”Attenuation & Rotation” (chapter 3)

Homodyne and Heterodyne receivers (chapter 3)

A basic bit error probability analysis for optical fiber communications (chapter 7)

Mobile communications (chapter 9)

3.4.1 Low-Rate QAM-Type of Input Signals

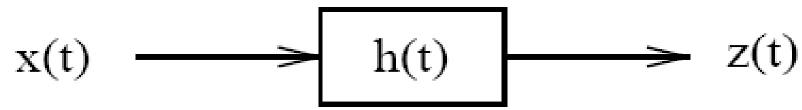


Figure 3.11: Bandpass filtering.

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \qquad (3.103)$$

$$\tilde{x}(t) = x_I(t) + jx_Q(t) \qquad (3.104)$$

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \tag{3.103}$$

$$z(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)Re\{\tilde{x}(t-\tau)e^{j\omega_c(t-\tau)}\}d\tau =$$

$$= Re\left\{e^{j\omega_c t}\int_{-\infty}^{\infty} h(\tau)\tilde{x}(t-\tau)e^{-j\omega_c \tau}d\tau\right\} \tag{3.105}$$

3 assumptions:

1) The duration of the impulse response $h(t)$ can be considered to be equal to T_h. This means that essentially all the energy in $h(t)$ is assumed to be contained within the time interval $0 \leq t \leq T_h$.

2) The input signal is assumed to be a QAM-type of signal with duration $T = T_s$:

$$x(t) = \begin{cases} 0 & , \quad t < 0 \\ A\cos(\omega_c t) - B\sin(\omega_c t) = \sqrt{A^2 + B^2}\cos(\omega_c t + \nu) & , \quad 0 \leq t \leq T_s \\ 0 & , \quad t > T_s \end{cases} \tag{3.106}$$

3) $T_s > T_h$ (“low” signaling rate).

$$\tilde{x}(t) = \begin{cases} A + jB = \sqrt{A^2 + B^2}\ e^{j\nu} & , \quad 0 \le t \le T_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{3.108}$$

$\boldsymbol{T_h \le t \le T_s}$ **:**

$$\begin{aligned} z(t) &= Re\left\{ e^{j\omega_c t} \int_0^{T_h} h(\tau) \sqrt{A^2 + B^2}\ e^{j\nu} e^{-j\omega_c \tau} d\tau \right\} = \\ &= Re\{\sqrt{A^2 + B^2}\ e^{j\nu} \cdot H(f_c) e^{j\omega_c t}\} = \\ &= |H(f_c)| \sqrt{A^2 + B^2} \cos(\omega_c t + \nu + \phi(f_c)) = A_z \cos(\omega_c t) - B_z \sin(\omega_c t) \end{aligned} \tag{3.109}$$

Hence, a QAM-signal at the output in this time interval!

However, **attenuation and rotation** compared with the input! Compare with the input x(t) in (3.106)!

$$\begin{aligned} A_z + jB_z &= (A + jB) H(f_c) = \sqrt{A^2 + B^2} |H(f_c)| e^{j(\nu + \phi(f_c))} = \\ &= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \end{aligned} \tag{3.110}$$

$$A_z + jB_z = (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu+\phi(f_c))} = \\ = (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \quad (3.110)$$

A COMPACT MODEL WITH A COMPLEX CHANNEL PARAMETER!!

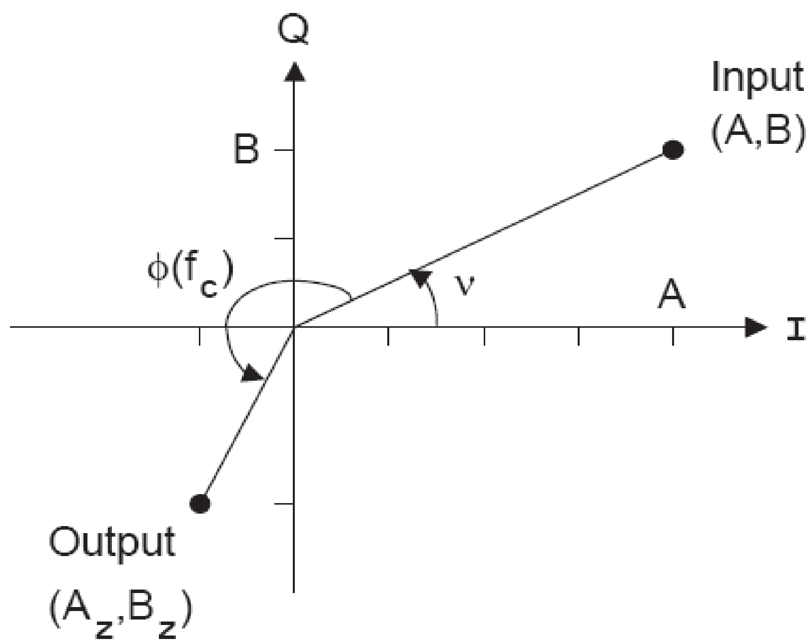


Figure 3.13: Illustrating that the input I-Q amplitudes (A,B) are scaled and rotated by the channel $H(f)$, see (3.109) and (3.110).

$$
z(t) = \begin{cases} 0 & , \quad t < 0 \\ \text{“non-stationary transient” starting interval} & , \quad 0 \le t \le T_h \\ |H(f_c)|\sqrt{A^2+B^2}\ \cos(\omega_c t + \nu + \phi(f_c)) & , \quad T_h \le t \le T_s \\ \text{“non-stationary transient” ending interval} & , \quad T_s \le t \le T_s + T_h \\ 0 & , \quad t > T_s + T_h \end{cases}
$$

$$
\text{and within } T_h \le t \le T_s,\ A_z + jB_z = (A + jB)H(f_c) \tag{3.111}
$$

An important result here is that the input QAM signal $x(t)$ in (3.106) is changed to a new QAM signal by $|H(f_c)|$ and $\phi(f_c)$ in the interval $T_h \le t \le T_s$, see also Figure 3.13 and (3.110) how the I-Q components are changed. Furthermore, in OFDM applications the signaling rate $1/T_s$ is low such that $T_s \gg T_h$, and many QAM signals with different carrier frequencies are sent in parallel. ***Due to linearity, the result in (3.111) can be applied to each QAM signal in the OFDM signal by replacing f_c with f_n.*** In OFDM applications the receiver uses the time interval $\Delta_h \le t \le T_s$ for detection of the output QAM signals, and the duration of this observation interval is denoted $T_{obs} = T_s - \Delta_h$ (compare with (2.110) on page 51, and $T_h \le \Delta_h$).

