Answers to exam in ETTN01, Jan 12, 2016.

1.

a) True, since

$$\rho = \frac{r_c \log_2(M)}{c} = 6/c \text{ for both cases.}$$

b) True, since $\underline{r} = \underline{z} + \underline{w}$

c) False, since also the same time interval is used, e.g., $0 \le t \le T_s$.

d) False, since

$$f_{samp} = N\tfrac{1}{\Delta} > W_{OFDM} = K\tfrac{1}{\Delta}$$

N integer

e) False, the signal space used in the receiver is sometimes larger.
E.g., 1-dim BPSK sent, att & rot. imply 2 dimensions (I & Q) in the receiver.
Also diversity is often received in several dimensions.

2a) $d_{min}^2 = 2/7$, $\mathcal{E}_b/N_0 = 100$

i) $Pr\{m_1|m_0\} = Pr\{-\frac{3D_{min}}{2} \le w_1 \le -\frac{D_{min}}{2}, -\frac{D_{min}}{2} \le w_2 \le \frac{D_{min}}{2}\} =$

$= \left(Q\left(\frac{D_{min}}{2\sqrt{N_0/2}}\right) - Q\left(\frac{3D_{min}}{2\sqrt{N_0/2}}\right)\right)\left(1 - 2Q\left(\frac{D_{min}}{2\sqrt{N_0/2}}\right)\right) =$

$= \left(Q\left(\sqrt{d_{min}^2 \frac{\mathcal{E}_b}{N_0}}\right) - Q\left(\sqrt{9 d_{min}^2 \frac{\mathcal{E}_b}{N_0}}\right)\right)\left(1 - 2Q\left(\sqrt{d_{min}^2 \frac{\mathcal{E}_b}{N_0}}\right)\right) =$

$= \left(Q(5.3452) - Q(16.036)\right)\left(1 - 2Q(5.3452)\right) \approx$

$\approx 4.5 \cdot 10^{-8}$

ii) $D_{0,2}^2 = 2 D_{min}^2$

$Pr\{\underline{r}$ closer to m_2 than to $m_0 | m_0$ sent$\} =$

$= Q\left(\sqrt{\frac{D_{0,2}^2}{2N_0}}\right) = Q\left(\sqrt{2 d_{min}^2 \frac{\mathcal{E}_b}{N_0}}\right) = Q(7.56) \approx$

$\approx 21.3 \cdot 10^{-15}$

iii) Yes it can happen.

E.g., if $\underline{r} = \begin{pmatrix} -3D_{min}/4 \\ 3D_{min}/4 \end{pmatrix}$

2b) $\underline{z}_j$ can be any of the signals (vectors)

$(\pm a, \pm a, \pm a, \pm a, \pm a)$

$\Rightarrow D_{min}^2 = 4a^2$, $\underline{\underline{\mathcal{E}_s = 5a^2}} = 5\mathcal{E}_b$

$d_{min}^2 = \frac{D_{min}^2}{2\mathcal{E}_b} = \underline{\underline{2}}$

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a)

$\varsigma = 8$ bps/Hz

Shannon test:

$$\left(\frac{E_b}{N_0}\right)_{min} = \frac{2^8-1}{8} = 31.9 \ (\approx 15 \text{ dB})$$

This value is too close to (and even larger than) the intended region for E_b/N_0.
So, ς has to be reduced.
If the intended region for E_b/N_0 could be increased to larger values of E_b/N_0, then the decrease in ς could be less.

b) i)

+ Higher bit rates ("waterfilling")
- Requires a feed-back link from the receiver, to send channel info.

ii)

If the message probabilities are the same then ML and MAP are identical.
Otherwise, P_s is smaller for MAP, but it needs to know P_i and N_0.
If N_0 "small" then MAP $\rightarrow$ ML.

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a)

i) The person is correct since,

$$\rho = \frac{r_c \sum_{k=0}^{K-1} \log_2(M_k)}{(T_{cp}+T_{obs})K f_\Delta} \le \frac{r_c K \log_2(M_{max})}{(T_{cp} f_\Delta + 1)K} < r_c \log_2(M_{max})$$

$(T_{cp}=0)$

ii)

$$q_0 = -300$$

So, $X_0 = 0$, $X_1 = 1024\, a_{301}$

$$X_2 = X_3 = \ldots = X_{1022} = 0$$

$$X_{1023} = 1024\, a_{299}$$

b)

i) $$x_n = a_{301} e^{j 2\pi n/N} + a_{299} e^{j 2\pi (N-1) n/N}, \quad n = 0, 1, \ldots, N-1$$

ii) $$s_I(t) + j s_Q(t) = \sum_k a_k e^{j 2\pi q_k f_\Delta t} = a_{299} e^{-j 2\pi f_\Delta t}, \quad 0 \le t \le T_s$$

$$s_I(t) = a_{299,Re} \cos(2\pi f_\Delta t) + a_{299,Im} \sin(2\pi f_\Delta t)$$

$$s_Q(t) = a_{299,Im} \cos(2\pi f_\Delta t) - a_{299,Re} \sin(2\pi f_\Delta t)$$

iii) $$s(t) = a_{299,Re} \cos(2\pi (f_{rc} - f_\Delta) t + \phi) - a_{299,Im} \sin(2\pi (f_{rc} - f_\Delta) t + \phi)$$

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a) Some points on the curve are: $(1, 100\cdot 10^{-3})$, $(2, 38.46\cdot 10^{-3})$, $(5, 6.17\cdot 10^{-3})$, $(8, 1.95\cdot 10^{-3})$, $(10, 1.1\cdot 10^{-3})$. P_b decreasis as the diversity parameter K increasis.

b) Replacing $\sin^2(\theta)$ with the value 1 upper bounds the integrand, and the integral is then easy to determine (=upper bound).

c) $t_{coh} \approx \frac{1}{B_D}$, $f_{coh} \approx \frac{1}{T_m}$

Example: $T_s \ll t_{coh}$, $W \ll f_{coh}$

See also the compendium.