In study period Vt-2 2010 you have the opportunity to study the course:

ETT062 Principles of Spread Spectrum Multiple Access Communications, 7.5 hp, A-level.
Spread spectrum techniques means that a much larger bandwidth is used for communication than what normally is required with conventional digital communication methods. With this technique high-performing schemes can be obtained, and it is very interesting in special applications.
Examples of important applications are the 3G system and the GPS system.

This course considers multi-user digital communication systems based on spread spectrum techniques.
The aim of this course is to give very good knowledge concerning principles, concepts, functioning, performance and limitations for such communication systems.

Examples of what is included in the course are:
What is meant by spread spectrum and how does it work?
Why multi-user communication systems based on spread spectrum techniques?
What is meant by CDMA? How does it work and why is it so popular (3G, GPS,…)?
The evolution 3.5G, 4G, 5G,…..

Recommended qualifications: ETT051 Digital Communications.
If you have any questions concerning this course, please contact Göran Lindell, room E:2360 email: goran.lindell@eit.lth.se.

If the channel is slowly changing, then the coherence time is large. Note that $z_I(t+\tau)$ and $z_I(t)$ (also $z_Q(t+\tau)$ and $z_Q(t)$) are correlated over time-intervals τ (much) smaller than the coherence time t_{coh}. Hence, input signals within such intervals are therefore affected similarly by the fading channel. On the other hand, input signals that are separated in time by (much) more than t_{coh}, are affected differently by the channel, and at the output of the channel they become essentially independent of each other. If the former case apply (time flat fading), for a given time-interval, then we say that the channel is **time-nonselective**, and if the latter case apply, then the channel is said to be **time-selective**.

The **coherence bandwidth** $\boldsymbol{f_{coh}}$ of the channel is defined as the width of the autocorrelation function $\tilde{c}_{z,freq}(f_\Delta)$, see Figure 9.3. Note that frequencies within a frequency-interval (much) smaller than the coherence bandwidth f_{coh} are correlated, and they are affected similarly by the fading channel. On the other hand, two frequencies that are separated by (much) more than f_{coh}, are affected differently by the channel, and they are essentially independent of each other. If the former case apply (frequency flat fading), for a given frequency-interval, then we say that the channel is **frequency-nonselective**, and if the latter case apply, then the channel is said to be **frequency-selective**.

$$\begin{aligned} z_I(t) + jz_Q(t) &= \frac{1}{2}\,(s_I(t) + js_Q(t))(H_I + jH_Q) = \\ &= e_s(t)e^{j\theta_s(t)} \cdot ae^{j\phi} = e_z(t)e^{j\theta_z(t)} \end{aligned} \quad (9.37)$$

$$\boxed{z(t) = ae_s(t)\cos(\omega_c t + \theta_s(t) + \phi)} \quad (9.38)$$

$$p_a(x) = \frac{2x}{b}\,e^{-x^2/b}, \quad x \geq 0 \quad \text{(Rayleigh distribution)} \quad (9.39)$$

where,

$$E\{a\} = \frac{1}{2}\,\sqrt{\pi b} \quad (9.40)$$

$$E\{a^2\} = b \quad (9.41)$$

and,

$$p_\phi(y) = \begin{cases} 1/2\pi & , \quad -\pi \leq y \leq \pi \\ 0 & , \quad \text{otherwise} \end{cases} \quad (9.42)$$

If we assume uncoded equally likely binary signals over a Rayleigh fading channel ($z_1(t) = as_1(t), z_0(t) = as_0(t)$), then the bit error probability of the ideal coherent ML receiver is ($0 < d^2 = \frac{D^2_{s_1,s_0}}{2E_{b,sent}} \leq 2$)

$$\boxed{\boldsymbol{P_b = \int_0^\infty \mathrm{Pr}\{\mathrm{error}|a\} p_a(x) dx = E\{\mathrm{Pr}\{\mathrm{error}|a\}\}}} \qquad (9.43)$$

