Answers & Hints to the exam in the course Digital Communications, Advanced Course (ETTN01), December 14, 2010, 08-13.

Problem 1

a) True, since $R_b/R_s = 6$ in both cases.

b) True, since $T_s = 8T_b/3$. Alternatively: Each coded bit carries 2/3 of an information bit.

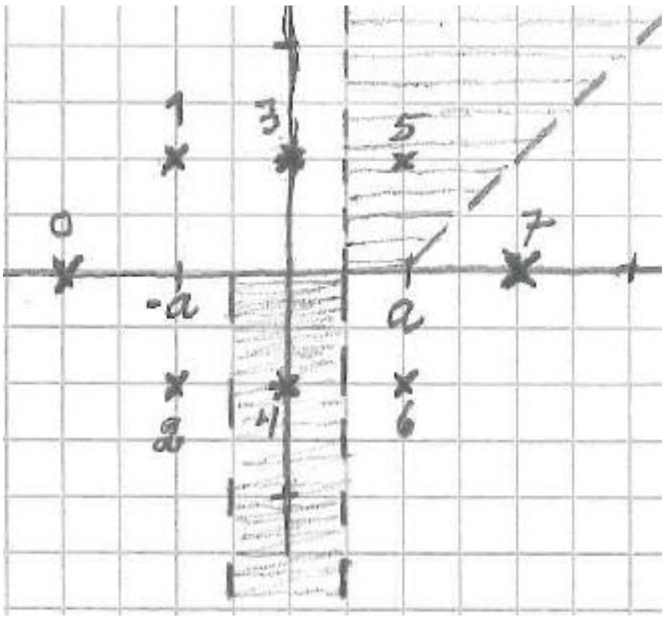
c) False, since $C_{max} = 1.4427$ Mbps which is smaller than 1.5 Mbps.

d) True.

e) False, diversity helps indeed in this case.

Problem 2

ai)



aii) The average received energy per bit is found to be $0.75a^2$, and $D^2_{min} = a^2$. So, $d^2_{min} = 2/3$.

bi) Let $d = -a$ correspond to the signal point $z_0 = (0.4a, 0, 0.6a, -0.4a)$.

Similarly, let $d = a$ correspond to the signal point $z_1 = (-0.4a, 0, -0.6a, 0.4a) = -z_0$.

Then we find that $D^2_{r,z_0} = 0.79a^2$ and $D^2_{r,z_1} = 0.87a^2$. Hence, the decision is $\hat{d} = -a$, i.e. that $-a$ was sent.

bii) We find that $D^2_{z_0,z_1} = 2.72a^2$. So, $P_b = Q(\sqrt{1.36a^2/N_0})$.

Problem 3

a) $C/W = 4$ implies that $P_z/N_0 = 15W = 75 \cdot 10^6$. If the bandwidth is increased to $W_1 = 4W = 20$ MHz, then the capacity C_1 will be,

$C_1 = W_1 log_2(1 + \frac{P_z}{N_0 W_1}) = 4W log_2(19/4) = 4W 2.248 = 45$ Mbps, and $\rho_1 = C_1/W_1 = 2.248$ bps/Hz.

b)$C_2 = W log_2(1 + \frac{P_z}{N_0 W}) = 4W 2.248 = W log_2(509.17)$. Hence, $P_z/N_0 = 508.17W = 2.54 \cdot 10^9$.($\rho_2 = C_2/W = 8.99$ bps/Hz).

c) In a) the bandwidth used is quite large which is a disadvantage. On the other hand the capacity is kept the same as in b).

In b) the received signal power is significantly stronger. We can use a higher bandwidth efficiency, and a smaller bandwidth than in a). The case in b) could represent a certain communication range, and the case in a) could represent a longer communication range.

Problem 4

ai) The path in the trellis is: state 13, state 6, state 3, state 9.

The corresponding output sequence of signal alternatives is: $s_3(t), s_3(t), s_2(t)$.

aii) The bandwidth efficiency R_b/W will be doubled which is good. However, the implementation will be slightly more complicated since we have to get a rate 1/2 encoder and a 16-ary constellation to work together.

b) See the compendium.

Problem 5

$P_b \approx \frac{M^{N_t}}{y^{N_r}} \leq 10^{-6}$

Hence, $\frac{M^{N_t/N_r}}{y} \leq 10^{-6/N_r}$.

Using that $R_b = N_t log_2(M)/T_s$ in the given exression for y, and re-arranging, we obtain that,

$\frac{L^3}{a} \leq \frac{10^{-6/N_r} 1.5 P_t T_s/N_0}{(M-1)N_t M^{N_t/N_r}}$

So, $M = 4$ gives L_{max}, implying that,

$\frac{L^3_{max}}{a} = \frac{10^{-6/N_r} 0.5 P_t T_s/N_0}{N_t 4^{N_t/N_r}}$

and $R_b = N_t 2/T_s$.

5i) With $N_t = N_r = 1$ (SISO) we obtain that,

$\frac{L^3_{max}}{a} = \frac{10^{-6} P_t T_s/N_0}{8}$

So, $L_{max} = 5 \cdot 10^{-3}(aP_t T_s/N_0)^{1/3}$, and $R_{b,L_{max}} = 2/T_s$.

5ii) With $N_t = 2$ and $N_r = 4$ we obtain that,

$\frac{L^3_{max}}{a} = \frac{10^{-6/4} P_t T_s/N_0}{8}$

So, $L_{max} = 0.158(aP_t T_s/N_0)^{1/3}$, and $R_{b,L_{max}} = 4/T_s$. This means twice as high bitrate and almost 32 times longer range compared with SISO.

5iii) With $N_t = 2$, $N_r = 4$ and $L = L_{max}/16$ we obtain:

$\frac{L^3}{a} = \frac{L^3_{max}}{a} \cdot \frac{1}{16^3}$

$\frac{L^3}{a} = \frac{10^{-6/4} P_t T_s/N_0}{8} \cdot \frac{1}{16^3} \leq \frac{10^{-6/4} 1.5 P_t T_s/N_0}{(M-1)2M^{1/2}}$

From the inequality we find that,

$(M-1)\sqrt{M} \leq 8 \cdot 16^3 \cdot 3/4 = 24576$

This gives us that $M = 256$ and that $R_b = 16/T_s$ (8 times higher than SISO).

The diversity gain obtained by increasing N_r here gives us a significantly longer communication distance. Increasing also N_t increases the bit rate.