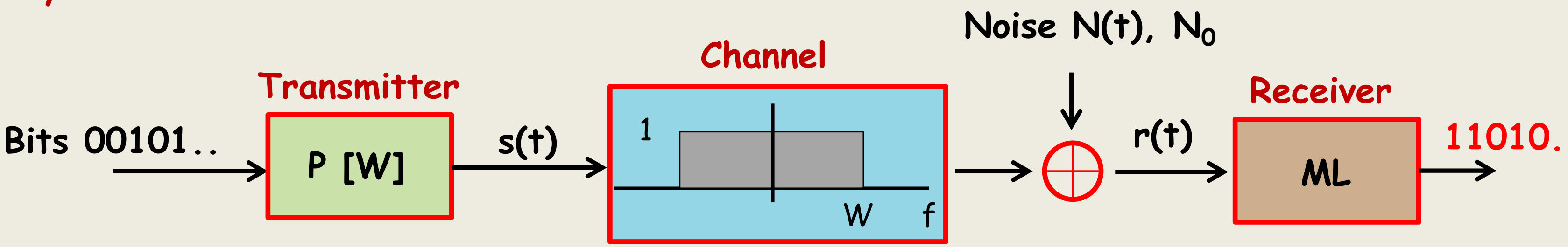
The bitrate is: K/T [bit/sec]



Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



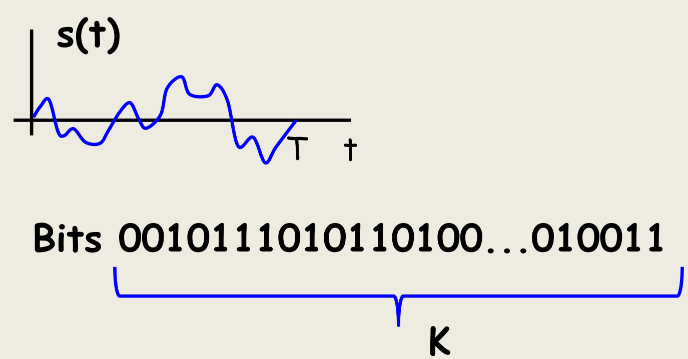
Interpretation of capacity:

Given a transmission of length T (seconds)

And a number of bits K

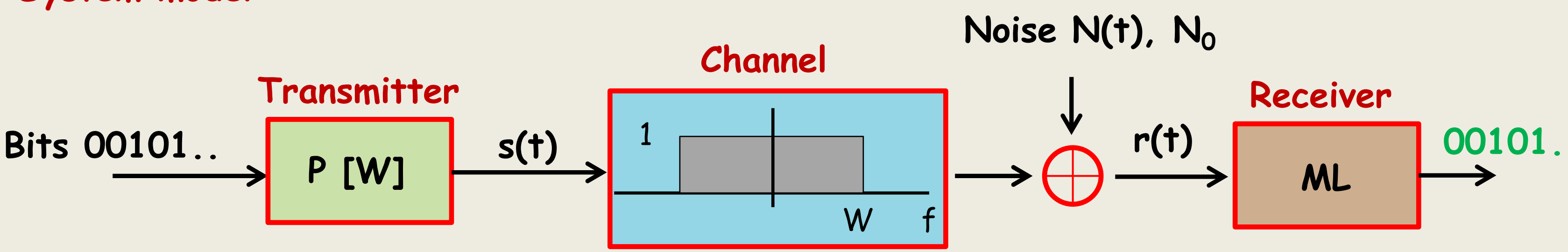
The bitrate is: K/T [bit/sec]

If K/T is too high, then many errors

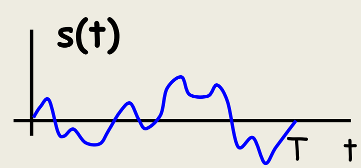


Extension to continuous channel (Shannon '48)

System model:



Interpretation of capacity:
Given a transmission of length T (seconds)



And a number of bits K

Bits $\underbrace{0010111010110100\ldots010011}_{K}$

The bitrate is: K/T [bit/sec]

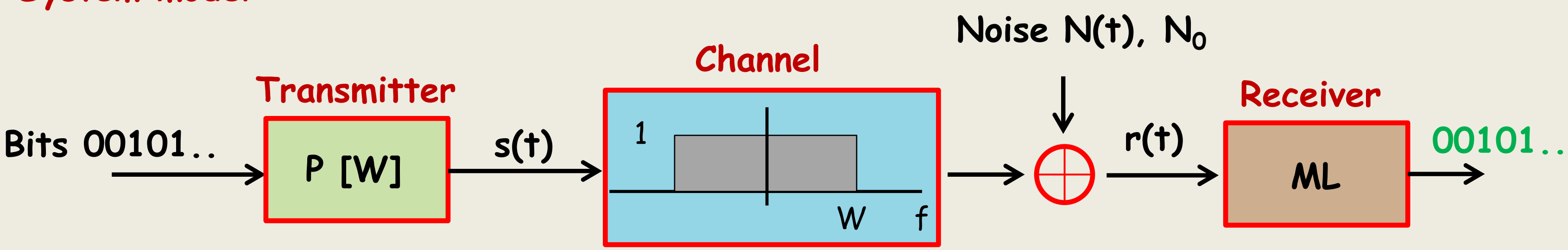
If K/T is too high, then many errors
Shannon proved: Possible to have NO ERRORS if,

1) $T \to \infty$ 2) $\lim_{T\to\infty} \frac{K}{T} = C = W \log_2\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

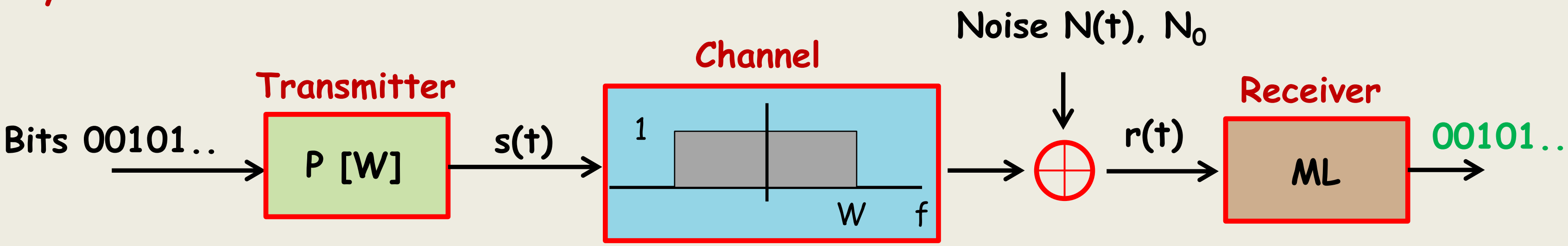
Facts:

1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

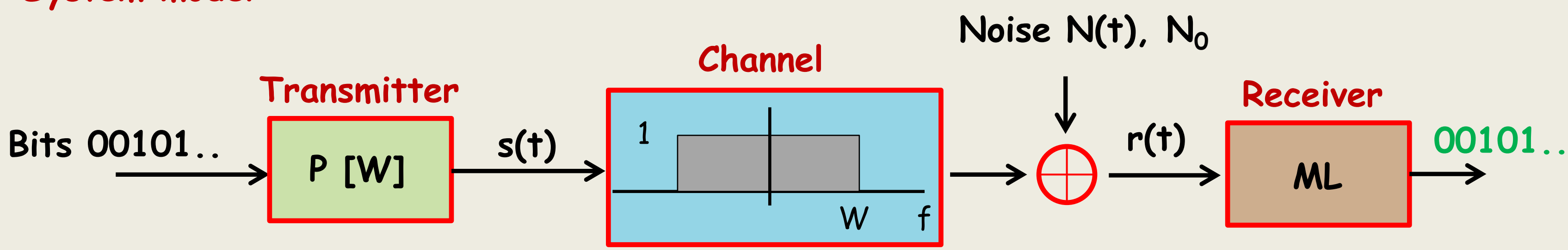
Facts:

1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



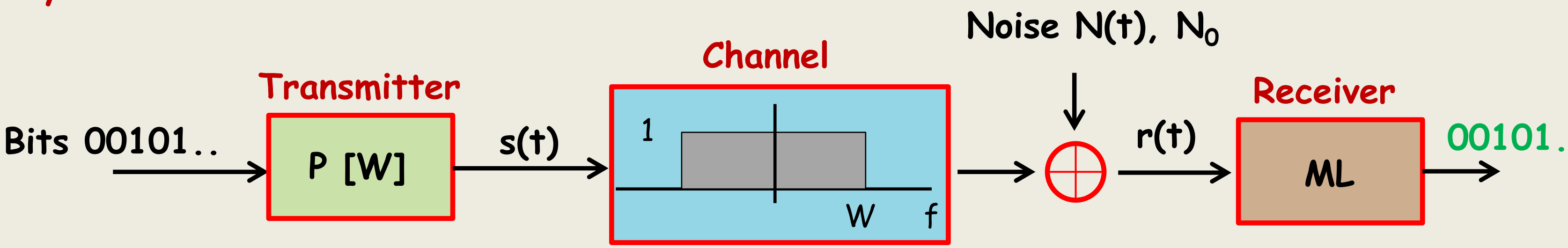
$$C = W \log_2\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

Facts:

1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)
3. There is no parameter called $d^2_{\min}$

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W} \right)$$

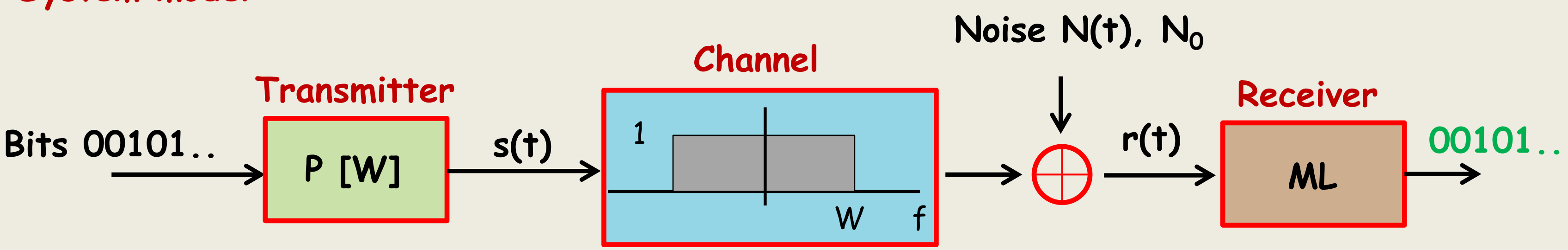
Facts:

1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)
3. There is no parameter called d^2_{min}
4. When W grows:

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W} \right)$$

Grows linearly

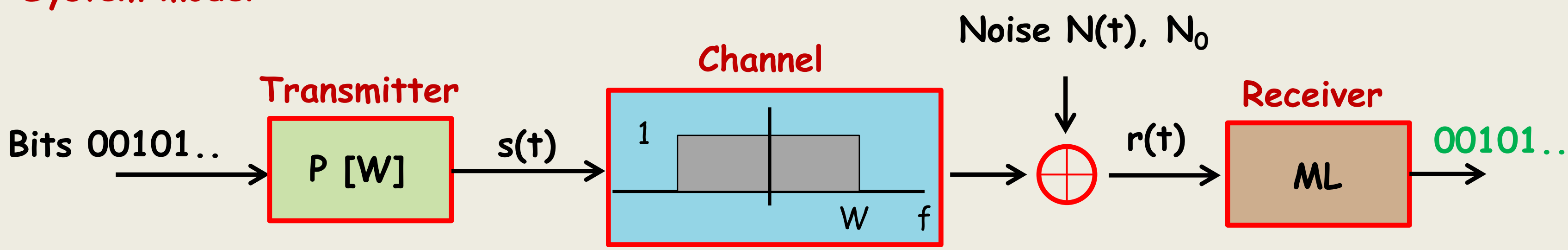
Facts:

1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)
3. There is no parameter called $d^2_{\min}$

4. When W grows:

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

Grows linearly

decreases

Facts:

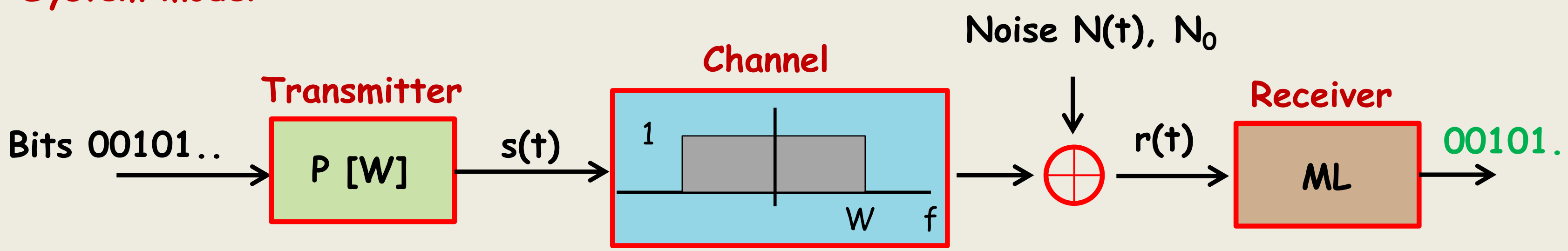
1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)
3. There is no parameter called $d^2_{\min}$

4. When W grows:

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



Decreases logarihtmically

$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

Grows linearly

Facts:

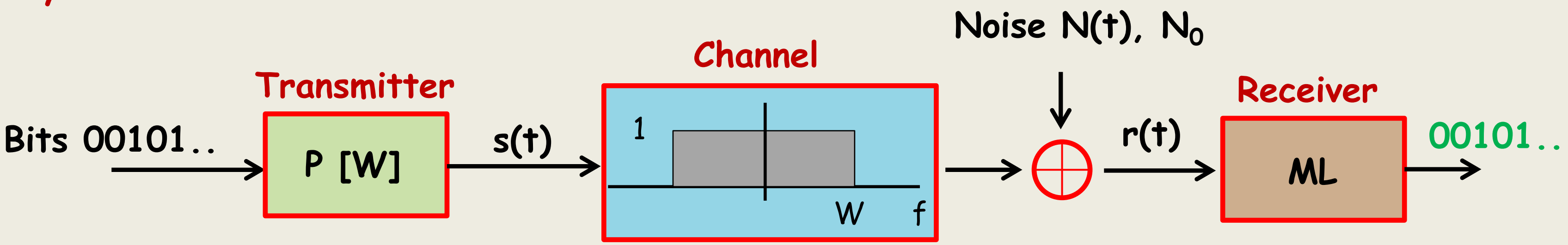
1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)
3. There is no parameter called $d^2_{\min}$

4. When W grows:

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

Grows

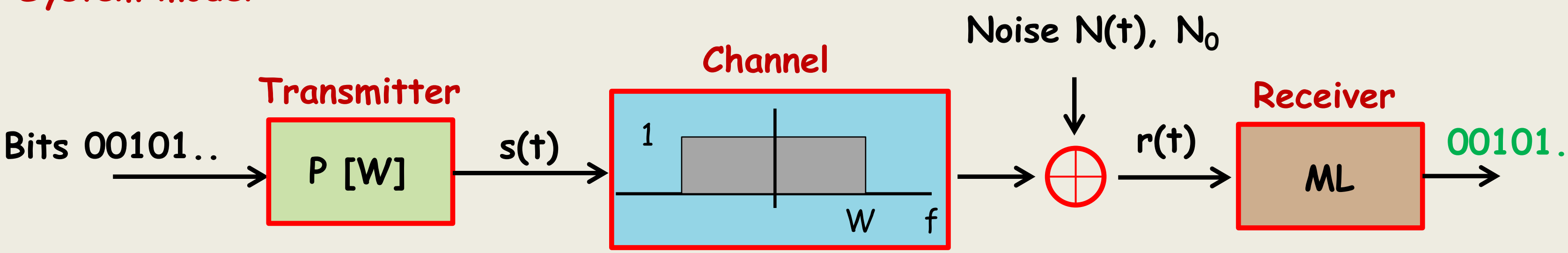
Facts:

1. C is not power, nor bandwidth efficiency
   (C is not dimensionless)
2. Not easy to reach C
   (i.e., to find a set of s(t) signals)
3. There is no parameter called $d^2_{\min}$
4. When W grows: C grows

Lecture 4: Capacity

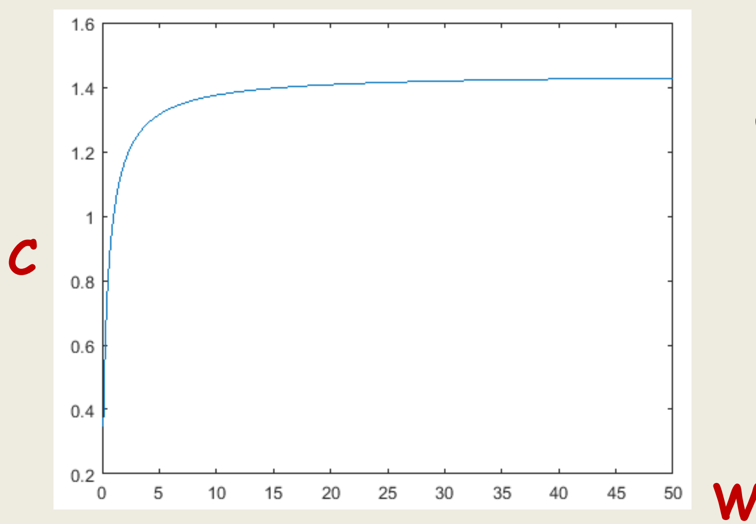
Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

Example, P/N_0 =1

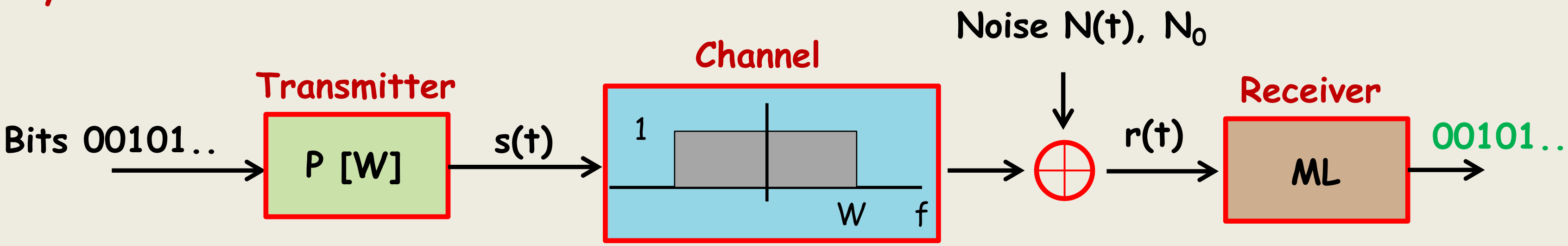


But it grows to a limit

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

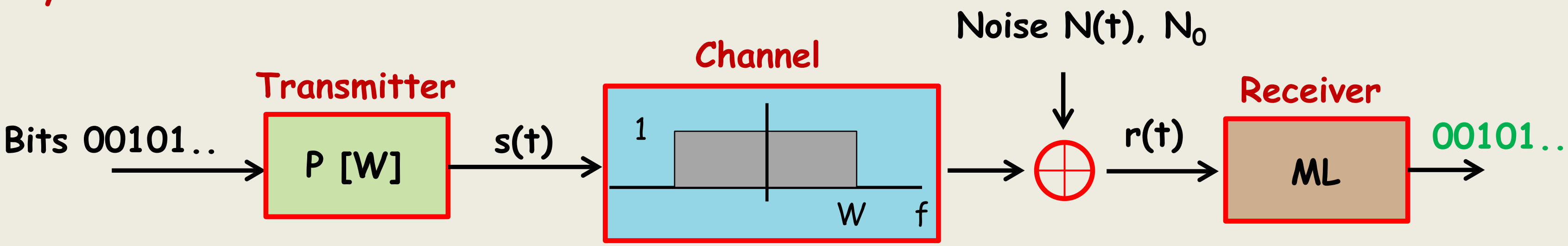
Standard limit

$$\lim_{x \to \infty} x \ln \left(1 + \frac{A}{x}\right) = A$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

