In study period Vt-2 2010 you have the opportunity to study the course:

ETT062 Principles of Spread Spectrum Multiple Access Communications, 7.5 hp, A-level.
Spread spectrum techniques means that a much larger bandwidth is used for communication than what normally is required with conventional digital communication methods. With this technique high-performing schemes can be obtained, and it is very interesting in special applications.

Examples of important applications are the 3G system and the GPS system.

This course considers multi-user digital communication systems based on spread spectrum techniques. The aim of this course is to give very good knowledge concerning principles, concepts, functioning, performance and limitations for such communication systems.

Examples of what is included in the course are:
What is meant by spread spectrum and how does it work?
Why multi-user communication systems based on spread spectrum techniques?
What is meant by CDMA? How does it work and why is it so popular (3G, GPS,…)?
The evolution 3.5G, 4G, 5G,…..

Recommended qualifications: ETT051 Digital Communications.
If you have any questions concerning this course, please contact Göran Lindell, room E:2360, email: goran.lindell@eit.lth.se.

$$\boxed{\begin{aligned} &P_b \approx Q(\varrho) \\ &\varrho = \sqrt{\mu_1} - \sqrt{\mu_0} \end{aligned}} \qquad (7.39)$$

$$\boxed{\begin{aligned} \mu_0 &= E\{\mathcal{N}_{T_b} | m_0 \text{ sent}\} = \int_0^{T_b} \left(\frac{\eta}{hf}\, p_0 + \mathcal{I}_d \right) dt = \mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\, p_0 T_b \\ \mu_1 &= E\{\mathcal{N}_{T_b} | m_1 \text{ sent}\} = \mu_0 + \frac{\eta\lambda}{hc} \int_0^{T_b} p(t) dt = \mu_0 + \frac{\eta\lambda}{hc} \cdot \mathcal{E}_p \end{aligned}} \qquad (7.34)$$

$$\mathcal{I}_d = i_d / q$$

7.3.2 Additive Noise

Consider the receiver in Figure 7.8a, and assume now that noise is introduced by the amplifier. This means that the decision variable ξ will contain a noisy component, here denoted by U,

$$\xi = y(T_b) = Aq\mathcal{N}_{T_b} + U \tag{7.40}$$

$$\begin{aligned} P_F &= Prob\{\mathcal{N}_{T_b} + w > \alpha | m_0 \text{ sent}\} = \qquad (7.43) \\ &= Prob\left\{\frac{\mathcal{N}_{T_b} + w - \mu_0}{\sqrt{\mu_0 + \sigma_w^2}} > \frac{\alpha - \mu_0}{\sqrt{\mu_0 + \sigma_w^2}} | m_0 \text{ sent}\right\} \approx Q\left(\frac{\alpha - \mu_0}{\sqrt{\mu_0 + \sigma_w^2}}\right) \end{aligned}$$

$$\boxed{\begin{aligned} &P_b \approx Q(\varrho) \\ &\varrho = \sqrt{\mu_1 + \sigma_w^2} - \sqrt{\mu_0 + \sigma_w^2} = \frac{\mu_1 - \mu_0}{\sqrt{\mu_0 + \sigma_w^2} + \sqrt{\mu_1 + \sigma_w^2}} \end{aligned}} \tag{7.46}$$

$$\varrho = \frac{\frac{\eta\lambda}{hc}\,\mathcal{P}_p T_b}{\sqrt{\mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\,p_0 T_b + k_\sigma T_b} + \sqrt{\mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\,(p_0 T_b + \mathcal{P}_p T_b) + k_\sigma T_b}} \tag{7.47}$$

$$\mathcal{P}_p = \mathcal{E}_p / T_b$$

$$\frac{\mathcal{P}_{p,1}}{\sqrt{R_{b,1}}} = \frac{\mathcal{P}_{p,2}}{\sqrt{R_{b,2}}} \tag{7.48}$$

Increasing L: Attenuation & dispersion (longer pulses) may increase Pb too much!

EXAMPLE 7.7

Consider an ideal on-off optical fiber communication system with no ISI. The transmitted optical power for logical "1" equals -10 [dBm], and zero for logical "0". Assume that the attenuation in the fiber is 2.5 [dB/km], and that the so-called dispersion parameter σ *for the pulse* $p(t)$*, per kilometer, equals* $\sigma_\lambda \cdot 70$ *[ps/km], where* σ_λ *is the so-called spectral width of the light source in [nm].*

To limit the consequences of inter-symbol interference (ISI) it is required that the length of the fiber must be such that $\sigma R_b \leq 0.25$.