3.4.2 Group and Phase Delay

General narrowband input signal:

Let us approximate $H(f)$, around the carrier frequency f_c, with

$$H(f) \approx |H(f_c)|e^{j(\phi(f_c)+(f-f_c)\phi'(f_c))}, \quad f_c - W_{lp} \leq f \leq f_c + W_{lp} \tag{3.113}$$

according to (3.112). Before doing this let us define the **group delay** $\boldsymbol{\tau_g}$ and the **phase delay** $\boldsymbol{\tau_\phi}$ as

$$\tau_g = -\frac{1}{2\pi}\frac{d\phi(f)}{df}\bigg|_{f=f_c} \tag{3.117}$$

$$\tau_\phi = -\frac{\phi(f_c)}{2\pi f_c} \tag{3.118}$$

The final expression for the output signal $z(t)$ is then obtained as,

$$\begin{aligned} z(t) &= |H(f_c)|\ x_I(t-\tau_g)\cos(2\pi f_c(t-\tau_\phi)) - \\ &\quad -|H(f_c)|\ x_Q(t-\tau_g)\sin(2\pi f_c(t-\tau_\phi)) = \\ &= |H(f_c)|\ e_x(t-\tau_g)\cos(\omega_c t + \theta_x(t-\tau_g) + \phi(f_c)) = \\ &= Re\{e_x(t-\tau_g)e^{j\theta_x(t-\tau_g)} \cdot H(f_c) \cdot e^{j\omega_c t}\} \end{aligned} \tag{3.119}$$

where it is assumed that the input signal $x(t)$ is,

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) = \\ &= e_x(t)\cos(2\pi f_c t + \theta_x(t)) \end{aligned} \tag{3.120}$$

$$\boxed{\begin{aligned} z(t) &= |H(f_c)|\; x_I(t-\tau_g)\cos(2\pi f_c(t-\tau_\phi))- \\ &\quad -|H(f_c)|\; x_Q(t-\tau_g)\sin(2\pi f_c(t-\tau_\phi)) = \\ &= |H(f_c)|\; e_x(t-\tau_g)\cos(\omega_c t+\theta_x(t-\tau_g)+\phi(f_c)) = \\ &= Re\{e_x(t-\tau_g)e^{j\theta_x(t-\tau_g)}\cdot H(f_c)\cdot e^{j\omega_c t}\} \end{aligned}} \qquad (3.119)$$

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) = \\ &= e_x(t)\cos(2\pi f_c t + \theta_x(t)) \end{aligned} \qquad (3.120)$$

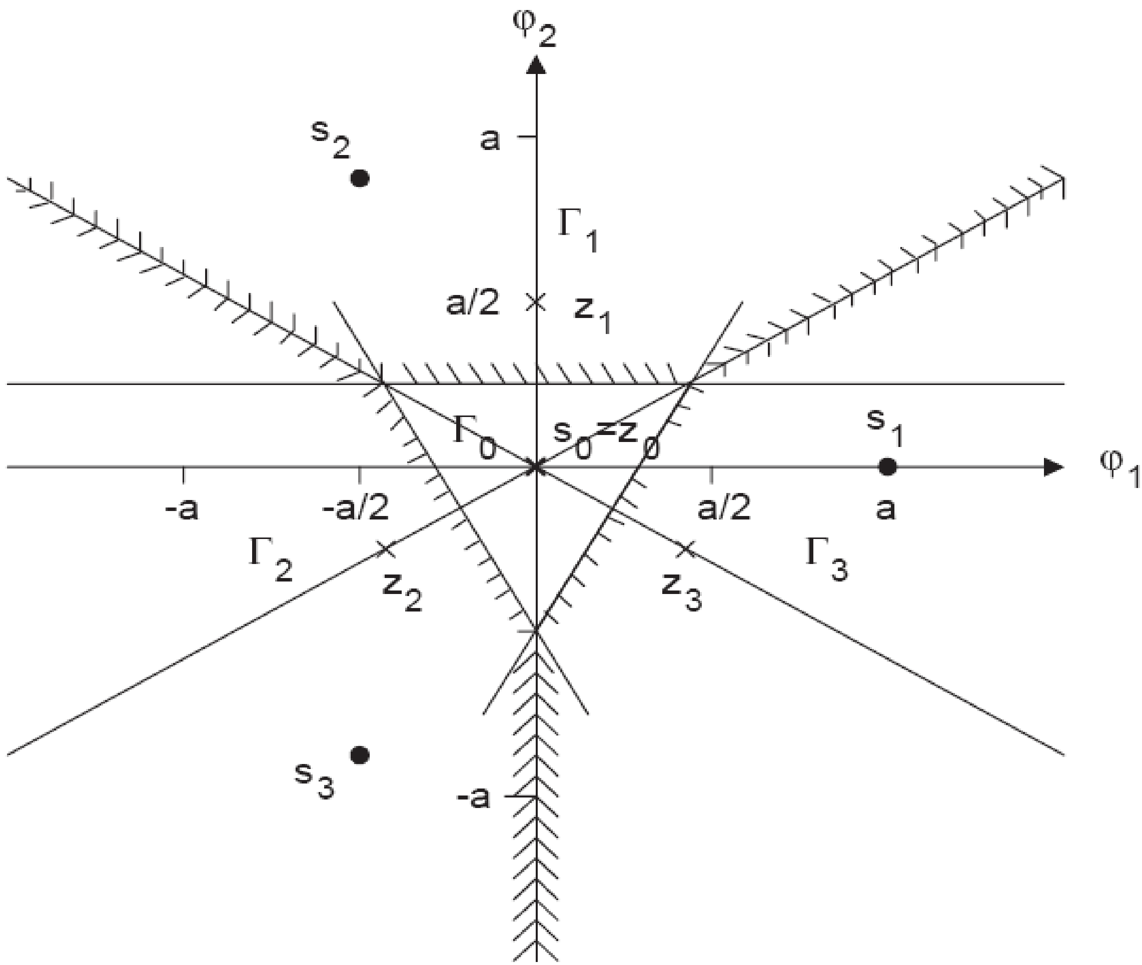
If $z(t)$ is expressed as,

$$z(t) = z_I(t)\cos(2\pi f_c t) - z_Q(t)\sin(2\pi f_c t) \qquad (3.124)$$

then the quadrature components of $z(t)$ satisfy,

$$z_I(t) + jz_Q(t) = (x_I(t-\tau_g) + jx_Q(t-\tau_g))H(f_c) \qquad (3.125)$$

EXAMPLE 3.17 EXAMPLE 5.4



3.4.3 N-Ray Channel Model

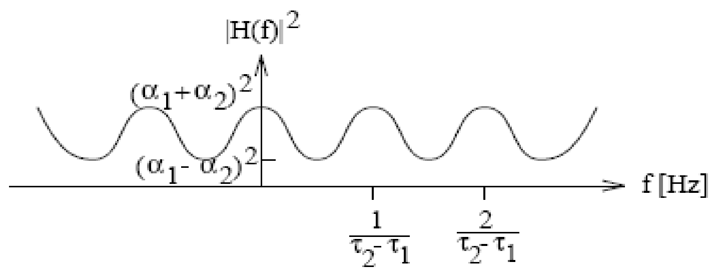
$$z(t) = x(t) * \underbrace{\left(\sum_{i=1}^{N} \alpha_i \delta(t - \tau_i) \right)}_{\text{Impulse response } h(t)} = \sum_{i=1}^{N} \alpha_i x(t - \tau_i) \qquad (3.126)$$

$$H(f) = \mathcal{F}\{h(t)\} = \sum_{i=1}^{N} \alpha_i e^{-j2\pi f \tau_i} \qquad (3.128)$$

So, **H(fc)** is easy to find!