Standard limit

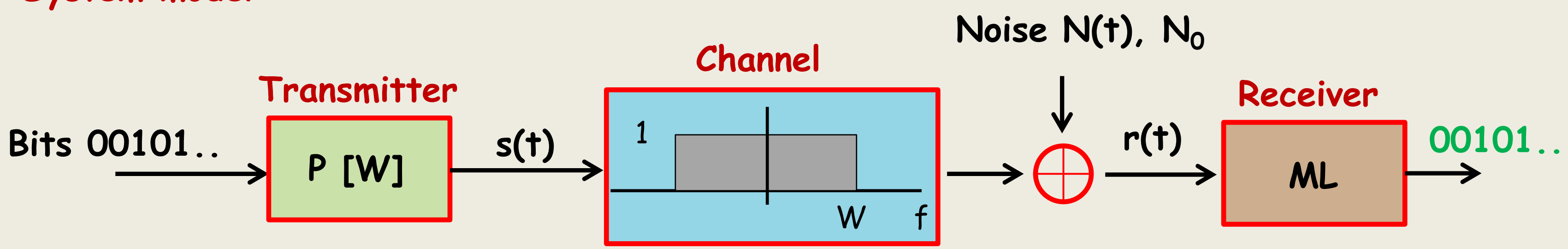
Identify x with W

$$\lim_{x\to\infty} x \ln\left(1 + \frac{A}{x}\right) = A$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = W \log_2\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

<u>**Standard limit**</u>

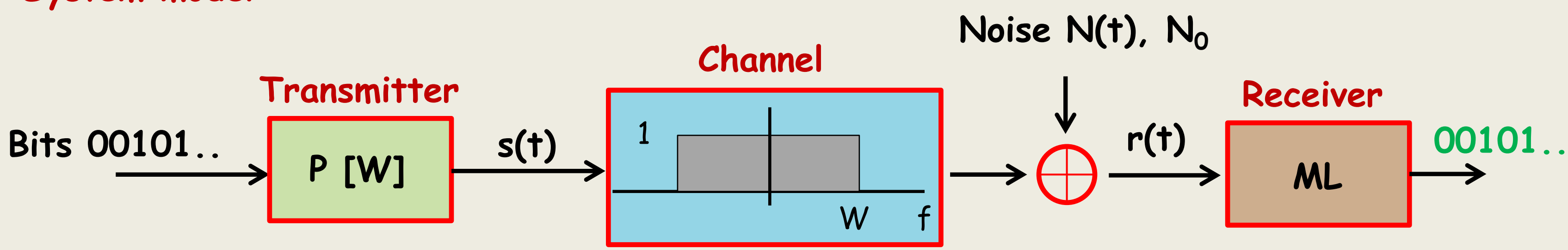
Identify x with W
Identify A with P/N_0

$$\lim_{x\to\infty} x \ln\left(1 + \frac{A}{x}\right) = A$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

<u>Standard limit</u>

$$\lim_{x \to \infty} x \ln\left(1 + \frac{A}{x}\right) = A$$

Identify x with W

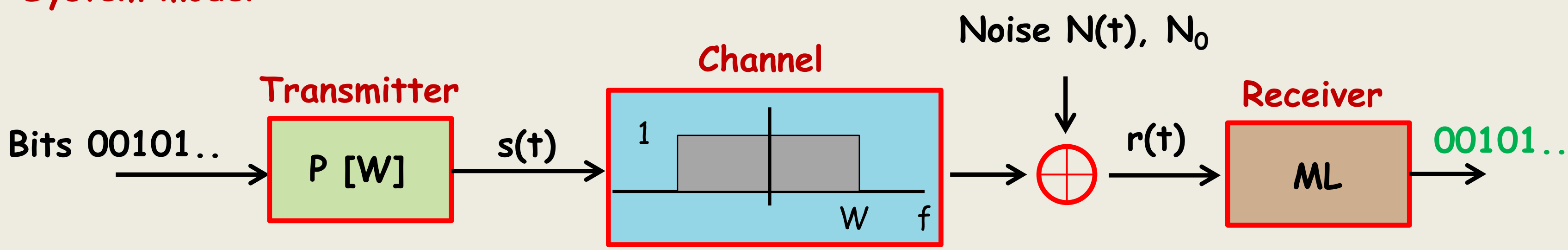
Identify A with P/N_0

Express $\log_2(x)$ as $\ln(x)/\ln(2)$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

Standard limit

$$\lim_{x \to \infty} x \ln\left(1 + \frac{A}{x}\right) = A$$

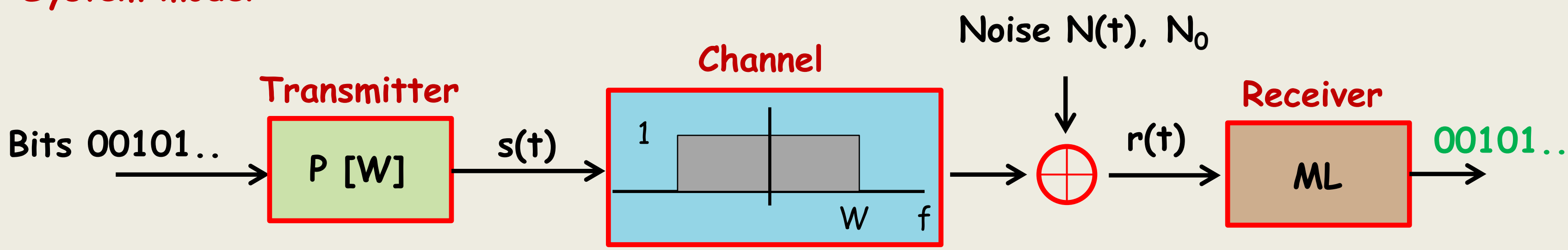
Carry out limit

$$C_{\max} = \lim_{W \to \infty} \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right) =$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

Standard limit

$$\lim_{x\to\infty} x \ln\left(1 + \frac{A}{x}\right) = A$$

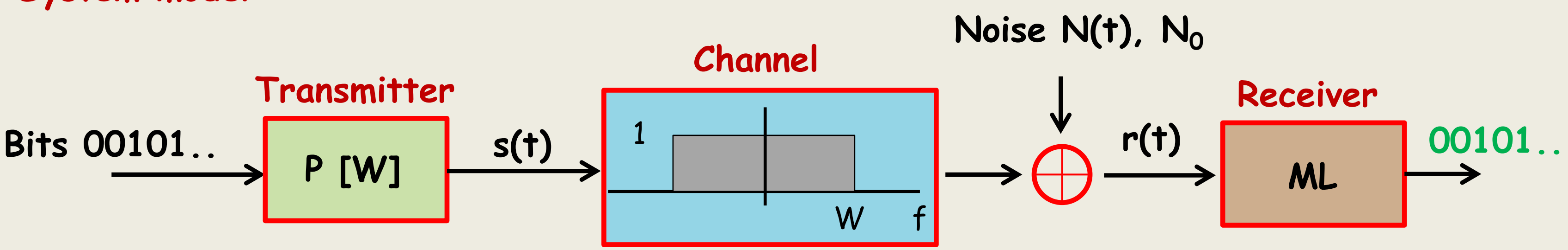
Carry out limit

$$C_{\max} = \lim_{W\to\infty} \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right) = \frac{\mathcal{P}}{N_0 \ln(2)}$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:



$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

What is the limit?

Standard limit

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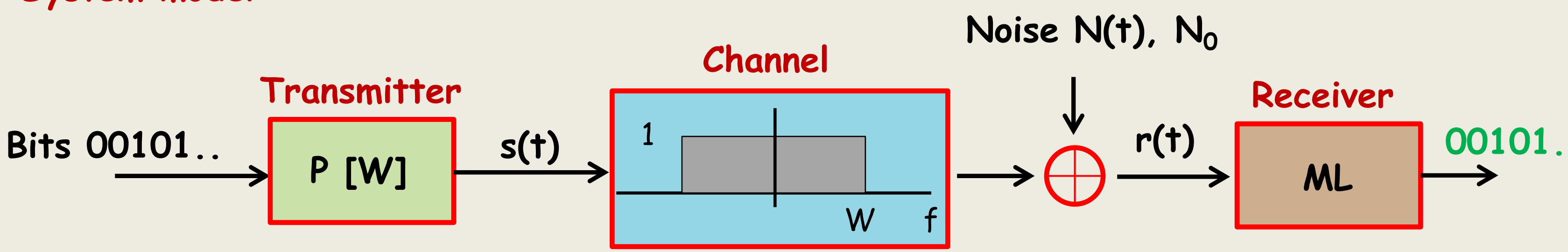
Carry out limit

$$C_{\max} = \lim_{W\to\infty} \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right) = \frac{\mathcal{P}}{N_0 \ln(2)}$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

System model:

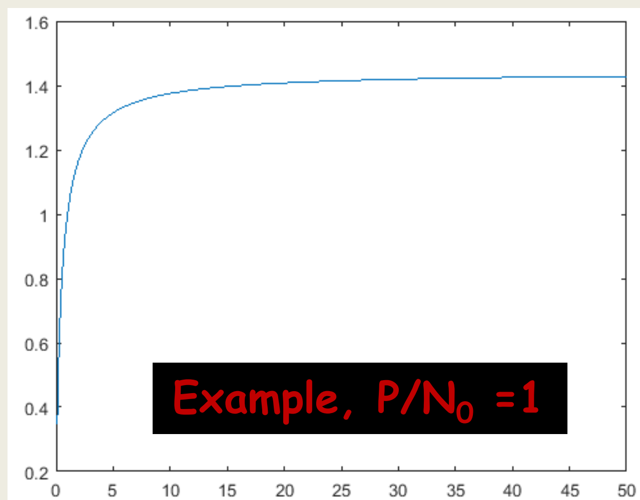


$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

<u>Standard limit</u>

$$\lim_{x\to\infty} x \ln\left(1 + \frac{A}{x}\right) = A$$

C



W

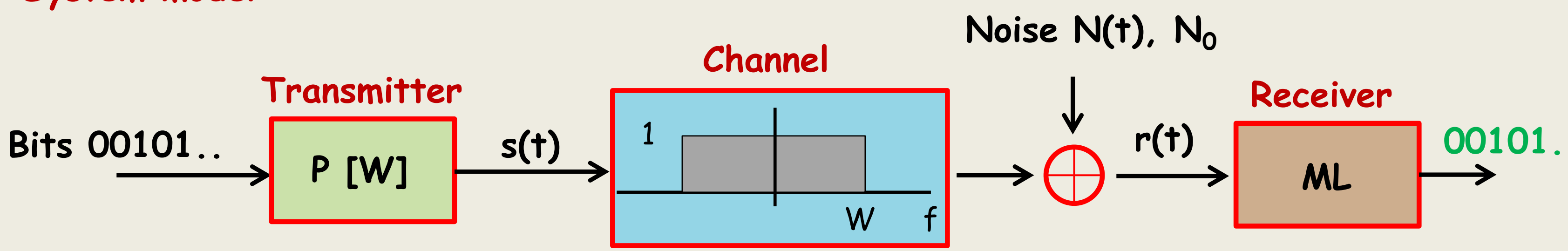
But it grows to a limit

$$C_{\max} = \frac{\mathcal{P}}{N_0 \ln(2)}$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

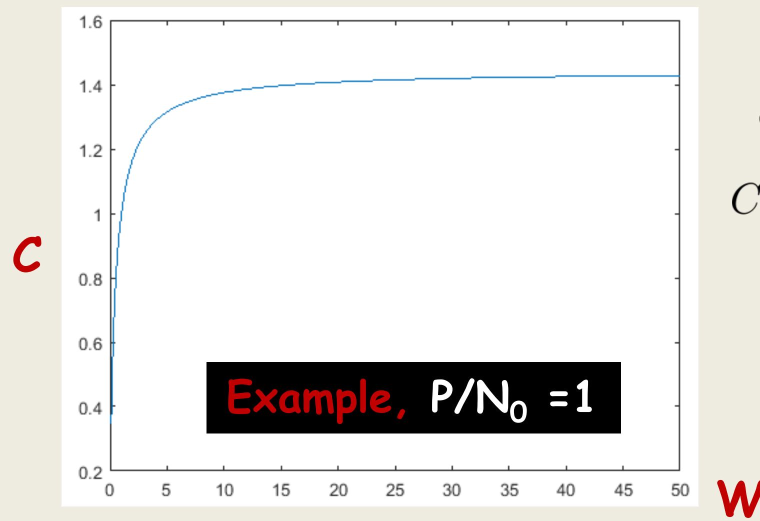
System model:



$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

<u>Standard limit</u>

$$\lim_{x\to\infty} x \ln\left(1 + \frac{A}{x}\right) = A$$



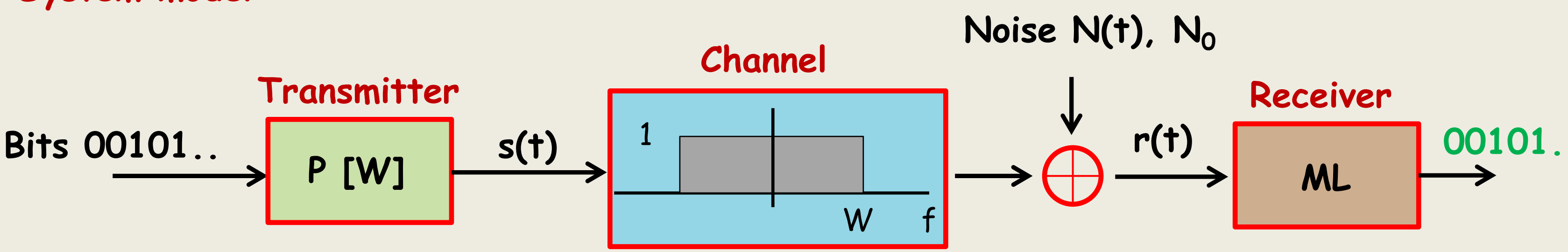
But it grows to a limit

$$C_{\max} = \frac{\mathcal{P}}{N_0 \ln(2)}$$

$$= \frac{1}{\ln(2)}$$

Extension to continuous channel (Shannon '48)

System model:



$$C = \frac{W}{\ln(2)} \ln\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

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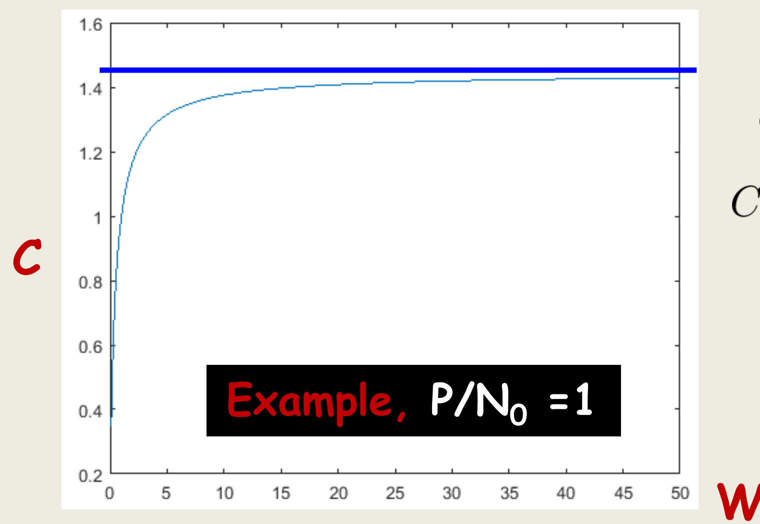

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$$= 1.4427$$

Lecture 4: Capacity

Extension to continuous channel (Shannon '48)

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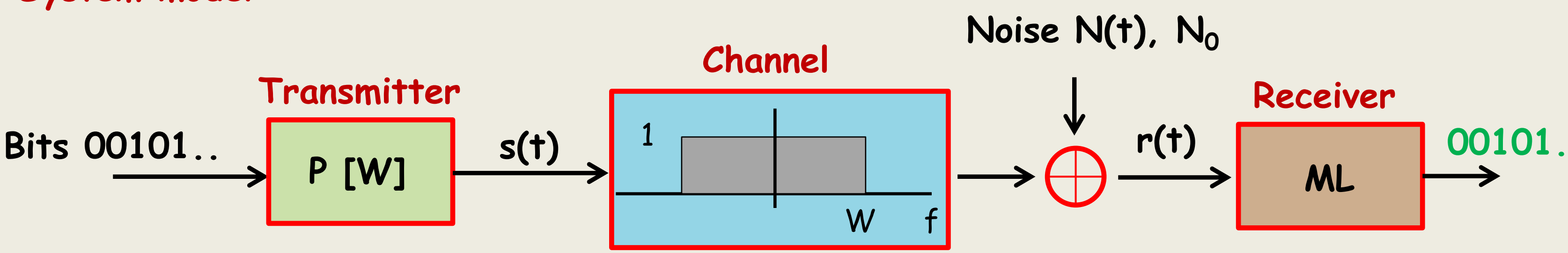

Summary

1. We stated that the capacity of the above is $C = W \log_2\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$ bits/second

2. We proved that for infinite bandwidth, the capacity is $C_{\max} = \frac{\mathcal{P}}{N_0 \ln(2)}$

Lecture 4: Capacity

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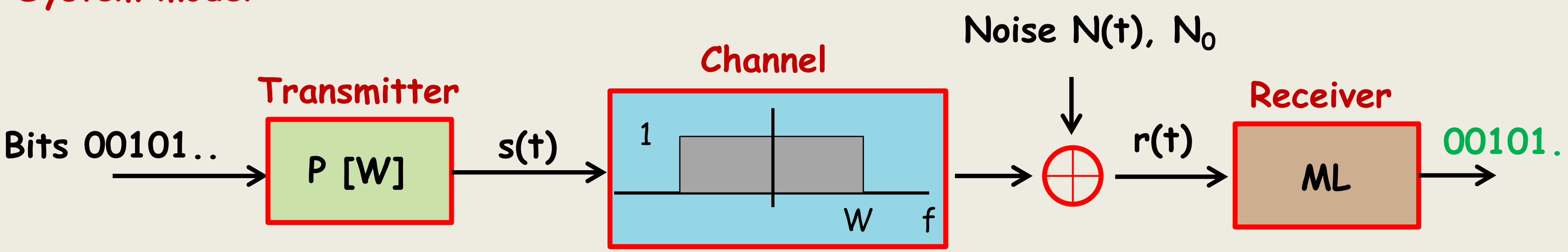

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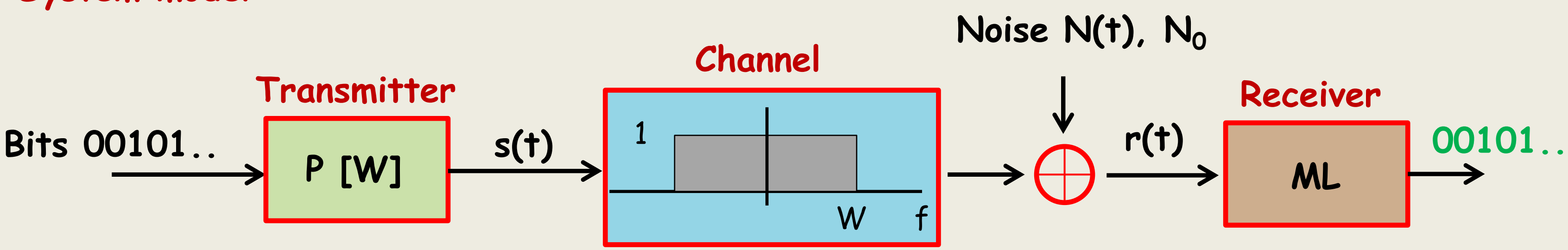

Bandwidth efficiency

By definition, $\frac{C}{W} = \log_2\left(1 + \frac{\mathcal{P}}{N_0 W}\right)$

Lecture 4: Capacity

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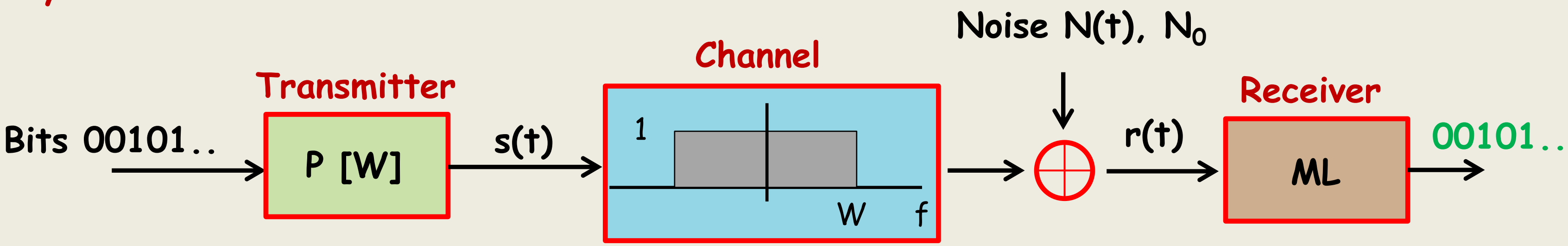