It is also required that the length of the fiber must be such that the power $\mathcal{P}_p$ *in the pulse* $p(t)$ *is large enough to achieve a given level of performance* P_b*, e.g.* $P_b = 10^{-9}$.
If the first condition is dominating, we say that the system is ***dispersion-limited****, otherwise it is* ***noise-limited****.*
If $R_b = 8$ *[Mb/s],* $P_b = 10^{-9}$ *is obtained with* $\mathcal{P}_p = 10^{-7}$ *[W], and the system is then dispersion-limited with a maximal fiber length of 11.16 [km].*
Find the maximal length of the fiber if the bit rate is reduced to $R_b = 6$ *[Mb/s].* $P_b = 10^{-9}$ *is required.*

Solution:

$-10\ [dBm] \Leftrightarrow 10^{-4}\ [W]$

$R_b = 6\ [Mb/s]$:

Attenuation:

$$P_{out} \geq P_{p,6} \Rightarrow 10\log\ \frac{P_{in}}{P_{out}} = L\alpha \leq 10\log\ \frac{P_{in}}{P_{p,6}}$$

$$10\log\left(\frac{10^{-4}}{\mathcal{P}_{p,6}}\right) \geq 2.5L\ ,\quad \mathcal{P}_{p,6} = \mathcal{P}_{p,8}\sqrt{\frac{6}{8}}\ ,\quad \mathcal{P}_{p,8} = 10^{-7}\ [W]$$

$$\Rightarrow L \leq \frac{10}{2.5}\ \log\left(\frac{10^{-4}}{10^{-7}} \cdot \sqrt{\frac{8}{6}}\right) = 12.25\ [km]$$

Dispersion:

$$\sigma R_b = \sigma_\lambda \cdot 70 \cdot 10^{-12} \cdot L \cdot 6 \cdot 10^6 \leq 0.25$$

But, $\sigma_\lambda \cdot 70 \cdot 10^{-12} \cdot 11.16 \cdot 8 \cdot 10^6 = 0.25$ *(at* $R_b = 8$ *[Mb/s]).*

$$\Rightarrow L \leq 11.16 \cdot \frac{8}{6} = 14.88\ [km]$$

So, at $R_b = 6$ *[Mb/s], the system is* ***noise-limited*** *with maximal length of the fiber equal to 12.25 [km].* □