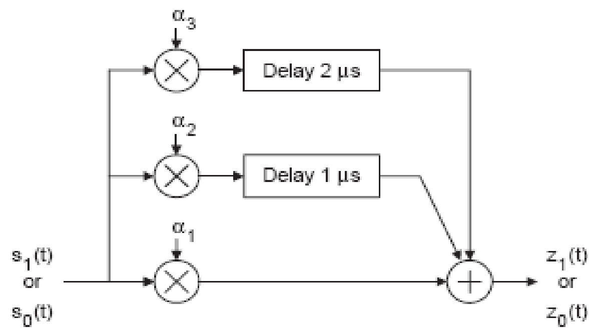
EXAMPLE 3.20

Rough sketch:



It is seen in this figure that the two signal paths add constructively or destructively (fading) depending on the frequency. Furthermore, if $\alpha_1 \approx \alpha_2$ then $|H(f)|$ is very close to zero at certain frequencies (so-called deep fades)!

EXAMPLE 3.19



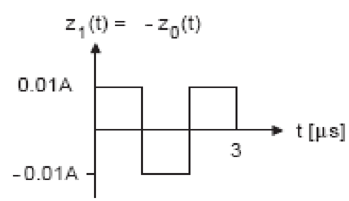
The signal $z_i(t) = s_i(t) * h(t)$ *is the output signal corresponding to the input signal* $s_i(t)$, $i = 0, 1$. *Determine and sketch* $z_0(t)$ *and* $z_1(t)$ *if* $\alpha_1 = 0.01$, $\alpha_2 = -0.01$, *and* $\alpha_3 = 0.01$.

Your conclusions concerning choice of bit rate to avoid overlapping signal alternatives after the channel?

Solution:

$$z_\ell(t) = \sum_{i=1}^{3} \alpha_i s_\ell(t - \tau_i) = 0.01 s_\ell(t) - 0.01 s_\ell(t - 10^{-6}) + 0.01 s_\ell(t - 2 \cdot 10^{-6}), \;\; \ell = 0, 1$$

yields,



Observe that the signal alternatives are changed significantly by the channel (filtering), and that the duration of both signal alternatives is increased from 1 [µs] before the channel, to 3[µs] after the channel!

If the bit rate is reduced to at most $10^6/3$ bps, *then no overlap of signal alternatives will exist after the channel.* □

3.5.1 Additive Interference

Assume that the information carrying bandpass signal $z(t)$ is corrupted by an additive bandpass signal $w(t)$, resulting in the bandpass signal $y(t)$, see Figure 3.16a,

$$y(t) = z(t) + w(t) \tag{3.129}$$

3.5.2 Multiplicative Interference

Now assume that the information carrying bandpass signal $z(t)$ is corrupted by a multiplicative baseband signal $w(t)$, resulting in the bandpass signal $y(t)$, see Figure 3.17,

$$y(t) = z(t)w(t) = z_I(t)w(t)\cos(2\pi f_c t) - z_Q(t)w(t)\sin(2\pi f_c t) \tag{3.138}$$

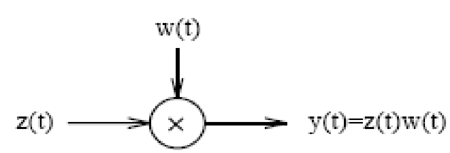


Figure 3.17: Multiplicative (baseband) interference $w(t)$.

If the interference $w(t)$ is small ($\ll 1$), then the signal $y(t)$ is heavily attenuated, and this is often referred to as **signal fading**.

5.34 Consider a communication system where N_t M-ary QAM signals are sent simultaneously (from N_t antennas). The n:th transmitted M-ary QAM signal is denoted $s_n(t)$,

$$s_n(t) = A(n)g(t)\cos(\omega_c t) - B(n)g(t)\sin(\omega_c t) \tag{5.133}$$

for $n = 1, 2, \ldots, N_t$. Note that the same carrier frequency is used for all N_t transmitted QAM signals!

The receiver is assumed to have N_r receiving antennas. The received signal $r_k(t)$ at the k:th receiving antenna is here modelled as

See (3.109)-(3.110)!

$$\begin{aligned} r_k(t) = \sum_{n=1}^{N_t} \Big(\left[H_{k,n}^{Re} A(n) - H_{k,n}^{Im} B(n)\right] g(t)\cos(\omega_c t) - \\ - \left[H_{k,n}^{Re} B(n) + H_{k,n}^{Im} A(n)\right] g(t)\sin(\omega_c t)\Big) + w_k(t) \end{aligned} \tag{5.134}$$

for $k = 1, 2, \ldots, N_r$. The variables $H_{k,n}^{Re}$ and $H_{k,n}^{Im}$ models how the n:th transmitted QAM signal is received at the k:th receiving antenna (attenuation and rotation of the I-Q components).

After I and Q demodulation of $r_k(t)$ to baseband, the receiver obtains the noisy signal space coordinates, here collected in r_k as

$$\begin{aligned} r_k = \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re} A(n) - H_{k,n}^{Im} B(n)\right)}_{\text{received } I \text{ component}} + j \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re} B(n) + H_{k,n}^{Im} A(n)\right)}_{\text{received } Q \text{ component}} \\ + \underbrace{(w_k^{Re} + j w_k^{Im})}_{\text{due to AWGN}} \end{aligned} \tag{5.135}$$

Note that complex notation ($j^2 = -1$) is used in (5.135)!

Let us now introduce the complex notations:

See (3.110)!

$$\begin{aligned} d_n &= A(n) + jB(n) \\ \alpha_{k,n} &= H_{k,n}^{Re} + jH_{k,n}^{Im} \\ w_k &= w_k^{Re} + j w_k^{Im} \end{aligned} \tag{5.136}$$

Then (5.135) can be formulated as,

$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k \quad , k = 1, 2, \ldots, N_r \qquad (5.137)$$

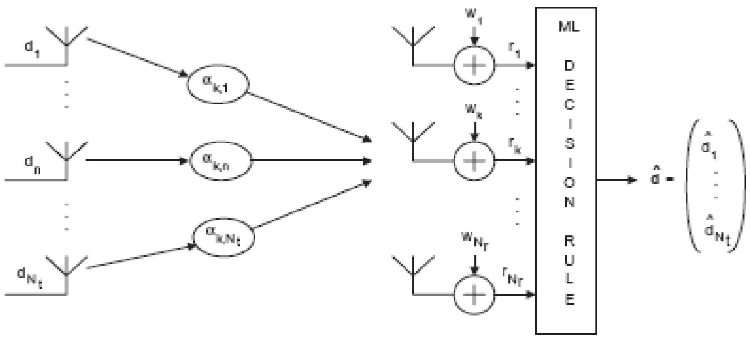
A compact formulation is now obtained as

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w} \qquad (5.138)$$

where the $N_r \times N_t$ matrix $\boldsymbol{A}$ contains the channel coefficients $\{\alpha_{k,n}\}$. The relationship in (5.138) is a basic model in so-called multiple-input multiple-output (MIMO) systems.

