Lecture 2: MAP receiver and signal space

Any bandpass signal can be written as follows (from dig com course, p.118)

$$s(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

where $x_I(t)$ and $x_Q(t)$ are: **1)** real-valued, **2)** low pass, **3)** bandwidth << f_c

Lecture 2: MAP receiver and signal space

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Let us now choose $x_I(t)$ and $x_Q(t)$ as **PAM** signals, i.e.,

$$x_I(t) = A_\ell g(t)$$
$$x_Q(t) = B_\ell g(t) \quad \ell = 0, \ldots, M-1$$

Lecture 2: MAP receiver and signal space

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We get

$$s_\ell(t) = A_\ell g(t)\cos(2\pi f_c t) - B_\ell g(t)\sin(2\pi f_c t)$$

Lecture 2: MAP receiver and signal space

Let us now rewrite this as

$$s_\ell(t) = A_\ell\sqrt{E_g/2}\phi_1(t) - B_\ell\sqrt{E_g/2}\phi_2(t)$$

where

$$\phi_1(t) = \frac{g(t)\cos(2\pi f_c t)}{\sqrt{E_g/2}} \qquad \phi_2(t) = -\frac{g(t)\sin(2\pi f_c t)}{\sqrt{E_g/2}}$$

$$s_\ell(t) = A_\ell g(t)\cos(2\pi f_c t) - B_\ell g(t)\sin(2\pi f_c t)$$

Lecture 2: MAP receiver and signal space

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Define $s_{\ell,1} = A_\ell\sqrt{E_g/2}$ $s_{\ell,2} = B_\ell\sqrt{E_g/2}$

$$s_\ell(t) = A_\ell g(t)\cos(2\pi f_c t) - B_\ell g(t)\sin(2\pi f_c t)$$

Lecture 2: MAP receiver and signal space

We get

$$s_\ell(t) = s_{\ell,1}\phi_1(t) + s_{\ell,2}\phi_2(t)$$

where

$$\phi_1(t) = \frac{g(t)\cos(2\pi f_c t)}{\sqrt{E_g/2}} \qquad \phi_2(t) = -\frac{g(t)\sin(2\pi f_c t)}{\sqrt{E_g/2}}$$

Lecture 2: MAP receiver and signal space

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Continuous wafefrom, representing message ℓ

Lecture 2: MAP receiver and signal space

We get

Basis functions

$$s_\ell(t) = s_{\ell,1}\phi_1(t) + s_{\ell,2}\phi_2(t)$$

where

$$\phi_1(t) = \frac{g(t)\cos(2\pi f_c t)}{\sqrt{E_g/2}} \qquad \phi_2(t) = -\frac{g(t)\sin(2\pi f_c t)}{\sqrt{E_g/2}}$$

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Lecture 2: MAP receiver and signal space

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$$s_\ell(t) = s_{\ell,1}\phi_1(t) + s_{\ell,2}\phi_2(t)$$

Expansion coefficients, representing message ℓ

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Checkpoint: Is both $\phi_1(t)$ and $\phi_2(t)$ needed?

Lecture 2: MAP receiver and signal space

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Basis functions

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$$\phi_1(t) = \frac{g(t)\cos(2\pi f_c t)}{\sqrt{E_g/2}} \qquad \phi_2(t) = -\frac{g(t)\sin(2\pi f_c t)}{\sqrt{E_g/2}}$$

Continuous wafefrom, representing message ℓ

Checkpoint: Is both $\phi_1(t)$ and $\phi_2(t)$ needed?

YES, since $\phi_1(t) \neq \alpha\phi_2(t)$

Lecture 2: MAP receiver and signal space

We get

$$s_\ell(t) = s_{\ell,1}\phi_1(t) + s_{\ell,2}\phi_2(t)$$

Basis functions

Expansion coefficients, representing message ℓ

where

$$\phi_1(t) = \frac{g(t)\cos(2\pi f_c t)}{\sqrt{E_g/2}} \qquad \phi_2(t) = -\frac{g(t)\sin(2\pi f_c t)}{\sqrt{E_g/2}}$$

Continuous wafefrom, representing message ℓ

Checkpoint: Is basis orthonormal ?

Lecture 2: MAP receiver and signal space

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Basis functions

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Expansion coefficients, representing message ℓ

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Continuous wafefrom, representing message ℓ

Checkpoint: Is basis orthonormal ?