Chapter 9

An Introduction to Time-varying Multipath Channels

$$\boxed{z(t) = \sum_{n} \alpha_n(t) s(t - \tau_n(t))} \qquad (9.1)$$

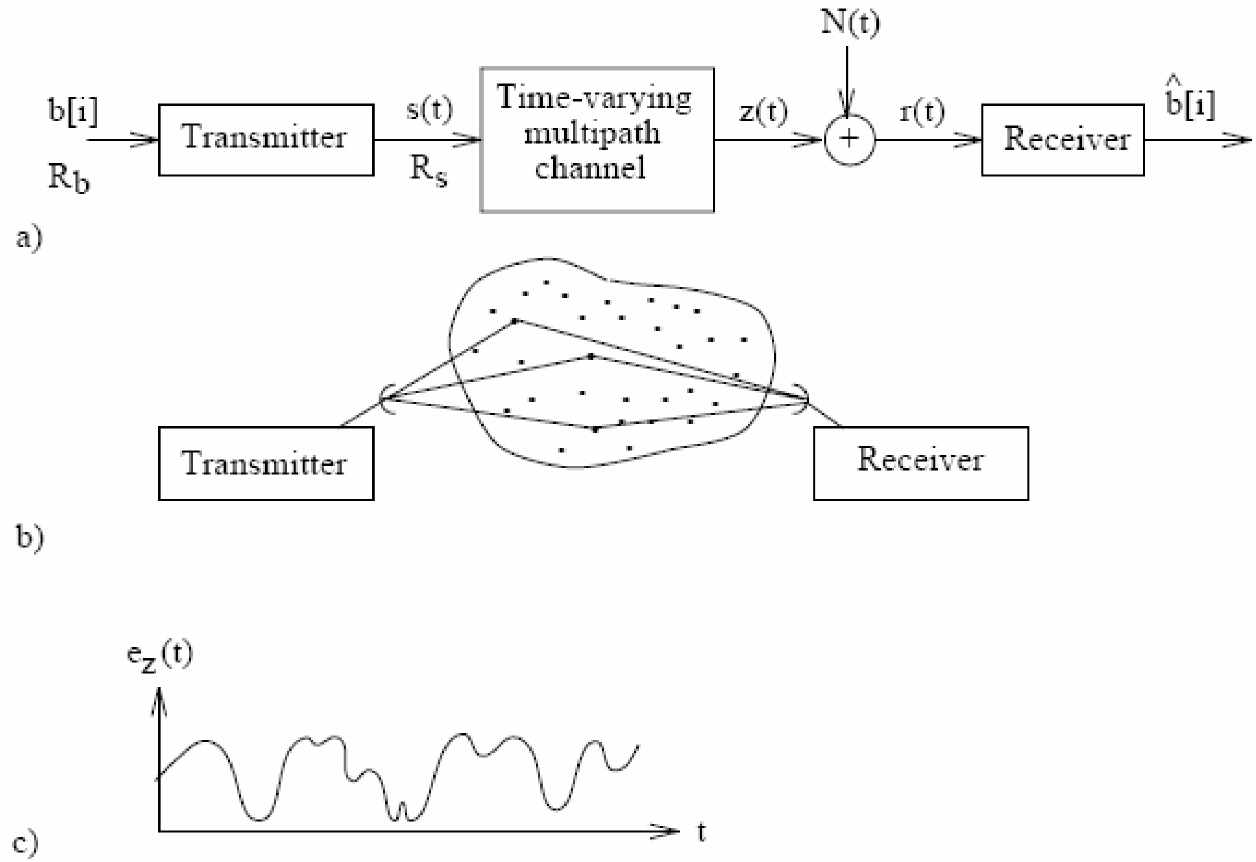


Figure 9.1: a) The digital communication system; b) A scattering medium; c) Illustrating the fading envelope $e_z(t)$.

$$s(t) = \cos((\omega_c + \omega_1)t) \ , \quad -\infty \le t \le \infty \tag{9.2}$$

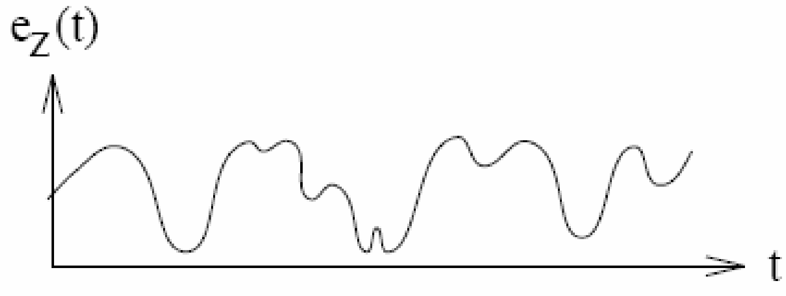
$$\begin{aligned}
z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) = \\
&= \underbrace{\left[\sum_n \alpha_n(t) \cos((\omega_c + \omega_1)\tau_n(t))\right]}_{z_I(t) = \tilde{H}_{Re}(f_1,t)/2} \cos((\omega_c + \omega_1)t) - \\
&\quad - \underbrace{\left[\sum_n \alpha_n(t) \sin(-(\omega_c + \omega_1)\tau_n(t))\right]}_{z_Q(t) = \tilde{H}_{Im}(f_1,t)/2} \sin((\omega_c + \omega_1)t) \\
&= z_I(t) \cos((\omega_c + \omega_1)t) - z_Q(t) \sin((\omega_c + \omega_1)t) \\
&= e_z(t) \cos((\omega_c + \omega_1)t + \theta_z(t))
\end{aligned} \tag{9.3}$$

Compare with the time-invariant QAM-result:

$$\begin{aligned}
A_z + jB_z &= (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu + \phi(f_c))} = \\
&= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c))
\end{aligned} \tag{3.110}$$

$$s(t) = \cos((\omega_c + \omega_1)t) \ , \quad -\infty \le t \le \infty \tag{9.2}$$

$$\begin{aligned} z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) = \\ &= e_z(t) \cos((\omega_c + \omega_1)t + \theta_z(t)) \end{aligned} \tag{9.3}$$



