EXAMPLE 5.23

Assume a binary communication system with equiprobable antipodal signal alternatives,

$$s_1(t) = -s_0(t) = \sum_{k=1}^{K} g_k(t), \quad 0 \le t \le T_b$$

Let $E_{b,sent}$ *denote the average transmitted energy per information bit, i.e.* $E_{s_1} = E_{s_0} = E_{b,sent}$. *It is also assumed that the individual pulses* $g_k(t)$ *are such that*

$$\int_0^{T_b} g_i(t) g_j(t) = \begin{cases} E_{b,sent}/K & , \quad i = j \\ 0 & , \quad i \neq j \end{cases}$$

We can therefore define (sent) basis functions as,

$$\phi_k(t) = \frac{g_k(t)}{\sqrt{E_{b,sent}/K}} \; , \; k = 1, 2, \ldots, K$$

and the signal energy $E_{b,sent}/K$ *is sent in each of the* K *dimensions.*

Observe that the situation studied in this example applies to several kinds of diversity, e.g., time- and/or frequency-diversity, depending on how the pulses $g_k(t)$ *are chosen.*

The communication channel is assumed to be such that the received signal alternatives are,

$$z_1(t) = -z_0(t) = \sum_{k=1}^{K} \alpha_k g_k(t) = \sum_{k=1}^{K} \underbrace{\alpha_k \sqrt{\frac{E_{b,sent}}{K}}}_{z_{1,k}} \; \phi_k(t)$$

and they are disturbed by AWGN $N(t)$ *with power spectral density* $R_N(f) = N_0/2$. *Note that the channel coefficients* $\{\alpha_k\}_{k=1}^{K}$ *multiply the signal in each dimension, respectively. The ideal ML receiver is used and it is assumed that perfect estimates of the channel coefficients are available to the receiver.*

a) Assume that the channel parameters $\{\alpha_k\}_{k=1}^{K}$ *are known to the receiver. Determine an expression of* P_b *that includes* $E_{b,sent}$.

b) Suggest a receiver structure for the case in a).

Solution:

a)

$$P_b = Q\left(\sqrt{2\mathcal{E}_b/N_0}\right)$$

$$\mathcal{E}_b = \frac{E_{z_0}+E_{z_1}}{2} = E_{z_0} = E_{z_1} = \sum_{k=1}^{K} z_{j,k}^2 = \frac{E_{b,sent}}{K}\sum_{k=1}^{K}\alpha_k^2$$

Hence, we obtain that

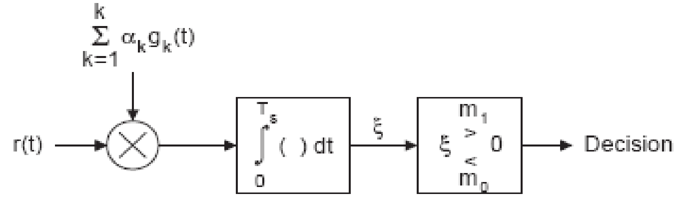
$$P_b = Q\left(\sqrt{\frac{2E_{b,sent}}{N_0 K}\sum_{k=1}^{K}\alpha_k^2}\right)$$

Note that here a K-fold diversity is obtained, in the sense that signal energy from all K dimensions (or "sub-channels") is efficiently collected and used in the decision process.
Note also that

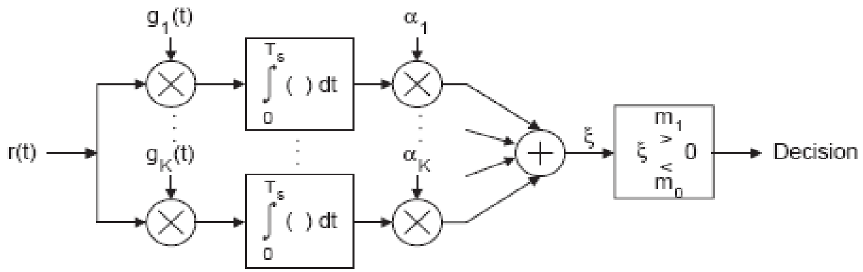
$$D^2_{s_1,s_0} = 4E_{b,sent}$$

$$D^2_{z_1,z_0} = 4E_z = \frac{4E_{b,sent}}{K}\sum_{k=1}^{K}\alpha_k^2 = \frac{D^2_{s_1,s_0}}{K}\sum_{k=1}^{K}\alpha_k^2$$