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Lecture 4: Capacity

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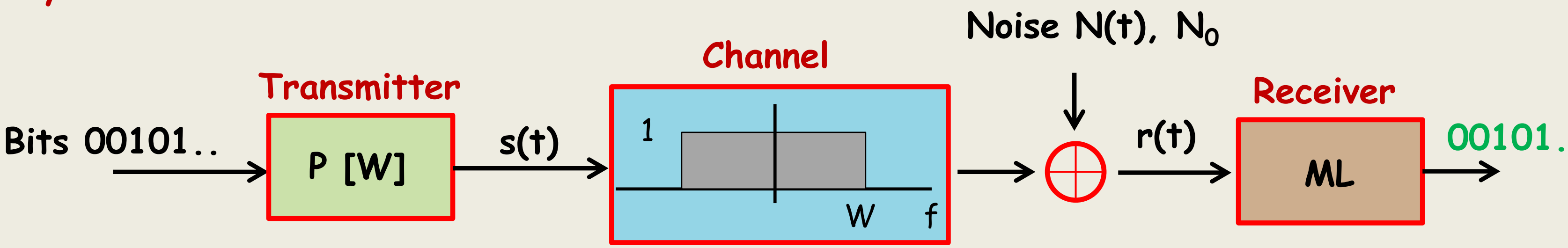

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Lecture 4: Capacity

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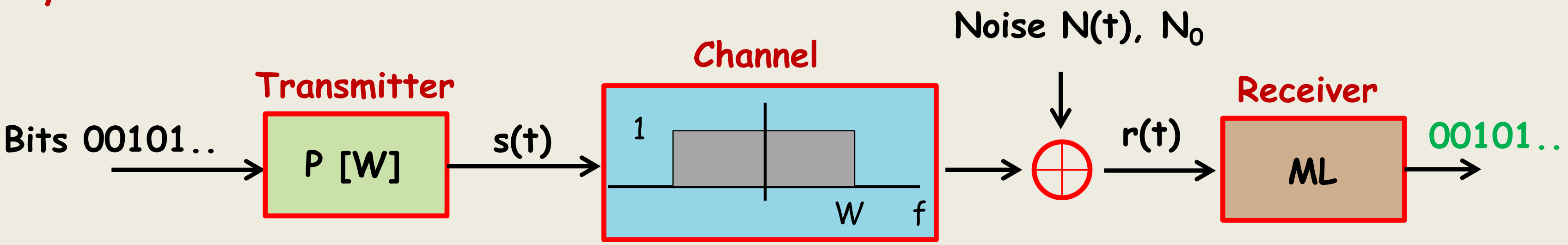

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Bandwidth vs. Power efficiency

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Lecture 4: Capacity

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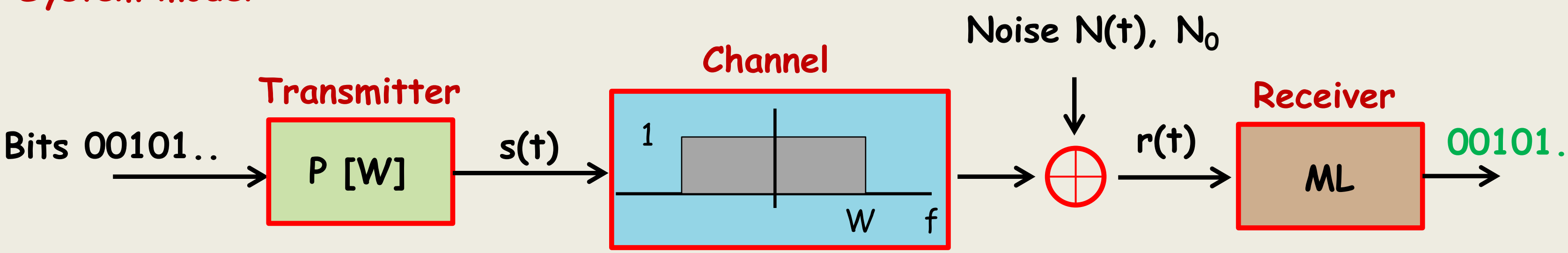

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Lecture 4: Capacity

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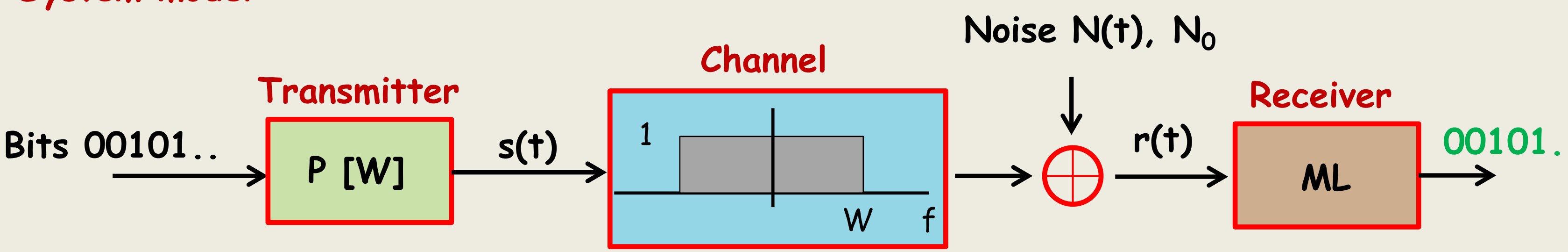

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Lecture 4: Capacity

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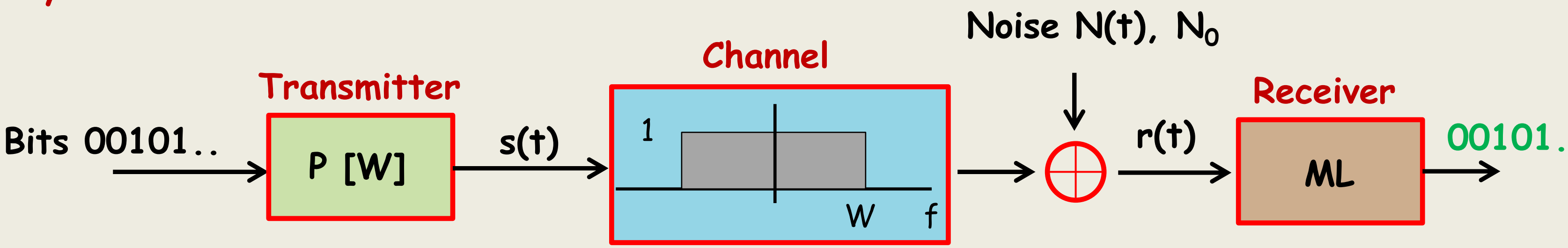

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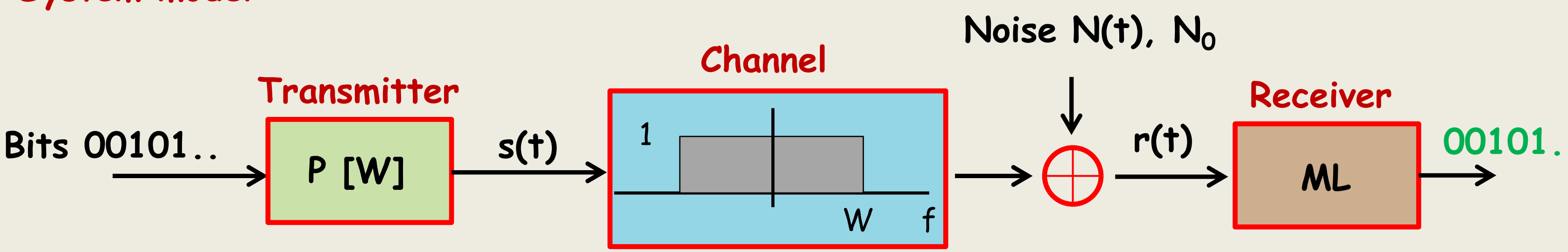

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Lecture 4: Capacity

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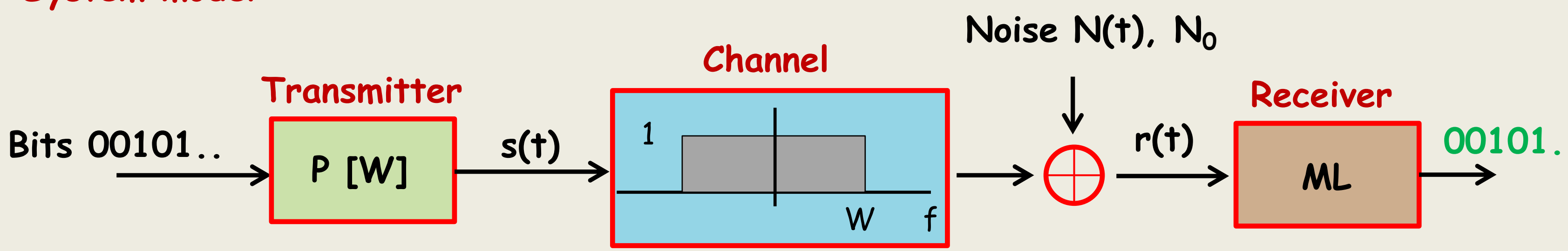

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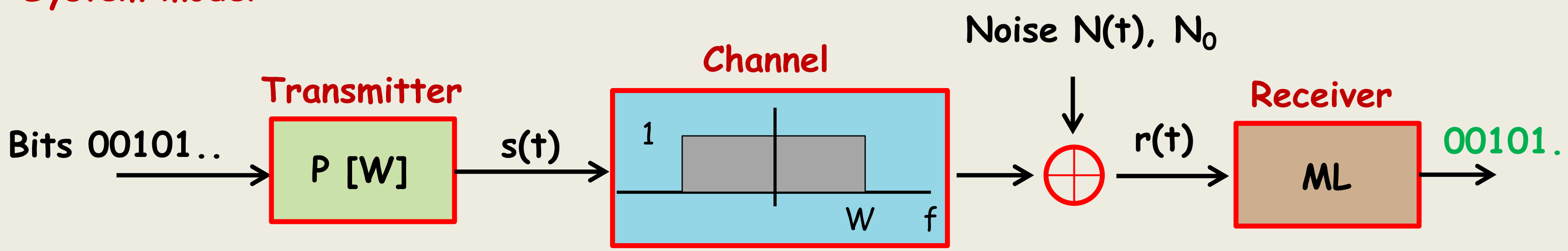

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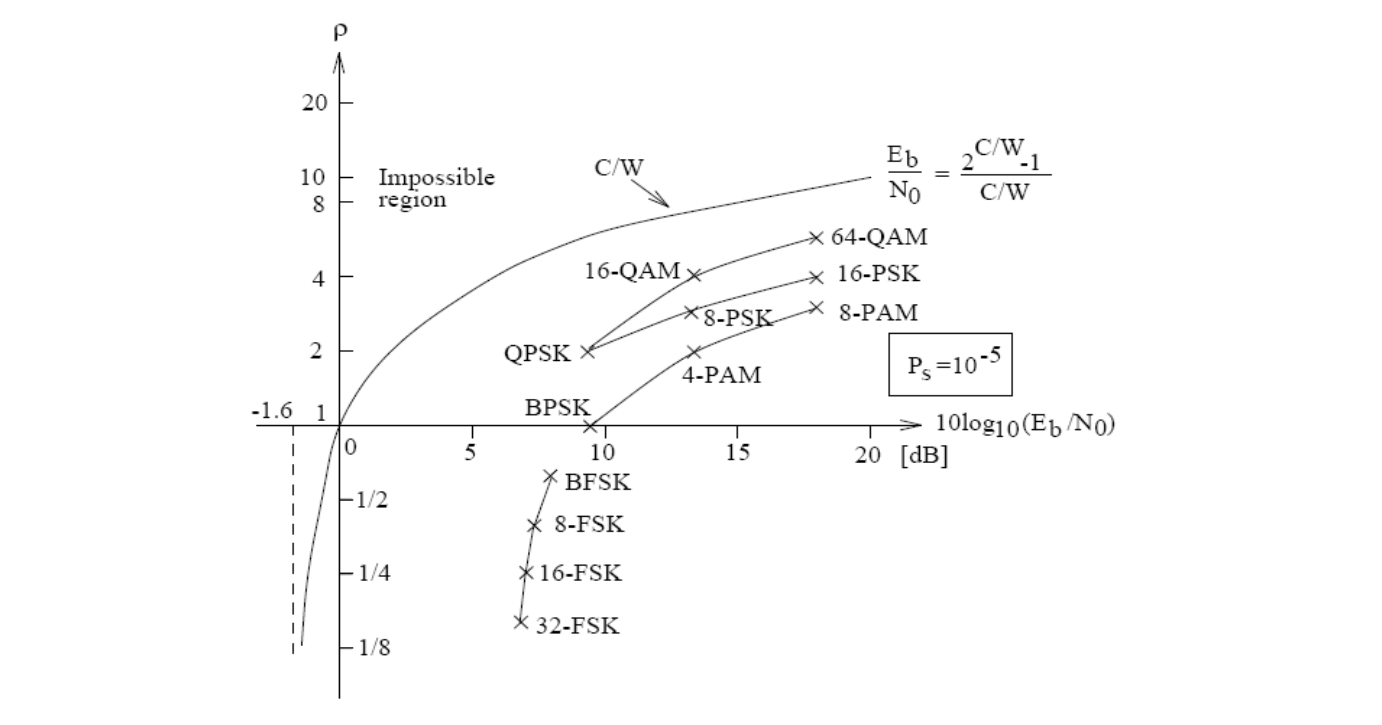
Thus $\boxed{\lim_{C/W \to 0} \frac{E_b}{N_0} \geq \ln(2)}$ $(= -1.6\text{dB})$

Lecture 4: Capacity



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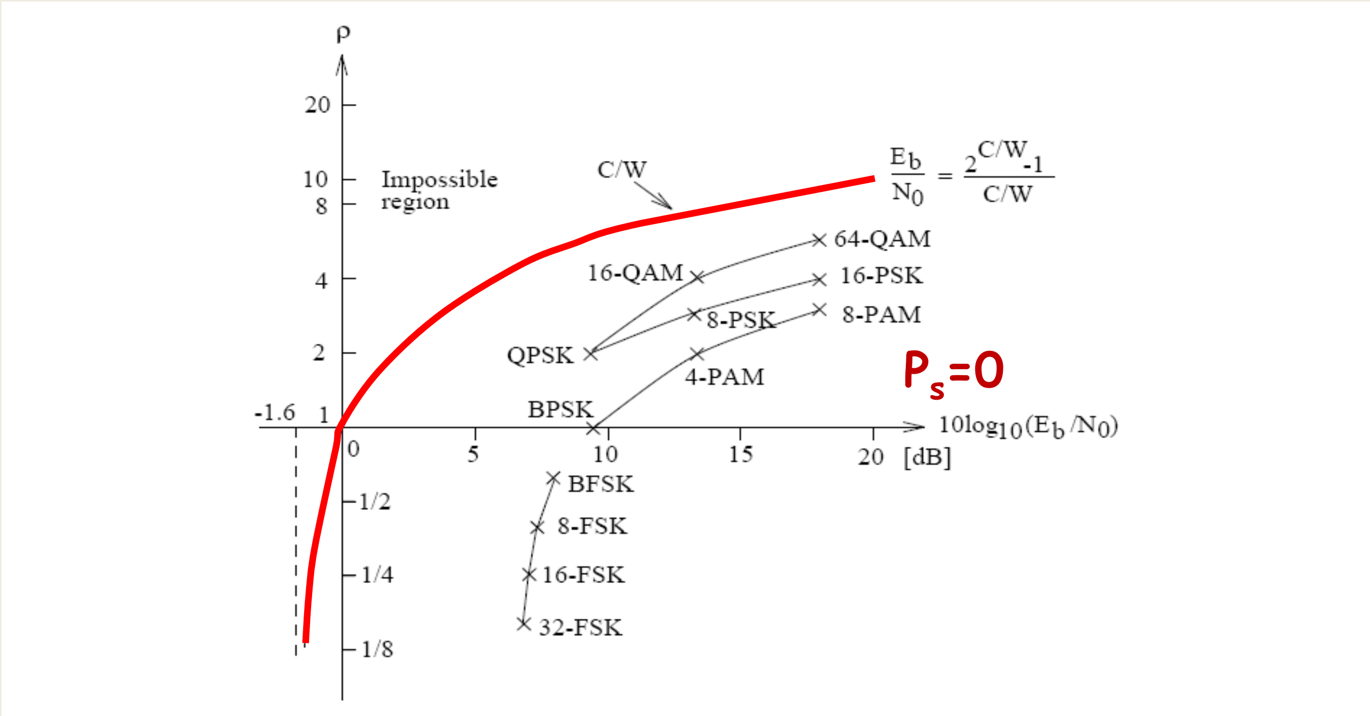

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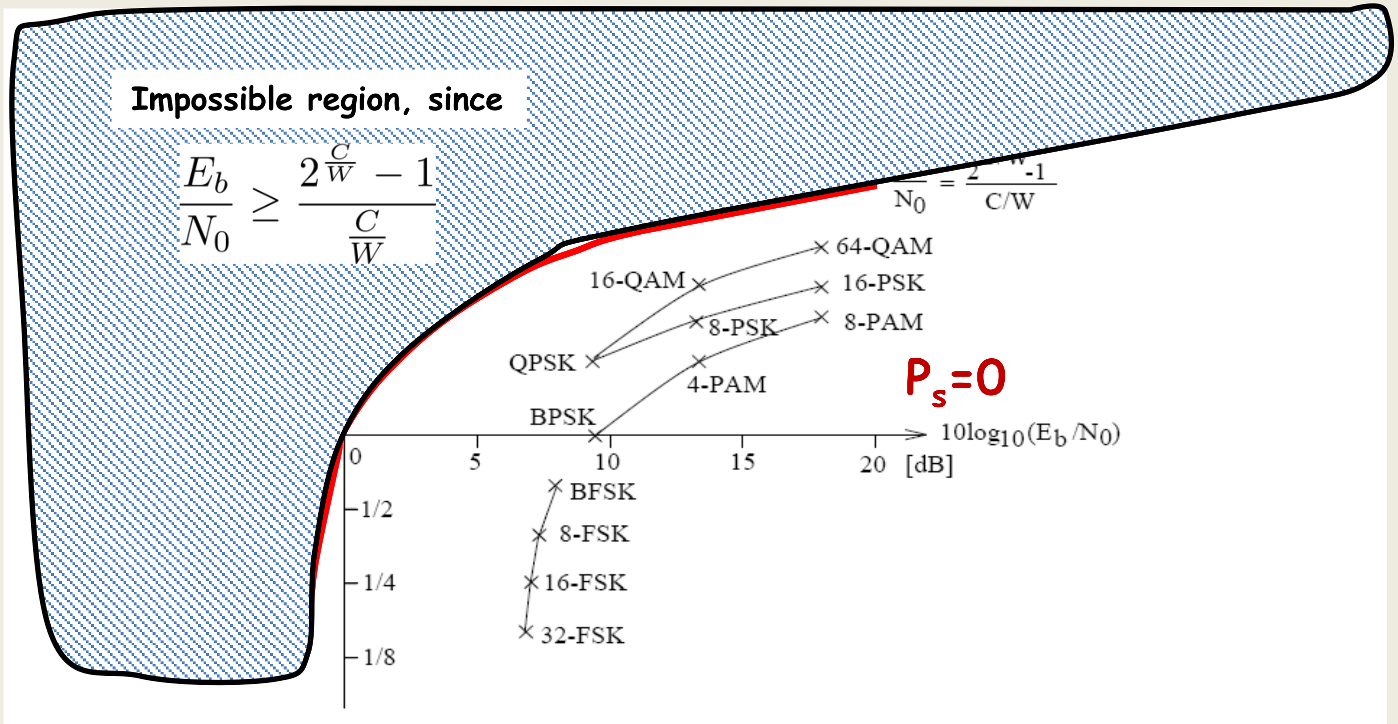

