Final exam in

Digital Communications, Advanced Course (ETTN01)

on Jan 12, 2016, 08–13.

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- During this final exam, you are allowed to use a calculator, the course compendium, and the lecture notes on OFDM.
- Each solution should be written on a separate sheet of paper.

  Please number each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.

  If You use results from the course literature, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

Good Luck!

Problem 1:

Determine for each of the five statements below if it is true or false.
Observe! As always, motivations to your answers should be given.

a) " A rate 6/7 encoder in combination with 128-ary bandpass PAM has the same bandwidth efficiency as uncoded 64-ary QAM."

b) "The signal space at the receiver appears in the output values from the correlaters in the receiver."

c) "In MIMO the N_t transmitted signals occupy the same frequency interval, but not the same time interval."

d) "If the high frequency OFDM signal has bandwidth W, then any sampling frequency (f_{samp}) larger than W can be used."

e) "With SISO the dimension of the signal space at the transmitter is the same as the dimension of the signal space at the receiver."

(10 points)

Problem 2:

a) Consider the 64-QAM signal constellation in Figure 5.12 on page 356 in the compendium. Let the signal point with coordinates $(D_{min}/2, D_{min}/2)$ represent message 0, i.e. m_0. Also, let the signal point with coordinates $(-D_{min}/2, D_{min}/2)$ represent message 1, i.e. m_1. Furthermore, let the signal point with coordinates $(-D_{min}/2, 3D_{min}/2)$ represent message 2, i.e. m_2.

Assume that message m_0 is sent in the three sub-problems below.

i) Calculate exactly the probability that the ML receiver decides that message m_1 was sent, if $\mathcal{E}_b/N_0$ is 20 dB.

ii) Calculate exactly the probability that the received noisy two-dimensional vector **r** is closer to message point m_2 than to message point m_0, if $\mathcal{E}_b/N_0$ is 20 dB.

iii) Can it happen that the received noisy two-dimensional vector **r** is closer to message point m_2 than to message point m_0, but the decision will be that message m_1 was sent.

b) Consider a signal constellation (equally likely signals), such that the signal points are on all the 32 "corners" of a five-dimensional "cube". A signal point is always such that each of the five coordinates can take the value +a, or -a.

Calculate the average symbol (signal) energy, and d^2_{min}.

(10 points)

Problem 3:

a) A relatively new company has developed an application which needs the bit rate 8 Mbps, and requires a bandwidth of 1 MHz.

The company is thinking of a trellis coded QAM SISO communication system solution, and it would like you to develope this idea in more technical details for the current application. Furthermore, the application is aimed to be used in an environment where $\mathcal{E}_b/N_0$ is 13-16 dB.

What are your recommendations/advice to the company?

b)

i) Consider a communication situation when the transmitter knows the multi-path channel (i.e., H(f) and/or h(t)).

Explain advantages and disadvantages with channel knowledge at the transmitter.

ii) Compare the ML receiver with the MAP receiver concerning advantages and disadvantages in general. Also, for which situations do these receivers produce the same, or approximatly the same, symbol error probability?

(10 points)

Problem 4:

The notation in the OFDM lecture notes, Version 150202, should be used!

a)

i) A person claims that the bandwidth efficiency for OFDM never exceeds the value $r_c log_2(M_{max})$ bps/Hz, where M_{max} is the largest value in the set $(M_0, M_1, ..., M_{K-1})$.

Determine if the person is correct or not.

ii) Consider an OFDM system with K=601, and with a given f_Δ. Furthermore, N=1024. Several tests are performed on the OFDM transmitter to validate its functionality. One such test is studied below.

Assume that only the two sent signal points a_{299} and a_{301} are non-zero. Hence, in this test-scenario all the other 599 signal points are intentionally set to zero.

Determine, the input sequence to the IDFT.

b) Continued from aii).

i) Calculate the IDFT output values x_n explicitly, for the input sequence determined in aii).

ii) For simplicity, let us here assume that $T_{CP} = 0$ and also that $a_{301} = 0$. Assuming ideal conditions (e.g., D/A), give explicit expressions for $s_I(t)$ and $s_Q(t)$ that appear in Equation (3.6) and in Figure 7 (page 27).

iii) Use $s_I(t)$ and $s_Q(t)$ obtained in bii) and calculate the OFDM signal $s(t)$ in Figure 8 (page 31). Appendeix B on page 611 in the compendium may here be useful.

(10 points)

Problem 5:

Consider the communication situation in Exampel 5.23 in the compendium. For given outcomes of the K channel parameters we know that

$P_b = Q(\sqrt{\frac{2E_{b,sent}}{N_0 K} \sum_{k=1}^{K} \alpha_k{}^2})$

In this problem it is assumed that the K channel parameters are independent zero-mean Gaussian random variables, each having variance σ^2. By using that $Q(x)$ can be expressed as, $Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2sin^2(\phi)}} d\phi$, $x \geq 0$, it is, by also using Eq. (3.165) in the compendium, straightforward to obtain an exact expression on the bit error probability as,

$P_b = E\{Q(\sqrt{\frac{2E_{b,sent}}{N_0 K} \sum_{k=1}^{K} \alpha_k{}^2})\} = \frac{1}{\pi} \int_0^{\pi/2} (\frac{1}{1+\frac{2\mathcal{E}_b}{N_0 K sin^2(\phi)}})^{K/2} d\phi < \frac{1}{2}(\frac{1}{1+\frac{2\mathcal{E}_b}{K N_0}})^{K/2}$

An upper bound on P_b is also given above.

a) Evaluate and plot the upper bound on P_b versus K, for $1 \leq K \leq 10$. Assume that $\mathcal{E}_b/N_0 = 12$. What are your conclusions?

b) Explain how the expression for the upper bound is obtained from the exact expression on P_b.

c) Give examples, the more the better, of parameters that roughly characterize important properties of time-varying channels. Also give examples of how these parameters may be used when selecting values for some important communication parameters.

(10 points)