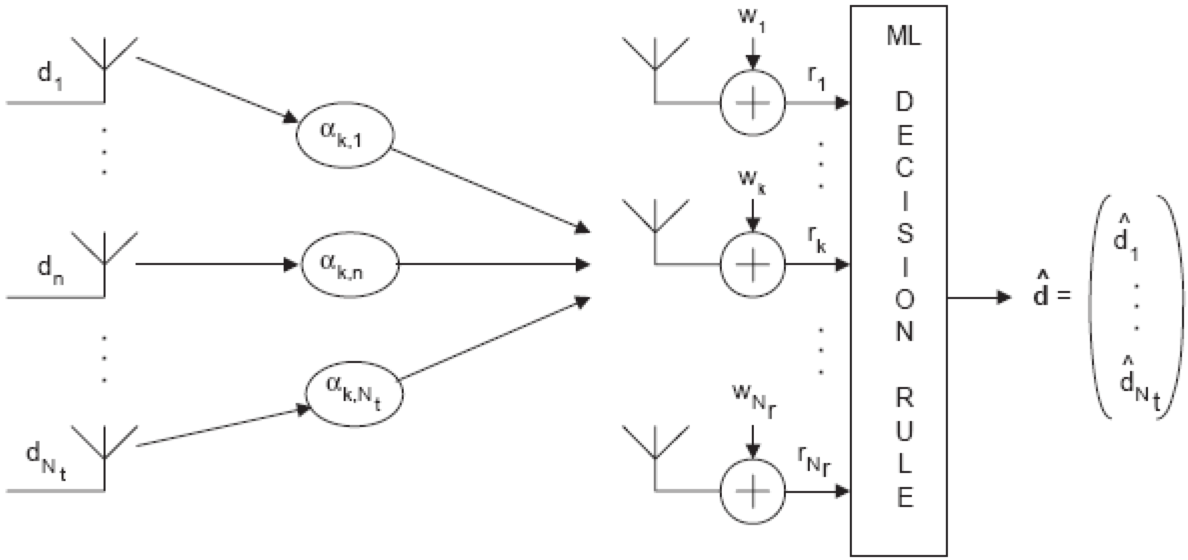
b) From Figure 4.10 on page 247 we obtain the receiver structure below (the constant 2 is ignored in the correlation below),



An equivalent receiver structure is also shown below,



MIMO MODEL



$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{Ad} + \boldsymbol{w}$$

Assume, e.g., that: Nt=1 and data symbol d1 is binary:+A or –A

5.4.1.1 An Example Illustrating Diversity Gains

Here we study the case when the channel parameters $\{\alpha_k\}_{k=1}^{K}$ have the following properties:

- They are assumed to be independent random variables, and only two values are possible for each α_k.
- Each α_k takes the value α_G ("Good") with probability P_G, and the value α_B ("Bad") with probability $P_B = 1 - P_G$.

$$\mathcal{E}_b = E\left\{\frac{E_{b,sent}}{K}\sum_{k=1}^{K}\alpha_k^2\right\} = E_{b,sent}E\{\alpha_k^2\} =$$
$$= E_{b,sent}(\alpha_G^2 P_G + \alpha_B^2(1-P_G)) \qquad (5.84)$$

$$P_b = E\left\{P_{b|\{\alpha_k\}_{k=1}^{K}}\right\} = E\left\{Q\left(\sqrt{\frac{2E_{b,sent}}{N_0 K}\sum_{k=1}^{K}\alpha_k^2}\right)\right\} =$$
$$= E\left\{Q\left(\sqrt{\frac{2}{\alpha_G^2 P_G + \alpha_B^2(1-P_G)}\cdot\frac{\mathcal{E}_b}{N_0}\cdot\frac{1}{K}\sum_{k=1}^{K}\alpha_k^2}\right)\right\} \qquad (5.85)$$

$$P_b = \sum_{n=0}^{K} \binom{K}{n} P_G^n (1-P_G)^{K-n} \\ Q\left(\sqrt{\frac{2}{P_G + (1-P_G)\alpha_B^2/\alpha_G^2} \cdot \frac{n + (K-n)\alpha_B^2/\alpha_G^2}{K} \cdot \frac{\mathcal{E}_b}{N_0}}\right) \tag{5.90}$$