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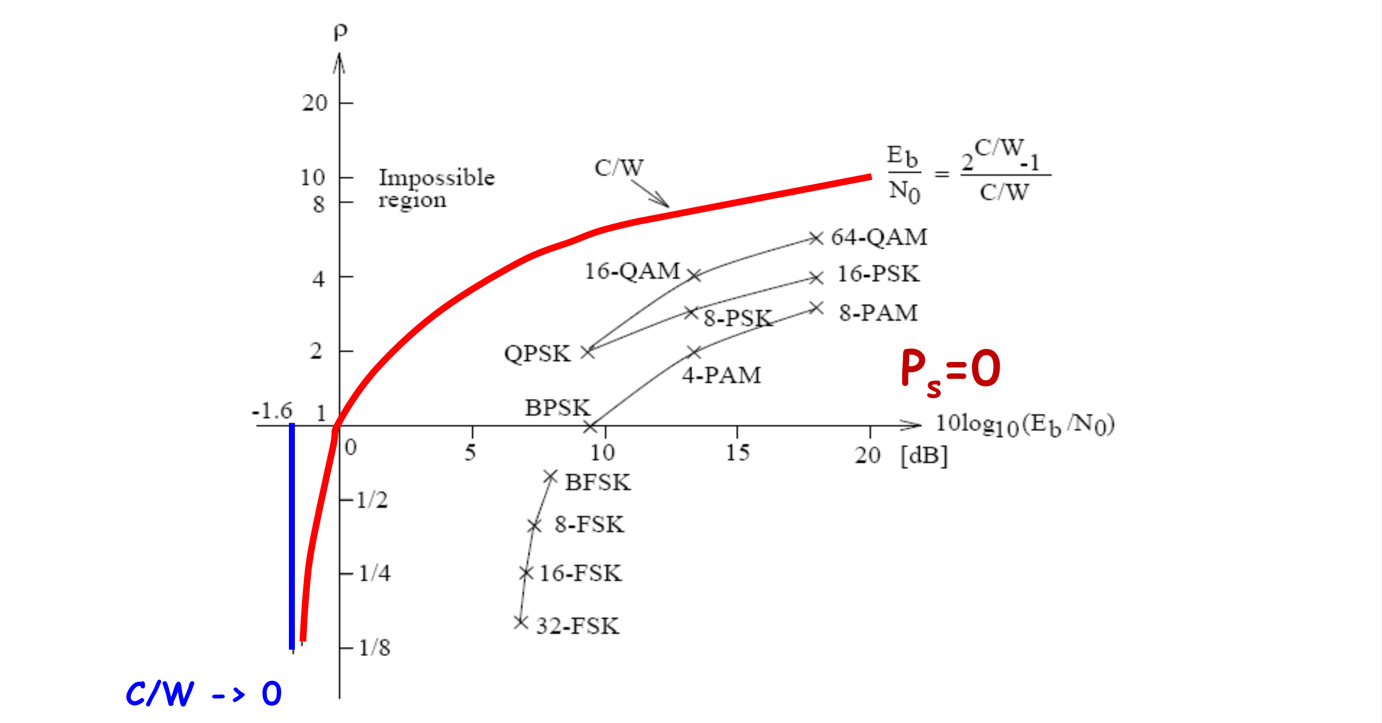
NOTE: Plot does not tell what the capacity is in bit/sec

Lecture 4: Capacity



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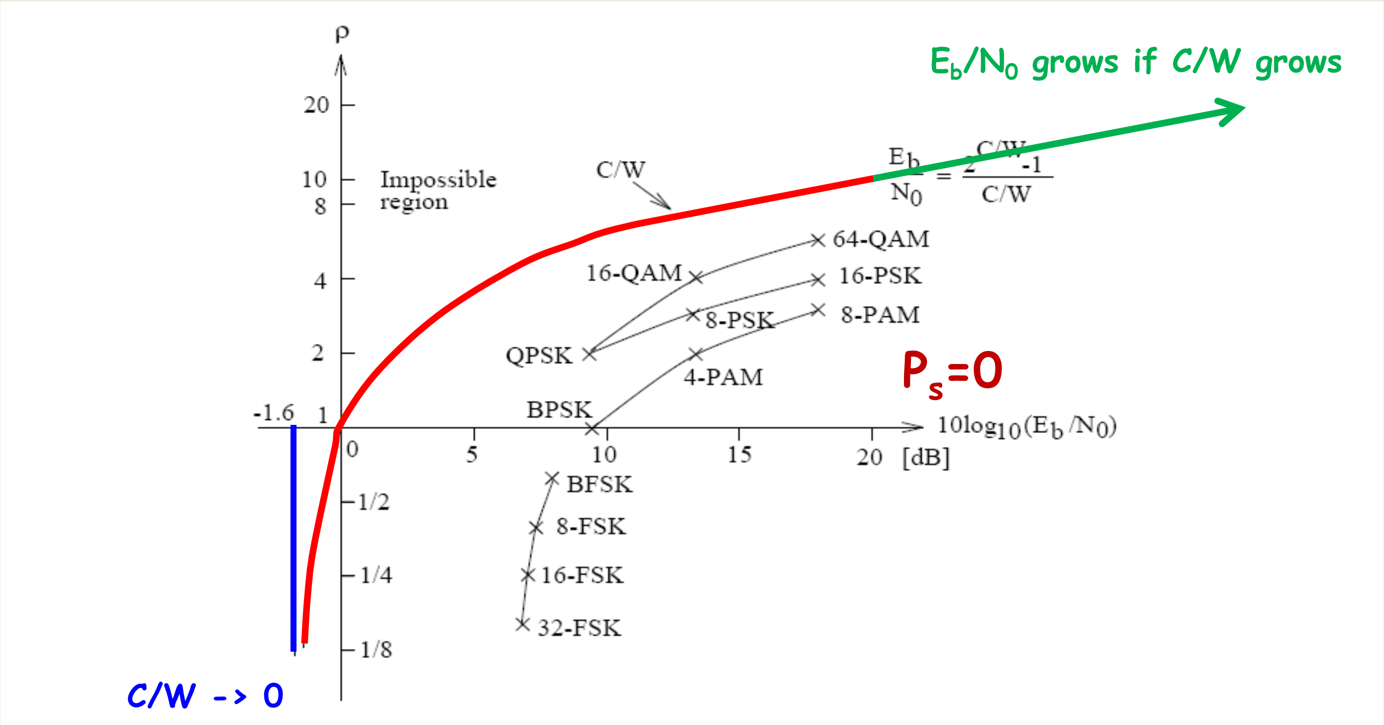

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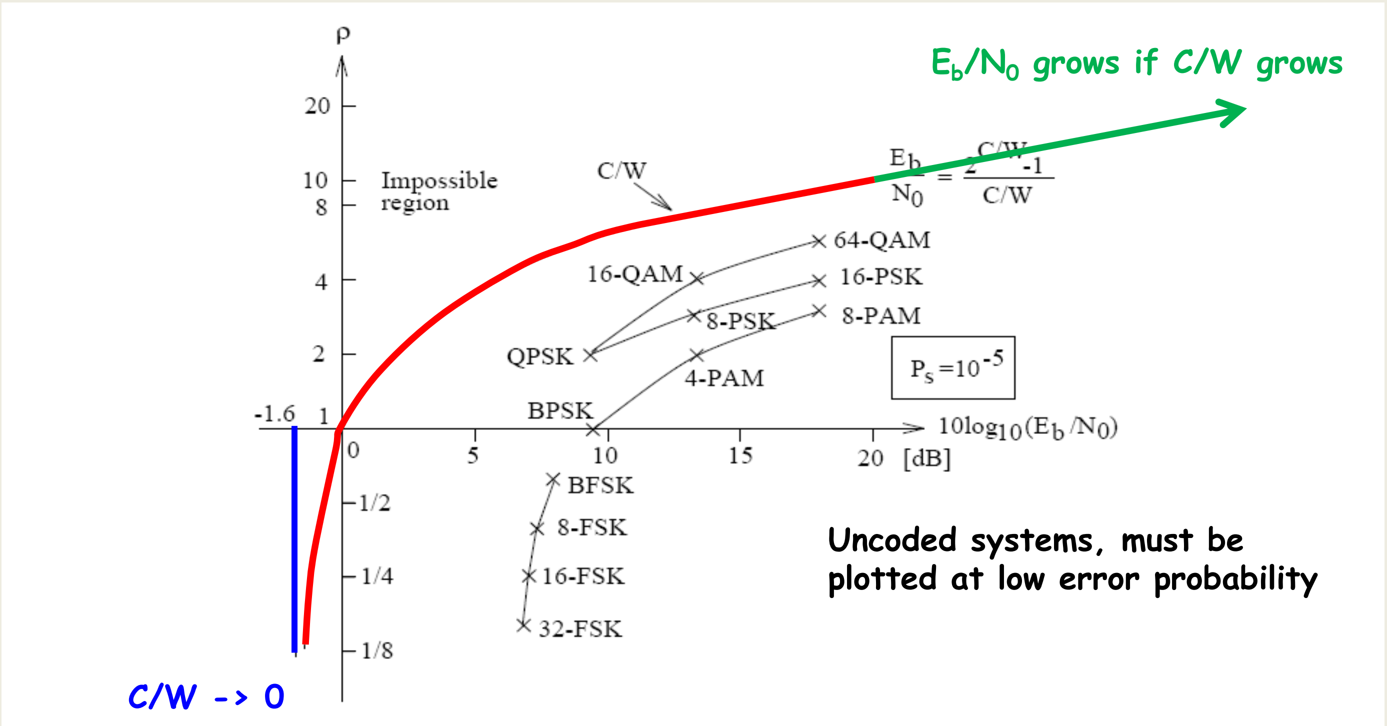

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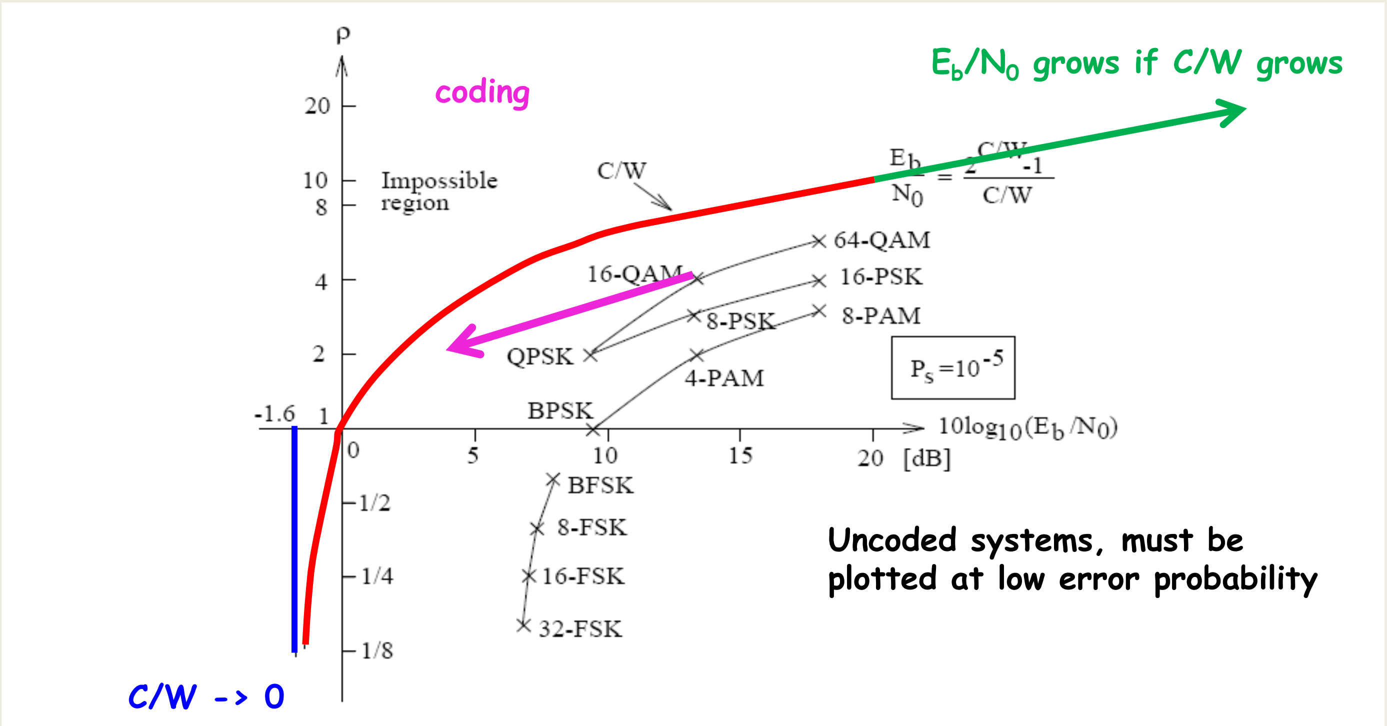

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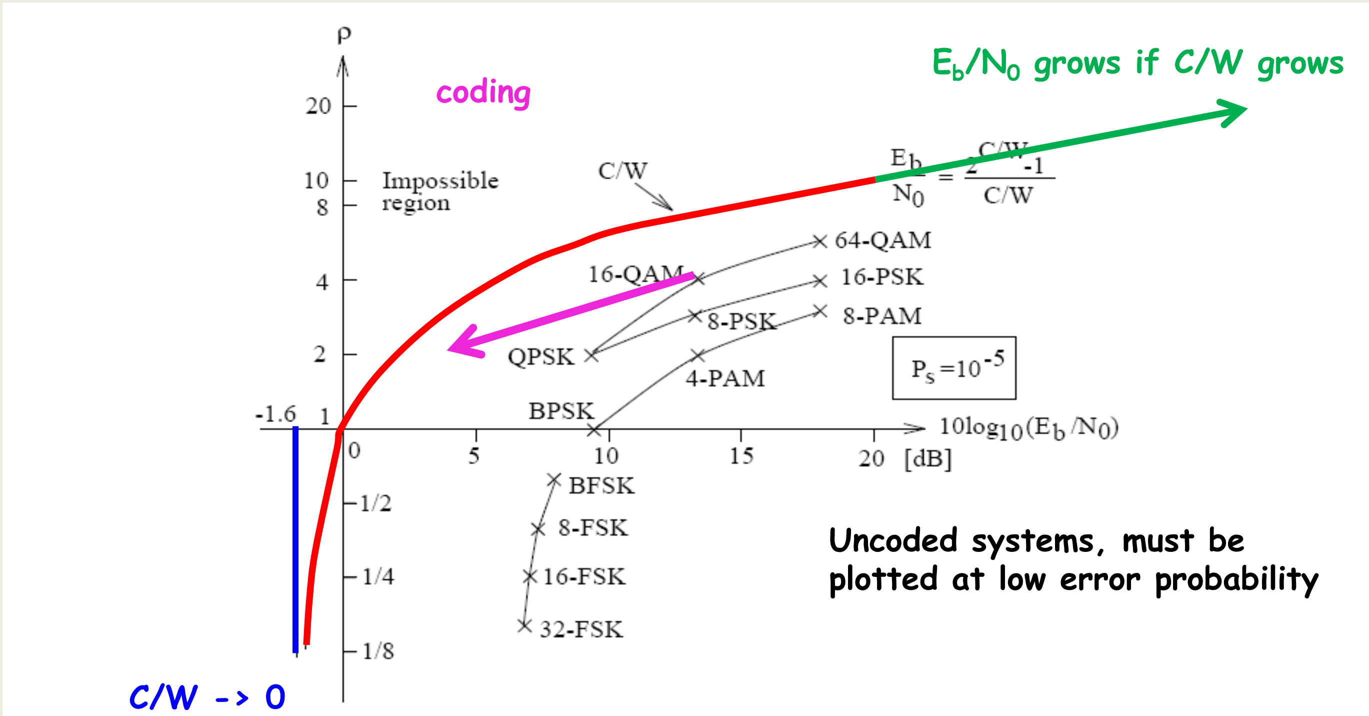

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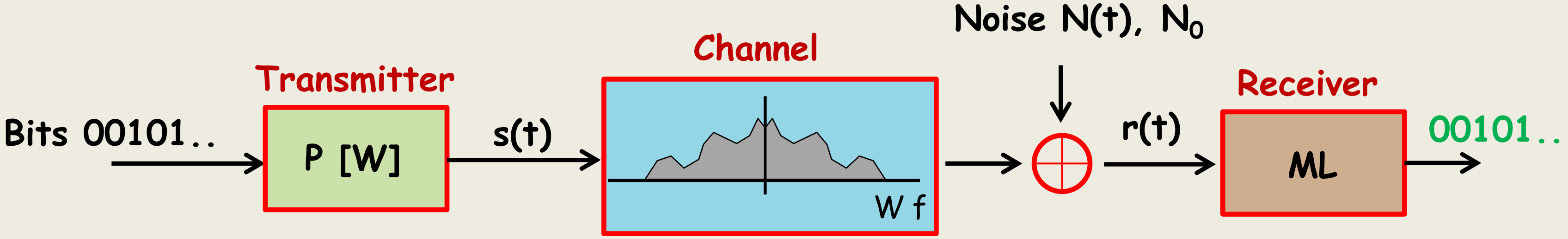
Lecture 4: Capacity

Extension to frequency dependent channel

System model:



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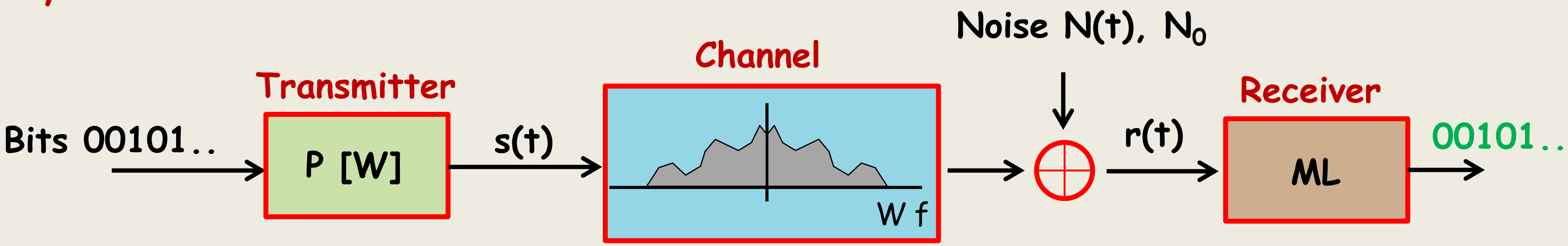
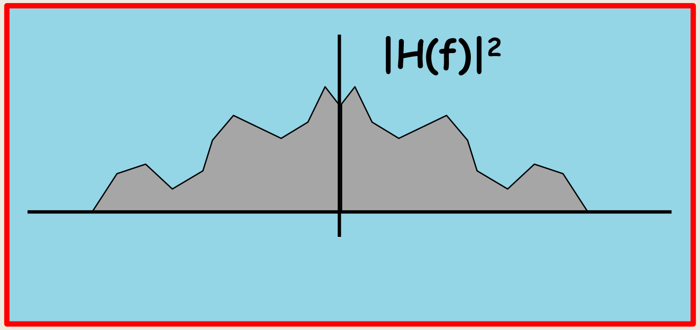