1) $\int \phi_1^2(t)\mathrm{d}t = 1$

2) $\int \phi_2^2(t)\mathrm{d}t = 1$

Prove this at home

Lecture 2: MAP receiver and signal space

We get

Basis functions

$$s_\ell(t) = s_{\ell,1}\phi_1(t) + s_{\ell,2}\phi_2(t)$$

Expansion coefficients, representing message ℓ

where

$$\phi_1(t) = \frac{g(t)\cos(2\pi f_c t)}{\sqrt{E_g/2}} \qquad \phi_2(t) = -\frac{g(t)\sin(2\pi f_c t)}{\sqrt{E_g/2}}$$

Continuous wafefrom, representing message ℓ

Checkpoint: Is basis orthonormal ? Yes

1) $\int \phi_1^2(t)\mathrm{d}t = 1$

2) $\int \phi_2^2(t)\mathrm{d}t = 1$

3) $\int \phi_1(t)\phi_2(t)\mathrm{d}t = 0$

Proofs requires usage of $f_c >> 1$

Lecture 2: MAP receiver and signal space

Generalization

$$s_\ell(t) = \sum_n^N s_{\ell,n}\phi_n(t) = s_{\ell,1}\phi_1(t) + \ldots + s_{\ell,N}\phi_N(t)$$

In general, the signal set can require >2 basis functions

Lecture 2: MAP receiver and signal space

Generalization

$$s_\ell(t) = \sum_{n}^{N} s_{\ell,n}\phi_n(t) = s_{\ell,1}\phi_1(t) + \ldots + s_{\ell,N}\phi_N(t)$$

In general, the signal set can require >2 basis functions

Question: For a signal set of below form, can N>2?

$$s_\ell(t) = x_{I,\ell}(t)\cos(2\pi f_c t) - x_{Q,\ell}(t)\sin(2\pi f_c t)$$

Lecture 2: MAP receiver and signal space

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Question: For a signal set of below form, can N>2?

$$s_\ell(t) = x_{I,\ell}(t)\cos(2\pi f_c t) - x_{Q,\ell}(t)\sin(2\pi f_c t)$$

Yes! We only derived that for PAM components we get N=2

Lecture 2: MAP receiver and signal space

Generalization

$$s_\ell(t) = \sum_n^N s_{\ell,n}\phi_n(t) = s_{\ell,1}\phi_1(t) + \ldots + s_{\ell,N}\phi_N(t)$$

In general, the signal set can require >2 basis functions

Question: For a single signal of below form, can N>2?

$$s(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

Lecture 2: MAP receiver and signal space

Generalization

$$s_\ell(t) = \sum_n^N s_{\ell,n}\phi_n(t) = s_{\ell,1}\phi_1(t) + \ldots + s_{\ell,N}\phi_N(t)$$

In general, the signal set can require >2 basis functions

Question: For a single signal of below form, can N>2?

$$s(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

There is no need to!!!

We can write $s(t) = s\phi_1(t)$

where $\phi_1(t) = s(t)/\sqrt{E_s}, \qquad s = \sqrt{E_s}$

Lecture 2: MAP receiver and signal space

Definition

The dimensionality of a signal set $\mathcal{S} = \{s_\ell(t)\}_{\ell=0}^{M-1}$ is the smallest number N such that each signal in $\mathcal{S}$ can, for some orthonormal set of basis functions $\{\phi_n(t)\}_{n=1}^{N}$ be expressed as

$$s_\ell(t) = \sum_n^N s_{\ell,n}\phi_n(t) = s_{\ell,1}\phi_1(t) + \ldots + s_{\ell,N}\phi_N(t)$$

Lecture 2: MAP receiver and signal space

Why is it so important ?

Lecture 2: MAP receiver and signal space

Why is it so important ? Because most properties are not dependent on the basis functions!

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Energy

$$E_\ell = \int s_\ell^2(t)\mathrm{d}t =$$

By definition

Lecture 2: MAP receiver and signal space

Why is it so important ? Because most properties are not dependent on the basis functions!

Energy

$$E_\ell = \int s_\ell^2(t)\mathrm{d}t = \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]^2 \mathrm{d}t =$$

Use the basis expansion

Lecture 2: MAP receiver and signal space

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$$E_\ell = \int s_\ell^2(t)\mathrm{d}t = \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]^2 \mathrm{d}t$$

$$= \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]\left[\sum_{m=1}^{N} s_{\ell,m}\phi_m(t)\right]\mathrm{d}t =$$

Expand the power. Must use two different indeces

Lecture 2: MAP receiver and signal space

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$$= \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]\left[\sum_{m=1}^{N} s_{\ell,m}\phi_m(t)\right]\mathrm{d}t$$

$$= \int \sum_{m=1}^{N}\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)s_{\ell,m}\phi_m(t)\mathrm{d}t =$$

Reorder

Lecture 2: MAP receiver and signal space

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Energy

$$E_\ell = \int s_\ell^2(t)\mathrm{d}t = \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]^2 \mathrm{d}t$$

$$= \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]\left[\sum_{m=1}^{N} s_{\ell,m}\phi_m(t)\right] \mathrm{d}t$$

$$= \int \sum_{m=1}^{N}\sum_{n=1}^{N} s_{\ell,n}\phi_n(t) s_{\ell,m}\phi_m(t)\mathrm{d}t$$

Pull out constnants not dependent on t

$$= \sum_{m=1}^{N}\sum_{n=1}^{N} s_{\ell,n}s_{\ell,m}\int \phi_n(t)\phi_m(t)\mathrm{d}t =$$

