EXAM in Advanced Digital communications, ETTN10

Monday April 9

- Write clearly. If I cannot read what you write, I'll consider it as not written at all. My decision on this matter is final. You cannot argue later that I should have been able to read it.
- It is important that all steps in your derivations are provided. Answers of the type "yes" or "5" will count as 0 points.
- If anything is missing from the problem description, you have to introduce suitable notation, variables, etc, on your own.
- In a problem of the form: "Text 1. a) Text 2, Question, b) Text 3, Question etc", only "Text 1" and "Text 3" applies to the question in b), not "Text 2".
- Allowed tools: course book, OFDM notes, lab manual and pocket calculator.
- Duration: 5 hours.

Problem 1 (10p)

In a digital communication system the transmitter has two possible signal alternatives, $s_0(t) = -s_1(t)$, $0 \le t \le T_s$ where

$$s_0(t) = s_1\phi_1(t) + s_2\phi_2(t).$$

The values of s_k are free for the designer to choose. In this problem there is an intended user, as well as an eavesdropper. An eavesdropper is a terminal that can hear the transmission and wants to steal the data. Thus, the transmission should be such that only the intended user can recover the data, but the eavesdropper should not be able to do so.

The received signal at the intended user is

$$r_I(t) = s_k(t) + n_I(t)$$

and at the eavesdropper

$$r_E(t) = \alpha s_k(t) + n_E(t).$$

Both noise processes are white Gaussian with spectral density $N_0/2$.

It is known that

$$\int_0^{T_s} \phi_m(t)\phi_n(t)\mathrm{d}t = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

and that $\phi_n(t) = 0$, $\forall n$ for $t < 0$ and $t > T_s$.

a) (1p) Assume optimal receivers at both the intended user and the eavesdropper. Derive the bit error rates at both the intended user and the eavesdropper. Your answer should be in terms of the variables given above.

b) (2p) Assume $\alpha = 0.5$. Assume that if the error probability, P_B, satisfies $P_B > 10^{-1}$, then the message is considered lost, and that if $P_B < 10^{-3}$, it is considered fully recovered. Is it possible to design s_1 and s_2 such that the message is recovered at the intended user, but lost at the eavesdropper? Motivate your answer.

c) (1p) Assume that it is known that the eavesdropper decodes the signal by first constructing

$$r_E = \int_0^{T_s} r_E(t)(\phi_1(t) + \phi_2(t))\mathrm{d}t$$

and then throws the signal $r_E(t)$ away. I.e., it can decode the message only on the basis of r_E. Suggest a suitable signal design for the transmitter.

d) (2p) Assume that it is known that the eavesdropper decodes the signal by first constructing

$$r_E = \int_0^{T_s} r_E(t)(\phi_1(t) + \phi_2(t))\mathrm{d}t$$

and then throws the signal $r_E(t)$ away. Further, assume that the intended user decodes the signal by first constructing

$$r_I = \int_0^{T_s} r_E(t)\phi_1(t)\mathrm{d}t$$

and then throws the signal $r_E(t)$ away. Design a suitable signal design for the transmitter and compute the error probability at the intended user.

e) (2p) Assume that it is known that the eavesdropper decodes the signal by first constructing

$$r_E = \int_0^{T_s} r_E(t)(\phi_1(t) + \phi_2(t))\mathrm{d}t$$

and then throws the signal $r_E(t)$ away. Further, assume that the intended user decodes the signal by first constructing

$$r_I = \int_0^{T_s} r_E(t)(\phi_1(t) - \phi_2(t))\mathrm{d}t$$

and then throws the signal $r_E(t)$ away. Design a suitable signal design for the transmitter and compute the error probability at the intended user.

f) (2p) Assume that it is known that the eavesdropper decodes the signal by first constructing

$$r_E = \int_0^{T_s} r_E(t)(\phi_1(t) + \phi_2(t))\mathrm{d}t$$

and then throws the signal $r_E(t)$ away. Further, assume that the intended user decodes the signal optimally. Design a suitable signal design for the transmitter and compute the error probability at the intended user.

Problem 2 (10p)

It is known that for a transmitted power spectral density $R(f)$ the highest Shannon capacity over a frequency flat channel is given by (eq. 5.67)

$$C = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{2R(f)}{N_0}\right) \mathrm{d}f.$$

Although it is known that the capacity formula only applies to Gaussian inputs, we assume in this problem that it applies to PAM constellations as well.

We assume that the transmitted signal is given by

$$s(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$$

where a_k are M-PAM symbols with average energy E_s. Further, the data symbols a_k have been encoded with an outer code with rate $0 < R < 1$. $T = 1/8000$ and the Fourier transform of $p(t)$ satisfies $|P(f)|^2 = 1/8000,\ |f| \leq 4000$ and $|P(f)|^2 = 0,\ |f| > 4000$. From basic theory, it is known that

$$R(f) = \beta|P(f)|^2$$

for some value β. The transmit power is P.

a) (1p) Provide a formula that connects P and $R(f)$. Simplify the formula as far as possible for the given parameter values.

