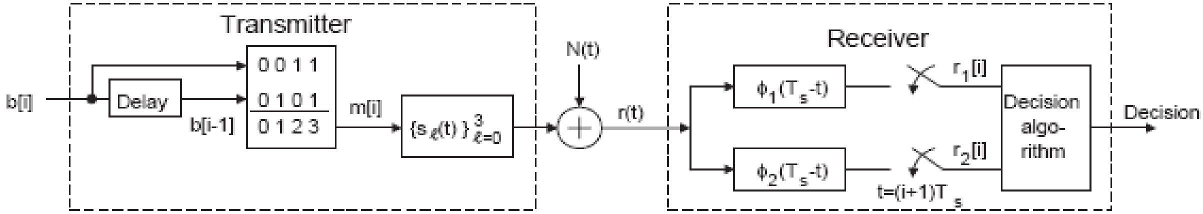
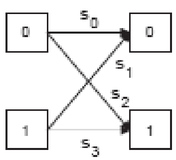
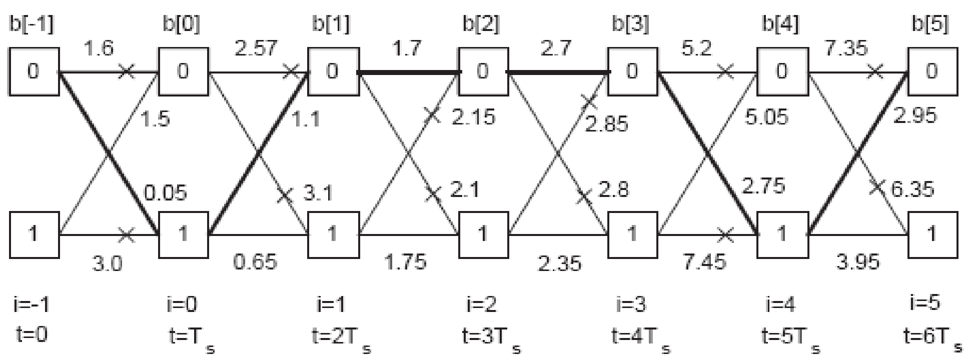
EXAMPLE 8.16





The table below gives the squared Euclidean distance increments $D^2_{inc}[i] = (r_1[i] - s_{j,1})^2 + (r_2[i] - s_{j,2})^2$, obtained between $\mathbf{r}[i]$ and $\mathbf{s}_j$, $j = 0, 1, 2, 3$. The noise $N(t)$ is AWGN.

i :	0	1	2	3	4	5
$\mathbf{s}_0$	1.6	1.07	0.6	1.0	2.5	2.3
$\mathbf{s}_1$	1.5	1.05	1.5	1.1	2.7	0.2
$\mathbf{s}_2$	0.05	1.6	1.0	1.1	0.05	1.3
$\mathbf{s}_3$	3.0	0.6	1.1	0.6	5.1	1.2



$$r(t) = z(t) + N(t) = \sum_{n=-\infty}^{\infty} x_{m[n]}(t - nT_s) + N(t), \quad -\infty \leq t \leq \infty \tag{8.54}$$

$$x_\ell(t) = s_\ell(t) * h(t), \quad \ell = 0, 1, \ldots, \mathcal{M}_{tra} - 1 \tag{8.55}$$

$$\int_{-\infty}^{\infty} (r(t) - z(t))^2 dt \tag{8.56}$$

maximize the expression,

$$\int_{-\infty}^{\infty} \left(r(t)z(t) - \frac{z^2(t)}{2} \right) dt \tag{8.57}$$

$$\int_{-\infty}^{\infty}\left(r(t)z(t)-\frac{z^2(t)}{2}\right)dt=\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} r(t)x_{m[n]}(t-nT_s)dt-$$

$$-\frac{1}{2}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty} x_{m[n]}(t-nT_s)x_{m[\ell]}(t-\ell T_s)dt=$$

$$=\sum_{n=-\infty}^{\infty}\underbrace{\int_{-\infty}^{\infty} r(t)x_{m[n]}(t-nT_s)dt}_{y_{m[n]}[n]}-$$

$$-\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\int_{-\infty}^{\infty} x_{m[n]}(t-nT_s)x_{m[\ell]}(t-\ell T_s)dt=$$

$$=\sum_{n=-\infty}^{\infty} y_{m[n]}[n]-\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\int_{-\infty}^{\infty} x_{m[n]}(\xi)x_{m[\ell]}(\xi+(n-\ell)T_s)d\xi=$$

$$=\sum_{n=-\infty}^{\infty} y_{m[n]}[n]-\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\underbrace{[x_{m[n]}(t)*x_{m[\ell]}(-t)]_{t=(\ell-n)T_s}}_{p_{m[n],m[\ell]}[\ell-n]} \qquad (8.58)$$

