

# Final exam in Digital Communications, Advanced Course (ETTN01)



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on December 19, 2013, 14–19.

- During this final exam, you are allowed to use a calculator and the textbook.
- Each solution should be written on a separate sheet of paper.  
Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.  
If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

**Good Luck!**

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## Problem 1:

Determine for each of the five statements below if it is true or false.

*Observe! As always, motivations to your answers should be given.*

- “ A rate  $3/4$  encoder in combination with 32-ary PSK has a smaller bandwidth efficiency than a rate  $4/5$  encoder with 16-ary QAM.”
- “An encoder in combination with M-QAM can improve both energy efficiency and bandwidth efficiency, compared to uncoded M-QAM.”
- “If the received signal constellation is M-PSK with equally likely signal alternatives, then the implementation of the ML-receiver can be significantly simplified, compared with the receiver for M-QAM (also with equally likely signal alternatives).”
- “Channel knowledge is always needed at the transmitter and at the receiver.”
- “Attenuation and rotation can be compensated for (equalized) at the receiver, such that the symbol error probability remains approximately unchanged.”

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(10 points)

**Problem 2:**

Consider a communication link where the eight equally likely received signal alternatives are represented in a one-dimensional signal space as:

$$\mathbf{z}_0 = -\mathbf{z}_7 = a$$

$$\mathbf{z}_1 = -\mathbf{z}_6 = 2a$$

$$\mathbf{z}_2 = -\mathbf{z}_5 = 3a$$

$$\mathbf{z}_3 = -\mathbf{z}_4 = 7a$$

where  $a$  is a positive value. AWGN and ML symbol receiver are assumed.

Calculate the exact value of the symbol error probability if  $\mathcal{E}_b/N_0$  is 20 dB.

(10 points)

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**Problem 3:**

a)

Consider the basic Shannon capacity equation. Strongly connected to this equation are several important parameters, e.g.,  $SNR = P_z/(N_0W)$  and  $\mathcal{E}_b/N_0$ .

Determine and sketch  $\mathcal{E}_b/N_0$  versus  $SNR$ , for  $10 \leq SNR \leq 130$ .

What are your conclusions?

b)

Explain in detail with your own words:

i) What is meant by “a time-selective channel”?

ii) Why is diversity needed in Rayleigh fading channels?

iii) Determine at least two important properties of CPM-signals.

(10 points)

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**Problem 4:**

a)

Consider the transmitter in Figure 8.6a) on page 510 in the compendium. It is here assumed that the received signal  $r(t)$  is  $r(t) = -0.5s(t) + N(t)$ , where  $N(t)$  is AWGN.

It is known that the accumulated squared Euclidean distances below are saved by the VA after processing the received vector  $r[i]$ .

The saved value at state 0,1,...,7 is,  $6.5a^2, 6.6a^2, 6.8a^2, 6.4a^2, 6.7a^2, 6.55a^2, 6.75a^2$ , and  $6.45a^2$ , respectively.

Assume now that the next received vector  $r[i+1]$  is such that  $r_1[i+1] = 0$  and  $r_2[i+1] = 0.25a$ .

Calculate what will be stored by the VA in state number 4 after processing the received vector  $r[i+1]$ .

b)

Consider the transmitter in Figure 8.6a) on page 510 in the compendium. A person claims that the corresponding VA at the receiver side will never produce a signal sequence decision that contains the sub-sequence:  $s_7, s_2, s_5, s_1, s_0, s_2$ .

Determine if the person is correct, or not.

(10 points)

**Problem 5:**

Consider a communication link where the eight equally likely received signal alternatives are represented in a two-dimensional signal space as ("tr" means transpose):

$$\mathbf{z}_0 = -\mathbf{z}_2 = (6a, 6a)^{tr}$$

$$\mathbf{z}_1 = -\mathbf{z}_3 = (-6a, 6a)^{tr}$$

$$\mathbf{z}_4 = -\mathbf{z}_6 = (3a, 0)^{tr}$$

$$\mathbf{z}_5 = -\mathbf{z}_7 = (0, 3a)^{tr}$$

where  $a$  is a positive value. AWGN and ML symbol receiver are assumed.

i) Determine the decision region for message 4.

ii) Assume that message 4 is sent, and also that the noise component  $w_2 = -2a$ . For which values of  $w_1$  will then a symbol error be obtained? The more precise your answer is, the better.

iii) Assume that message 3 is sent, and also that the noise component  $w_2 = -2.5a$ . For which values of  $w_1$  will then a symbol error be obtained? The more precise your answer is, the better.

(10 points)