Frequency response of channel



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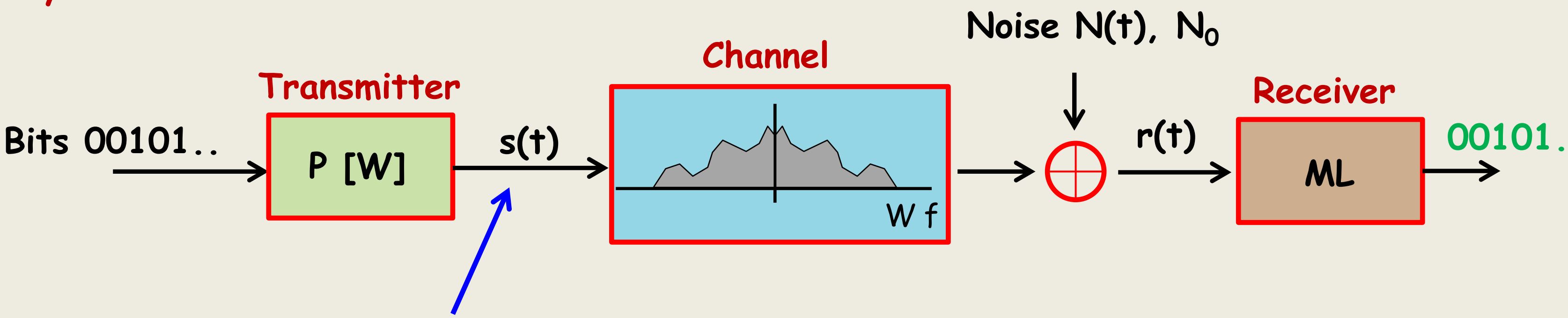
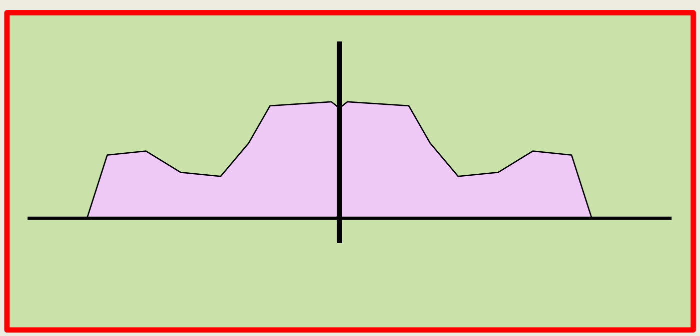


Signal also have frequency representation



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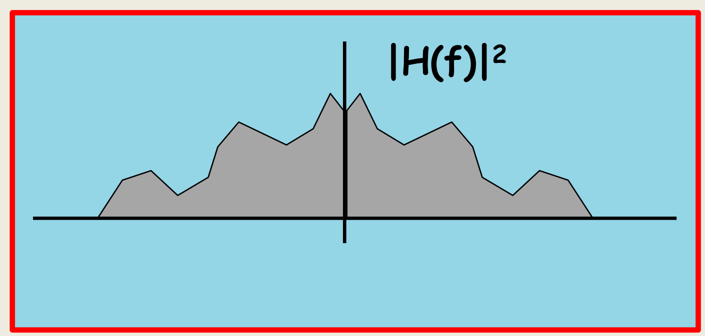

Lecture 4: Capacity

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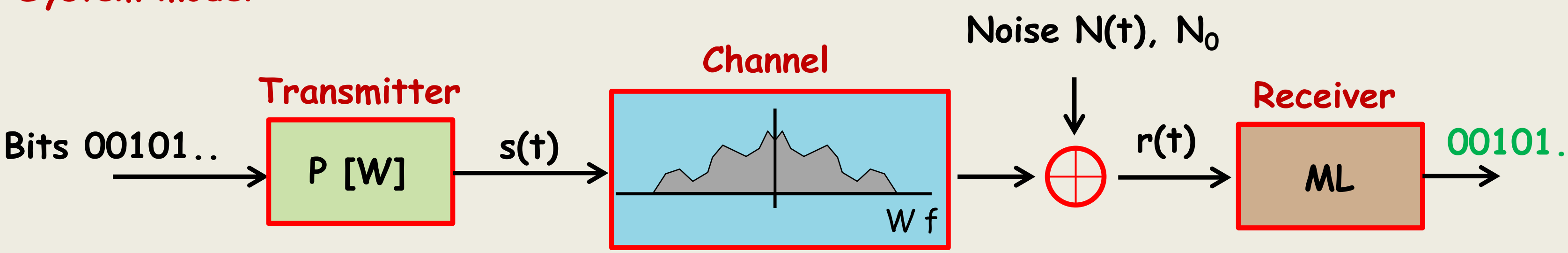
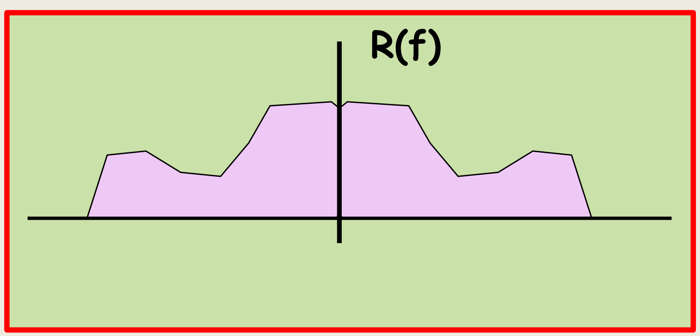


Power spectral density is what matters



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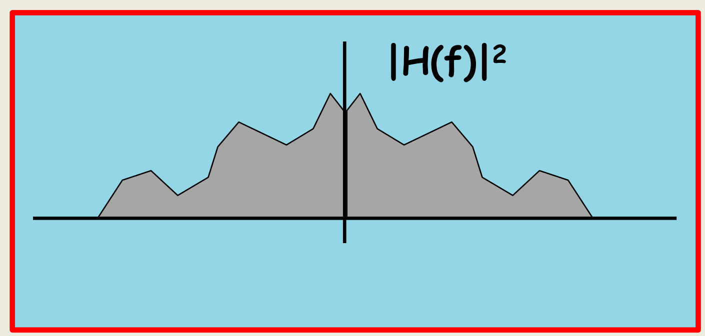

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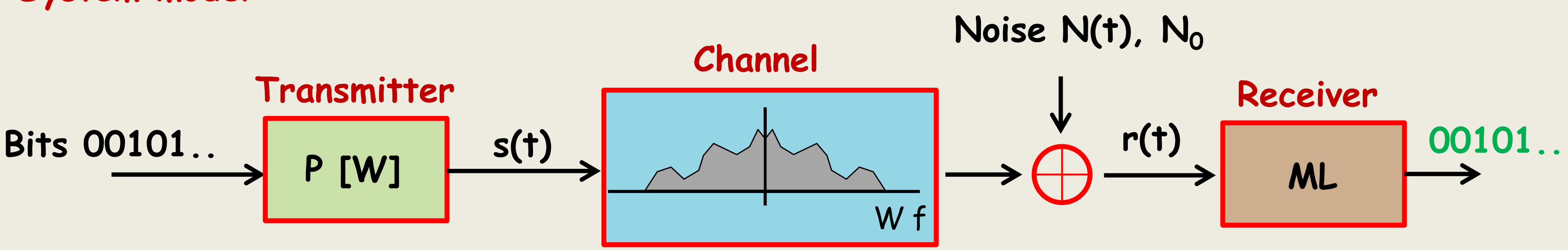

Constraint on R(f) ?

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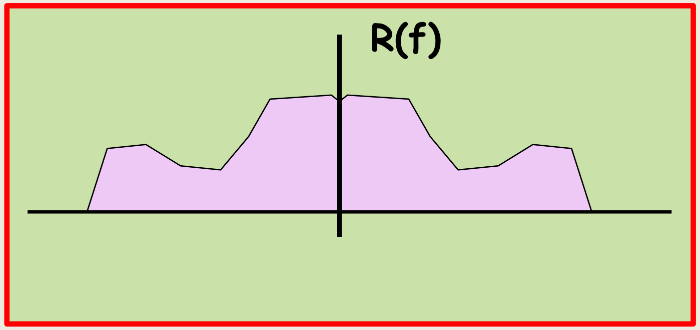

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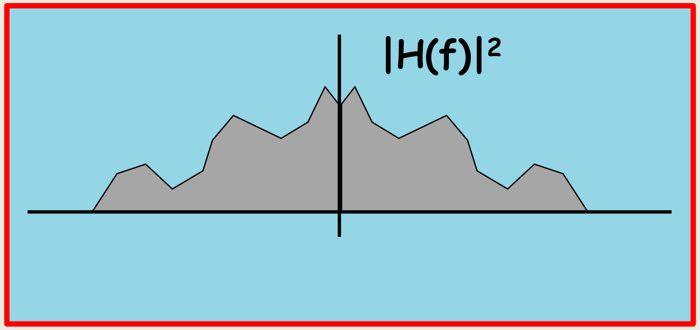

Lecture 4: Capacity

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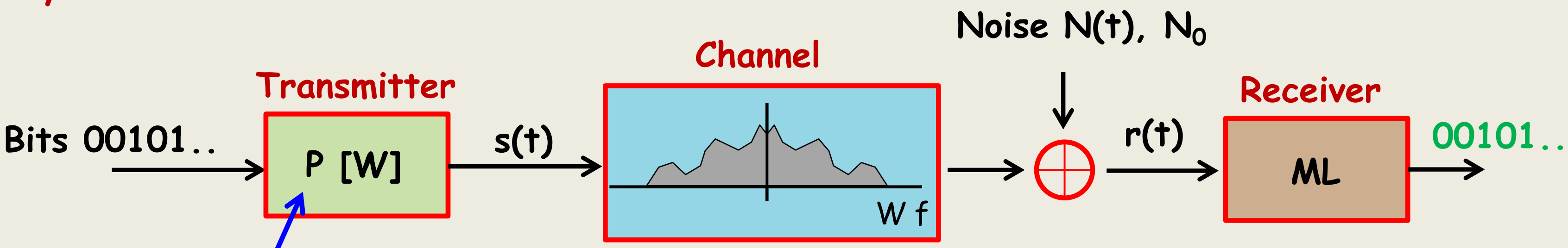
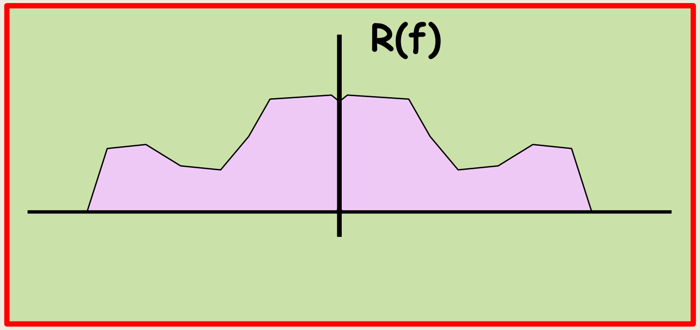


Constraint on R(f) ? $\int_{-\infty}^{\infty} R(f)\mathrm{d}f = P$

Power spectral density is what matters



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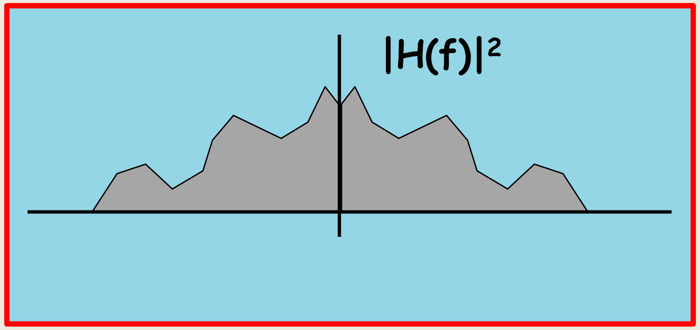

Lecture 4: Capacity

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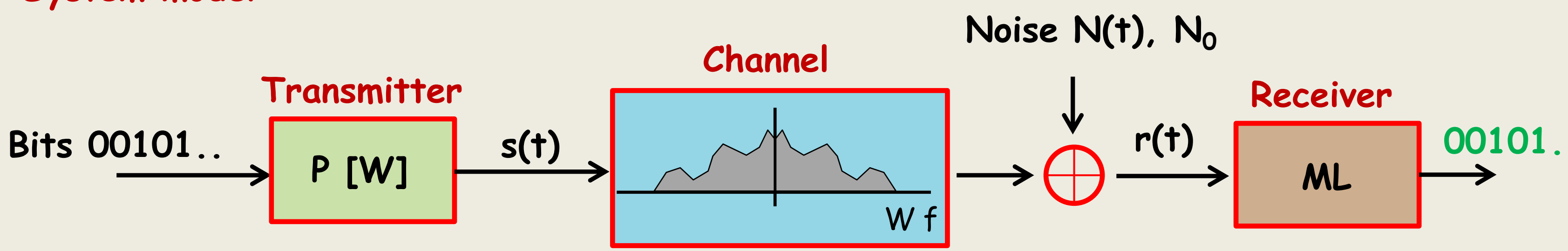

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If deep fade here

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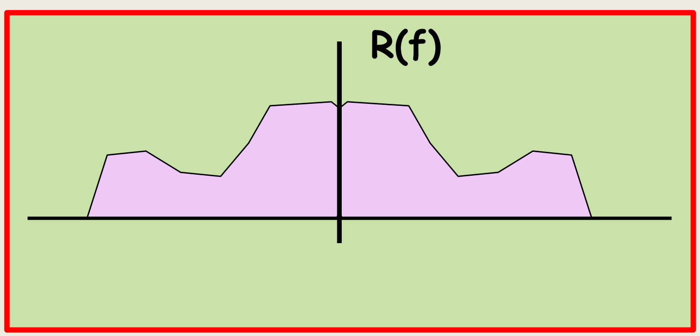
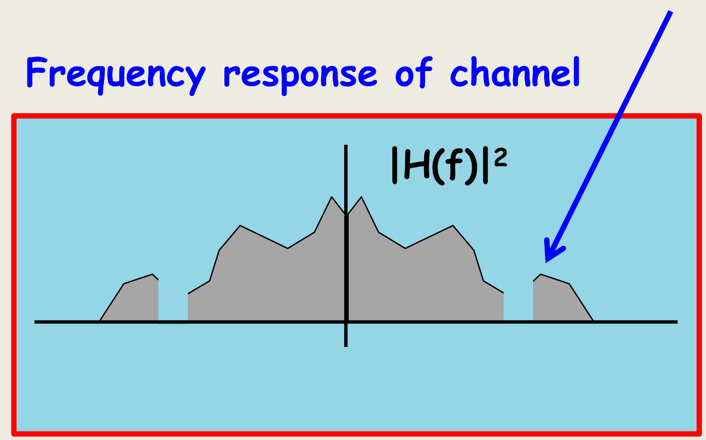


Frequency response of channel



Lecture 4: Capacity

Extension to frequency dependent channel

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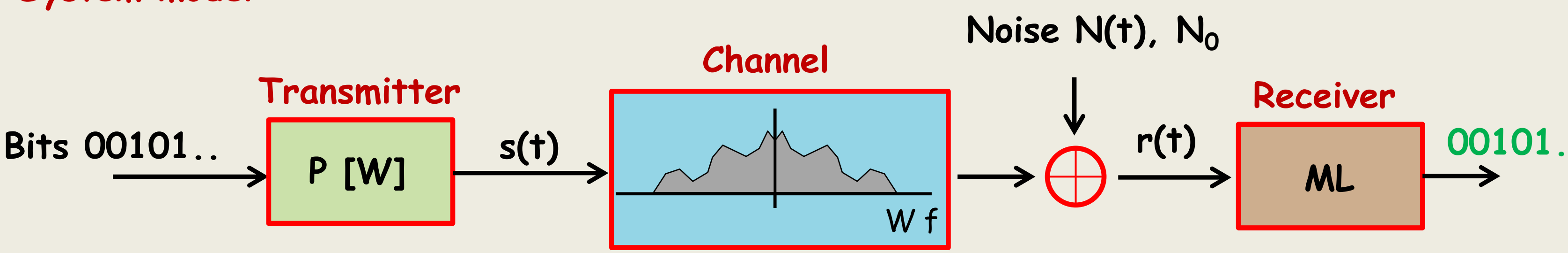

Stupidity to put power here

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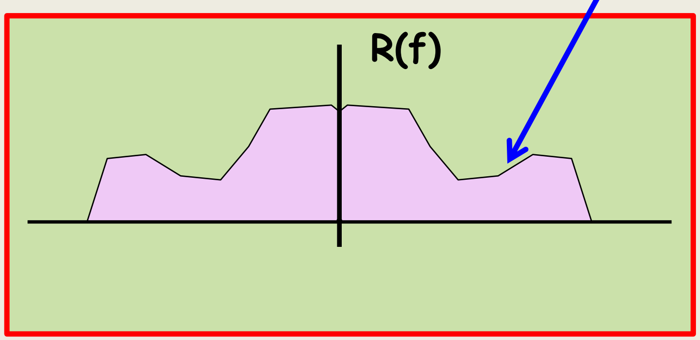
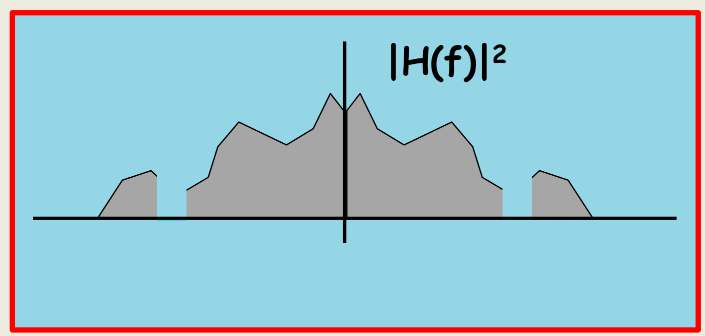
Power spectral density is what matters

Frequency response of channel





Lecture 4: Capacity

Extension to frequency dependent channel

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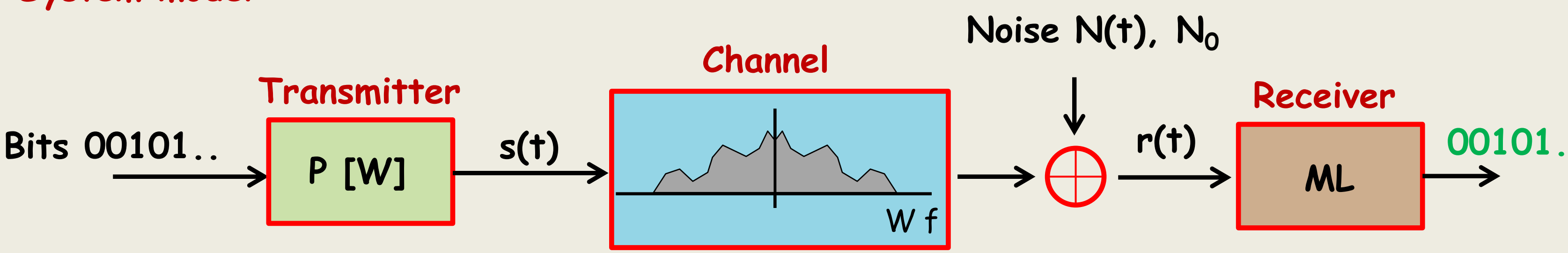
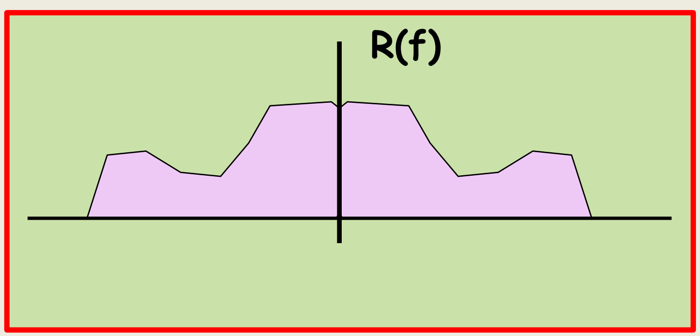


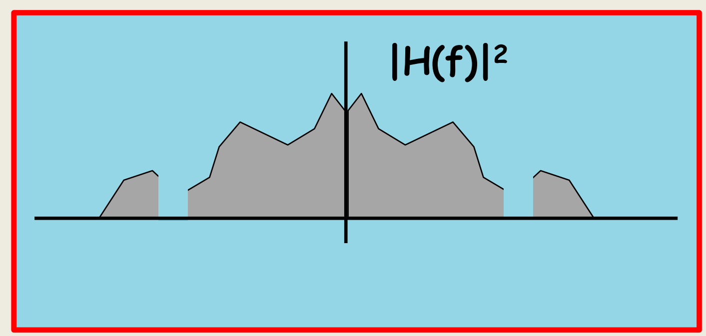
Conclusion: We should optimize the left plot, for the given right plot