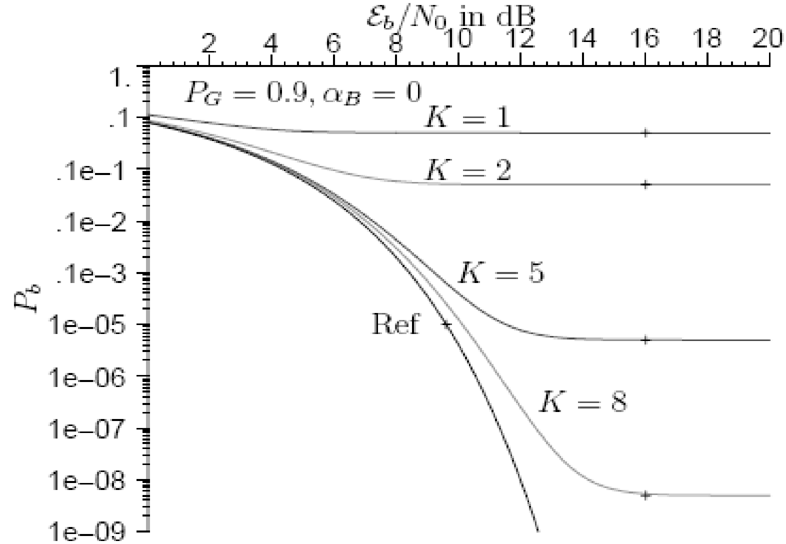


Figure 5.22: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $P_G = 0.9$ and $\alpha_B = 0$, with $K = 1, 2, 5, 8$.

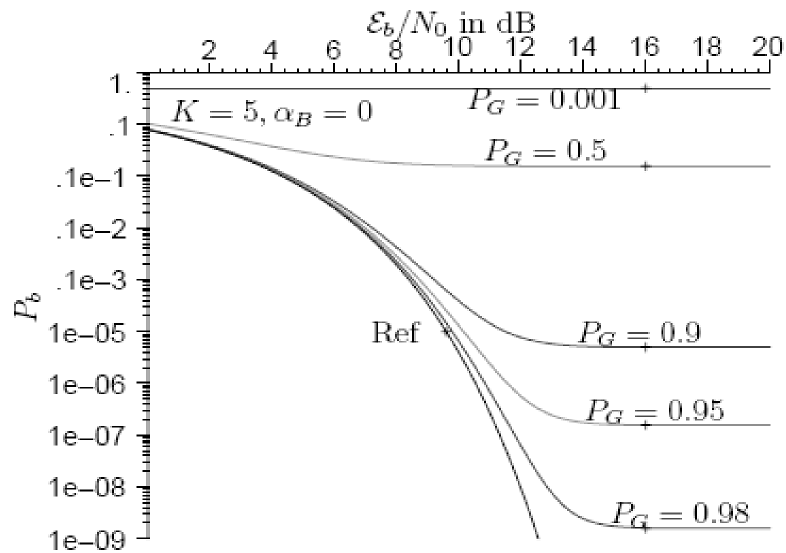


Figure 5.23: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $K = 5$ and $\alpha_B = 0$, with $P_G = 0.001, 0.5, 0.9, 0.95, 0.98$.

