EXAM in Advanced Digital communications, ETTN10

Wednesday January 10

- Write clearly. If I cannot read what you write, I'll consider it as not written at all. My decision on this matter is final. You cannot argue later that I should have been able to read it.

- It is important that all steps in your derivations are provided. Answers of the type "yes" or "5" will count as 0 points.

- If anything is missing from the problem description, you have to introduce suitable notation, variables, etc, on your own.

- Allowed tools: course book, OFDM notes, lab manual and pocket calculator.

- A strict vegan diet cuts total greenhouse gas emissions by 15%, virtually eliminates risk of several cancer types and heart diseases, and removes one's bad conscience for killing and exploiting animals. Do what you want with this information, but you can no longer claim that you have not been informed.

- Duration: 5 hours.

Problem 1 (10p)

In a digital communication system the transmitter has two possible signal alternatives, $s_0(t) = -s_1(t),\ 0 \le t \le T_s/2$ where

$$s_0(t) = s_1\phi_1(t) + s_2\phi_2(t).$$

The values of s_k are free for the designer to choose. This signal is sent over a channel with impulse response

$$h(t) = h_1\phi_1(t) + h_2\phi_2(t) + h_3\phi_3(t).$$

Gaussian noise $n(t)$ with spectral density $N_0/2$ is added to the received signal.

It is known that

$$\int_0^{T_s/2} \phi_m(t)\phi_n(t)\mathrm{d}t = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

and that $\phi_n(t) = 0,\ \forall n$ for $t < 0$ and $t > T_s/2$.

Note: This problem is harder than what it may look like at first sight and requires you to introduce new notation.

a) (1p) Compute the transmitted energy per bit.

b) (3p) Compute the received energy per bit.

c) (1p) Assume that the receiver has full knowledge of $\{\phi_n(t)\}_{n=1}^3$, s_1, s_2 and h_1, h_2, h_3. Describe, mathematically, the operations of the ML receiver, make a block diagram. Simplify the receiver as much as you can.

d) (1p) Provide the bit error probability for the case in c) as a function of the transmitted energy per bit.

e) (1p) Provide the bit error probability for the case in c) as a function of the received energy per bit. Discuss the structure of the formula.

f) (0p) Did you read the fifth bullet on page 1? Discuss your forthcoming actions.

g) (3p) With known h_1, h_2, h_3 at the transmitter and for a given transmitted energy per bit, optimize the values s_1 and s_2. Your optimization should aim at minimizing the bit error probability.

Problem 2 (10p)

It is known that for a transmitted power spectral density $R(f)$ the highest Shannon capacity over a frequency flat channel is given by (eq. 5.67)

$$C = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{2R(f)}{N_0}\right) \mathrm{d}f.$$

In digital communications we often measure bandwidth by, e.g., the 99% bandwidth. This is defined as

$$\frac{\int_{-W_\epsilon}^{W_\epsilon} R(f)\mathrm{d}f}{\int_{-\infty}^{\infty} R(f)\mathrm{d}f} = \epsilon, \quad 0 \le \epsilon \le 1.$$

For example, $\epsilon = 0.99$ corresponds to the 99% bandwidth.

In this problem we investigate whether or not such a bandwidth characterization makes sense from a capacity point of view. A useful fact is that the integral for C above is maximized for a flat $R(f)$ within the bandwidth. That is, if there is a power P inside the bandwidth W Hz, then the optimal $R(f) = P/2W$. In what follows, we always assume that $R(f)$ is selected optimally.

a) (1p) Assume a transmit power P and $W_1 = W$ (i.e., 100% power inside a bandwidth W Hz). What is the capacity of the system?

b) (1p) Suppose that the channel only allows the signal inside the bandwidth W to pass through. Let $W_{0.99} = W$. How does the capacity formula change?