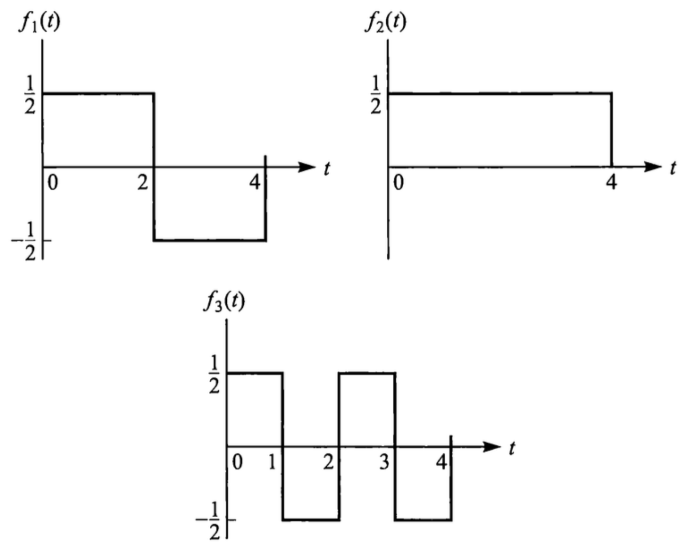
b) (1p) Provide a formula that connects P and E_s. Simplify the formula as far as possible for the given parameter values.

c) (2p) Solve the integral for C and manipulate the result so that it is given in terms of P/N_0.

d) (3p) Provide a bound for R in terms of P/N_0 for $M = 2$ and $M = 4$. Show this bound in a plot where R is on the y-axis and P/N_0 is on the x-axis. Clarify whether the bound is an upper or lower bound. The bound should be based on the capacity formula.

e) (3p) For $M = 2$ and $R = 1/2$ and the minimum P/N_0 that achieves capacity. What is the error probability of the coded symbols a_k? That is, assume a receiver that optimally decodes a_k, but disregards the code.

Problem 3 (12p)
Consider the three signals below.



a) (1p) Derive, mathematically, the dimensionality of the signal space. Provide an orthonormal set of basis functions.

b) (1p) Assume a signal

$$x(t) = \begin{cases} -1 & 0 \le t < 1 \\ 1 & 1 \le t < 3 \\ -1 & 3 \le t < 4 \end{cases}$$

Is it possible to express the signal $x(t)$ as a linear combination of the orthonormal basis functions? If yes, provide the weighting coefficients.

c) (1p) Provide another set of three signals that is equivalent to the set in the figure in a signal space sense.

d) (2p) Assume that a fourth signal $f_4(t) = 0$ is added. Would the normalized minimum distance increase, remain the same, or increase?

in e) - g), the term "energy efficiency" means error probability at a given transmitted energy per bit.

e) (1p) The signal constellation in the figure is in fact transmitting a DC-level. That is, we can write $f_k(t) = d(t) + s_k(t)$ for some $d(t) \neq 0$ and where $s_1(t) + s_2(t) + s_3(t) = 0$. Discuss the impact on energy efficiency due to this DC level.

f) (3p) Remove the DC level and derive a relationship between the energy efficiency of the original constellation and the one after DC level removal.

g) (3p) Add a fourth signal $f_4(t)$ such that there is no DC level transmitted. Comment on the energy efficiency of the new constellation.

Problem 4 (10p)

Suppose that the binary signal $\pm s(t)$ is transmitted over a fading channel and the received signal is

$$r(t) = \pm a s(t) + z(t),$$

where $z(t)$ is zero-mean white Gaussian noise.

The channel gain a is specified by the probability density function

$$p(a) = \begin{cases} 0.1, & a = 0 \\ 0.9, & a = 2 \end{cases}$$

a) (3p) Determine the average probability of error P_B.

b) (1p) What value does P_B approach as the SNR approaches infinity?

c) (5p) Suppose that the same signal is transmitted on two statistically independently fading channels with gains $a_1$and a_2, where

$$p(a_k) = \begin{cases} 0.1, & a_k = 0 \\ 0.9, & a_k = 2 \end{cases} \qquad k = 1, 2$$

The noises on the two channels are statistically independent and identically distributed. The demodulator employs a matched filter for each channel and simply adds the two filter outputs to form the decision variable. Determine the average P_B.

d) (1p) For the case in c) what value does P_B approach as the SNR approaches infinity?

Problem 5 (8p)
Consider (2.3) in the OFDM compendium. This equation gives the (baseband equivalent) OFDM signal without the CP:

$$x(t) = \sum_{k=0}^{K-1} a_k e^{j2\pi g_k f_\Delta t}, \quad 0 \le t \le T_{obs}.$$

a) (3p) Suppose a transmit power P is available. What is the average energy in the symbols a_k?

b) (1p) Suppose we intend to transmit 16QAM data symbols using an OFDM system and that $N = K$. In OFDM there is also, as we have studied, variables denoted by X_k and others by x_n (see for example (2.18)). Which of X_k, x_n, and a_k are 16QAM symbols?

c) (1p) Suppose a transmit power P is available and that $N = K$. What is the average energy in the symbols X_k?

The next 3 questions do not relate to the text before problem a)

d) (1p) In (2.18) , the value of N refers to sampling. There is also sampling in the receiver. Is it important that this sampling is the same as that in the transmitter?

e) (1p) Shortly after (6.29) it is written that (6.29) is "extremely important" (also (6.28), but we do not care about that here). Explain why you think that the author found this equation so important.

f) (1p) Consider a case where the channel satisfies $h(t) = \delta(t)$. What advantages, if any, does OFDM offer compared with single carrier systems?