Lecture 2: MAP receiver and signal space

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$$E_\ell = \int s_\ell^2(t)\mathrm{d}t = \int \left[\sum_{n=1}^{N} s_{\ell,n}\phi_n(t)\right]^2 \mathrm{d}t$$

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$$= \sum_{m=1}^{N}\sum_{n=1}^{N} s_{\ell,n}s_{\ell,m}\int \phi_n(t)\phi_m(t)\mathrm{d}t$$

=0, if n≠m

=1, if n=m

Lecture 2: MAP receiver and signal space

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Lecture 2: MAP receiver and signal space

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Lesson learned

1. Energy does not depend on basis functions
2. True only for orthonormal basis functions
3. All signal sets with equal expansion coefficients have same energies

Lecture 2: MAP receiver and signal space

Why is it so important ? Because most properties are not dependent on the basis functions!

Euclidian distances

$$D_{i,j}^2 = \int (s_i(t) - s_j(t))^2 \mathrm{d}t =$$

By definition

Lecture 2: MAP receiver and signal space

Why is it so important ? Because most properties are not dependent on the basis functions!

Euclidian distances

$$D_{i,j}^2 = \int (s_i(t) - s_j(t))^2 \mathrm{d}t = \int \left[\sum_{n=1}^{N} (s_{i,n} - s_{j,n}) \phi_n(t) \right]^2 \mathrm{d}t$$

Basis expansion

Lecture 2: MAP receiver and signal space

Why is it so important ? Because most properties are not dependent on the basis functions!

Euclidian distances

$$D_{i,j}^2 = \int (s_i(t) - s_j(t))^2 \mathrm{d}t = \int \left[\sum_{n=1}^{N} (s_{i,n} - s_{j,n}) \phi_n(t) \right]^2 \mathrm{d}t$$

Recall energy computation

$$E_\ell = \int s_\ell^2(t) \mathrm{d}t = \int \left[\sum_{n=1}^{N} s_{\ell,n} \phi_n(t) \right]^2 \mathrm{d}t = \sum_{n=1}^{N} s_{\ell,n}^2$$

Lecture 2: MAP receiver and signal space

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The same setup...

Recall energy computation

$$E_\ell = \int s_\ell^2(t) \mathrm{d}t = \int \left[\sum_{n=1}^{N} s_{\ell,n} \phi_n(t) \right]^2 \mathrm{d}t = \sum_{n=1}^{N} s_{\ell,n}^2$$

Lecture 2: MAP receiver and signal space

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$$= \sum_{n=1}^{N} (s_{i,n} - s_{j,n})^2$$

Lessons learned

1. Euclidian distances do not depend on basis functions
2. True only for orthonormal basis functions
3. All signal sets with equal expansion coefficients have same distances

Lecture 2: MAP receiver and signal space

Why is it so important ? Because most properties are not dependent on the basis functions!

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Lessons learned

1. Euclidian distances do not depend on basis functions
2. True only for orthonormal basis functions
3. All signal sets with equal expansion coefficients have same distances
4. THEREFORE, SAME PERFORMANCE

Lecture 2: MAP receiver and signal space

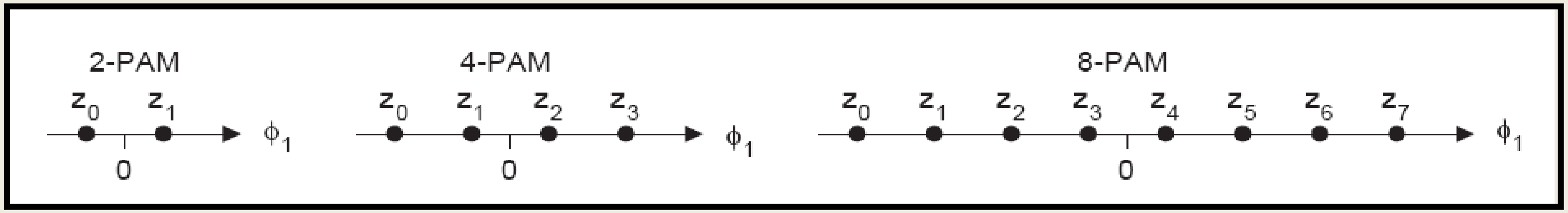
To study performance of a communication systems, it is sufficient to study its signal space decomposition

Lessons learned

1. Euclidian distances do not depend on basis functions
2. True only for orthonormal basis functions
3. All signal sets with equal expansion coefficients have same distances
4. THEREFORE, SAME PERFORMANCE