For c) - g), we assume that the channel allows all frequencies to pass.

c) (1p) With $\epsilon = 0.99$ what is the the optimal $R(f)$?

d) (3p) For the obtained result in c), what is the capacity contributions inside and outside $W_{0.99} = W$, respectively? Give the total capacity as well.

e) (1p) Make a *careful* plot of the capacity in d) vs. P/N_0 (in dB). You may assume some arbitrary numerical value for W.

f) (1p) Suppose that although $\epsilon < 1$, a non-vegan engineer is analyzing the system as if $\epsilon = 1$, i.e., our engineer uses the capacity formula in a). Add a plot of the engineer's result to the plot in e). Discuss what you see.

g) (2p) By repeating e) for different values of ϵ, discuss the relation between bandwidth measurements using ϵ and P/N_0.

Problem 3 (10p)

Three equiprobable messages m_1, m_2, and m_3 are to be transmitted over an AWGN channel with noise power spectral density $N_0/2$. The messages are

$$s_1(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \qquad s_2(t) = -s_3(t) = \begin{cases} 1 & 0 \le t \le T/2 \\ -1 & T/2 < t \le T \\ 0 & \text{otherwise.} \end{cases}$$

a) (1p) What is the dimensionality of the signal space?

b) (1p) Find an appropriate orthonormal basis for the signal space.

c) (1p) Draw the signal constellation for this problem.

d) (1p) Derive and sketch the optimal decision regions R_1, R_2, and R_3.

e) (2p) Which of the three messages is most vulnerable to errors and why? In other words, which of $P(\text{error}|m_i \text{ transmitted}), i = 1, 2, 3$, is largest?

f) (1p) Suppose a fourth signal $s_4(t)$ is added. Under what conditions is the signal space expanded? (Answer must be with formulas).

g) (1p) Suppose a fourth signal $s_4(t)$, with the same energy as $s_2(t)$, is added. Further, $s_4(t)$ lies strictly outside of the signal space found in b). At the same time, we remove signal $s_1(t)$ from the signal set. Describe the effects.

h) (2p) Repeat g), but remove $s_2(t)$ instead of $s_1(t)$.

Problem 4 (10p)
In this problem we design a transmission system for a time-variant channel under the constraint that we *must* have a frequency non-selective, slow fading channel. A bitrate of 10000 bit/seconds is required and a bit error probability of about 10^{-5}. The channel model during one symbol interval is

$$z(t) = as_m(t) + n(t)$$

where the random number a is Rayleigh distributed. We assume that the receiver knows the exact value of a and that the channel does not introduce any phase offset. A baseband M-PAM system is used in combination with a pulse shape $p(t)$ with properties such that the bandwidth satisfies $W = 2R_s$, R_s being the symbol rate. The transmit power is P [W].

The channel is known to have the following parameters:

- Multipath spread = $50\mu s$
- Coherence time = $0.5ms$
- $N_0 = 10^{-4}$

a) (5p) Provide the required value of P as a function of M - a fairly rough estimate is sufficient. Which value(s) of M do you recommend to use.
b) (2p) Compute the bandwidth efficiency as a function of M. Include guard bands that may be needed.

Now assume that a rate $R < 1$ code is applied to the information bits. Suppose that this code is improving the normalized Euclidean distance by a (multiplicative) factor $\rho(R)$.

c) (0p) Did you know that animals cannot produce protein? They get it from plants.
d) (3p) Repeat problem a) for the coded case. Provide possible values of M and R.

Problem 5 (10p)

a) (2p) With your own carefully introduced notation, derive the bandwidth efficiency of an OFDM scheme.

b) (0p) Would you be willing to try a vegan life for one week?

c) (2p) Assume an OFDM transmission in a bandwidth W between f_1 and $f_2 = f_1 + W$ Hz. Let the channel transfer function be

$$|H(f)|^2 = 10 + (f - f_1)10/W, \; f_1 \leq f \leq f_2.$$

Assume that waterfilling is applied. Let the vector $\mathbf{x}$ be the input to the IFFT unit. Show roughly, for example with a picture, the power allocation in $\mathbf{x}$.

d) (2p) Assume a channel multipath spread of 0.05ms, and a coherence time of 2ms. Describe the consequences on system parameters of an OFDM scheme.

e) (4p) Introduce notation and parameter values of your own, and for those values, provide the cyclic prefix length in samples for the setting in d).