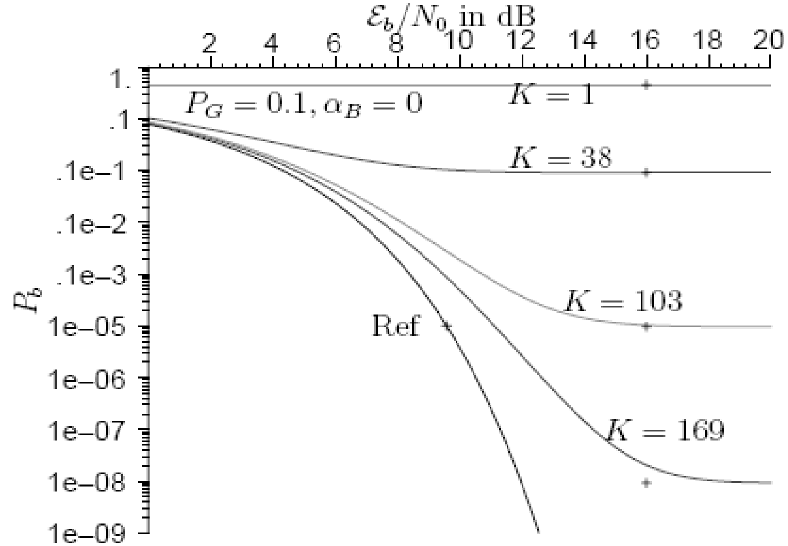


Figure 5.24: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $P_G = 0.1$ and $\alpha_B = 0$, with $K = 1$, 38, 103, 169.

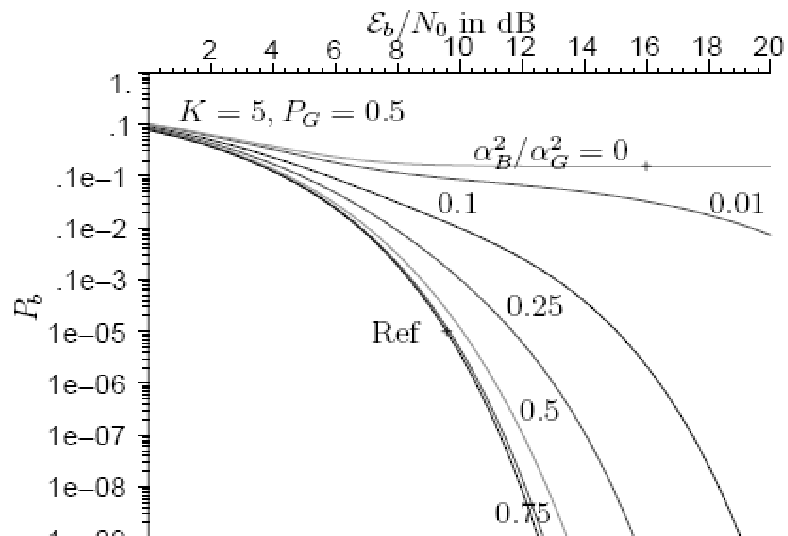


Figure 5.25: The bit error probability versus $\mathcal{E}_b/N_0$ for the case $K = 5$ and $P_G = 0.5$, with $\alpha_B^2/\alpha_G^2 = 0$, 0.01, 0.1, 0.25, 0.5, 0.75.