The MIMO model is illustrated in the figure below,

MIMO MODEL



$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w}$$

64-QAM+Nt=8 (48bits): ML symbol decision rule ....?

3.6 Receivers for Bandpass Signals

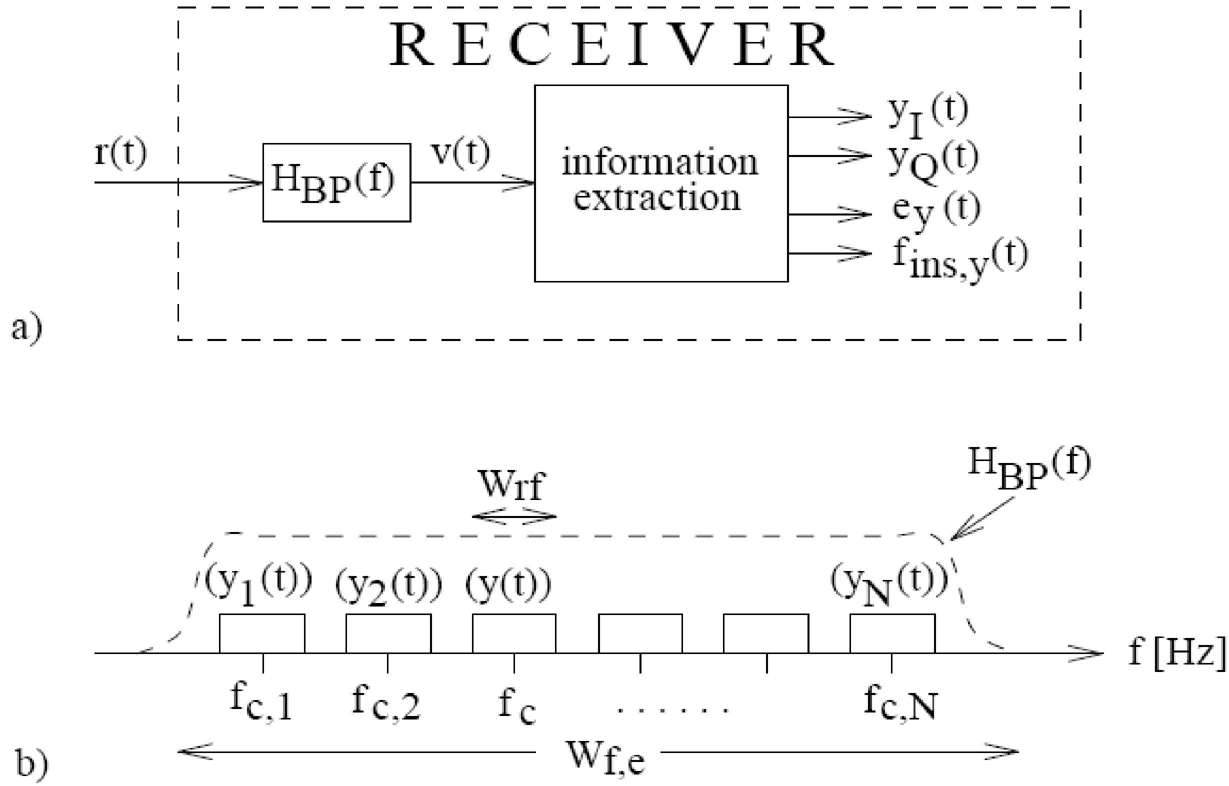


Figure 3.24: a) General blockdiagram of the receiver. b) Illustrating the location of the desired signal $y(t)$ in the frequency domain.

$$\boxed{v(t) = y(t) + y_1(t) + y_2(t) + \ldots + y_N(t)} \quad (3.166)$$

$$\boxed{v(t) = y(t) + y_1(t) + y_2(t) + ... + y_N(t)} \tag{3.166}$$

y(t) is the desired signal and it is located at the carrier frequency fc.

$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t) = e_y(t)\cos(2\pi f_c t + \theta_y(t)) \tag{3.167}$$

We want to extract y(t) from v(t)!

<u>Homodyne reception:</u>

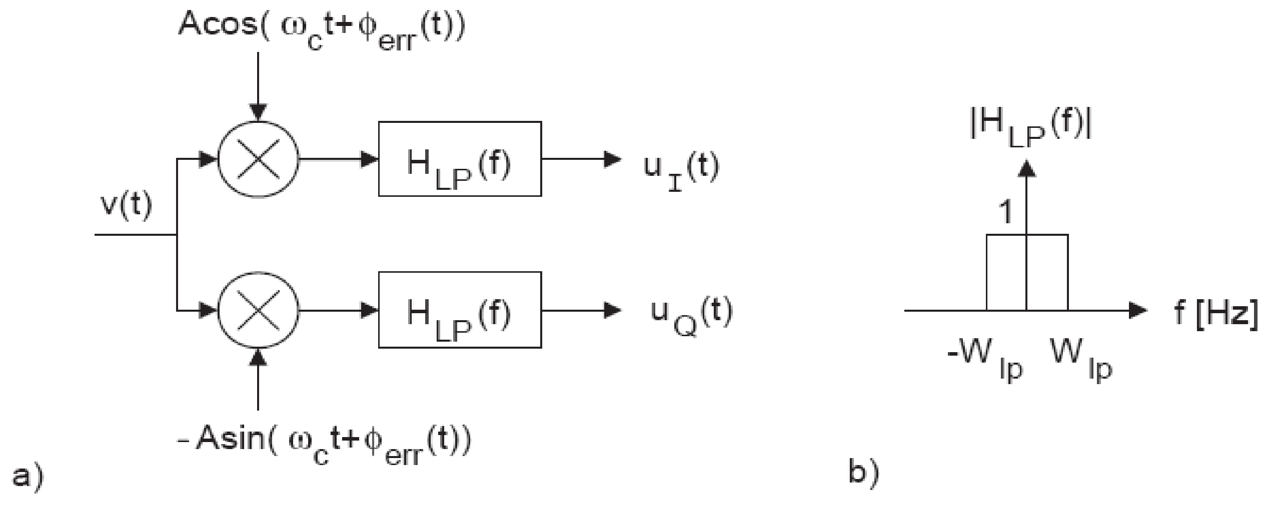


Figure 3.25: a) Homodyne reception. Here a non-ideal phase function $\phi_{err}(t)$ is included. b) Amplitude function of the lowpass filters.