$$P_b = \int_0^\infty Q(\sqrt{d^2 x^2 E_{b,sent}/N_0}) \ \frac{2x}{b} \ e^{-x^2/b} dx =$$

$$= -e^{-x^2/b} \ Q(x\sqrt{d^2 E_{b,sent}/N_0})\Big]_0^\infty - \int_0^\infty (-e^{-x^2/b})$$

$$\left(\frac{-\sqrt{d^2 E_{b,sent}/N_0}}{\sqrt{2\pi}} \ e^{-\frac{x^2 d^2 E_{b,sent}/N_0}{2}} \right) dx =$$

$$= \frac{1}{2} - \sqrt{d^2 E_{b,sent}/N_0} \cdot \beta \underbrace{\int_0^\infty \frac{e^{-x^2/2\beta^2}}{\beta\sqrt{2\pi}} dx}_{1/2} \qquad (9.44)$$

$$\mathcal{E}_b = E\{a^2\}E_{b,sent} = bE_{b,sent} \tag{9.45}$$

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{d^2\mathcal{E}_b/N_0}{2 + d^2\mathcal{E}_b/N_0}}\right) = \frac{1}{2 + d^2\mathcal{E}_b/N_0 + \sqrt{2 + d^2\mathcal{E}_b/N_0}\sqrt{d^2\mathcal{E}_b/N_0}}$$

$$\overset{\mathcal{E}_b/N_0 \text{ “large”}}{\downarrow} \approx \frac{1}{2d^2\mathcal{E}_b/N_0} \tag{9.46}$$

where $d^2 = 2$ for antipodal signals and $d^2 = 1$ for orthogonal signals.
Observe the dramatic increase in P_b due to the Rayleigh fading channel. P_b is no longer exponentially decaying in $\mathcal{E}_b/N_0$, it now decays essentially as $(\mathcal{E}_b/N_0)^{-1}$!

EXAMPLE 9.1

Assume that equally likely, binary orthogonal FSK signals, with equal energy, are sent from the transmitter. Hence, $s_i(t) = \sqrt{2E_{b,sent}/T_b}\cos(2\pi f_i t)$ *in* $0 \le t \le T_b$, $i = 0, 1$.

These signals are communicated over a Rayleigh fading channel, i.e. the received signal is (see (9.38)),

$$r(t) = a\sqrt{2E_{b,sent}/T_b}\cos(2\pi f_i t + \phi) + N(t)$$

Assume that the incoherent receiver in Figure 5.28 on page 397 is used. From (5.109) it is known that for a given value of a,

$$P_b = \frac{1}{2}\ e^{-a^2 E_{b,sent}/2N_0}$$

since $a^2 E_{b,sent}$ *then is the average received energy per bit.*

For the Rayleigh fading channel, and the same receiver, P_b *can be calculated by using (9.43),*

$$\boxed{P_b = \int_0^\infty \Pr\{error|a = x\}p_a(x) = E\{\Pr\{error|a\}\}}$$

$$E\{\Pr\{error|a\}\} = E\left\{\frac{1}{2}\ e^{-a^2 E_{b,sent}/2N_0}\right\} =$$

$$E\left\{\frac{1}{2}\ e^{-a_1^2 E_{b,sent}/2N_0}\right\} \cdot E\left\{e^{-a_2^2 E_{b,sent}/2N_0}\right\}$$

$$P_b = \frac{1/2}{1 + \frac{E_{b,sent}}{N_0} \cdot \frac{E\{a^2\}}{2}} = \frac{1}{2 + \mathcal{E}_b/N_0}$$

Observe the dramatic increase in P_b due to the Rayleigh fading channel. P_b is no longer exponentially decaying in $\mathcal{E}_b/N_0$, it now decays essentially as $(\mathcal{E}_b/N_0)^{-1}$! *As an example, assuming $\mathcal{E}_b/N_0 = 1000$ (30 dB), we obtain*

$$P_b = \begin{cases} 0.5e^{-500} \approx 3.6 \cdot 10^{-218} & , \quad AWGN \\ (1002)^{-1} \approx 10^{-3} & , \quad Rayleigh{+}AWGN \end{cases}$$

DIVERSITY IS NEEDED!

