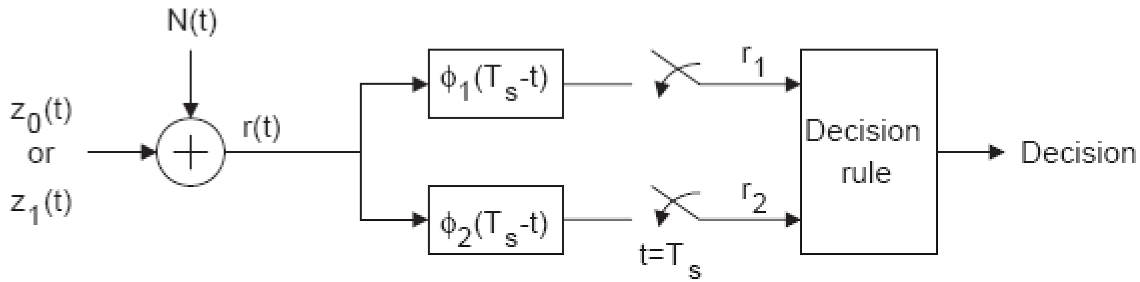
5.29

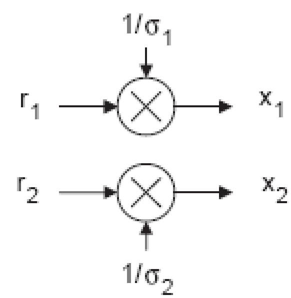
$$z_0(t) = \sqrt{E_0} \cdot \underbrace{\frac{\sqrt{2}\, g(t) \cos(\omega_0 t)}{\sqrt{E_g}}}_{\phi_1(t)}$$

$$z_1(t) = \sqrt{E_1} \cdot \underbrace{\frac{\sqrt{2}\, g(t) \cos(\omega_1 t)}{\sqrt{E_g}}}_{\phi_2(t)}$$

$N(t)$ is a stationary Gaussian random process, but here **the noise is stronger at f_0 than at f_1!** This results in that the noise component w_1 (in r_1) has larger variance than w_2, i.e. $\sigma_1^2 \geq \sigma_2^2$. We assume that w_1 and w_2 can be considered to be independent stochastic random variables.

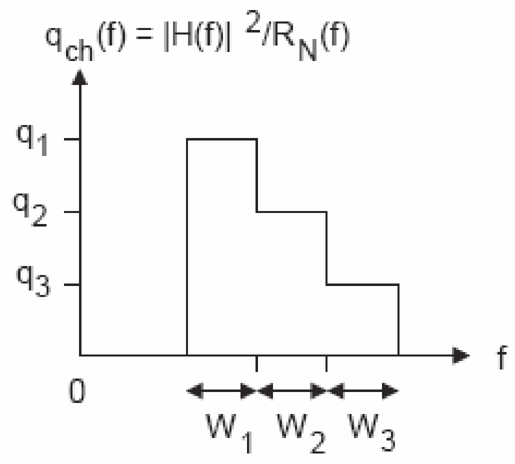


If $\sigma_1^2 \geq \sigma_2^2$, then we can transform this situation to an equal variance situation, see below.



The noise components in x_1 and x_2 both have variance 1. Observe that we do not loose optimality, since the transform $(r_1, r_2) \rightarrow (x_1, x_2)$ is reversible. The optimal decision rule, based on (x_1, x_2), is to calculate the Euclidean distances between (x_1, x_2) and $(z_{i,1}/\sigma_1, z_{i,2}/\sigma_2)$, and pick as the decision the closest one. We have seen that this decision rule is significantly better. However, σ_1 and σ_2 must be estimated (known) by the receiver.

5.30



The total transmitted signal power is here denoted $P_{tot} = \sum_{n=1}^{3} P_{sent,n}$.

$$R_{b,n} = \rho_{BPSK,n} W_n \log_2\left(1 + \frac{3/2}{\chi \rho_{BPSK,n}} \cdot \frac{P_{sent,n}}{W_n} \cdot q_n\right) \ , \ n = 1, 2, 3$$

Here we are interested in how P_{tot} should be distributed over the three channels to obtain a high combined information bit rate, i.e. a high value of $R_b = \sum_{n=1}^{3} R_{b,n}$.