3.6.2 Heterodyne Reception

Basically, the carrier frequency fc of the desired signal y(t) is first changed to a lower, so called intermediate frequency.
After that homodyne detection is used.

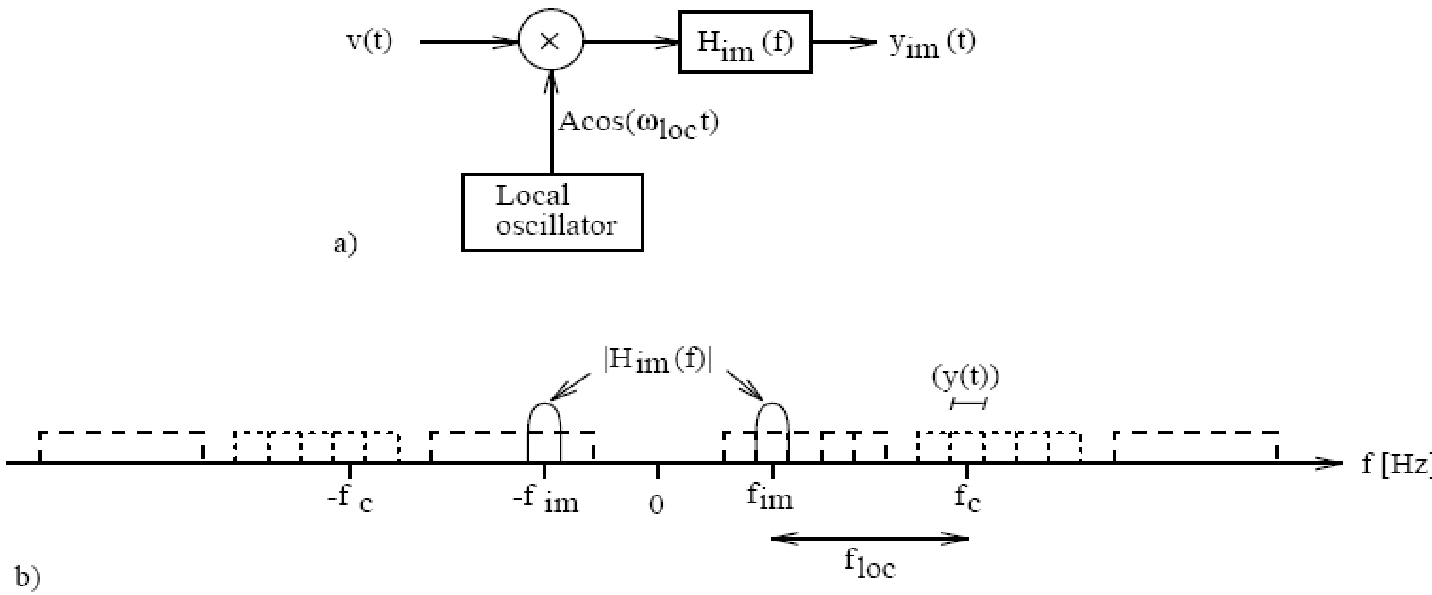


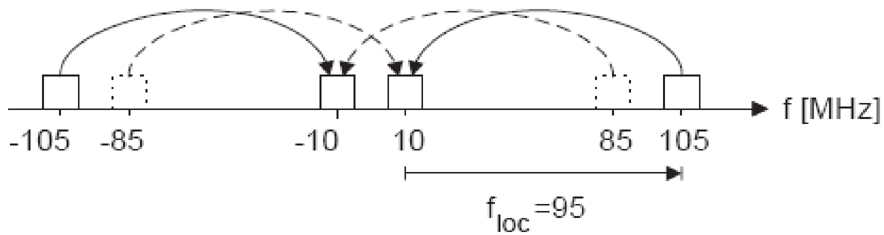
Figure 3.29: a) Mixing down to an intermediate frequency f_{im} [Hz].
b) Illustrating parameters in heterodyne reception.

EXAMPLE 3.26

Assume that $f_{im} = 10$ MHz *in Figure 3.29a. A user wants to tune in a station centered at 105* MHz.

Find the frequency of the local oscillator, and the corrsponding image frequency.

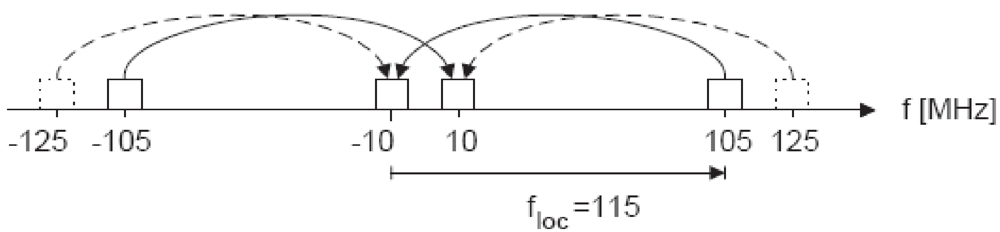
Solution:



From the figure above it is concluded that $f_{loc} = 95$ [MHz] *and* $f_{image} = 85$ [MHz].

Comment:

In principle we may alternatively here choose $f_{loc} = f_c + f_{im} = 105 + 10 = 115$ [MHz]. *This is illustrated in the figure below*



It is seen that in this case the image frequency equals 125 MHz *which is above the carrier frequency of the desired signal* $f_c = 105$ [MHz].

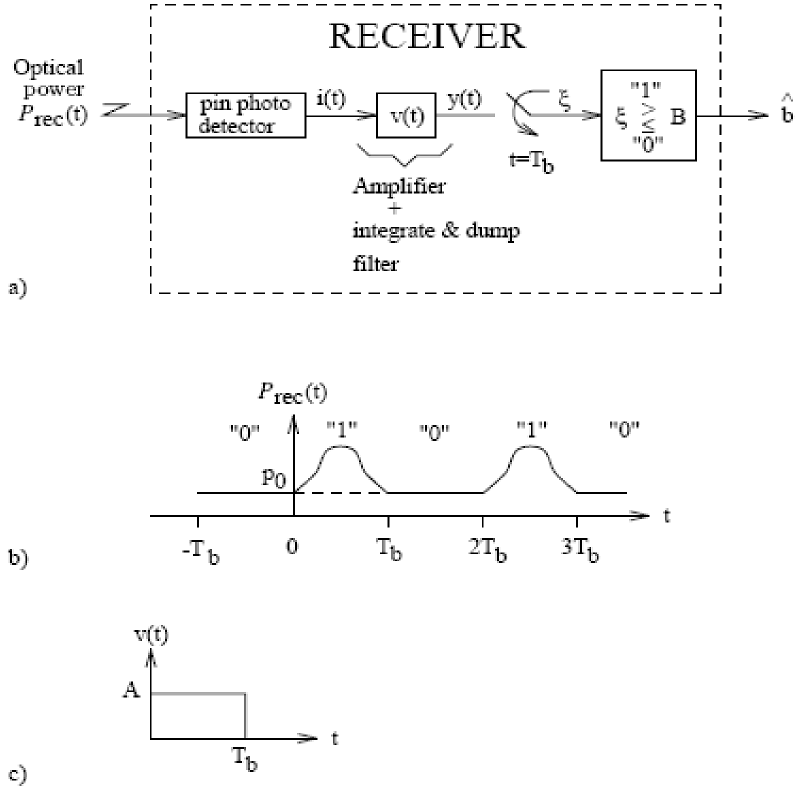
A consequence of using $f_{loc} = f_c + f_{im}$ *is that the output signal* $y_{im}(t)$ *from the intermediate filter is (ignoring possible signals from the image frequency),*