Lecture 2: MAP receiver and signal space

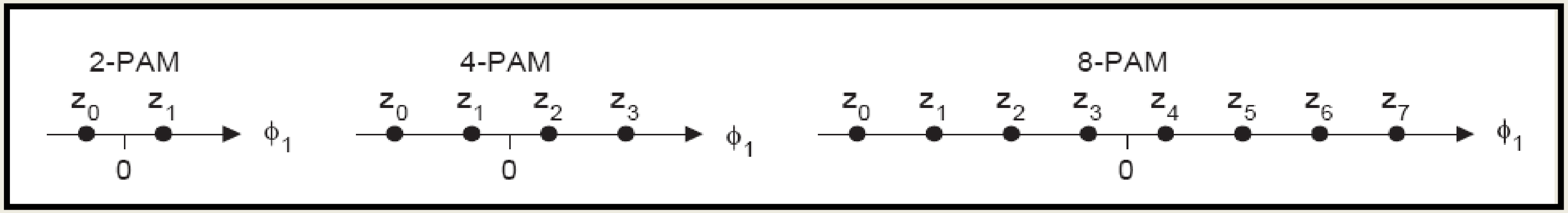
Examples



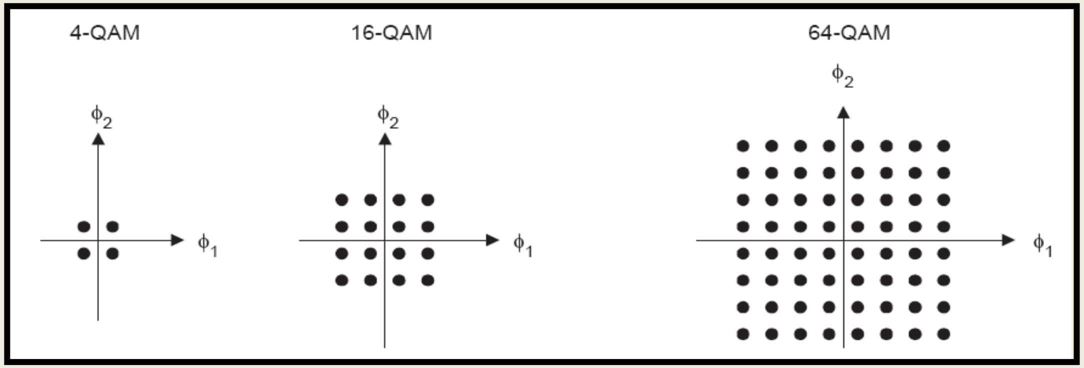
N = ??

Lecture 2: MAP receiver and signal space

Examples



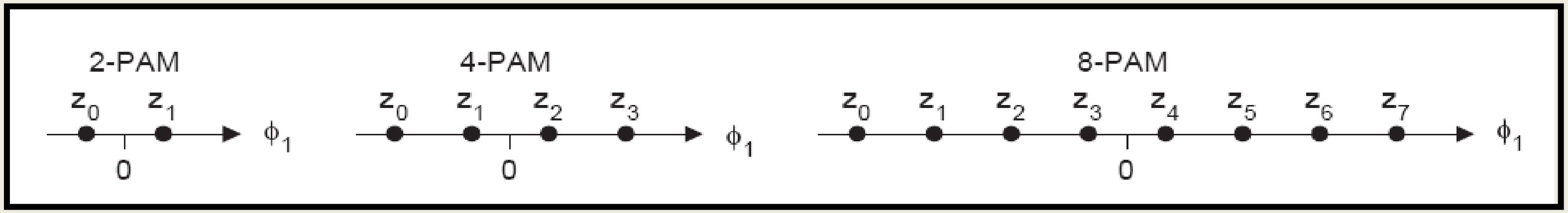
$N = 1$



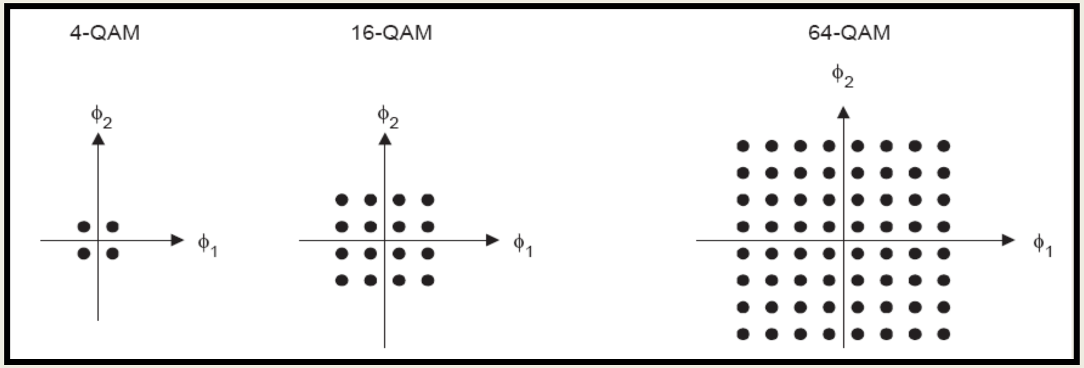
N = ??

