Digital Communications - Advanced Course
Answers/Hints to exam 20/12 - 2012.

1

a) True, since $2^{15} = 8^{5}$ sequences (for a given startstate).

b) False, since 4 input bits give 1 QAM signal.

c) False, its recursive structure makes it suitable in many real time applications.

d) True, since the VA deletes sequences at an early stage in the decoding process. The VA performs MLSD.

e) True, since the same info reaches the receiver through several independent paths (or dimensions).

2.

a) P_A = Pr{ message 2 decided | message 1 sent } =

$$= \Pr\left\{ W_1 < \frac{D_{min}}{2},\ W_2 < -\frac{D_{min}}{2} \right\} = \left(1 - Q\left(\sqrt{\frac{D^2_{min}}{2N_0}}\right)\right) Q\left(\sqrt{\frac{D^2_{min}}{2N_0}}\right) =$$

$$= Q\left(\sqrt{E/N_0}\right) - Q^2\left(\sqrt{E/N_0}\right) = Q(3.72) - Q^2(3.72) \approx 10^{-4}$$

b)

P_B = Pr{ message 3 decided | message 1 sent } =

$$= \Pr\left\{ W_1 > \frac{D_{min}}{2},\ W_2 < -\frac{D_{min}}{2} \right\} = Q^2\left(\sqrt{E/N_0}\right) \approx 10^{-8}$$

c) Symmetry gives that,

$$P_S = 2P_A + P_B$$

3.

a) It is found that: $\frac{(P_Z/N_0)_1}{W_1} = 15$, $\frac{(P_Z/N_0)_2}{W_2} = 3$, $\frac{W_1}{W_2} = \frac{1}{2}$.

Then $\frac{(P_Z/N_0)_1}{(P_Z/N_0)_2} = \frac{5}{2}$

b) Please see the compendium.

4.

a) The sequence of states is found to be:

$$\boxed{6}\underset{\binom{0}{1}}{\text{——}}\boxed{3}\underset{\binom{0}{0}}{\text{——}}\boxed{7}\underset{\binom{0}{1}}{\text{——}}\boxed{2}\underset{\binom{1}{0}}{\text{——}}\boxed{5}\underset{\binom{1}{1}}{\text{——}}\boxed{6}$$

Hence, 3 + 10 = 13 information bits are known.

b) Input pair of bits, such that $b_1[n] = 1$, will change the output signal alternative, compared with the original encoder.

The output signal sequence is:

$$\underset{\uparrow}{s_5},\ s_3,\ \underset{\uparrow}{s_2},\ s_3,\ \underset{\uparrow}{s_4}$$

changed

c) It is found that the sequence in b) can not be generated by the encoder in a).

Hence, the two set are not identical.

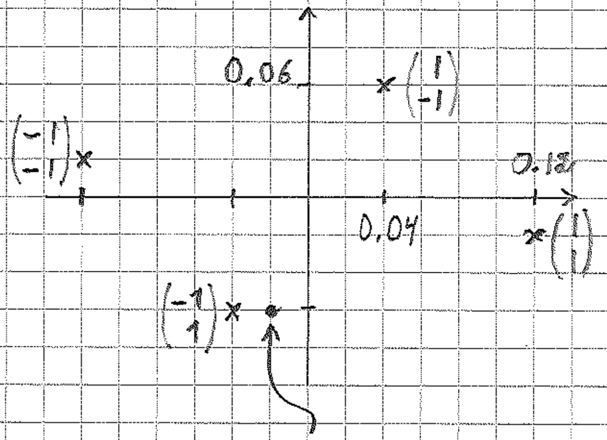
5.

a) The received signal point $\underline{z}$ corresponding to $d_1 = 1$ and $d_2 = -1$ is:

$$\underline{z} = \frac{1}{100}\begin{pmatrix}8 & 4\\ 2 & -4\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} = \frac{1}{100}\begin{pmatrix}4\\ 6\end{pmatrix}$$

If instead $d_1 = d_2 = 1$ we get the signal point:

$$\frac{1}{100}\begin{pmatrix}8 & 4\\ 2 & -4\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{100}\begin{pmatrix}12\\ -2\end{pmatrix}$$



If $\underline{r} = \begin{pmatrix}r_1\\ r_2\end{pmatrix} = \begin{pmatrix}-0.02\\ -0.06\end{pmatrix}$ then the decision is $\hat{\underline{d}} = \begin{pmatrix}-1\\ 1\end{pmatrix}$.

b) $x = (1\ 1)\,\underline{r} = \frac{d_1}{10} + w_1 + w_2$

So, x can be used to make a sub-optimal decision on d_1, but not on d_2 (since x does not depend on d_2).