$$y_{im}(t) = [e_y(t)\cos(\omega_c t + \theta_y(t))\cos((\omega_c + \omega_{im})t)] * h_{im}(t) =$$

$$= \left[\frac{e_y(t)}{2}\;\cos(\omega_{im} t - \theta_y(t))\right] * h_{im}(t)$$

So, we obtain a change of sign in the phase. This can be compensated for at a later stage in the receiver. □

Fig. 7.8:



”0”: po
”1”: po+p(t)

Received optical power.

$$\mathcal{P}_{rec}(t) = p_0 + \sum_{i=-\infty}^{\infty} m[i]p(t - iT_b), \quad m[i] \in \{0,1\}, \quad -\infty \le t \le \infty \qquad (7.31)$$

$$\begin{aligned} \xi &= y(T_b) = \int_{-\infty}^{\infty} i(\tau)v(T_b - \tau)d\tau = A\int_0^{T_b} i(\tau)d\tau = \\ &= A\int_0^{T_b} (i_r(t) + i_d(t))dt = Aq\mathcal{N}_{T_b} \qquad (7.32) \end{aligned}$$

q=charge of an electron.
id(t)=”dark current”.

Bit error probability:

$$\begin{aligned}
P_b &= P_0 \underbrace{Prob\{\text{error}|m_0 \text{ sent}\}}_{P_F} + P_1 \underbrace{Prob\{\text{error}|m_1 \text{ sent}\}}_{P_M} \\
&= P_0 Prob\{\xi > B|m_0 \text{ sent}\} + P_1 Prob\{\xi \leq B|m_1 \text{ sent}\} = \\
&= P_0 Prob\{\mathcal{N}_{T_b} > (B/Aq)|m_0 \text{ sent}\} + \\
&\quad + P_1 Prob\{\mathcal{N}_{T_b} \leq (B/Aq)|m_1 \text{ sent}\} \qquad (7.33)
\end{aligned}$$

$$\begin{aligned}
P_F &= Prob\{\mathcal{N}_{T_b} > \alpha|m_0 \text{ sent}\} = \sum_{n=\alpha+1}^{\infty} \frac{\mu_0^n e^{-\mu_0}}{n!} \\
P_M &= Prob\{\mathcal{N}_{T_b} \leq \alpha|m_1 \text{ sent}\} = \sum_{n=0}^{\alpha} \frac{\mu_1^n e^{-\mu_1}}{n!} \\
\alpha &= B/Aq \qquad (7.35)
\end{aligned}$$

Exact expressions!

We need the averages!

$$\boxed{\begin{aligned} &P_b \approx Q(\varrho) \\ &\varrho = \sqrt{\mu_1} - \sqrt{\mu_0} \end{aligned}} \qquad (7.39)$$

$$\mu_0 = E\{\mathcal{N}_{T_b} | m_0 \text{ sent}\} = \int_0^{T_b} \left(\frac{\eta}{hf}\, p_0 + \mathcal{I}_d \right) dt = \mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\, p_0 T_b$$

$$\mu_1 = E\{\mathcal{N}_{T_b} | m_1 \text{ sent}\} = \mu_0 + \frac{\eta\lambda}{hc} \int_0^{T_b} p(t) dt = \mu_0 + \frac{\eta\lambda}{hc} \cdot \mathcal{E}_p \qquad (7.34)$$

$$\mathcal{I}_d = i_d / q$$

$$\boxed{\begin{aligned} &P_b \approx Q(\varrho) \\ &\varrho = \sqrt{\mu_1 + \sigma_w^2} - \sqrt{\mu_0 + \sigma_w^2} = \frac{\mu_1 - \mu_0}{\sqrt{\mu_0 + \sigma_w^2} + \sqrt{\mu_1 + \sigma_w^2}} \end{aligned}} \qquad (7.46)$$

$$\varrho = \frac{\frac{\eta\lambda}{hc}\,\mathcal{P}_p T_b}{\sqrt{\mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\,p_0 T_b + k_\sigma T_b} + \sqrt{\mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\,(p_0 T_b + \mathcal{P}_p T_b) + k_\sigma T_b}} \qquad (7.47)$$

$$\mathcal{P}_p = \mathcal{E}_p / T_b$$

$$\frac{\mathcal{P}_{p,1}}{\sqrt{R_{b,1}}} = \frac{\mathcal{P}_{p,2}}{\sqrt{R_{b,2}}} \qquad (7.48)$$

Chapter 9

An Introduction to Time-varying Multipath Channels

$$\boxed{z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))} \qquad (9.1)$$