Lecture 2: MAP receiver and signal space

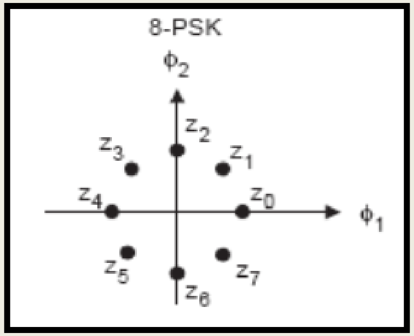
Examples



N = 1



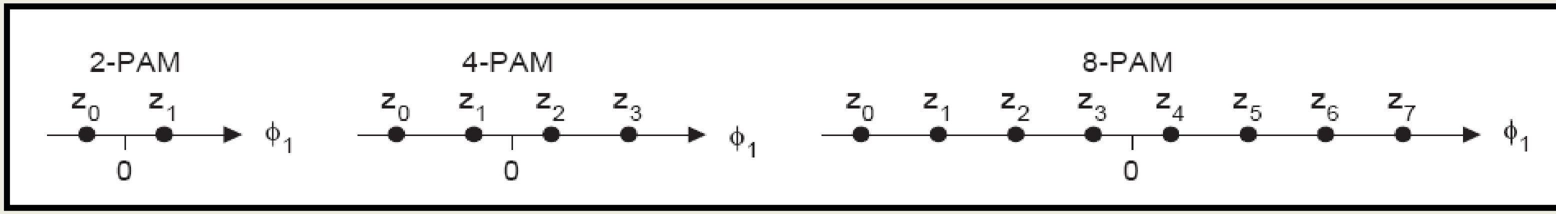
N = 2



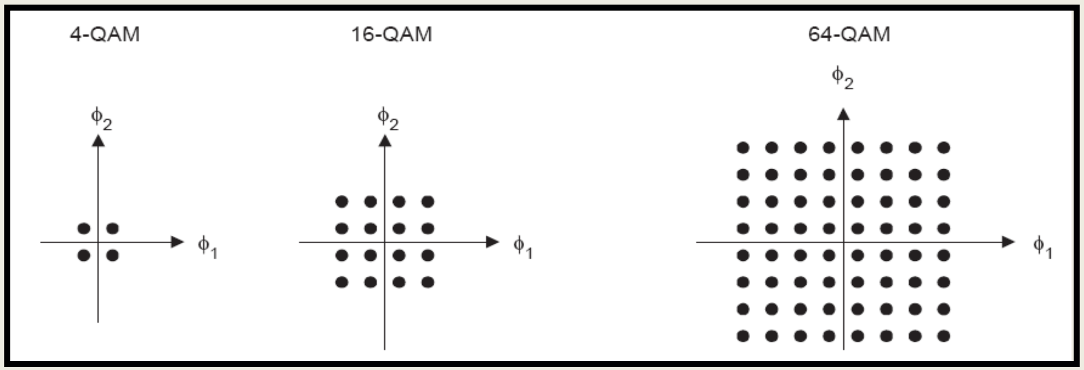
N = ??

Lecture 2: MAP receiver and signal space

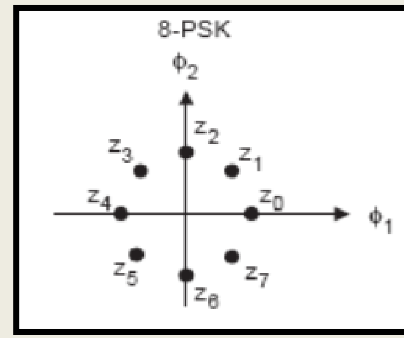
Examples



N = 1

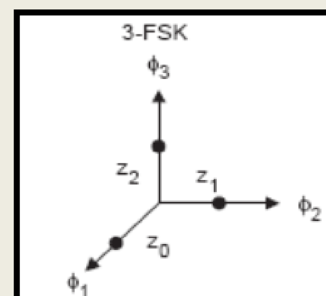


N = 2



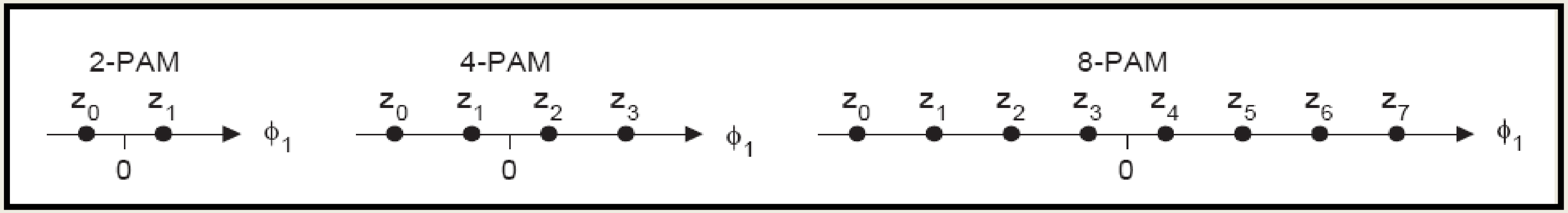
N = 2

N = ??