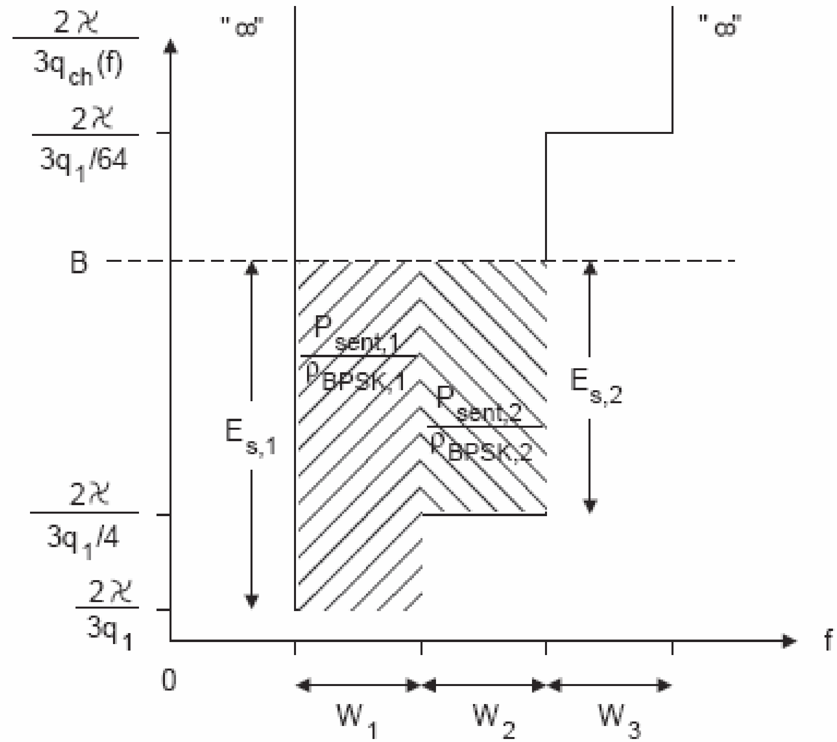
"In general all frequency bands should be used."
"If the sent signal power is small then only the best frequences are used."

Assume the following method to determine how large $P_{sent,n}$ should be:

$$P_{sent,n} = \begin{cases} \rho_{BPSK,n} W_n \left(B - \frac{2\mathcal{X}}{3q_n} \right) & , \quad B \geq \frac{2\mathcal{X}}{3q_n} \\ 0 & , \quad \text{otherwise} \end{cases}$$

The parameter B is chosen such that $P_{tot} = \sum_{n=1}^{3} P_{sent,n}$.

432 The algorithm is referred to as "water filling" type of algorithm, and it is illustrated in the figure below.



If P_{tot} is such that $\frac{2\mathcal{X}}{3q_1/4} \leq B \leq \frac{2\mathcal{X}}{3q_1/64}$ then the third channel is not used, i.e. $P_{sent,3} = 0$.

Start with a small B.
For increasing B calculate the sent signal power in each frequency band, until you have found out how the total sent signal power is distributed.

Hence, we have found that if $B = \frac{2\mathcal{X}}{3q_3}$ then the given value on P_{tot} is obtained, and P_{tot} should be distributed as:

$$
\begin{aligned}
P_{sent,1} &= \rho_{BPSK} \frac{W\mathcal{X}}{3q_1} \cdot 42 \\
P_{sent,2} &= \rho_{BPSK} \frac{W\mathcal{X}}{3q_1} \cdot 40 \\
P_{sent,3} &= 0
\end{aligned}
$$

$$R_b = \sum_{n=1}^{3} R_{b,n} = \rho_{BPSK} \frac{W}{3}(6+4) = \rho_{BPSK} \frac{W}{3} \cdot 10$$