Constraint on left plot is $\int_{-\infty}^{\infty} R(f)\mathrm{d}f = P$

Power spectral density is what matters

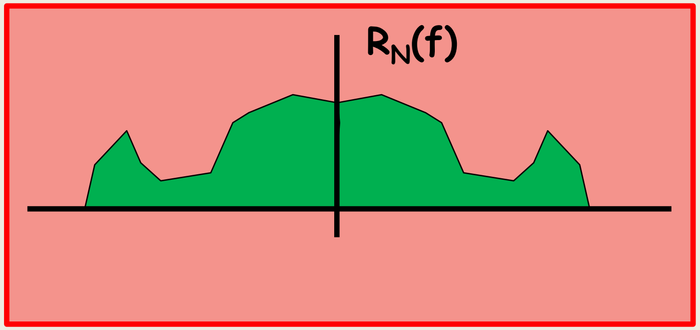


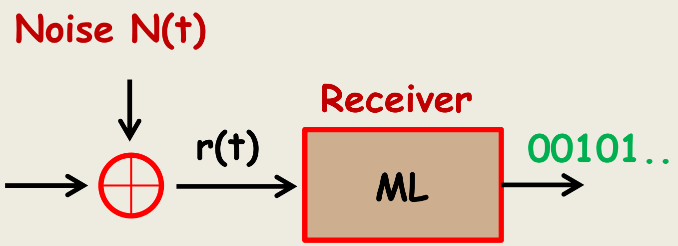
Frequency response of channel



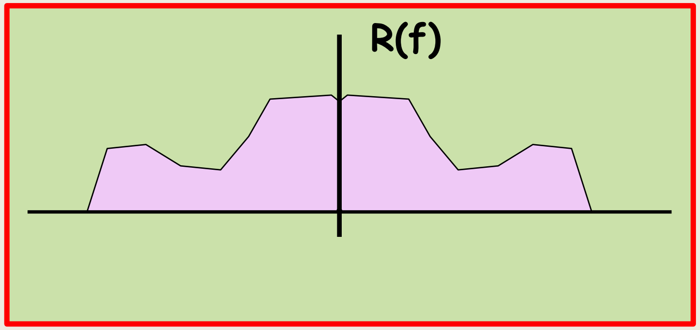
Lecture 4: Capacity

Frequency response of Noise

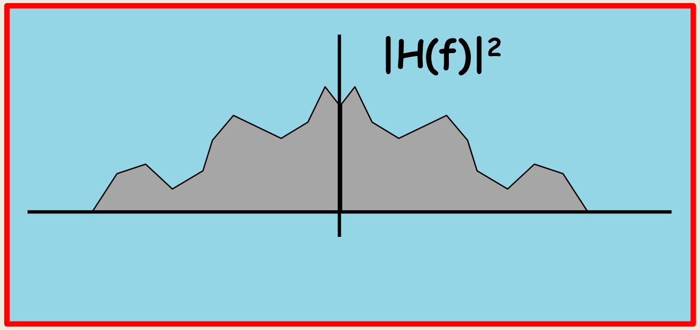




Power spectral density is what matters

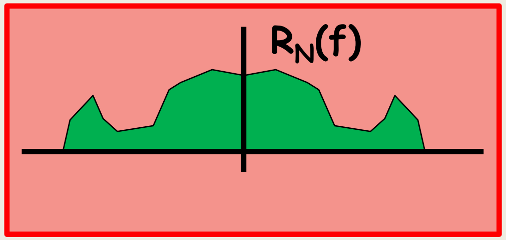


Frequency response of channel

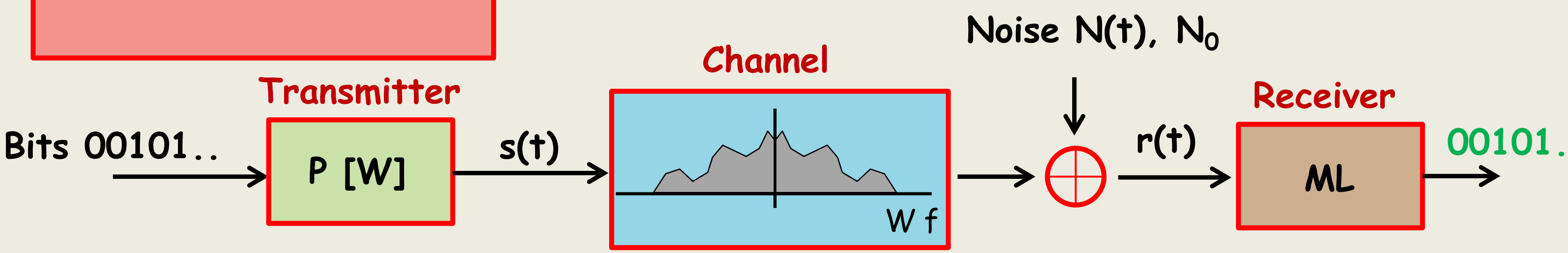


Lecture 4: Capacity

Frequency response of Noise



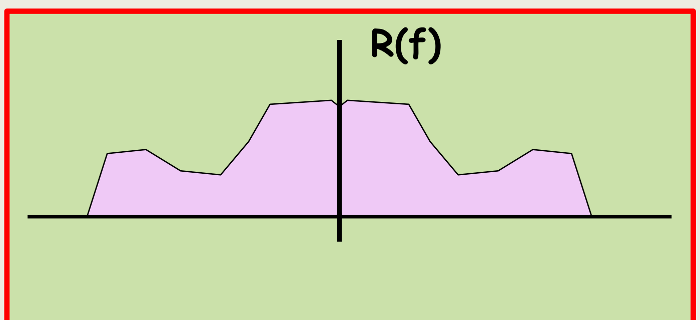
$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \text{Capacity}(|H(f)|^2, R_N(f), R(f))$$



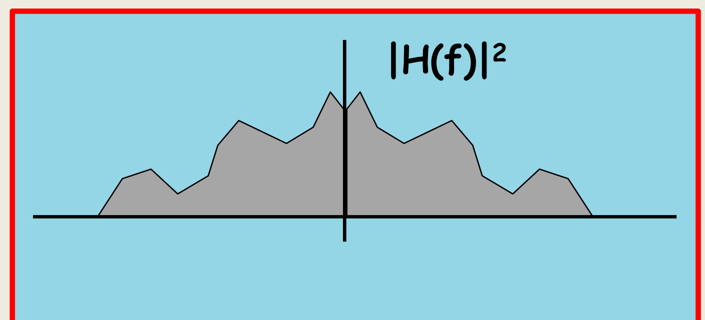
Conclusion: We should optimize the left plot, for the given right plot

Constraint on left plot is $\int_{-\infty}^{\infty} R(f)\mathrm{d}f = P$

Power spectral density is what matters

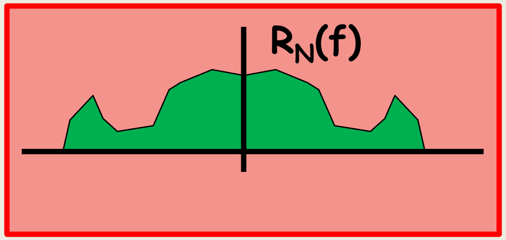


Frequency response of channel



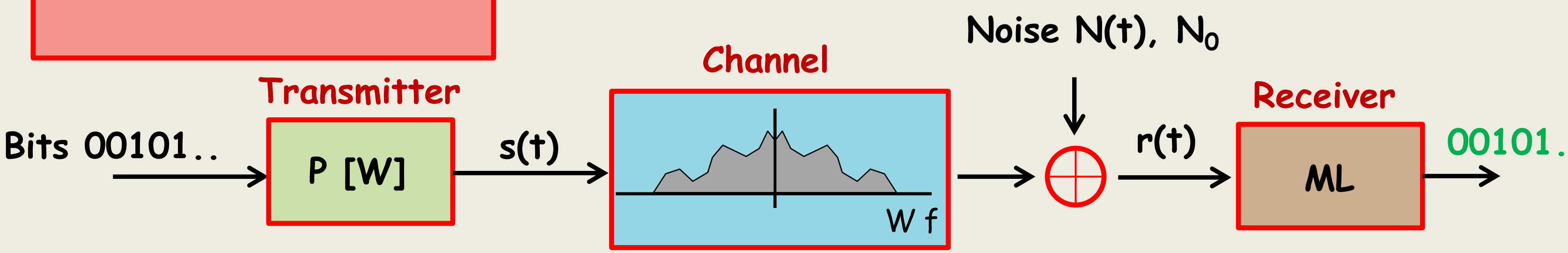
Lecture 4: Capacity

Frequency response of Noise



We need to find this formula

$$C = \max_{R(f): \int R(f) \mathrm{d}f = P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$



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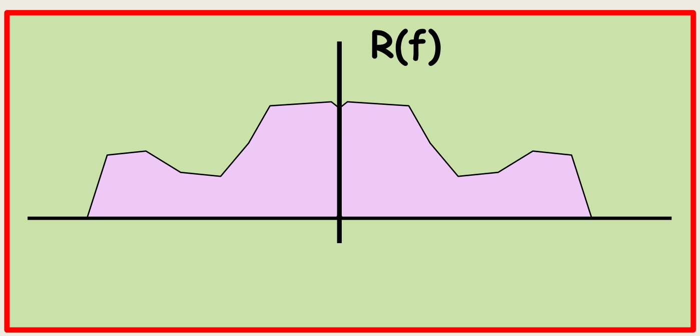
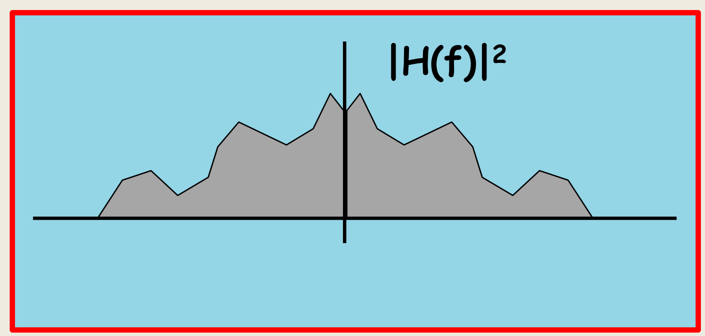
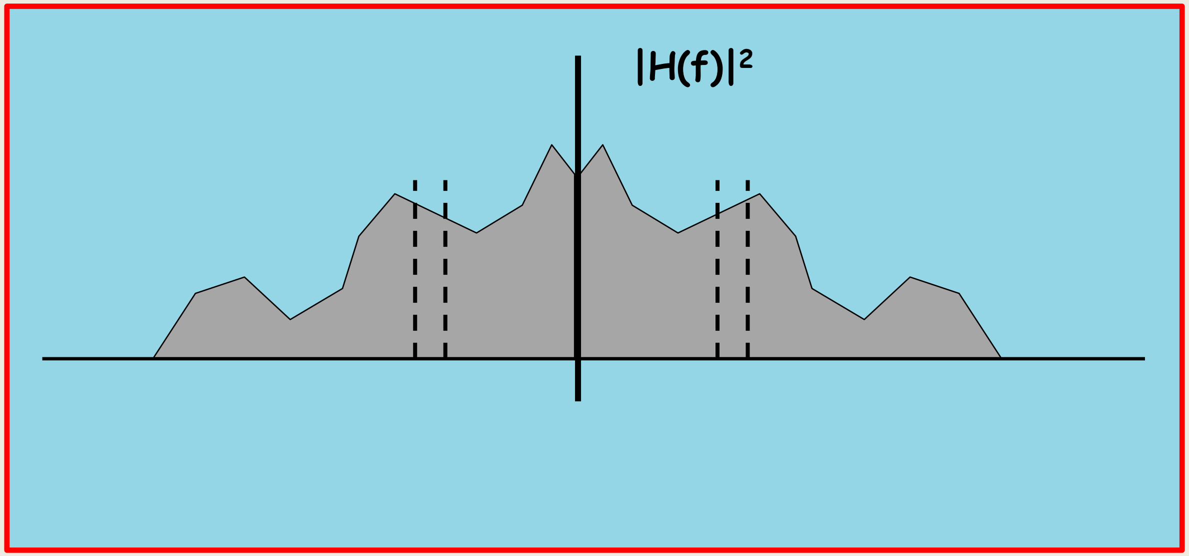


Frequency response of channel



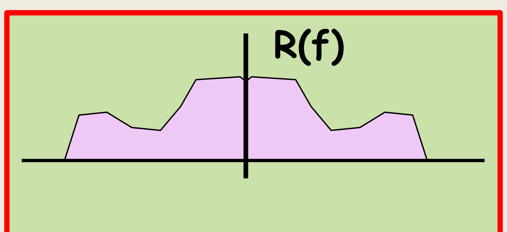
Lecture 4: Capacity

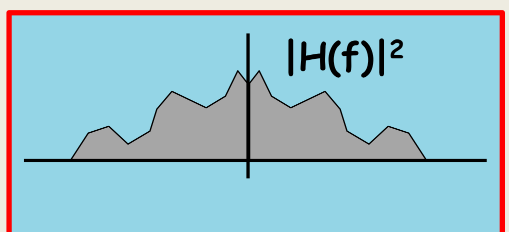
$$C = \max_{R(f): \int R(f) \mathrm{d}f = P} \text{Capacity}(|H(f)|^2, R_N(f), R(f))$$

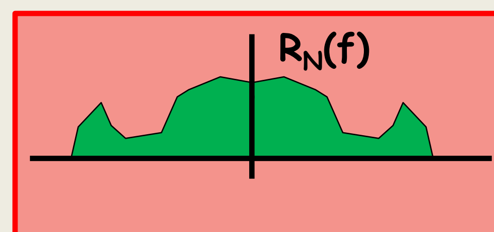


In this small piece
We can use

$$C = W \log_2 \left(1 + \frac{\mathcal{P}}{N_0 W}\right)$$

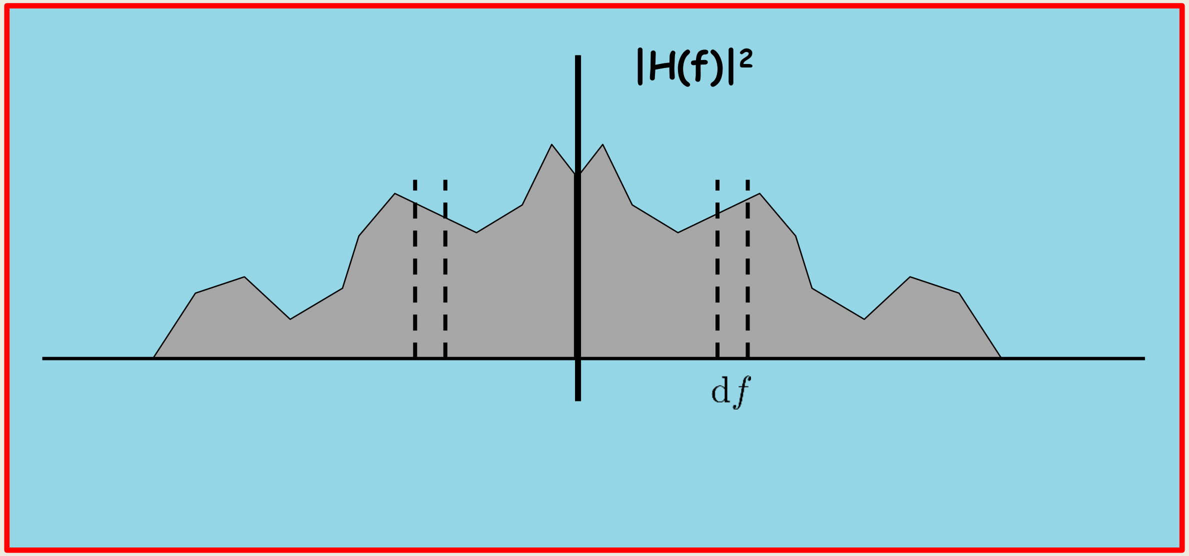






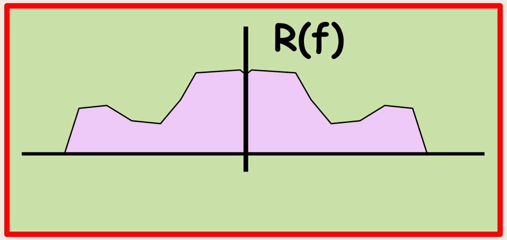
Lecture 4: Capacity

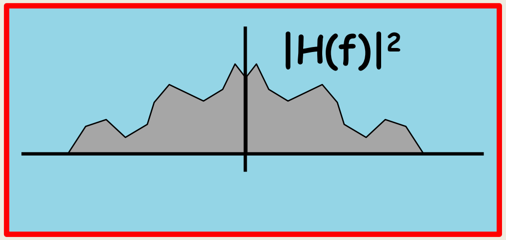
$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

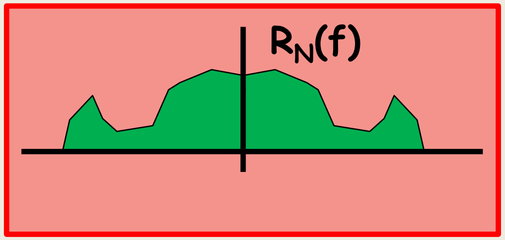


In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{N_0 \mathrm{d}f}\right)$$

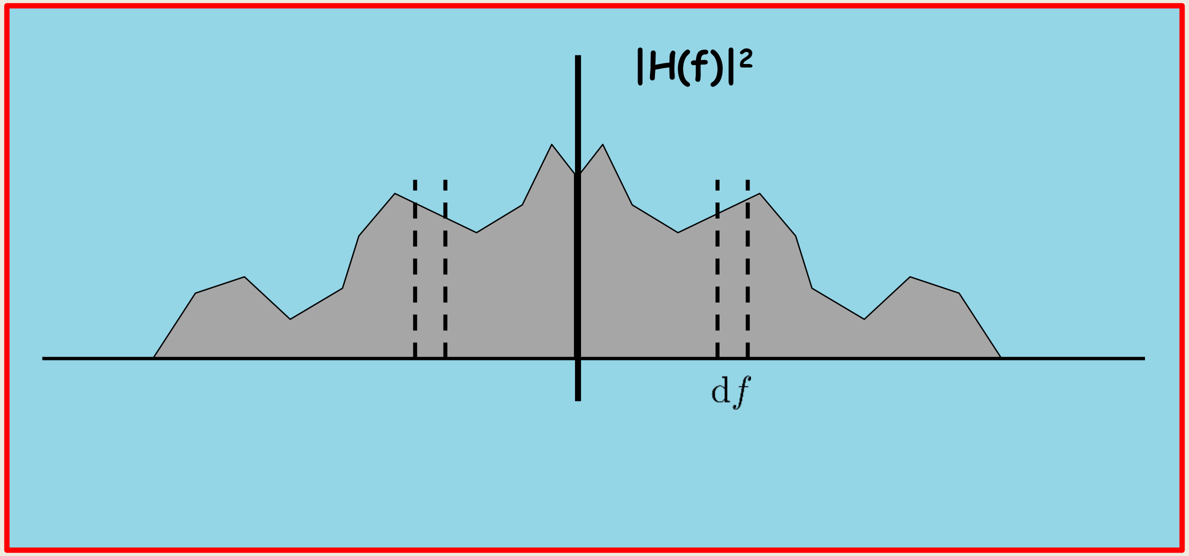






Lecture 4: Capacity

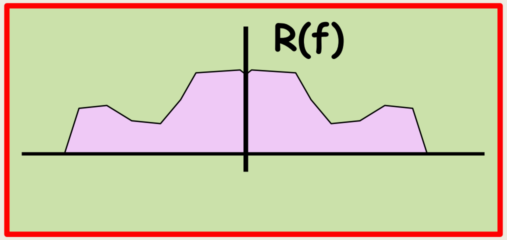
$$C = \max_{R(f):\int R(f)\mathrm{d}f=P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

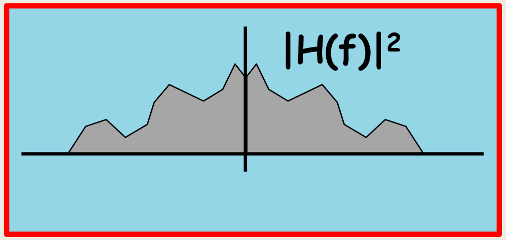


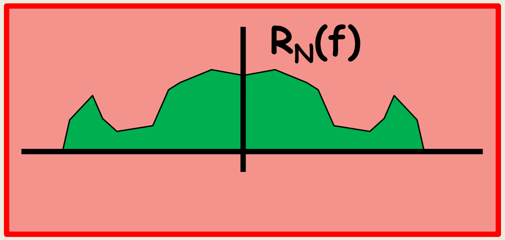
In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{N_0 \mathrm{d}f}\right)$$