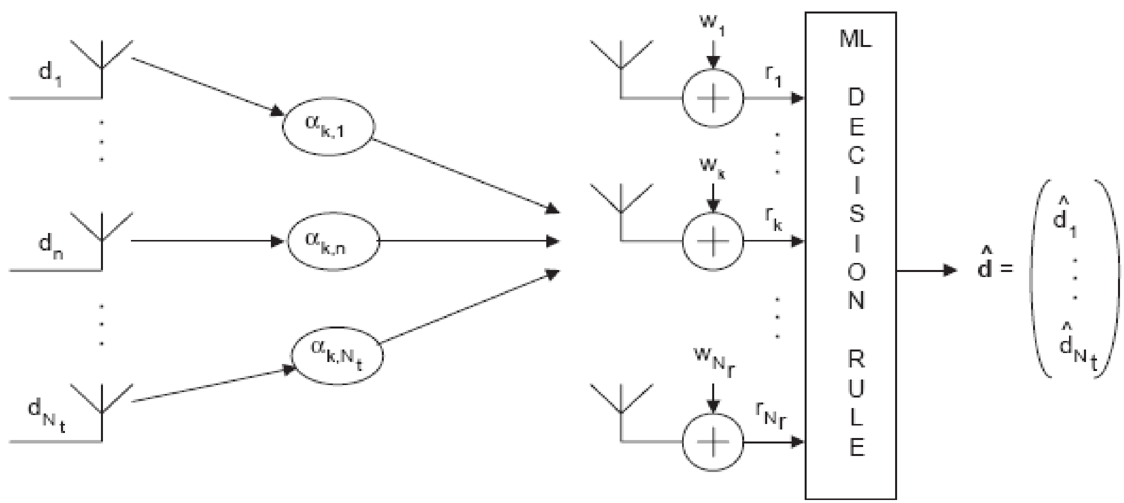
5.34 Consider a communication system where N_t M-ary QAM signals are sent simultaneously (from N_t antennas). The n:th transmitted M-ary QAM signal is denoted $s_n(t)$,

$$s_n(t) = A(n)g(t)\cos(\omega_c t) - B(n)g(t)\sin(\omega_c t) \qquad (5.133)$$

for $n = 1, 2, \ldots, N_t$. Note that the same carrier frequency is used for all N_t transmitted QAM signals!