Observe that the quadrature components $z_I(t)$ and $z_Q(t)$ in (9.3) are *time-varying*. Hence, the output signal $z(t)$ is *not* a pure sine wave with frequency $f_c + f_1$. ***This is a significant difference compared with the linear time-invariant channel.*** It is seen in (9.3) that the quadrature components depend

$$\begin{aligned} z(t) &= \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)(t-\tau_n(t))) = \\ &= z_I(t)\cos((\omega_c+\omega_1)t) - z_Q(t)\sin((\omega_c+\omega_1)t) \\ &= e_z(t)\cos((\omega_c+\omega_1)t+\theta_z(t)) \end{aligned}$$

Throughout this chapter it is assumed that $z_I(t)$ and $z_Q(t)$ may be modelled as baseband zero-mean wide-sense-stationary (WSS) *Gaussian random processes* (with variances $\sigma_I^2 = \sigma_Q^2 = \sigma^2$). This is a commonly used assumption when the number of scatterers is large, implying that central limit theorem arguments can be used [43], [65], [68], [39]. For a fixed value of t, this assumption leads to a Rayleigh-distributed envelope $e_z(t)$,

$$e_z(t) = \sqrt{z_I^2(t) + z_Q^2(t)} \tag{9.4}$$

$$p_{e_z}(x) = \frac{2x}{b}\, e^{-x^2/b}, \quad x \geq 0\ ,\ \text{Rayleigh distr.} \tag{9.5}$$

$$b = E\{e_z^2(t)\} = 2\sigma^2 = 2P_z \tag{9.6}$$

and a uniformly distributed phase $\theta_z(t)$ (over a 2π interval). The zero-mean assumption means that there is no deterministic signal path present in $z(t)$. If a

9.1.1 Doppler Power Spectrum and Coherence Time

$$\boxed{\begin{aligned} &R_{\mathcal{D}}(f) = \mathcal{F}(\tilde{c}_z(\tau)) \\ &\tilde{c}_z(\tau) = \frac{1}{2}\, E\{[z_I(t+\tau) + jz_Q(t+\tau)]\ [z_I(t) - jz_Q(t)]\} \\ &R_z(f) = \frac{1}{2}\, (R_{\mathcal{D}}(f+f_c+f_1) + R_{\mathcal{D}}(f-f_c-f_1)) \end{aligned}} \qquad (9.7)$$

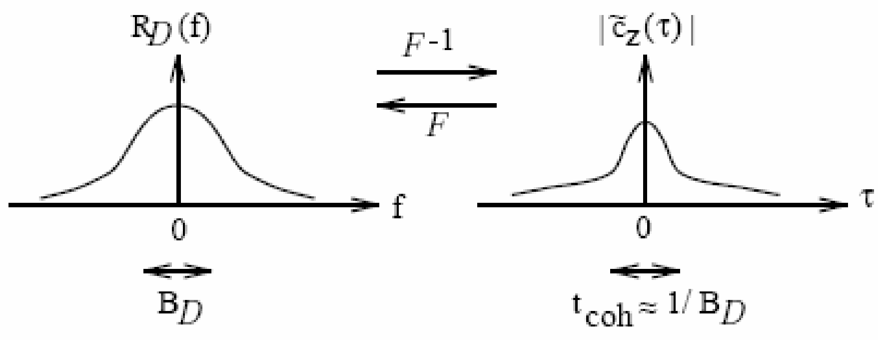


Figure 9.2: Illustrating the Fourier transform pair $\tilde{c}_z(\tau) \longleftrightarrow R_{\mathcal{D}}(f)$.

$$t_{coh} \approx 1/B_{\mathcal{D}} \qquad (9.8)$$

9.1.2 Coherence Bandwidth and Multipath Spread

$$z(t) = z(f_1, t) = \underbrace{\frac{1}{2}\ \tilde{H}_{Re}(f_1, t)}_{z_I(t)} \cos((\omega_c + \omega_1)t) - \underbrace{\frac{1}{2}\ \tilde{H}_{Im}(f_1, t)}_{z_Q(t)} \sin((\omega_c + \omega_1)t) \tag{9.9}$$