$$M_1 = 64,\ M_2 = 16$$

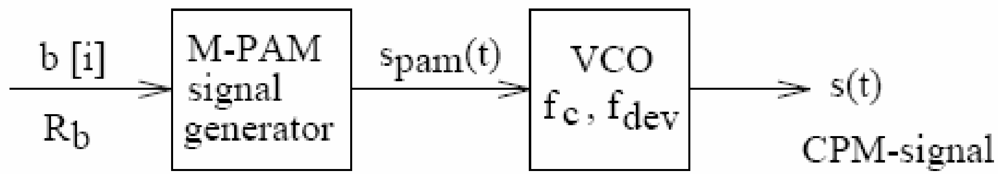
Assume now that the same total signal power instead is sent only in the best channel:

$$P_1 = P_{tot} = \frac{\mathcal{X}\, \rho_{BPSK} W/3}{q_1} \cdot 82, \quad P_2 = P_3 = 0$$

$$R_{b,1} = \rho_{BPSK} \frac{W}{3} \underbrace{\log_2\left(1 + \frac{3}{2} \cdot 82\right)}_{6.954}, \quad R_{b,2} = R_{b,3} = 0$$

<u>Observe the significant loss compared with waterfilling!!</u>

8.2.1 Continuous Phase Modulation (CPM)



(GSM, Bluetooth)

$$s_{pam}(t) = \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s), \quad -\infty \leq t \leq \infty \qquad (8.7)$$

$$\boxed{\begin{aligned} s(t) &= \sqrt{\frac{2E}{T_s}} \cos(\omega_c t + \theta(t)) \\ \theta(t) &= 2\pi h \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \end{aligned}}\ , \quad \text{CPM} \qquad (8.20)$$

Instantaneous frequency (”local frequency”):

$$\boxed{f_{ins}(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + f_{dev} s_{pam}(t) = f_c + f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s)} \qquad (8.12)$$

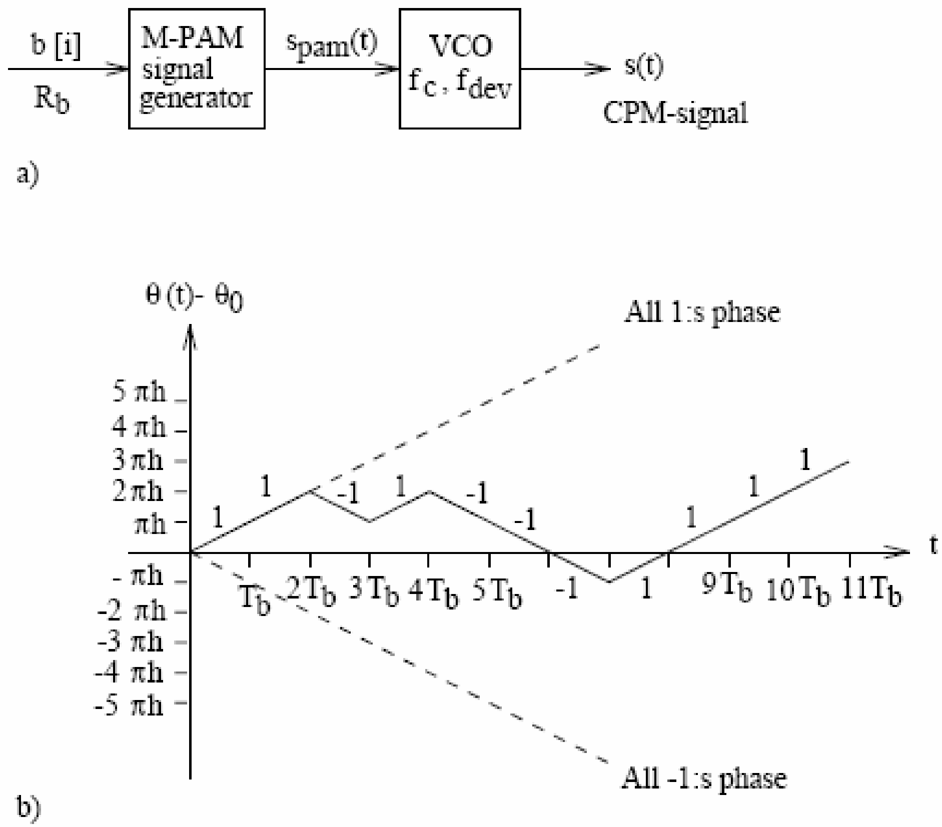


Figure 8.7: a) Generation of a CPM signal; b) Binary CPFSK, examples of phase functions.