9.2.1 M-PSK Signals Transmitted Over the Rayleigh Fading Channel

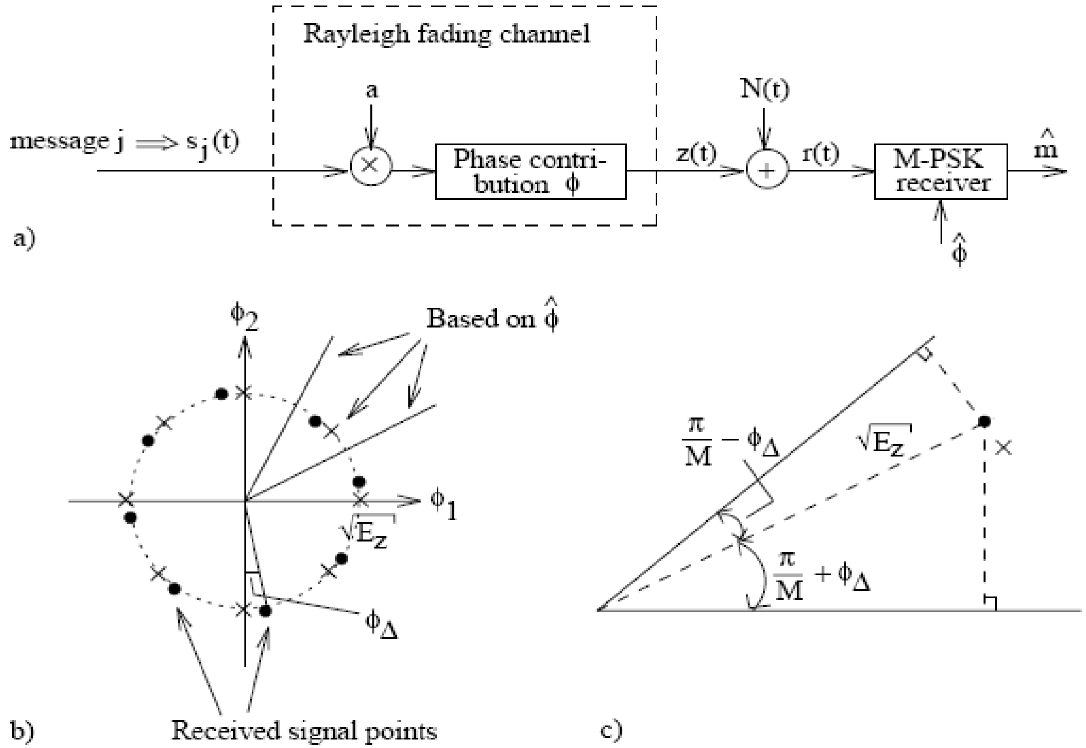

Figure 9.5: a) The Rayleigh fading channel; b) Illustrates how a phase estimation error rotates the signal points, •=true points, x=estimated points; c) Details of a decision region.

So, if message m_j is transmitted, then the input signal $r(t)$ to the receiver is,

$$r(t) = aA\cos(\omega_c t + \nu_j + \phi) + N(t)\ , \quad 0 \le t \le T_s \tag{9.47}$$

$$P_s \leq 1 - \sqrt{\frac{m}{2+m}} = \frac{2}{2+m+\sqrt{2+m}\,\sqrt{m}} \quad \underset{\approx}{\overset{m\text{``large''}}{\downarrow}} \quad \frac{1}{m},$$

$$m = E\{a^2\}\,\frac{2E_s}{N_0}\,\sin^2\left(\frac{\pi}{M} - |\phi_\Delta|\right) = \frac{\mathcal{E}_b}{N_0}\,2\log_2(M)\sin^2\left(\frac{\pi}{M} - |\phi_\Delta|\right)$$

$M - PSK$ (9.54)