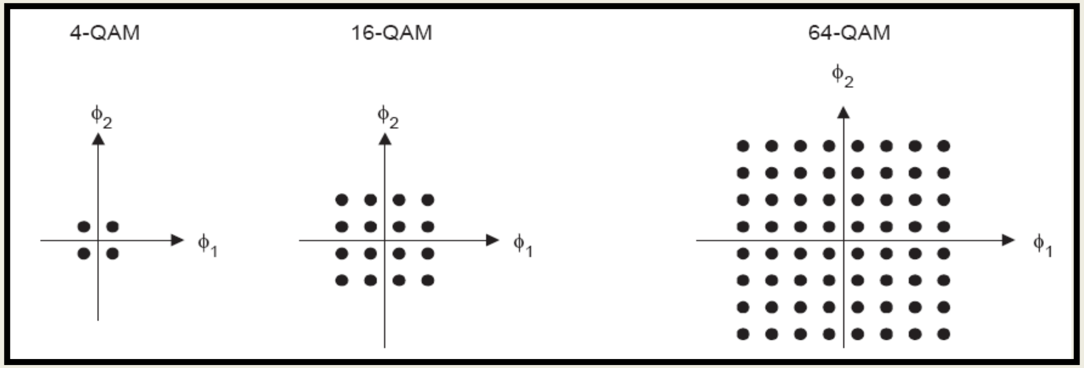


Lecture 2: MAP receiver and signal space

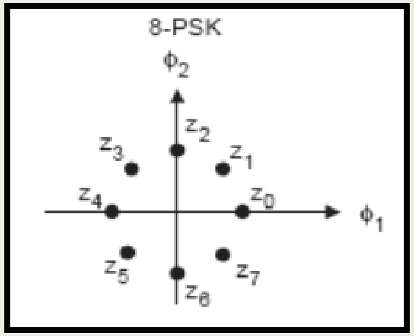
Examples



N = 1

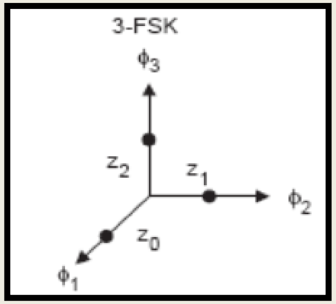


N = 2



N = 2

N = 3



Lecture 2: MAP receiver and signal space

Definition

The signal $s_j(t)$ is expressed in signal space as the vector $\mathbf{s}_j$

Lecture 2: MAP receiver and signal space

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Property

The signal space vector (or point) $\mathbf{s}_j$ carries all information relevant for performance evaluation

Lecture 2: MAP receiver and signal space

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The signal $s_j(t)$ is expressed in signal space as the vector $\mathbf{s}_j$

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The signal space vector (or point) $\mathbf{s}_j$ carries all information relevant for performance evaluation

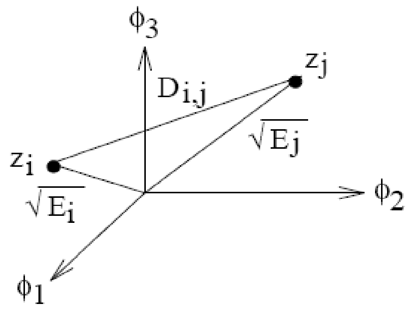


Figure 5.3: Illustrating E_ℓ and $D_{i,j}$ in signal space.

Lecture 2: MAP receiver and signal space

Definition

The signal $s_j(t)$ is expressed in signal space as the vector $\mathbf{s}_j$

Property

The signal space vector (or point) $\mathbf{s}_j$ carries all information relevant for performance evaluation

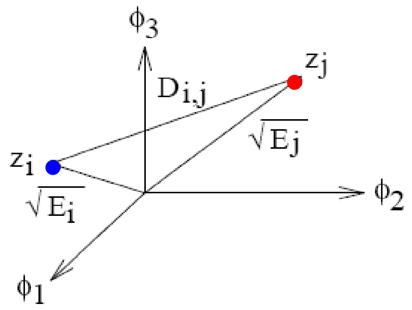
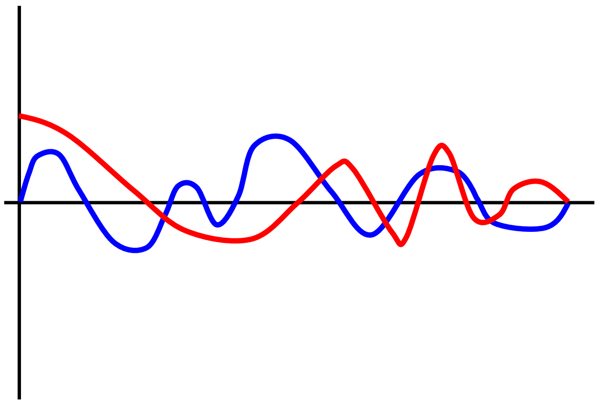


Figure 5.3: Illustrating E_ℓ and $D_{i,j}$ in signal space.