Note: Continuous phase implies that there is _memory_ in the signal!!

Examples of pulse shapes:

$$g_{rec}(t) = \begin{cases} 1/2LT_s & , \quad 0 \le t \le LT_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{8.9}$$

$$g_{rc}(t) = \begin{cases} [1 - \cos(2\pi t/LT_s)]/2LT_s & , \quad 0 \le t \le LT_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{8.10}$$

In these expressions, L is a positive integer which is greater or equal to 1. If $L = 1$, then the method is referred to as **full response** signaling, and if $L \geq 2$ then we have so-called **partial response** signaling, [2], [43].

Important special cases:

***M-ary CPFSK* (continuous phase frequency shift keying) means L=1 and a rectangular pulse shape.**

***MSK* (minimum shift keying)**
Means h=1/2 + binary CPFSK

$$\theta(t) = 2\pi f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \qquad (8.15)$$

$$q(t) = \int_0^t g(x)dx \qquad (8.14)$$

$$q_{rec}(t) = \int_0^t g_{rec}(x)dx = \begin{cases} 0 & , \quad t \le 0 \\ t/2LT_s & , \quad 0 \le t \le LT_s \\ 1/2 & , \quad t \ge LT_s \end{cases} \qquad (8.16)$$

$$q_{rc}(t) = \int_0^t g_{rc}(x)dx = \begin{cases} 0 & , \quad t \le 0 \\ \frac{t}{2LT_s} - \frac{1}{4\pi} \sin(2\pi t/LT_s) & , \quad 0 \le t \le LT_s \\ 1/2 & , \quad t \ge LT_s \end{cases} \qquad (8.17)$$

$$\boxed{\begin{aligned} s(t) &= \sqrt{\frac{2E}{T_s}}\;\cos(\omega_c t + \theta(t)) \\ \theta(t) &= 2\pi h \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \end{aligned}} \quad , \quad \text{CPM} \qquad (8.20)$$

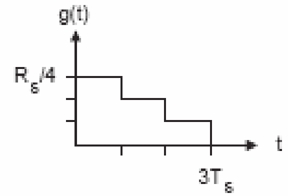
$$\boxed{f_{ins}(t) = f_c + \frac{1}{2\pi}\,\frac{d\theta(t)}{dt} = f_c + f_{dev} s_{pam}(t) = f_c + f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s)} \qquad (8.12)$$

Instantaneous frequency (”local frequency”) within a symbol interval:

$$\begin{aligned} f_{ins}(t) &= f_c + h \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s) \overset{\substack{\text{CPFSK} \\ \downarrow}}{=} f_c + \alpha_i\,\frac{hR_s}{2} = \\ &= f_c + \alpha_i\,\frac{hR_b}{2\log_2(M)}\;, \quad iT_s \le t \le (i+1)T_s \end{aligned} \qquad (8.22)$$

EXAMPLE 8.7

Assume binary CPM, where the pulse $g(t)$ is,



Which frequencies are possible in a symbol interval if the bit rate is 10 kbit/s and $h = 1/4$?

Solution:

Let us study the symbol interval $iT_s \leq t \leq (i+1)T_s$:

$$f_{ins}(t) \overset{(8.22)}{=} f_c + h \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s) =$$

$$= f_c + h \, \frac{R_s}{4} \left(\frac{\alpha_{i-2}}{3} + \alpha_{i-1} \, \frac{2}{3} + \alpha_i \right) = f_c + \frac{10^4}{16} \cdot x$$

where

$$x = \frac{\alpha_{i-2}}{3} + \alpha_{i-1} \, \frac{2}{3} + \alpha_i = \begin{cases} -2 & , \quad -1 \quad -1 \quad -1 \quad (\alpha_{i-2}, \alpha_{i-1}, \alpha_i) \\ 0 & , \quad -1 \quad -1 \quad 1 \\ -2/3 & , \quad -1 \quad 1 \quad -1 \\ 4/3 & , \quad -1 \quad 1 \quad 1 \\ -4/3 & , \quad 1 \quad -1 \quad -1 \\ 2/3 & , \quad 1 \quad -1 \quad 1 \\ 0 & , \quad 1 \quad 1 \quad -1 \\ 2 & , \quad 1 \quad 1 \quad 1 \end{cases}$$