What can be said about the output signal $z(t)$ if another frequency $f_2 = f_1 + f_\Delta$ is used, instead of f_1? Are different frequency-intervals, in the input signal spectrum, treated differently by the time-varying multipath channel? To answer these questions the correlation between $z(f_1, t)$ and $z(f_1 + f_\Delta, t)$ can be found by

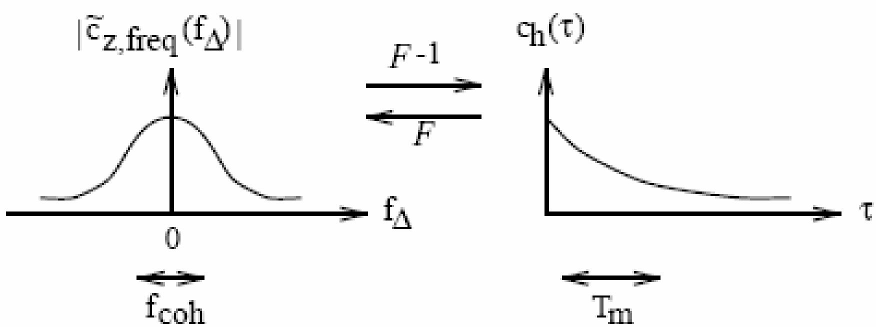


Figure 9.3: Illustrating the Fourier transform pair $c_h(\tau) \longleftrightarrow \tilde{c}_{z,freq}(f_\Delta)$.

$$\boxed{z(t) = \int_{-\infty}^{\infty} h(\tau, t) s(t - \tau) d\tau} \qquad (9.10)$$

delay power spectrum $c_h(\tau)$ (also multipath intensity profile) of the time-varying impulse response $h(\tau, t)$,

$$c_h(\tau) = E\left\{\frac{h^2(\tau, t)}{2}\right\} = \frac{1}{2}\, E\{h_I^2(\tau, t) + h_Q^2(\tau, t)\} = \frac{1}{2}\, E\{\tilde{h}(\tau, t)\tilde{h}^*(\tau, t)\} \quad (9.15)$$

An example of the delay power spectrum $c_h(\tau)$ is illustrated in Figure 9.3. The width of the delay power spectrum is referred to as the **multipath spread** of the channel and it is denoted by T_m. This is an important parameter since if T_m is too large, compared with e.g. the symbol time, then intersymbol interference can occur.

$$T_m \approx 1/f_{coh} \qquad (9.16)$$

9.2 Frequency-Nonselective, Slowly Fading Channel

$$T_s \ll t_{coh} \tag{9.27}$$

or equivalently,

$$B_{\mathcal{D}} \ll R_s \tag{9.28}$$

This means that the channel is **slowly fading**, which imply that it can be treated as a time-invariant channel within the coherence time.

In this subsection a frequency-nonselective channel is investigated. To obtain this situation it is required that the bandwidth of the transmitted signal, denoted W, is much smaller than the coherence bandwidth f_{coh} of the channel,

$$W \ll f_{coh} \tag{9.29}$$

or equivalently,

$$T_m \ll 1/W \tag{9.30}$$

$$\boxed{\tilde{z}(t) = \frac{1}{2}\int_{-\infty}^{\infty} \tilde{S}(f)\tilde{H}(f,t)e^{j2\pi ft}df} \tag{9.26}$$

$$\boxed{z_I(t) + jz_Q(t) = \frac{1}{2}\int_{-\infty}^{\infty}[S_I(f) + jS_Q(f)]\ [H_I(f,t) + jH_Q(f,t)]e^{j2\pi ft}df} \tag{9.33}$$

$$z_I(t) + jz_Q(t) = \frac{1}{2}\int_{-\infty}^{\infty}[S_I(f) + jS_Q(f)] \cdot (H_I + jH_Q)e^{j2\pi ft}df \tag{9.36}$$

$$\begin{aligned} z_I(t) + jz_Q(t) &= \frac{1}{2}\ (s_I(t) + js_Q(t))(H_I + jH_Q) = \\ &= e_s(t)e^{j\theta_s(t)} \cdot ae^{j\phi} = e_z(t)e^{j\theta_z(t)} \end{aligned} \tag{9.37}$$