Lecture 2: MAP receiver and signal space

Towards the MAP receiver

Assume that we have a noise free signal $r(t)$ at the input of the receiver

We know that this is one of $\{s_\ell(t)\}_{\ell=1}^{M-1}$ but we don't know exactly which one

How to exploit signal space to find out?

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The energy of the signal $r(t)$ is along the directions of the basis functions $\{\phi_n(t)\}_{n=1}^{N}$

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"Look" in the dimensions

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t$$

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"Look" in the dimensions

a is true message

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \int \left[\sum_{m=1}^{N} s_{a,m}\phi_m(t)\right]\phi_n(t)\mathrm{d}t$$

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$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \int \left[\sum_{m=1}^{N} s_{a,m}\phi_m(t)\right]\phi_n(t)\mathrm{d}t$$

$$= \sum_{m=1}^{N} s_{a,m} \int \phi_m(t)\phi_n(t)\mathrm{d}t$$

A simple reordering…

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$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \int \left[\sum_{m=1}^{N} s_{a,m}\phi_m(t)\right]\phi_n(t)\mathrm{d}t$$

$$= \sum_{m=1}^{N} s_{a,m}\int \phi_m(t)\phi_n(t)\mathrm{d}t = s_{a,n}$$

due to orthonormality

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In vector notation $\mathbf{r} = \mathbf{s}_a$

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In vector notation $\mathbf{r} = \mathbf{s}_a$

Receiver: $\hat{a} = \arg\min_{\ell} \|\mathbf{r} - \mathbf{s}_\ell\|^2 = a$

↑ Guaranteed since no noise

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Gaussian noise.
Variance $N_0/2$

Lecture 2: MAP receiver and signal space

Interlude: Gaussian random variables

Fact: Two Gaussian random variables are independent iff they are uncorrelated

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = s_{a,n} + w_n$$

Gaussian noise.
Variance $N_0/2$

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Question: Is w_i and w_j independent?

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$$\mathrm{E}(w_i w_j) =$$

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = s_{a,n} + w_n$$

Gaussian noise.
Variance $N_0/2$

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$$\mathrm{E}(w_i w_j) = \mathrm{E}\left(\int N(t)\phi_i(t)\mathrm{d}t \int N(t)\phi_j(t)\mathrm{d}t\right)$$

↑ Gaussian noise. Spectral density $N_0/2$

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = s_{a,n} + w_n$$

Gaussian noise.
Variance $N_0/2$

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$$= \mathrm{E}\left(\int\int N(t)\phi_i(t)N(\tau)\phi_j(\tau)\mathrm{d}t\mathrm{d}\tau\right)$$

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = s_{a,n} + w_n$$

Gaussian noise.
Variance $N_0/2$

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$$\begin{aligned}
\mathrm{E}(w_i w_j) &= \mathrm{E}\left(\int N(t)\phi_i(t)\mathrm{d}t \int N(t)\phi_j(t)\mathrm{d}t\right) \\
&= \mathrm{E}\left(\int\int N(t)\phi_i(t)N(\tau)\phi_j(\tau)\mathrm{d}t\mathrm{d}\tau\right) \\
&= \int\int \mathrm{E}\left(N(t)N(\tau)\right)\phi_i(t)\phi_j(\tau)\mathrm{d}t\mathrm{d}\tau
\end{aligned}$$

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&= \int\int \mathrm{E}\left(N(t)N(\tau)\right)\phi_i(t)\phi_j(\tau)\mathrm{d}t\mathrm{d}\tau \\
&= \frac{N_0}{2}\int \phi_i(t)\phi_j(t)\mathrm{d}t
\end{aligned}$$

The definition of white Gaussian noise

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$$= \int\int \mathrm{E}\left(N(t)N(\tau)\right)\phi_i(t)\phi_j(\tau)\mathrm{d}t\mathrm{d}\tau$$

$$= \frac{N_0}{2}\int \phi_i(t)\phi_j(t)\mathrm{d}t \quad = \frac{N_0}{2}\delta[i-j]$$

By orthonormality

Lecture 2: MAP receiver and signal space

Interlude: Gaussian random variables

Fact: Two Gaussian random variables are independent iff they are uncorrelated

Question: Is w_i and w_j independent? YES!!!

$$\begin{aligned}
\mathrm{E}(w_i w_j) &= \mathrm{E}\left(\int N(t)\phi_i(t)\mathrm{d}t \int N(t)\phi_j(t)\mathrm{d}t\right) \\
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\end{aligned}$$

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Assume that we have a noisy signal $r(t)$ at the input of the receiver

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How to exploit signal space to detect the message?

We can still "look" along the basis functions $r_n = \int r(t)\phi_n(t)\mathrm{d}t$

CRITICAL: Can we guarantee that this is optimal?