Hence, the possible frequencies in a symbol interval $iT_s \leq t \leq (i+1)T_s$ are:

$f_c - 1250 \; Hz$
$f_c - 833.33 \; Hz$
$f_c - 416.67 \; Hz$
f_c
$f_c + 416.67 \; Hz$
$f_c + 833.33 \; Hz$
$f_c + 1250 \; Hz$ □

$$\theta(t) = 2\pi f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \qquad (8.15)$$

The phase within the i:th symbol interval:

Phase = phase continuity + due to pulseoverlap + due to the current input datasymbol

$$\theta(t) = \pi h \sum_{n=-\infty}^{i-L} \alpha_n + \underbrace{2\pi h \sum_{n=i-L+1}^{i-1} \alpha_n q(t - nT_s)}_{\text{only if } L \geq 2} + 2\pi h \alpha_i q(t - iT_s) + \theta_0 \,,$$

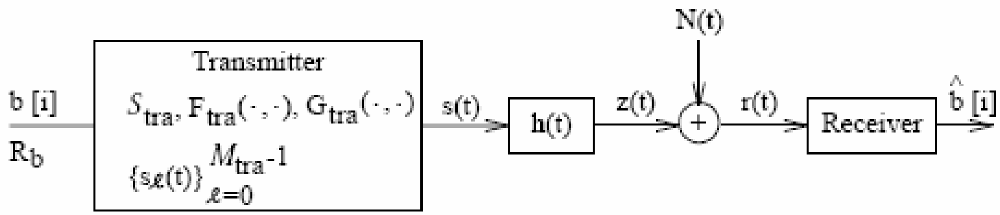
$$iT_s \leq t \leq (i+1)T_s \qquad (8.23)$$

The state of the CPM signal:

$$\boxed{\sigma[i] = \left(\left\{ \pi h \sum_{n=-\infty}^{i-L} \alpha_n \right\}_{\mathrm{mod} 2\pi} , \underbrace{\alpha_{i-L+1}, \alpha_{i-L+2}, \ldots, \alpha_{i-1}}_{\text{only if } L \geq 2} \right)} \qquad (8.25)$$

$$\sigma[i+1] = \left(\left\{ \pi h \sum_{n=-\infty}^{i-L} \alpha_n + \pi h \alpha_{i+1-L} \right\}_{\mathrm{mod} 2\pi} , \alpha_{i-L+2}, \alpha_{i-L+3}, \ldots, \alpha_i \right) \qquad (8.26)$$

8.2.2 The Filtered Channel Model



a)

$$s(t) = \sum_{n=-\infty}^{\infty} s_{m[n]}(t - nT_s), \quad -\infty \leq t \leq \infty \tag{8.29}$$

$$z(t) = s(t) * h(t) = \sum_{n=-\infty}^{\infty} x_{m[n]}(t - nT_s) \tag{8.30}$$

In general, the duration of the received signal alternatives $x_\ell(t)$ is several symbol intervals. Here it is assumed that **all signal alternatives $x_\ell(t)$ are equal to zero outside the interval $0 \leq t \leq L_x T_s$**, where the parameter L_x is a positive integer.

define the current state $\sigma_z[i]$ of the received signal $z(t)$ at $t = iT_s$ as,

$$\boxed{\sigma_z[i] = \{\sigma_{tra}[i - L_x + 1],\ \underbrace{b[i - L_x + 1],\ b[i - L_x + 2], \ldots, b[i-1]}_{\text{only if } L_x \geq 2}\}} \tag{8.35}$$