The MIMO model is illustrated in the figure below,

MIMO MODEL



$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w}$$

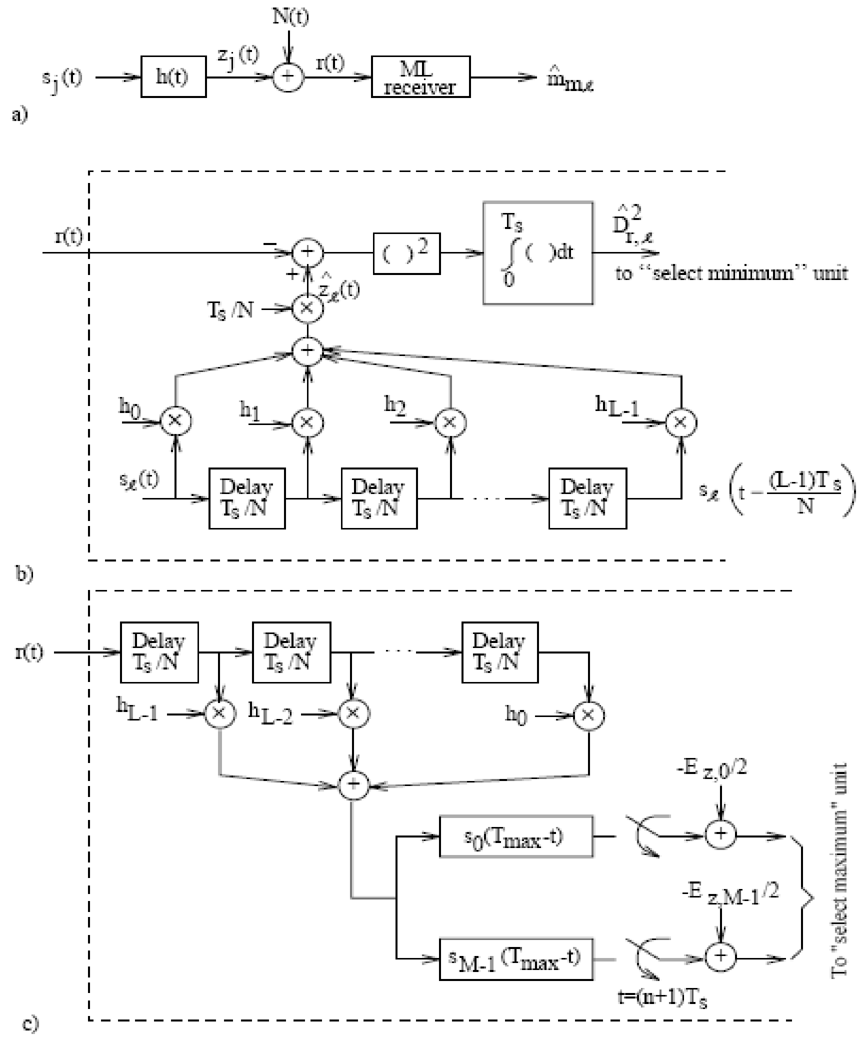


Figure 5.26:
a) The filtered channel model. b) Generation of the decision variable $\hat{D}^2_{r,\ell}$ using an L-ray approximation approach (delayed reference receiver structure). c) An alternative receiver structure based on (5.96) (RAKE receiver structure) .

5.4.4 Noncoherent Detection of M-ary FSK Signals

In this subsection noncoherent ML detection of equally likely, equal energy orthogonal M-ary FSK signals in AWGN is considered. Hence, it is here assumed that,

$$r(t) = z_j(t) + N(t) = \sqrt{2E/T_s}\ \cos(\omega_j t + \nu_j) + N(t),\ 0 \le t \le T_s \qquad (5.104)$$

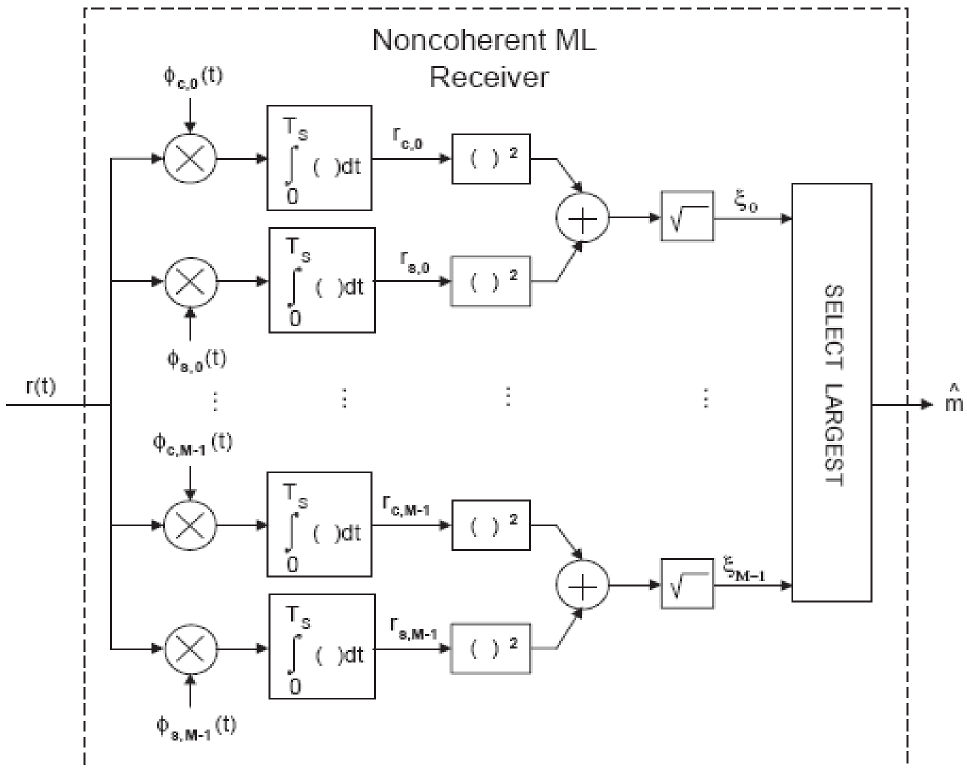


Figure 5.28: A correlator implementation of the noncoherent ML (symbol) receiver for equally likely, equal energy, orthogonal M-ary FSK signals in AWGN.