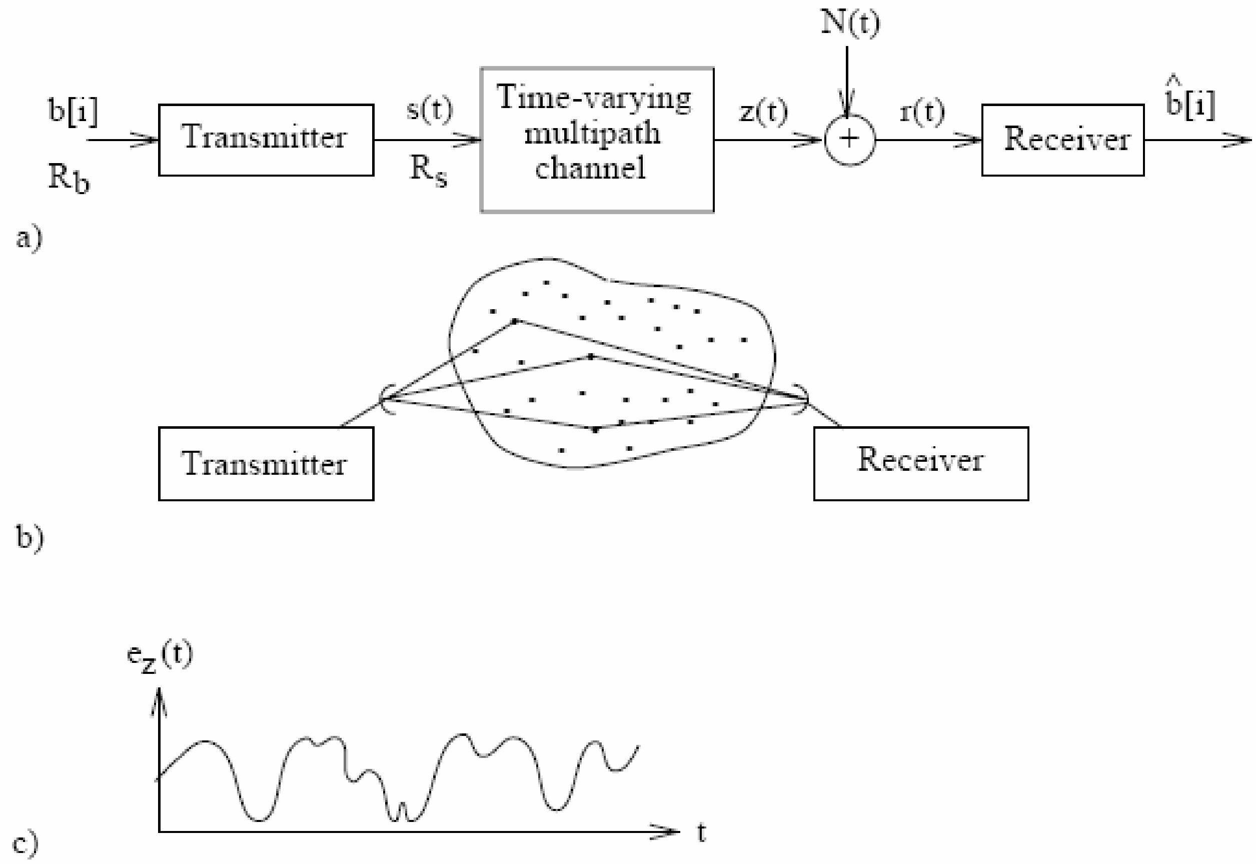
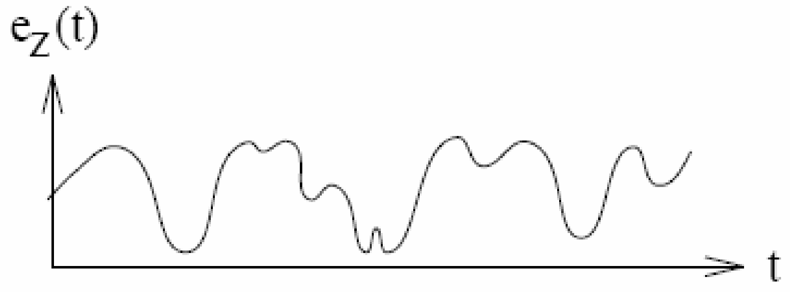


Figure 9.1: a) The digital communication system; b) A scattering medium; c) Illustrating the fading envelope $e_z(t)$.

$$s(t) = \cos((\omega_c + \omega_1)t) \ , \quad -\infty \le t \le \infty \tag{9.2}$$

$$\begin{aligned} z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) = \\ &= e_z(t) \cos((\omega_c + \omega_1)t + \theta_z(t)) \end{aligned} \tag{9.3}$$



Observe that the quadrature components $z_I(t)$ and $z_Q(t)$ in (9.3) are *time-varying*. Hence, the output signal $z(t)$ is *not* a pure sine wave with frequency $f_c + f_1$. ***This is a significant difference compared with the linear time-invariant channel.*** It is seen in (9.3) that the quadrature components depend

$$
\begin{aligned}
z(t) &= \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)(t-\tau_n(t))) = \\
&= z_I(t)\cos((\omega_c+\omega_1)t) - z_Q(t)\sin((\omega_c+\omega_1)t) \\
&= e_z(t)\cos((\omega_c+\omega_1)t+\theta_z(t))
\end{aligned}
$$

Throughout this chapter it is assumed that $z_I(t)$ and $z_Q(t)$ may be modelled as baseband zero-mean wide-sense-stationary (WSS) *Gaussian random processes* (with variances $\sigma_I^2 = \sigma_Q^2 = \sigma^2$). This is a commonly used assumption when the number of scatterers is large, implying that central limit theorem arguments can be used [43], [65], [68], [39]. For a fixed value of t, this assumption leads to a Rayleigh-distributed envelope $e_z(t)$,

$$e_z(t) = \sqrt{z_I^2(t) + z_Q^2(t)} \tag{9.4}$$

$$p_{e_z}(x) = \frac{2x}{b}\, e^{-x^2/b}, \quad x \geq 0\ , \text{ Rayleigh distr.} \tag{9.5}$$

$$b = E\{e_z^2(t)\} = 2\sigma^2 = 2P_z \tag{9.6}$$

and a uniformly distributed phase $\theta_z(t)$ (over a 2π interval). The zero-mean assumption means that there is no deterministic signal path present in $z(t)$. If a

9.1.1 Doppler Power Spectrum and Coherence Time

$$\boxed{\begin{aligned} R_{\mathcal{D}}(f) &= \mathcal{F}(\tilde{c}_z(\tau)) \\ \tilde{c}_z(\tau) &= \frac{1}{2}\, E\{[z_I(t+\tau) + jz_Q(t+\tau)]\ [z_I(t) - jz_Q(t)]\} \\ R_z(f) &= \frac{1}{2}\ (R_{\mathcal{D}}(f + f_c + f_1) + R_{\mathcal{D}}(f - f_c - f_1)) \end{aligned}} \qquad (9.7)$$

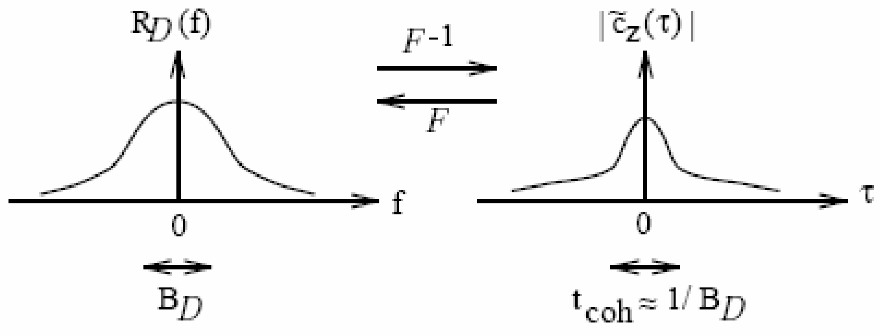


Figure 9.2: Illustrating the Fourier transform pair $\tilde{c}_z(\tau) \longleftrightarrow R_{\mathcal{D}}(f)$.

$$t_{coh} \approx 1/B_{\mathcal{D}} \qquad (9.8)$$

9.1.2 Coherence Bandwidth and Multipath Spread

$$z(t) = z(f_1,t) = \underbrace{\frac{1}{2}\,\tilde{H}_{Re}(f_1,t)}_{z_I(t)} \cos((\omega_c+\omega_1)t) - \underbrace{\frac{1}{2}\,\tilde{H}_{Im}(f_1,t)}_{z_Q(t)} \sin((\omega_c+\omega_1)t) \tag{9.9}$$

