Answers/Hints to exam January 12, 2015, Digital Communications – Advanced Course.

1

a) $\rho = r_c \log_2(M)/c = \begin{cases} 4/c, & 2/3 + 64\text{-PAM} \\ 3.5/c, & 7/8 + 16\text{-QAM} \end{cases}$

⇒ FALSE.

b) FALSE, since it is measured at the output of the matched filter.

c) TRUE since $\rho = \dfrac{\log_2(M)}{\frac{T_{CP}}{T_{obs}} + 1}$ bps/Hz.

d) FALSE since Rayleigh fading typically changes the decision regions (e.g. M-QAM).

e) FALSE, it is only the optimal symbol receiver if there is no memory in the sequence. With memory (coding) an ML sequence detection (decoding) algorithm is optimal.

2

a)

$$\Pr\{\hat{m}=m_4 \mid m_1 \text{ sent}\}=$$

$$=\Pr\{-3a \le w_1 \le -a,\ \ 3a \le w_2 \le 5a\}=$$

$$=\left(Q\left(\sqrt{\frac{2a^2}{N_0}}\right)-Q\left(\sqrt{\frac{18a^2}{N_0}}\right)\right)\left(Q\left(\sqrt{\frac{18a^2}{N_0}}\right)-Q\left(\sqrt{\frac{50a^2}{N_0}}\right)\right)=$$

$$=\left(Q(2.83)-Q(8.49)\right)\left(Q(8.49)-Q(14.4)\right)\approx 2.4\cdot 10^{-20}$$

b) $P_b = Q\left(\sqrt{d^2 \mathcal{E}_b/N_0}\right),\ \ \frac{\mathcal{E}_b}{N_0} = 18.197$

$$D^2 = \frac{29}{8}a^2,\quad \mathcal{E}_b = \frac{55}{16}a^2$$

$$d^2 = \frac{29/8}{55/8} = \frac{29}{55} \approx 0.527$$

$$P_b = Q(3.097) \approx 10^{-3}$$

3 a)

i) $\rho = r_c \log_2(M)/c \overset{\text{unc. 16-PSK}}{=} \frac{4}{c} = 4$ (p. 369)

$r_c = 4/5$ + 32-PSK and $r_c = \frac{2}{3}$ + 64-PSK

Both are examples with the same ρ as uncoded 16-PSK, but increased energy efficiency.

$\rho = 4$: $\left(\frac{E_b}{N_0}\right)_{min} = \frac{2^{\rho} - 1}{\rho} = 3.75$ (5.74 dB)

From Figure 5.17 16-PSK: $\frac{E_b}{N_0} \approx 17.9$ dB

$\Rightarrow$ Gain at most $\approx 17.9 - 5.74 = 12.16$ dB

ii) The purpose is to maximize the bit rate by distributing the fixed sent power optimally in the frequency domain.

Waterfilling can not be used when the channel is changing too fast since channel feedback info then is outdated.

b)

i) See the compendium.

ii) Frequency diversity is possible.

iii) Diversity in time (i.e. "repetition coding")

" " freq. (")

" by using several receive antennas.

4 a)

i) Since $K = 700$ a suitable and efficient choice is

$$f_{samp} = N f_\Delta = 1024 \cdot 7.5 \cdot 10^3 = 7.68 \text{ Msampel/s}$$

ii) $R_b = K \log_2(M)/T_s = 700 \cdot 4 \cdot 10^6/140 = 20 \text{ Mbps}$

iii) $X_0 = 1024 a_{-g_0} = 1024 a_{k_{rc}} = 1024 a_{349}$

b) The given frequency-domain samples in this problem correspond to a frequency-function $X(\nu)$ for which:

$X(-6/12) = 12 a_6 \quad (f = -6 f_\Delta)$

$X(-5/12) = 12 a_7 \quad (f = -5 f_\Delta)$

$X(-4/12) = X(-3/12) = X(-2/12) = X(-1/12) = 0 \quad (!)$

$X(k/12) = 12 a_k, \; k = 0, 1, \ldots, 7$

It is seen that within the fundamental interval $-0.5 \le \nu \le 0.5$ $(-f_{samp}/2 \le f \le f_{samp}/2)$ the QAM symbols a_k appears in a <u>non-ordered</u> fashion, and with <u>four zeroes</u> between a_7 and a_0.

Additional, costly, analog filtering is here needed after the D/A to obtain the correct high-frequency OFDM signal.

5

a)

i) $r_4 = \sum_{n=1}^{6} \alpha_{4,n} d_n + w_4$

A decision of the sent vector $\underline{d}$ should be based on the received vector $\underline{r}$, and not only on r_4.

ii)

$$\underline{r} = A\underline{d} + \underline{w} = \underline{z}(\underline{d}) + \underline{w}$$

The vector $\hat{\underline{d}}$ that minimizes $D^2_{\underline{r},\underline{z}(\underline{d})}$ will be the ML decision. There are 64 signal points $\underline{z}(\underline{d})$.

$\hat{\underline{d}} = (\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_6)^{tr}$.

b)

$$\underline{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_8 \end{pmatrix} = A \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} + \underline{w}$$, $2^{12} = 4096$ signal points

d_1 and d_2 are QPSK symbols from user 1.

d_3 and d_4 are 16-QAM symbols from user 2.

ML $\Rightarrow \hat{\underline{d}}$

$$\hat{\underline{d}} = \begin{pmatrix} \hat{d}_1 \\ \hat{d}_2 \\ \hat{d}_3 \\ \hat{d}_4 \end{pmatrix}$$

$\hat{d}_1, \hat{d}_2$ — 4 bits from user 1.

$\hat{d}_3, \hat{d}_4$ — 8 bits from user 2.