In this expression, $b[\ell]$ denotes the current k-tuple of information bits to the transmitter at time ℓT_s. So, in (8.35), the state $\sigma_z[i]$ consists of the state of the transmitter at time $t = (i - L_x + 1)T_s$, together with the $(L_x - 1)$ previous k-tuples of information bits. Hence, the number of states $\mathcal{S}_z$ is,

$$\mathcal{S}_z = \mathcal{S}_{tra} 2^{k(L_x - 1)} \tag{8.36}$$

8.3 ML Reception of Trellis-coded Signals in AWGN

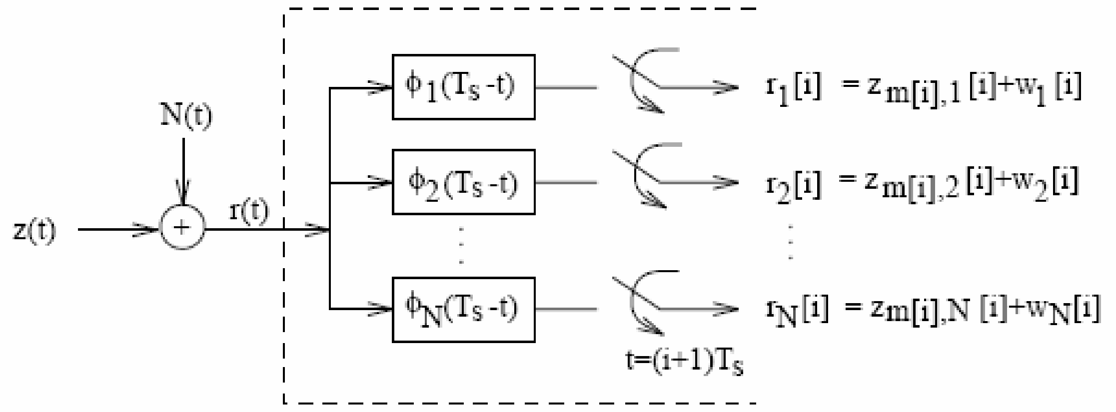


Figure 8.10: The first stage of the ML receiver.

Coherent ML decision rule:
"Choose as the decision the "message" corresponding to the signal $z(t)$ that is closest to the received signal $r(t)$ in signal space".

The difference in this subsection, compared with the situation considered in Chapter 4, is that the received information carrying signal $z(t)$ in (8.30) now contains dependency. Consequently, the waveforms received in different symbol intervals are here dependent, and this means that the ML receiver should observe the received signal $r(t)$,

$$r(t) = z(t) + N(t) \quad , \quad -\infty \leq t \leq \infty \tag{8.41}$$

over several symbol intervals before making a decision. In the special case when no such dependency exists, it was found in Chapter 5 that the MAP receiver had to observe the received signal $r(t)$ only over the current symbol interval, to make an optimum decision of the transmitted message.

Correlator-Outputs:

$$\boldsymbol{r}[i] = \begin{pmatrix} r_1[i] \\ r_2[i] \\ \vdots \\ r_N[i] \end{pmatrix} = \begin{pmatrix} z_{m[i],1}[i] \\ z_{m[i],2}[i] \\ \vdots \\ z_{m[i],N}[i] \end{pmatrix} + \begin{pmatrix} w_1[i] \\ w_2[i] \\ \vdots \\ w_N[i] \end{pmatrix} = \boldsymbol{z}_{m[i]}[i] + \boldsymbol{w}[i] \quad (8.44)$$

$$\ldots, \boldsymbol{r}[i-1], \boldsymbol{r}[i], \boldsymbol{r}[i+1], \ldots \quad (8.45)$$

ML: The received sequence of noisy signal points should be compared to all possible sequences

$$\ldots, \boldsymbol{z}_{m[i-1]}[i-1], \boldsymbol{z}_{m[i]}[i], \boldsymbol{z}_{m[i+1]}[i+1], \ldots \quad (8.46)$$

$$\begin{aligned} D^2_{\boldsymbol{r},\boldsymbol{z}} &= \sum_{n=-\infty}^{\infty} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right)^{tr} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right) = \\ &= \sum_{n=-\infty}^{\infty} \sum_{\ell=1}^{N} \left(r_\ell[n] - z_{m[n],\ell}[n]\right)^2 \end{aligned} \quad (8.47)$$

