ETTN01: Answers to Exam in Digital Communications - Advanced course, 13 December 2011, 08-13.

Problem 1.

a) True, since

$$\frac{R_b}{W} = r_c \log_2(M)/c = \begin{cases} \frac{5}{6} \cdot \frac{6}{c} = \frac{5}{c} \\ 1 \cdot \frac{5}{c} = \frac{5}{c} \end{cases}$$

b) False, since
the signal space is obtained only at the sampling time in the eye-diagram.

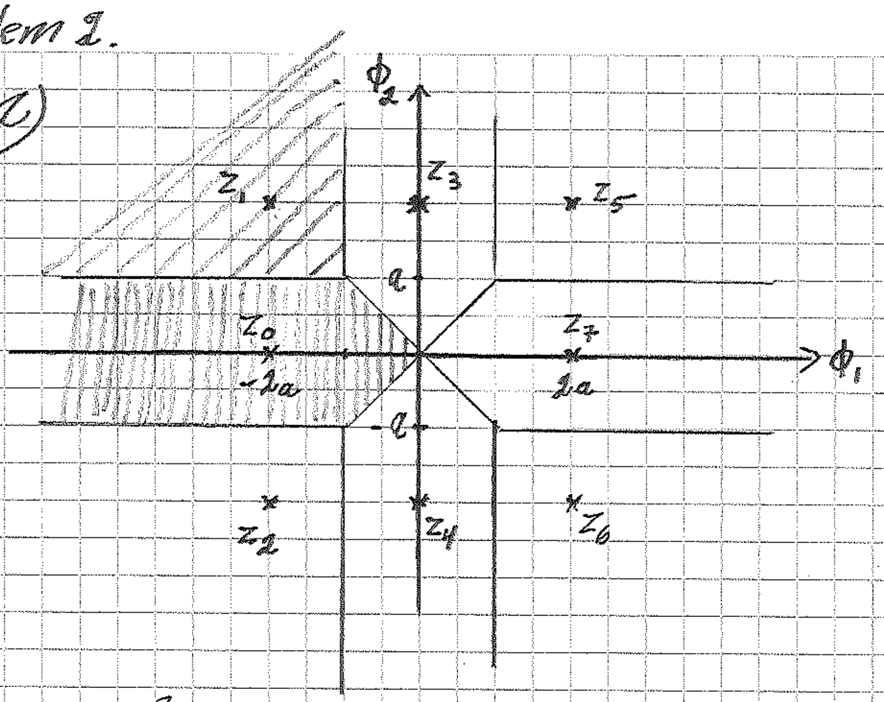
c) False, since
e.g. frequency diversity needs more bandwidth.

d) False, since
a negative value of the multiplication factor will rotate the decision regions 180°.

e) True, since
the signal in $nT_s \le t \le (n+1)T_s$ depends on both α_{n-1} and α_n. α_{n-1} can be considered as the state at $t = nT_s$. (Compare CPM).

Problem 2.

a i)



a ii) $D_{min}^2 = 4a^2$

$$E_s = \frac{1}{8}\left(4\cdot 4a^2 + 4\cdot 8a^2\right) = 6a^2 = 3E_b$$

$$d_{min}^2 = \frac{4a^2}{2\cdot 2a^2} = 1$$

b)

$$\text{Prob}\{\text{error} \mid m_6 \text{ sent}\} = 1 - \text{Prob}\{W_1 > -\frac{D_{min}}{2}, W_2 < \frac{D_{min}}{2}\} =$$
$$= 2Q\left(\sqrt{\frac{D_{min}^2}{2N_0}}\right) - Q^2\left(\sqrt{\frac{D_{min}^2}{2N_0}}\right) = 2Q(3.32) - Q^2(3.32) \approx 9.6\cdot 10^{-4}$$

c)

$$\text{Prob}\{\hat{m} = m_5 \mid m_6 \text{ sent}\} = \text{Prob}\{W_1 > -\frac{D_{min}}{2}, W_2 > \frac{3D_{min}}{2}\} =$$
$$= \left(1 - Q\left(\sqrt{\frac{D_{min}^2}{2N_0}}\right)\right) Q\left(\sqrt{\frac{9D_{min}^2}{2N_0}}\right) = (1 - Q(3.32))\, Q(9.95) \approx$$
$$\approx 2\cdot 10^{-23}$$

Problem 3.

a)

$$C = W \log_2 \left(1 + \frac{P_E}{N_0 W}\right)$$

If $W = 5$ MHz then $C = 50$ Mbps and this means that $\frac{P_E}{N_0} = 5.115 \cdot 10^9$.

Investigating the four cases $C/W = 1, 2, 3, 4$ we obtain the values below:

$\frac{C}{W} = 1 \Rightarrow C = 5.115$ Gbps

$\frac{C}{W} = 2 \Rightarrow C = 3.41$ Gbps

$\frac{C}{W} = 3 \Rightarrow C = 2.19$ Gbps

$\frac{C}{W} = 4 \Rightarrow C = 1.364$ Gbps

From these values a plot of C versus C/W can be sketched.

b) i) and ii): see the compendium.

iii) Adaptive OFDM is a good candidate.

Problem 4.

a)

True path: [3] —s_1— [5] —s_4— [4] —s_2— [6] —s_5— [7] —s_4— [6]

Decoded path: [3] —s_1— [5] —s_6— [0] —s_4— [2] —s_1— [1] —s_4— [6]

Bit errors: 0 1 1 0 0

So, two bit errors are made.

b)

i)

	s_1	s_4	s_2	s_5	s_4
	s_1	s_6	s_4	s_1	s_4
D^2_{symbol}	0	$2a^2$	$2a^2$	$4a^2$	0

$$D^2_{seq} = 2a^2 + 2a^2 + 4a^2 = 8a^2$$

ii) see the compendium

Problem 5.

a)

$N_r = 1 \Rightarrow \frac{E_b}{N_0}$ is 87.36 dB

$N_r = 4 \Rightarrow \frac{E_b}{N_0}$ is 37.82 dB

$N_r = 8 \Rightarrow \frac{E_b}{N_0}$ is 31.57 dB

Diversity gives significant gains in energy efficiency.

b)

$T_s = N_t \log_2(M) T_b$

The other possible cases are ($N_t = 2$, $M = 16$) and ($N_t = 4$, $M = 4$).

$N_t = 2$, $M = 16 \Rightarrow P_s \approx 2.6 \cdot 10^{-6}$

$N_t = 4$, $M = 4 \Rightarrow P_s \approx 1.7 \cdot 10^{-9}$ BEST!