What can be said about the output signal $z(t)$ if another frequency $f_2 = f_1 + f_\Delta$ is used, instead of f_1? Are different frequency-intervals, in the input signal spectrum, treated differently by the time-varying multipath channel? To answer these questions the correlation between $z(f_1,t)$ and $z(f_1+f_\Delta,t)$ can be found by

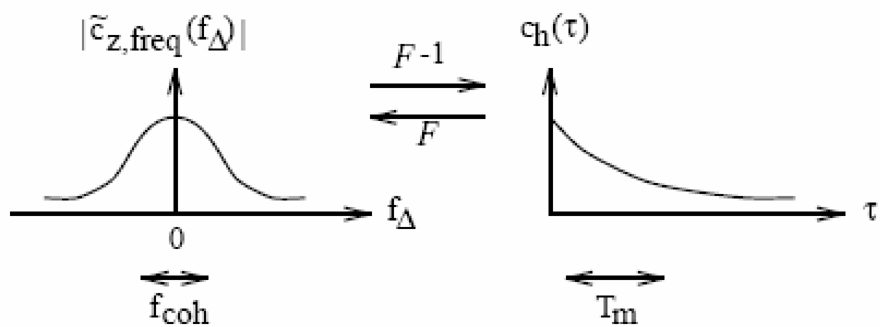


Figure 9.3: Illustrating the Fourier transform pair $c_h(\tau) \longleftrightarrow \tilde{c}_{z,freq}(f_\Delta)$.

$$z(t) = \int_{-\infty}^{\infty} h(\tau, t) s(t - \tau) d\tau \quad (9.10)$$

delay power spectrum $c_h(\tau)$ (also multipath intensity profile) of the time-varying impulse response $h(\tau, t)$,

$$c_h(\tau) = E\left\{\frac{h^2(\tau, t)}{2}\right\} = \frac{1}{2} E\{h_I^2(\tau, t) + h_Q^2(\tau, t)\} = \frac{1}{2} E\{\tilde{h}(\tau, t)\tilde{h}^*(\tau, t)\} \quad (9.15)$$

An example of the delay power spectrum $c_h(\tau)$ is illustrated in Figure 9.3. The width of the delay power spectrum is referred to as the **multipath spread** of the channel and it is denoted by T_m. This is an important parameter since if T_m is too large, compared with e.g. the symbol time, then intersymbol interference can occur.

$$T_m \approx 1/f_{coh} \quad (9.16)$$

9.2 Frequency-Nonselective, Slowly Fading Channel

$$T_s \ll t_{coh} \tag{9.27}$$

or equivalently,

$$B_{\mathcal{D}} \ll R_s \tag{9.28}$$

This means that the channel is **slowly fading**, which imply that it can be treated as a time-invariant channel within the coherence time.

In this subsection a frequency-nonselective channel is investigated. To obtain this situation it is required that the bandwidth of the transmitted signal, denoted W, is much smaller than the coherence bandwidth f_{coh} of the channel,

$$W \ll f_{coh} \tag{9.29}$$

or equivalently,

$$T_m \ll 1/W \tag{9.30}$$

$$
\begin{aligned}
z_I(t) + jz_Q(t) &= \frac{1}{2}\,(s_I(t) + js_Q(t))(H_I + jH_Q) = \\
&= e_s(t)e^{j\theta_s(t)} \cdot ae^{j\phi} = e_z(t)e^{j\theta_z(t)} \qquad (9.37)
\end{aligned}
$$

$$
\boxed{z(t) = ae_s(t)\cos(\omega_c t + \theta_s(t) + \phi)} \qquad (9.38)
$$

$$
p_a(x) = \frac{2x}{b}\,e^{-x^2/b}, \quad x \geq 0 \quad \text{(Rayleigh distribution)} \qquad (9.39)
$$

where,

$$
\begin{aligned}
E\{a\} &= \frac{1}{2}\,\sqrt{\pi b} \qquad (9.40) \\
E\{a^2\} &= b \qquad (9.41)
\end{aligned}
$$

and,

$$
p_\phi(y) = \begin{cases} 1/2\pi & , \quad -\pi \leq y \leq \pi \\ 0 & , \quad \text{otherwise} \end{cases} \qquad (9.42)
$$

If we assume uncoded equally likely binary signals over a Rayleigh fading channel ($z_1(t) = as_1(t), z_0(t) = as_0(t)$), then the bit error probability of the ideal coherent ML receiver is ($0 < d^2 = \frac{D^2_{s_1,s_0}}{2E_{b,sent}} \leq 2$)

$$\boxed{\boldsymbol{P_b = \int_0^\infty \Pr\{\text{error}|a\} p_a(x) dx = E\{\Pr\{\text{error}|a\}\}}} \qquad (9.43)$$

$$P_b = \int_0^\infty Q(\sqrt{d^2 x^2 E_{b,sent}/N_0})\ \frac{2x}{b}\ e^{-x^2/b} dx =$$

$$= -e^{-x^2/b}\ Q(x\sqrt{d^2 E_{b,sent}/N_0})\Big]_0^\infty - \int_0^\infty (-e^{-x^2/b})$$

$$\left(\frac{-\sqrt{d^2 E_{b,sent}/N_0}}{\sqrt{2\pi}}\ e^{-\frac{x^2 d^2 E_{b,sent}/N_0}{2}} \right) dx =$$

$$= \frac{1}{2} - \sqrt{d^2 E_{b,sent}/N_0} \cdot \beta \underbrace{\int_0^\infty \frac{e^{-x^2/2\beta^2}}{\beta\sqrt{2\pi}} dx}_{1/2} \qquad (9.44)$$

$$\mathcal{E}_b = E\{a^2\}E_{b,sent} = bE_{b,sent} \tag{9.45}$$

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{d^2\mathcal{E}_b/N_0}{2 + d^2\mathcal{E}_b/N_0}}\right) = \frac{1}{2 + d^2\mathcal{E}_b/N_0 + \sqrt{2 + d^2\mathcal{E}_b/N_0}\sqrt{d^2\mathcal{E}_b/N_0}}$$

$$\underset{\approx}{\overset{\mathcal{E}_b/N_0 \text{ “large”}}{\downarrow}} \quad \frac{1}{2d^2\mathcal{E}_b/N_0} \tag{9.46}$$

where $d^2 = 2$ for antipodal signals and $d^2 = 1$ for orthogonal signals.
Observe the dramatic increase in P_b due to the Rayleigh fading channel. P_b is no longer exponentially decaying in $\mathcal{E}_b/N_0$, it now decays essentially as $(\mathcal{E}_b/N_0)^{-1}$!

DIVERSITY IS NEEDED!