For flat noise,
$R_N(f) = N_0/2$

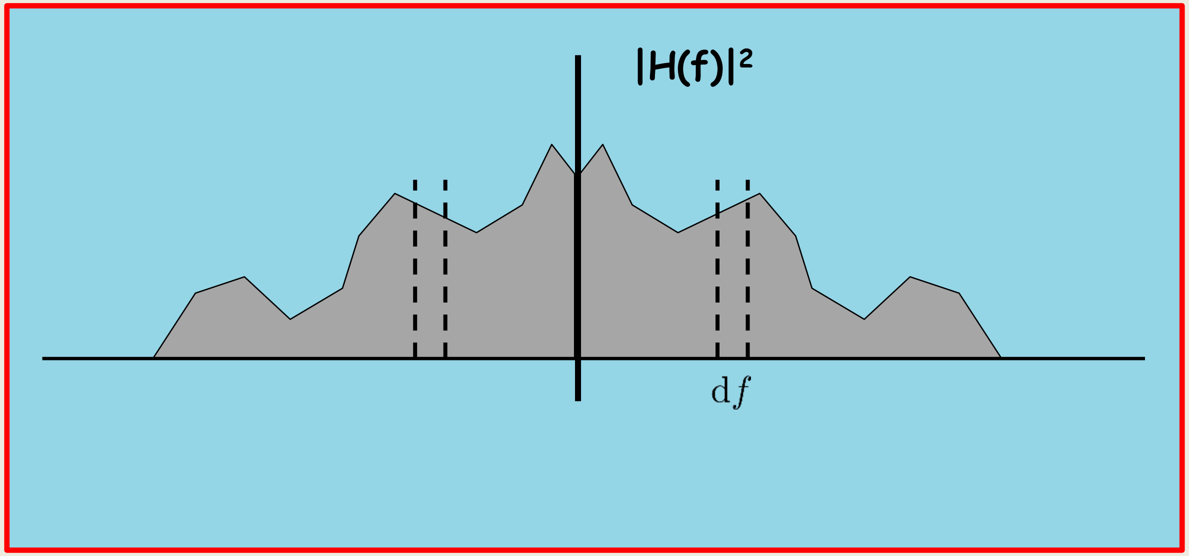






Lecture 4: Capacity

$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

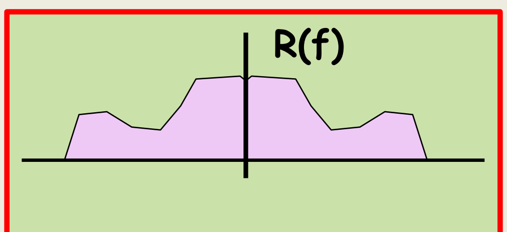


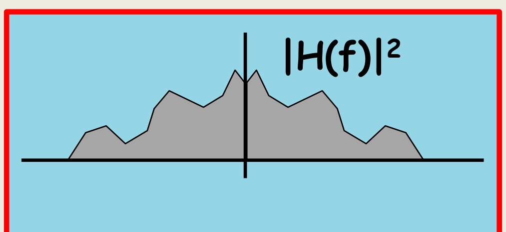
In this small piece
We can use

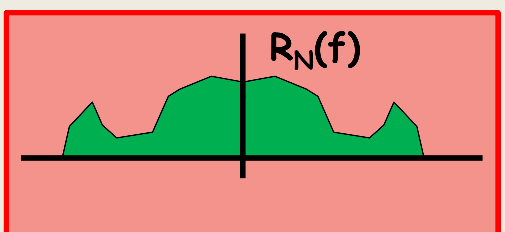
$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{N_0 \mathrm{d}f}\right)$$

$=2R_N(f)$

For flat noise,
$R_N(f) = N_0/2$

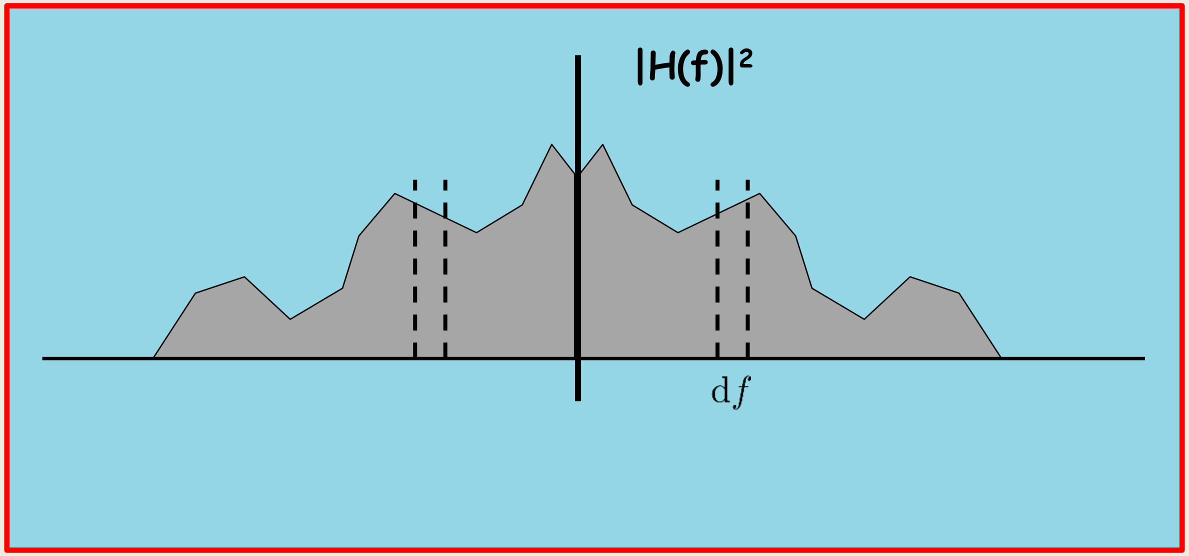






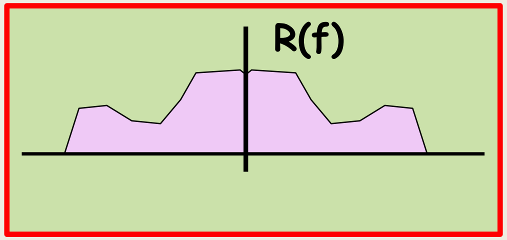
Lecture 4: Capacity

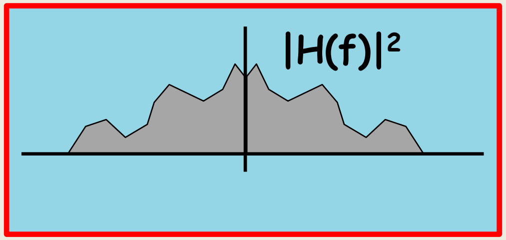
$$C = \max_{R(f):\int R(f)\mathrm{d}f=P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

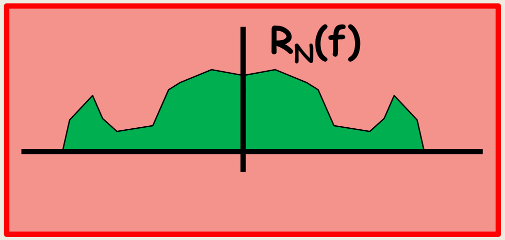


In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1+\frac{\mathcal{P}}{2R_N(f)\mathrm{d}f}\right)$$

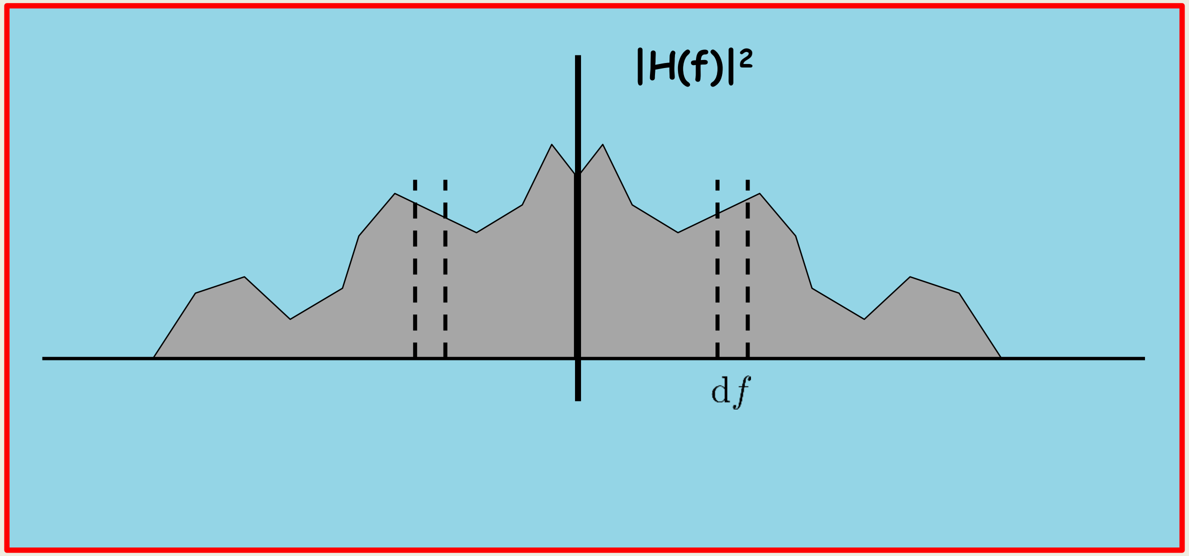






Lecture 4: Capacity

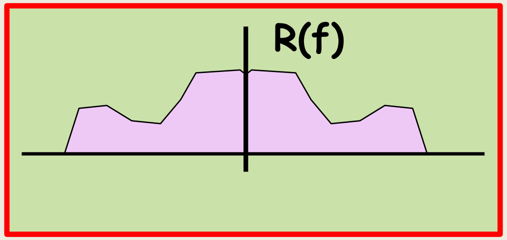
$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

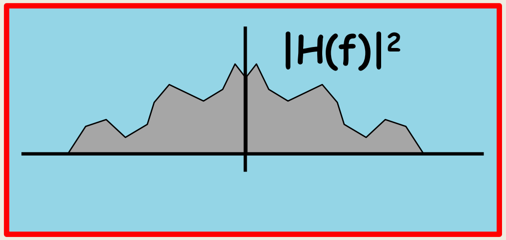


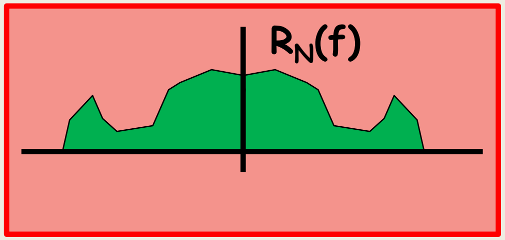
In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{2R_N(f)\mathrm{d}f}\right)$$

How much power do
we have?

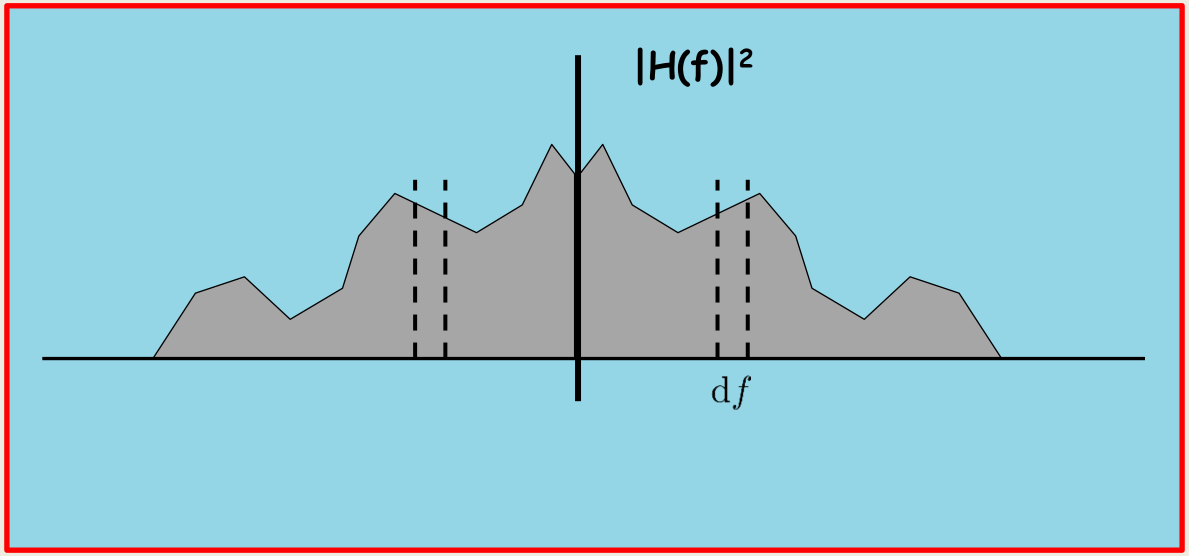






Lecture 4: Capacity

$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

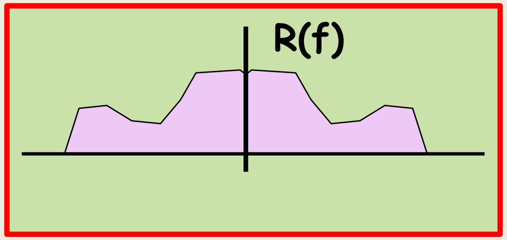


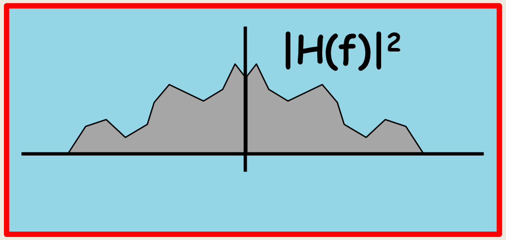
In this small piece
We can use

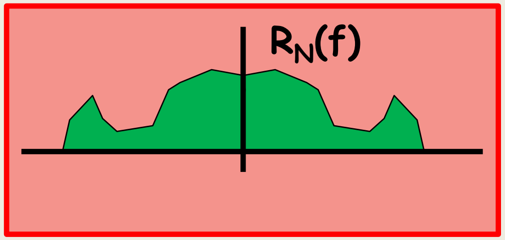
$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{2R_N(f)\mathrm{d}f}\right)$$

How much power do we have?

$$2\mathrm{d}f\, R(f)|H(f)|^2$$







Lecture 4: Capacity

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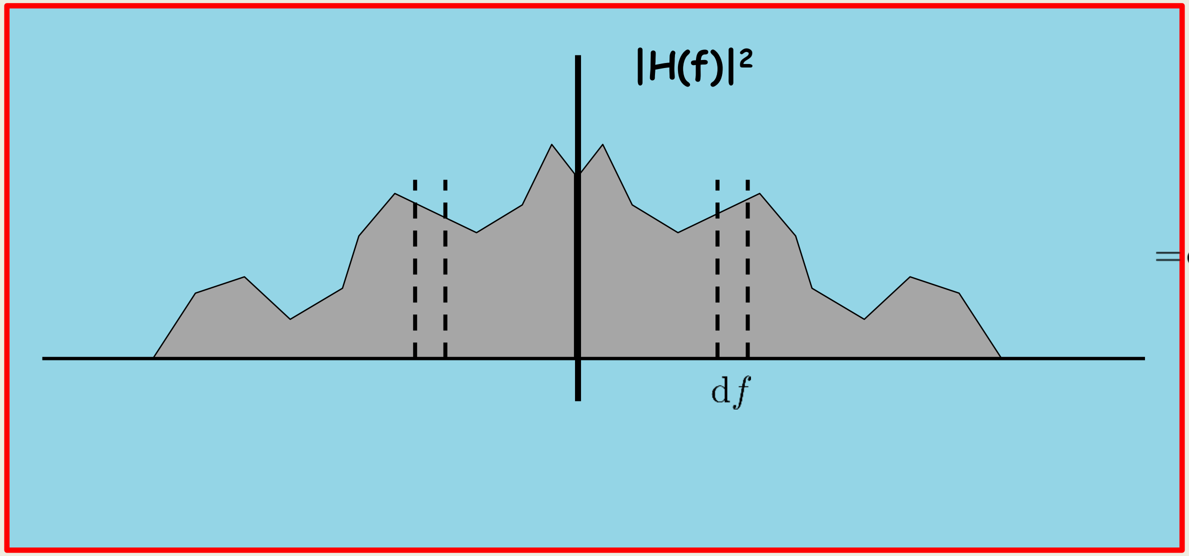
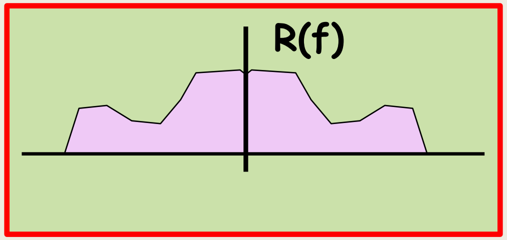
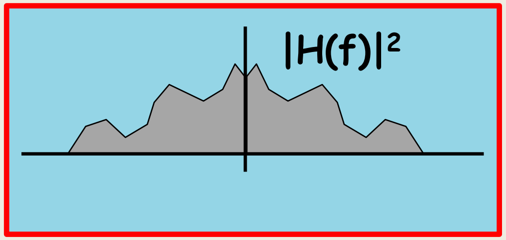


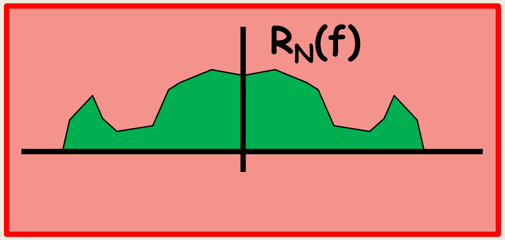
In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{2R_N(f)\mathrm{d}f}\right)$$

$$= \mathrm{d}f \log_2\left(1 + \frac{2R(f)|H(f)|^2\mathrm{d}f}{2R_N(f)\mathrm{d}f}\right)$$







Lecture 4: Capacity

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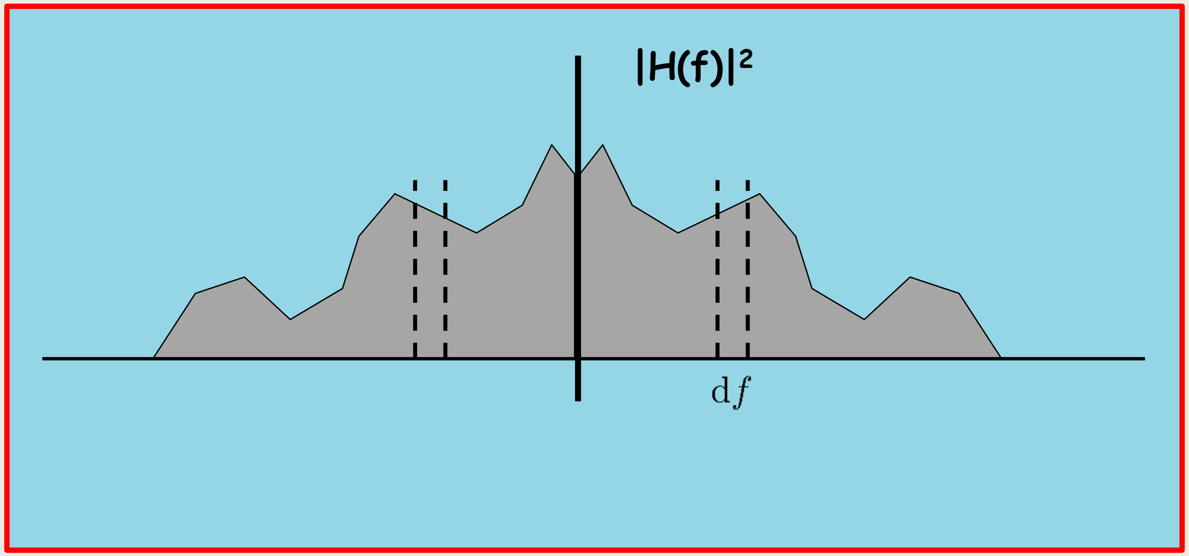
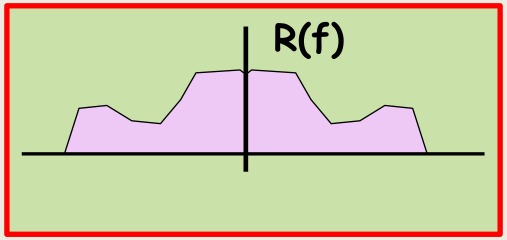
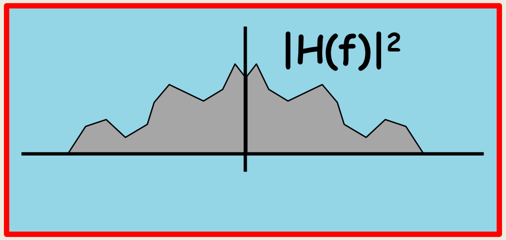