If $M = 2$, then the bit error probability for the receiver in Figure 5.28 equals,

$$P_b = \frac{1}{2}\, e^{-\mathcal{E}_b/2N_0},\ M = 2 \tag{5.109}$$

5.4.6 Additive Colored Gaussian Noise

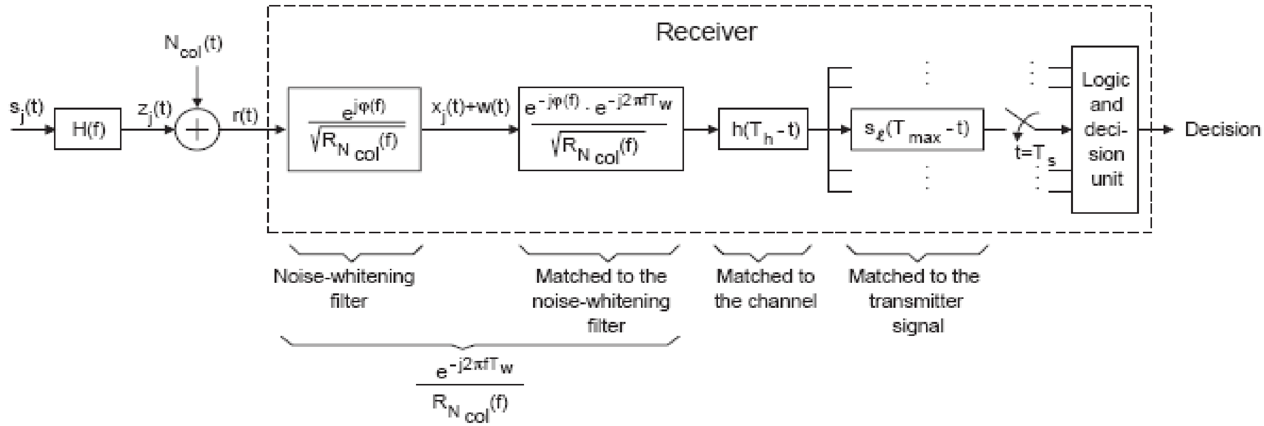


Figure 5.29: A possible receiver structure for detection of signals in colored noise $N_{col}(t)$.

