

# Final exam in Digital Communications, Advanced Course (ETTN01)



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on December 20, 2012, 08–13.

- During this final exam, you are allowed to use a calculator and the textbook.
- Each solution should be written on a separate sheet of paper.  
Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.  
If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

**Good Luck!**

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## Problem 1:

Determine for each of the five statements below if it is true or false.

*Observe! As always, motivations to your answers should be given.*

- “Over a  $5T_s$  time interval, a rate  $3/4$  encoder in combination with 16-ary QAM can generate  $8^5$  possible signal sequences.”
- “A rate  $2/3$  encoder in combination with 64-ary QAM does not work together.”
- “The Viterbi algorithm is not suited for real time applications.”
- “The Viterbi algorithm finds the most likely signal sequence but it does not explicitly check all possible signal sequences.”
- “Diversity makes the communication more reliable.”

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(10 points)

**Problem 2:**

Consider QPSK as given in Figure 5.10 on page 351 in the compendium. Equally likely signal alternatives, AWGN and ML symbol receiver are assumed.

a)

Assume that message 1 is sent.

Calculate the probability that the receiver decides that message 2 was sent if  $E/N_0$  is 11.41 dB.

b)

Assume that message 1 is sent.

Calculate the probability that the receiver decides that message 3 was sent if  $E/N_0$  is 11.41 dB.

c)

Denote the probability in a) as  $P_A$  and in b) as  $P_B$ .

How is the symbol error probability  $P_s$  (in eq. (5.39) in the compendium) related to  $P_A$  and  $P_B$ ?

(10 points)

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**Problem 3:**

a)

We have two communication applications. The first application has parameters  $W_1$  and  $(P_z/N_0)_1$ , and the second application has the parameters  $W_2$  and  $(P_z/N_0)_2$ . It is given that  $C_1/W_1 = 4$  bps/Hz, and that  $C_2/W_2 = 2$  bps/Hz.

Calculate the ratio  $(P_z/N_0)_1/(P_z/N_0)_2$  if it also is given that the capacities are the same, i.e.  $C_1 = C_2$ .

b)

Explain in detail with your own words:

i) What is the practical importance of the coherence time?

ii) What is the practical importance of the coherence bandwidth?

iii) Explain and motivate how to calculate the symbol error probability when the channel parameters are random variables (AWGN and ideal ML-receiver is assumed).

(10 points)

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**Problem 4:**

a) Consider the transmitter in Figure 8.6a) on page 510 in the compendium.

Over a  $5T_s$  time interval it is known that the sent sequence of signal alternatives is:  $s_1, s_3, s_6, s_3, s_0$ .

How many information bits are known since we know the given signal sequence?

Determine and plot the path in the trellis for the given signal sequence.

b) Here we modify the encoder part in Figure 8.6a) on page 510 in the compendium.

Remove the connection between  $b_1[i]$  and the modulo-2 adder. After removing this connection the first coded bit will be equal to  $b_2[i - 1]$ .

Assume now the same sequences  $b_1[i]$  and  $b_2[i]$  of input bits as in a).

Consider the same  $5T_s$  time interval as in a), and determine the output sequence of signal alternatives when the modified encoder is used instead.

c) Consider the set of all possible signal sequences that can be generated from the transmitter in Figure 8.6a).

Also consider the set of all possible signal sequences that can be generated from the modified transmitter in b).

Determine if these two sets (of signal sequences) are identical, or not.

(10 points)

**Problem 5:**

Consider a MIMO communication system with two transmitting antennas and two receiving antennas, see also Problem 5.34 in the compendium.

Each transmitting antenna uses conventional BPSK signals.

The signals from the two transmitting antennas are sent during the same time interval, and this interval has the duration  $T_s$ .

Here we consider the uncoded case which means that all input bits to the two transmitting antennas are independent and equally likely.

Slow Rayleigh fading, AWGN, and ideal ML reception is assumed. Furthermore, all individual channel links are assumed to be statistically independent.

Here we use the standard description that is used in Problem 5.34 in the compendium:

$$\mathbf{r} = \mathbf{A}\mathbf{d} + \mathbf{w} = \mathbf{z} + \mathbf{w}$$

For simplicity, all numbers are real in this problem.

Since BPSK is used, we can assume that,  $d_1 = 1$  or  $d_1 = -1$ , depending on the sent data from the first transmitting antenna.

Similarly, we can assume that,  $d_2 = 1$  or  $d_2 = -1$ .

In this problem we consider a specific outcome of the channel, where the elements in the channel matrix  $\mathbf{A}$  are:  $a_{1,1} = 8/100$ ,  $a_{1,2} = 4/100$ ,  $a_{2,1} = 2/100$ ,  $a_{2,2} = -4/100$ .

a) Make a detailed figure that shows the signal points in the receivers signal space.

Also determine the decision if  $r_1 = -0.02$  and  $r_2 = -0.06$ .

b) Assume that we create the number  $x$  as  $x = r_1 + r_2$ .

Is it reasonable to use  $x$  to make a decision on  $d_1$  or  $d_2$  (even though this would be a sub-optimal decision strategy)?

(10 points)