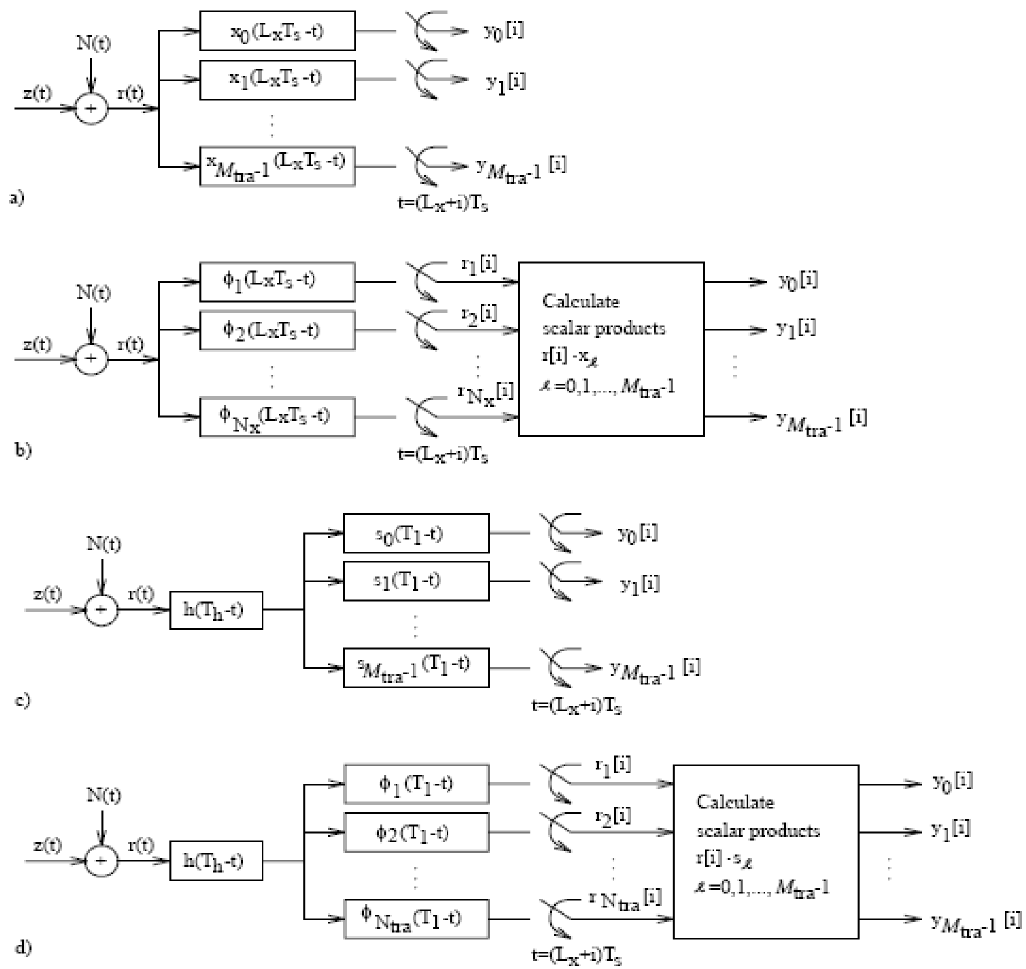


Figure 8.13: The first stage in the ML receiver. a) Filters matched to the signals $x_\ell(t)$; b) Filters matched to the N_x ($N_x \leq \mathcal{M}_{tra}$) basis functions of $\{x_\ell(t)\}_{\ell=0}^{\mathcal{M}_{tra}-1}$; c) Filters matched to $h(t)$ and to $s_\ell(t)$; d) Filters matched to $h(t)$ and to the N_{tra} basis functions of $\{s_\ell(t)\}_{\ell=0}^{\mathcal{M}_{tra}-1}$.

8.4 Bit Error Probability for the ML Receiver in AWGN

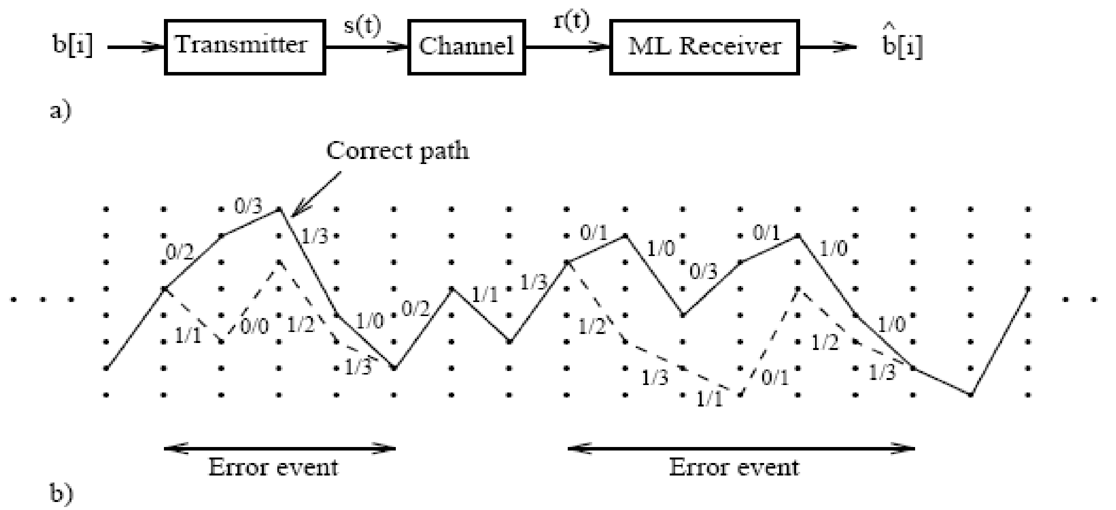


Figure 8.19: a) The digital communication system. b) Parts of the correct path (solid) and the decoded path (dashed) in the trellis.

Observe that by using trellis-coding, longer error events than in the uncoded case are possible. ***So, with trellis-coding there is a potential for larger Euclidean distances than in the uncoded case.***

It can be shown that as a first approximation, at high signal-to-noise ratios, the bit error probability can be approximated with the expression

$$P_b \approx cQ\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right) = cQ\left(\sqrt{d_{\min}^2 \mathcal{E}_b / N_0}\right) \qquad (8.102)$$

where $D_{\min}$ is the smallest Euclidean distance *in the set of all error events, and* $d_{\min}^2 = D_{\min}^2 / 2\mathcal{E}_b$.