Up to time t = (i+1)Ts:

Define the **accumulated** squared Euclidean distance $D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i]$ up to time $t = (i+1)T_s$ as,

$$D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i] = \sum_{n=-\infty}^{i} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right)^{tr} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right) \qquad (8.48)$$

Contribution over the i:th symbol interval:

Also define the squared Euclidean distance **increment** at time $t = (i+1)T_s$ as,

$$D^2_{inc}[i] = \left(\boldsymbol{r}[i] - \boldsymbol{z}_{m[i]}[i]\right)^{tr} \left(\boldsymbol{r}[i] - \boldsymbol{z}_{m[i]}[i]\right) = \sum_{\ell=1}^{N} \left(r_\ell[i] - z_{m[i],\ell}[i]\right)^2 \qquad (8.49)$$

Hence, $D^2_{inc}[i]$ is the squared Euclidean distance contribution obtained in the i:th symbol interval $iT_s \leq t \leq (i+1)T_s$.

Recursive calculation:

$$\boxed{D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i] = D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i-1] + D^2_{inc}[i]} \qquad (8.50)$$

$$\boxed{D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i] = D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i-1] + D^2_{inc}[i]} \tag{8.50}$$

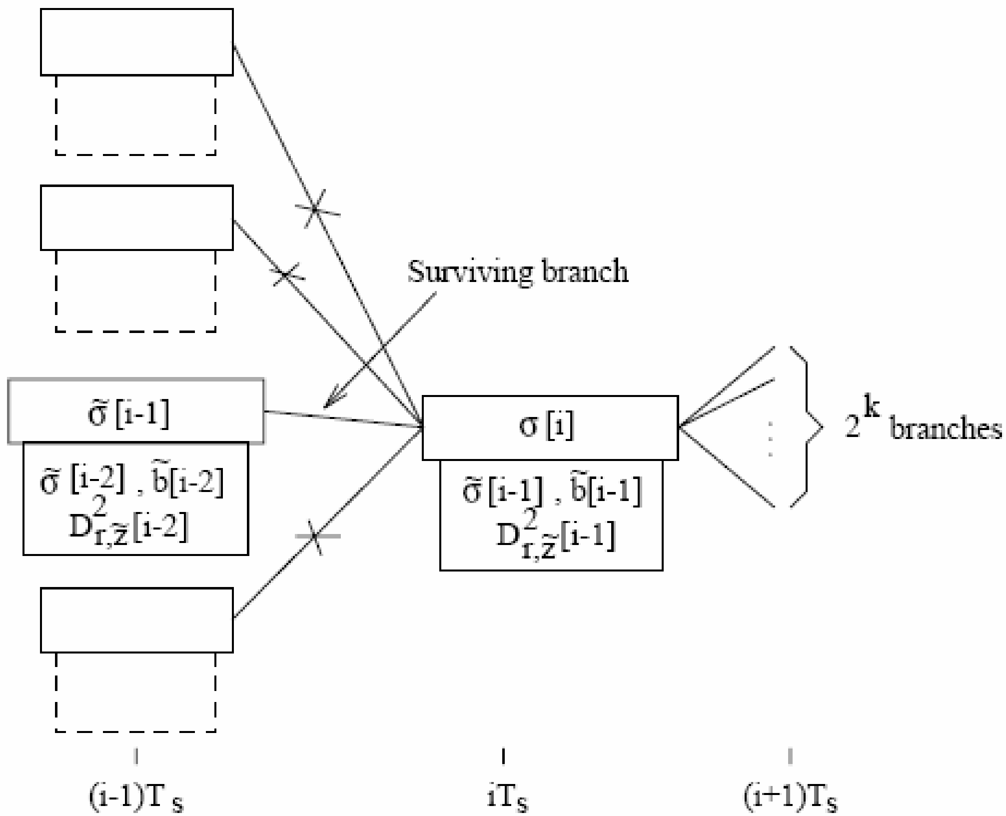


Figure 8.11: Illustrating how branches in the trellis are deleted (x) by the Viterbi algorithm.