We get, $r_n = \int r(t)\phi_n(t)\mathrm{d}t = s_{a,n} + w_n$

Gaussian noise.
Variance $N_0/2$
Independent over n

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$$r(t) = s_a(t) + N(t)$$

$\hat{r}(t) = \sum_{n=1}^{N} r_n\phi_n(t)$ Attempt to reconstruct r(t)

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$$r(t) = s_a(t) + N(t)$$

$$\hat{r}(t) = \sum_{n=1}^{N} r_n\phi_n(t) = \sum_{n=1}^{N}(s_{a,n} + w_n)\phi_n(t) = s_a(t) + \sum_{n=1}^{N} w_n\phi_n(t)$$

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$$r(t) = s_a(t) + N(t)$$

Identical ?

$$\hat{r}(t) = \sum_{n=1}^{N} r_n\phi_n(t) = \sum_{n=1}^{N}(s_{a,n} + w_n)\phi_n(t) = s_a(t) + \sum_{n=1}^{N} w_n\phi_n(t) = s_a(t) + \hat{N}(t)$$

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$r(t) = s_a(t) + N(t)$

Identical ? NO. Basis functions are basis for signal, not for the noise

$$\hat{r}(t) = \sum_{n=1}^{N} r_n\phi_n(t) = \sum_{n=1}^{N}(s_{a,n} + w_n)\phi_n(t) = s_a(t) + \sum_{n=1}^{N} w_n\phi_n(t) \quad = s_a(t) + \hat{N}(t)$$

Lecture 2: MAP receiver and signal space

Towards the MAP receiver

To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

We could guarantee optimiality if we could retrieve r(t) from r

$$r(t) = s_a(t) + N(t)$$

Identical ? NO. Basis functions are basis for signal, not for the noise

$$\hat{r}(t) = \sum_{n=1}^{N} r_n \phi_n(t) = \sum_{n=1}^{N} (s_{a,n} + w_n)\phi_n(t) = s_a(t) + \sum_{n=1}^{N} w_n \phi_n(t) \quad = s_a(t) + \hat{N}(t)$$

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To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

Now, look again in all dimensions,

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \begin{cases} s_{a,n} + w_n & 1 \leq n \leq N \\ w_n & n > N \end{cases}$$

We could guarantee optimiality if we could retrieve r(t) from r

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$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \begin{cases} s_{a,n} + w_n & 1 \leq n \leq N \\ w_n & n > N \end{cases}$$

With sufficiently many additional basis, we get

We could guarantee optimiality if we could retrieve r(t) from r

$r(t) = s_a(t) + N(t)$ Identical ? YES

$$\hat{r}(t) = \sum_{n=1}^{\infty} r_n \phi_n(t) = \sum_{n=1}^{\infty} (s_{a,n} + w_n)\phi_n(t) = s_a(t) + \sum_{n=1}^{\infty} w_n \phi_n(t) = s_a(t) + N(t)$$

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However, do we really need these extra basis functions ***for detection?***

Lecture 2: MAP receiver and signal space

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To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

Now, look again in all dimensions,

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \begin{cases} s_{a,n} + w_n & 1 \leq n \leq N \\ w_n & n > N \end{cases}$$

However, do we really need these extra basis functions *for detection?*

NO, since we proved earlier that the noise in different dimensions is independent

Lecture 2: MAP receiver and signal space

Towards the MAP receiver

To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

Now, look again in all dimensions,

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Although generated from the same noise N(t), these values could just as well have been generated from

1.
2.

Lecture 2: MAP receiver and signal space

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To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

Now, look again in all dimensions,

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \begin{cases} s_{a,n} + w_n & 1 \le n \le N \\ w_n & n > N \end{cases}$$

Although generated from the same noise N(t), these values could just as well have been generated from

1. A refrigerator somewhere in the Andromeda galaxy
2.

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To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

Now, look again in all dimensions,

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \begin{cases} s_{a,n} + w_n & 1 \leq n \leq N \\ w_n & n > N \end{cases}$$

Although generated from the same noise N(t), these values could just as well have been generated from

1. A refrigerator somewhere in the Andromeda galaxy
2. Matlab installed on my office computer

Lecture 2: MAP receiver and signal space

Towards the MAP receiver

To fix this, we can add more basis functions $\phi_n(t), \quad n > N$

Now, look again in all dimensions,

$$r_n = \int r(t)\phi_n(t)\mathrm{d}t = \begin{cases} s_{a,n} + w_n & 1 \leq n \leq N \\ w_n & n > N \end{cases}$$

Although generated from the same noise N(t), these values could just as well have been generated from

1. A refrigerator somewhere in the Andromeda galaxy
2. Matlab installed on my office computer

Clearly, a refrigerator in Andromeda cannot assist detection of systems on earth.

Lecture 2: MAP receiver and signal space

Towards the MAP receiver

So, for detection, it is sufficient to look in the N dimensions where the signal is

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Lecture 2: MAP receiver and signal space

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Write in vector notation $\mathbf{r} = \mathbf{s}_a + \mathbf{w}$

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