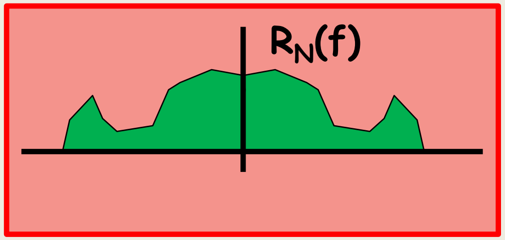
In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{2R_N(f)\mathrm{d}f}\right)$$

$$= \mathrm{d}f \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)$$







Lecture 4: Capacity

$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

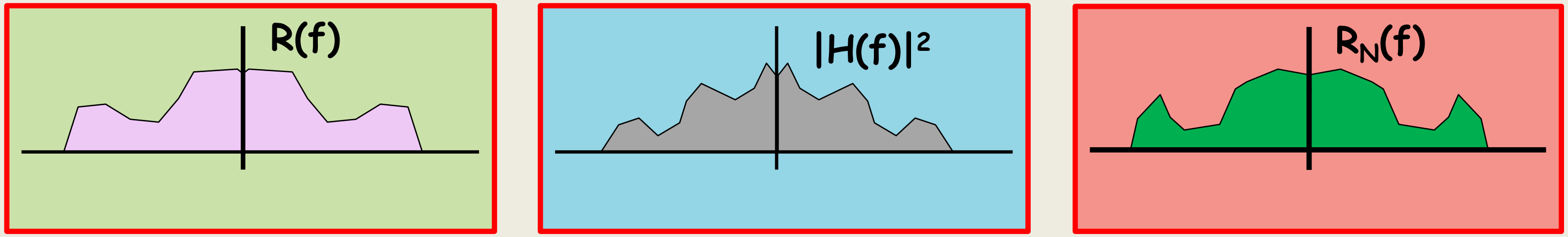
Sum up

$$\mathrm{Capacity}(|H(f)|^2, R_N(f), R(f)) = \int_0^\infty \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f$$

In this small piece
We can use

$$C = \mathrm{d}f \log_2\left(1 + \frac{\mathcal{P}}{2R_N(f)\mathrm{d}f}\right)$$

$$= \mathrm{d}f \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)$$

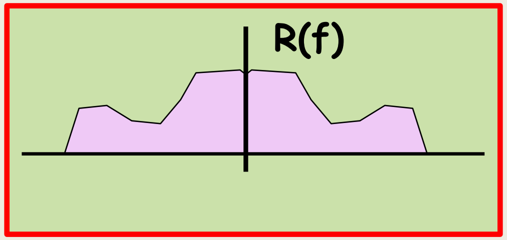


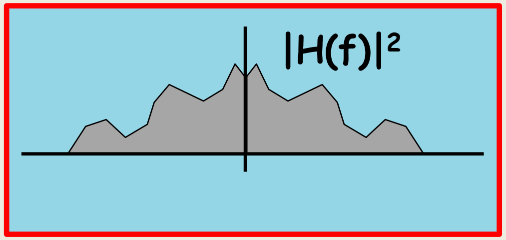
Lecture 4: Capacity

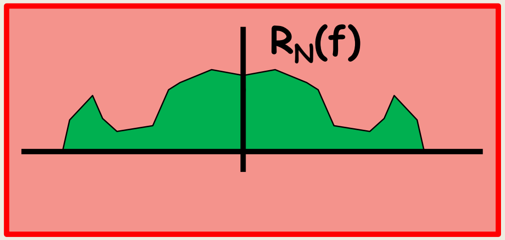
$$C = \max_{R(f):\int R(f)\mathrm{d}f=P} \mathrm{Capacity}(|H(f)|^2, R_N(f), R(f))$$

Sum up

$$\begin{aligned}\mathrm{Capacity}(|H(f)|^2, R_N(f), R(f)) &= \int_0^\infty \log_2\left(1+\frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f \\ &= \frac{1}{2}\int_{-\infty}^\infty \log_2\left(1+\frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f\end{aligned}$$

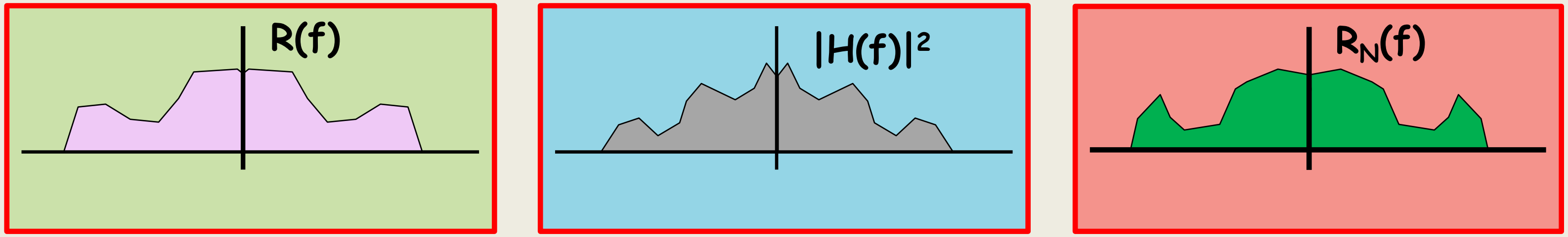






Lecture 4: Capacity

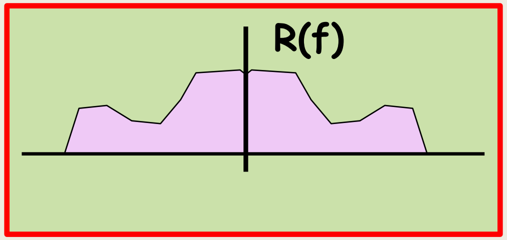
$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f$$

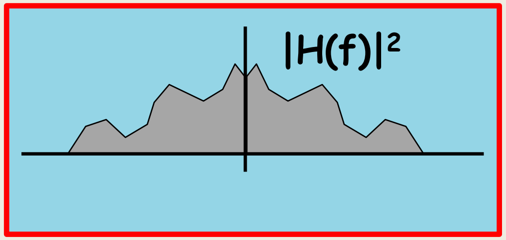


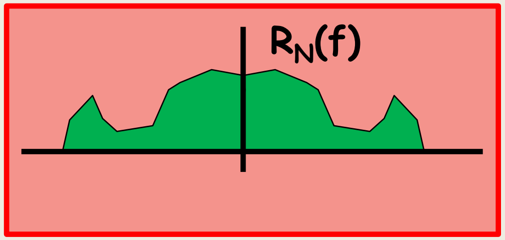
Lecture 4: Capacity

How to solve the below problem? WATERFILLING

$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f$$



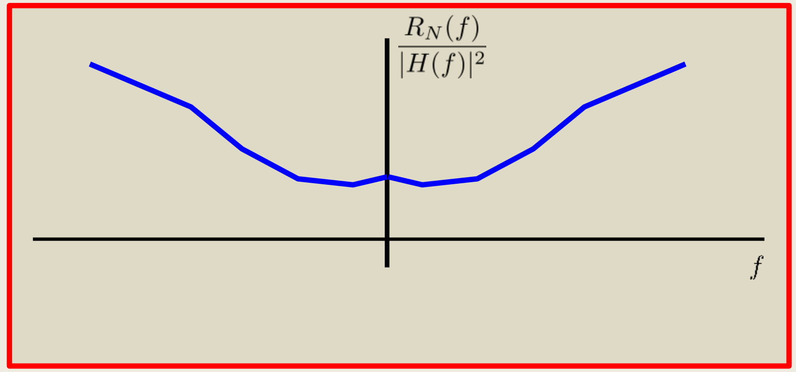




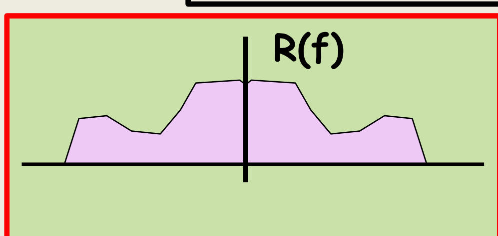
Lecture 4: Capacity

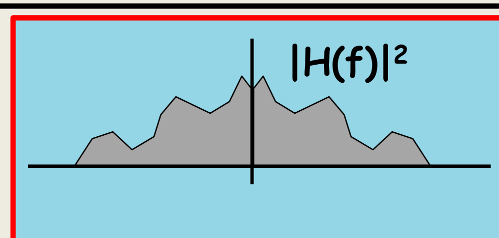
How to solve the below problem? WATERFILLING

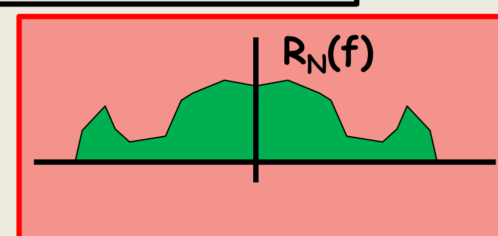
Step 1. Find and plot $\dfrac{R_N(f)}{|H(f)|^2}$



$$C = \max_{R(f):\int R(f)\mathrm{d}f=P} \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1+\frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f$$





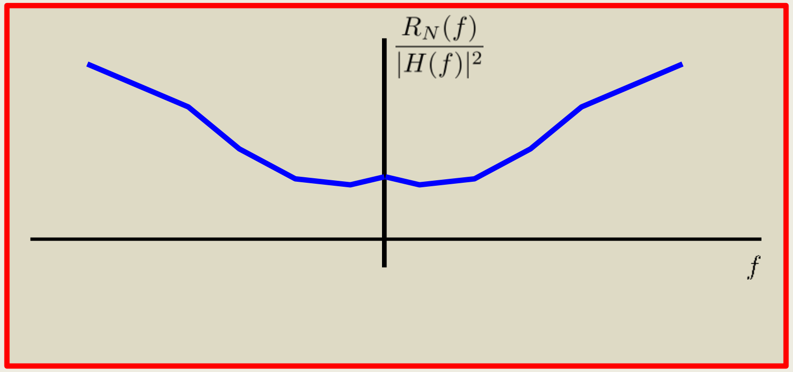


Lecture 4: Capacity

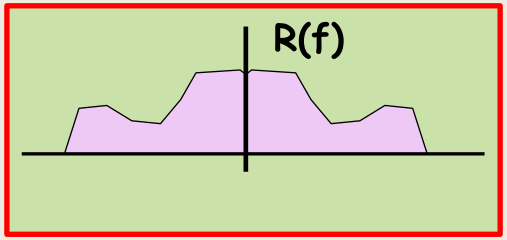
How to solve the below problem? WATERFILLING

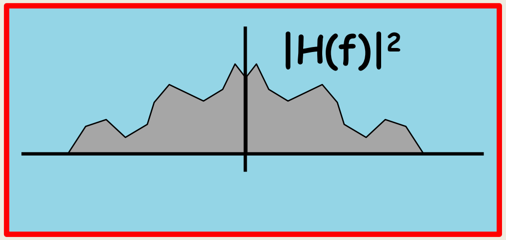
Step 1. Find and plot $\dfrac{R_N(f)}{|H(f)|^2}$

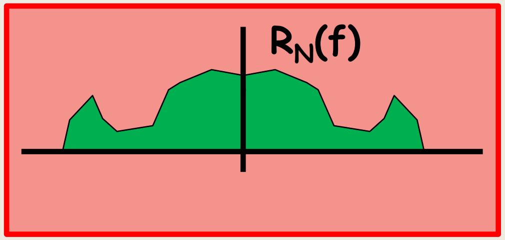
Step 2. Fill a bucket with P units of water



$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f$$







Lecture 4: Capacity

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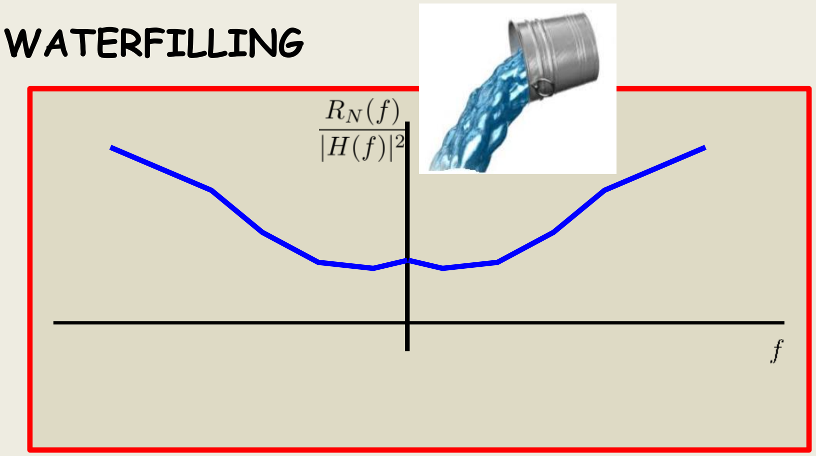
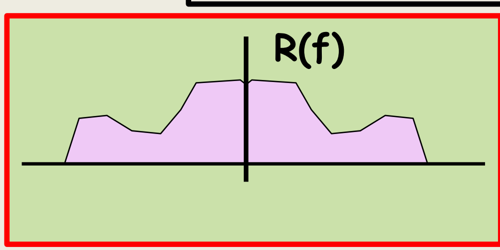
Step 1. Find and plot $\dfrac{R_N(f)}{|H(f)|^2}$

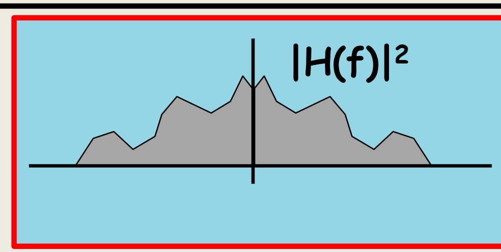


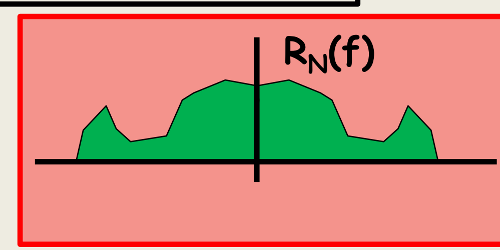
Step 2. Fill a bucket with P units of water

Step 3. Pour it in the shape

$$C = \max_{R(f):\int R(f)\mathrm{d}f = P} \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{R(f)|H(f)|^2}{R_N(f)}\right)\mathrm{d}f$$







Lecture 4: Capacity

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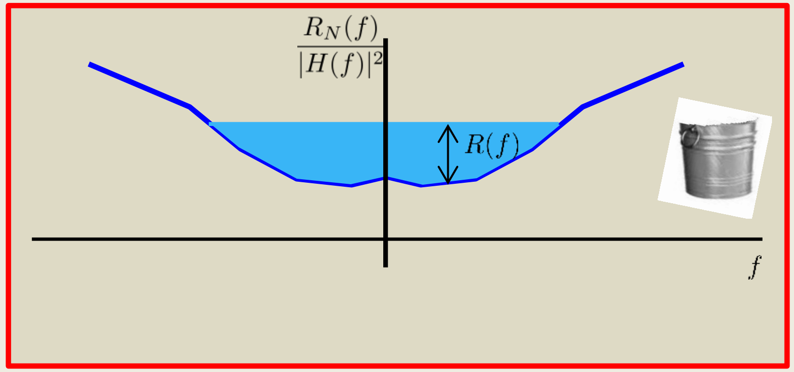
Step 1. Find and plot $\dfrac{R_N(f)}{|H(f)|^2}$

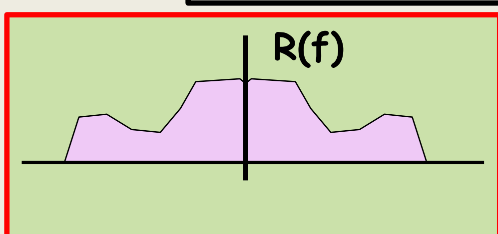


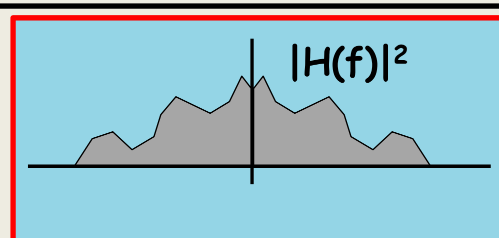
Step 2. Fill a bucket with P units of water

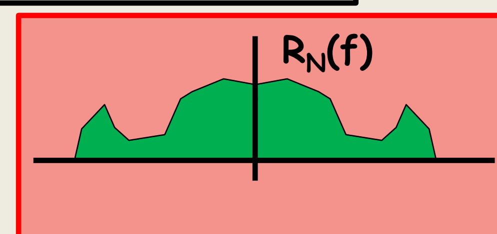
Step 3. Pour it in the shape

Step 4. R(f) is the water-level

$$C = \max_{R(f): \int R(f) \mathrm{d}f = P} \frac{1}{2} \int_{-\infty}^{\infty} \log_2 \left(1 + \frac{R(f)|H(f)|^2}{R_N(f)} \right) \mathrm{d}f$$







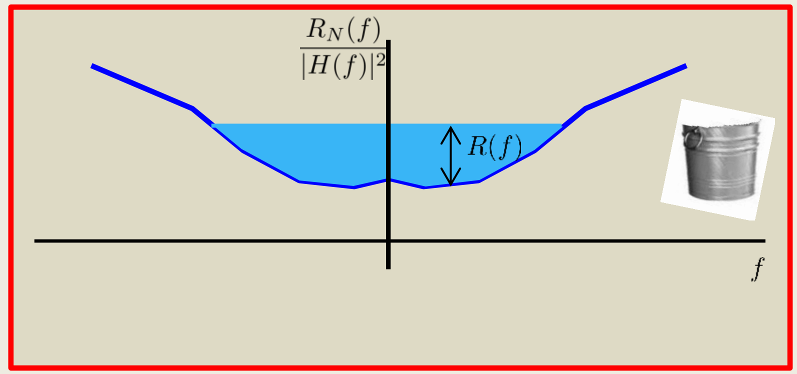
Lecture 4: Capacity

How to solve the below problem? WATERFILLING

Step 1. Find and plot $\dfrac{R_N(f)}{|H(f)|^2}$

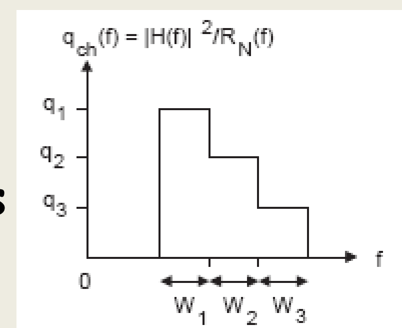
Step 2. Fill a bucket with P units of water

Step 3. Pour it in the shape



Step 4.
R(f) is the water-level

On Exam, |H(f)|^2 would be "nice", such as



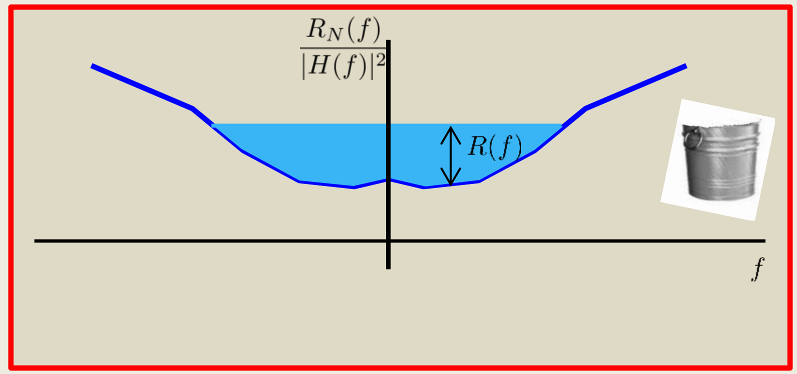
Lecture 4: Capacity

How to solve the below problem? WATERFILLING

Step 1. Find and plot $\dfrac{R_N(f)}{|H(f)|^2}$

Step 2. Fill a bucket with P units of water

Step 3. Pour it in the shape



Step 4. R(f) is the water-level

Observations:

1. Good channels get more power than bad
2. At very high SNRs, all channels get, roughly, the same power