$$R_{N_{col}}(f) = \frac{N_0}{2} + R_u(f) \tag{5.127}$$

Chapter 8

Trellis-coded Signals

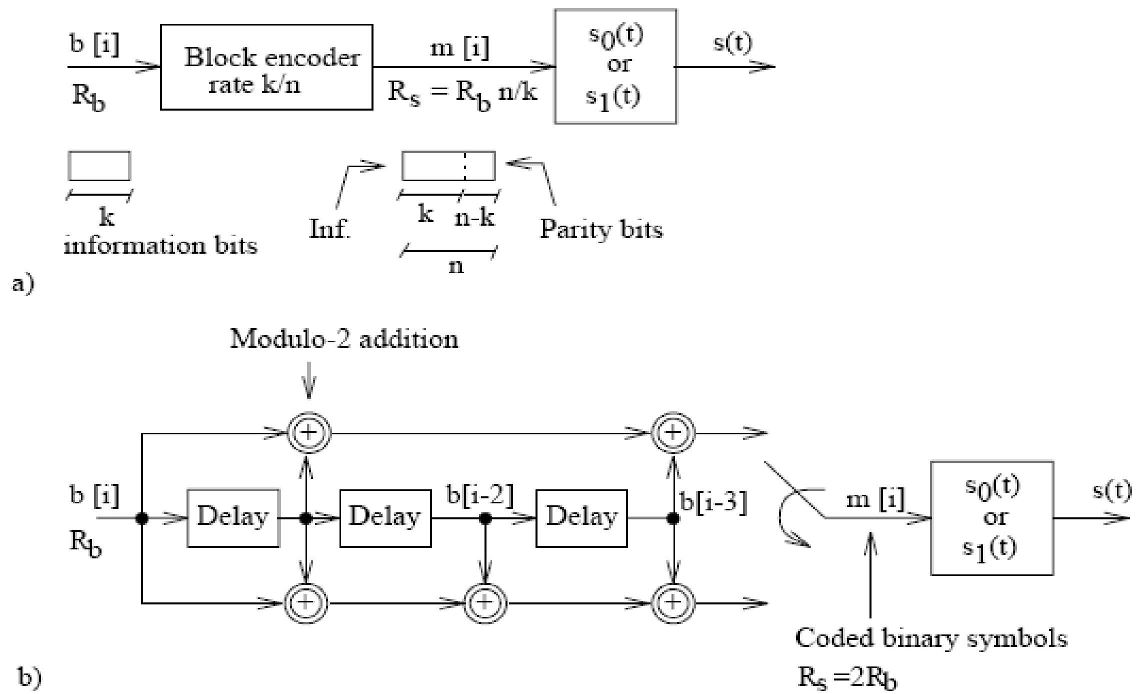


Figure 8.1: a) Block coding, $r_c = k/n$. b) Convolutional coding, $r_c = 1/2$.

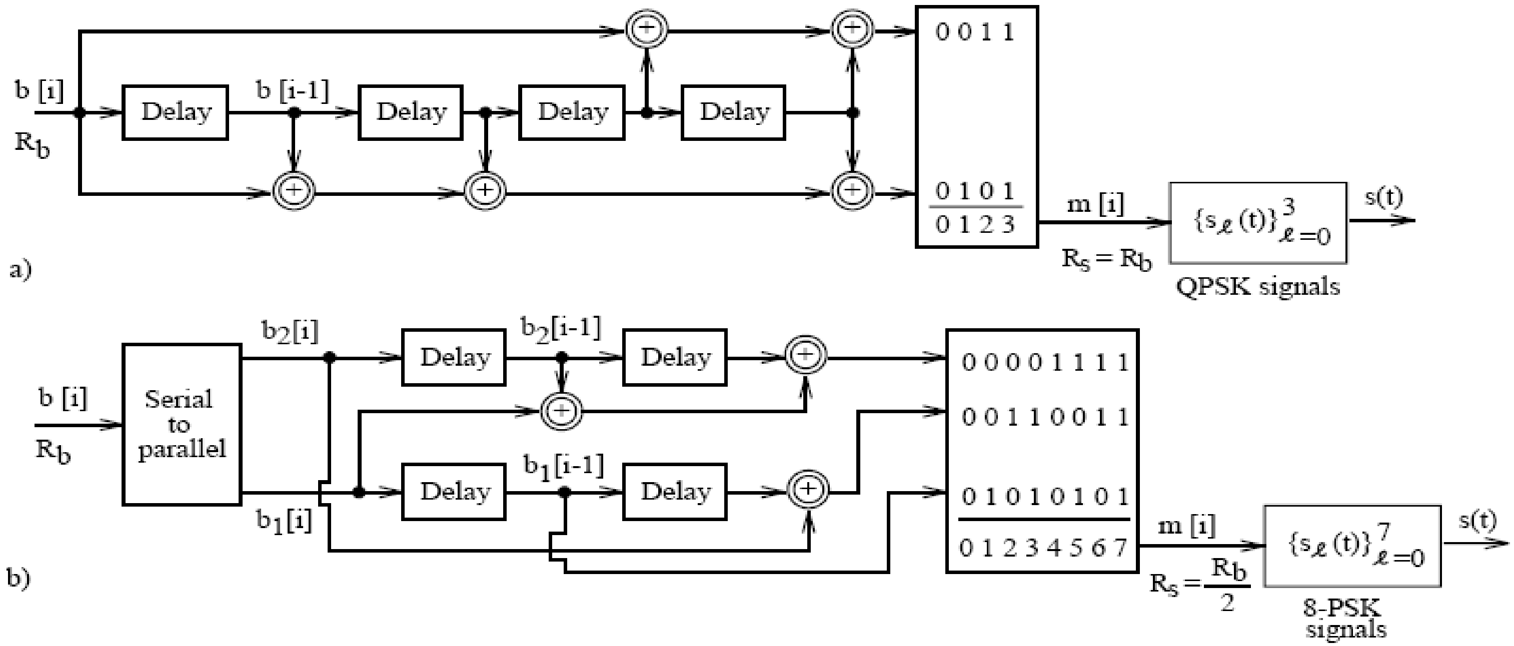
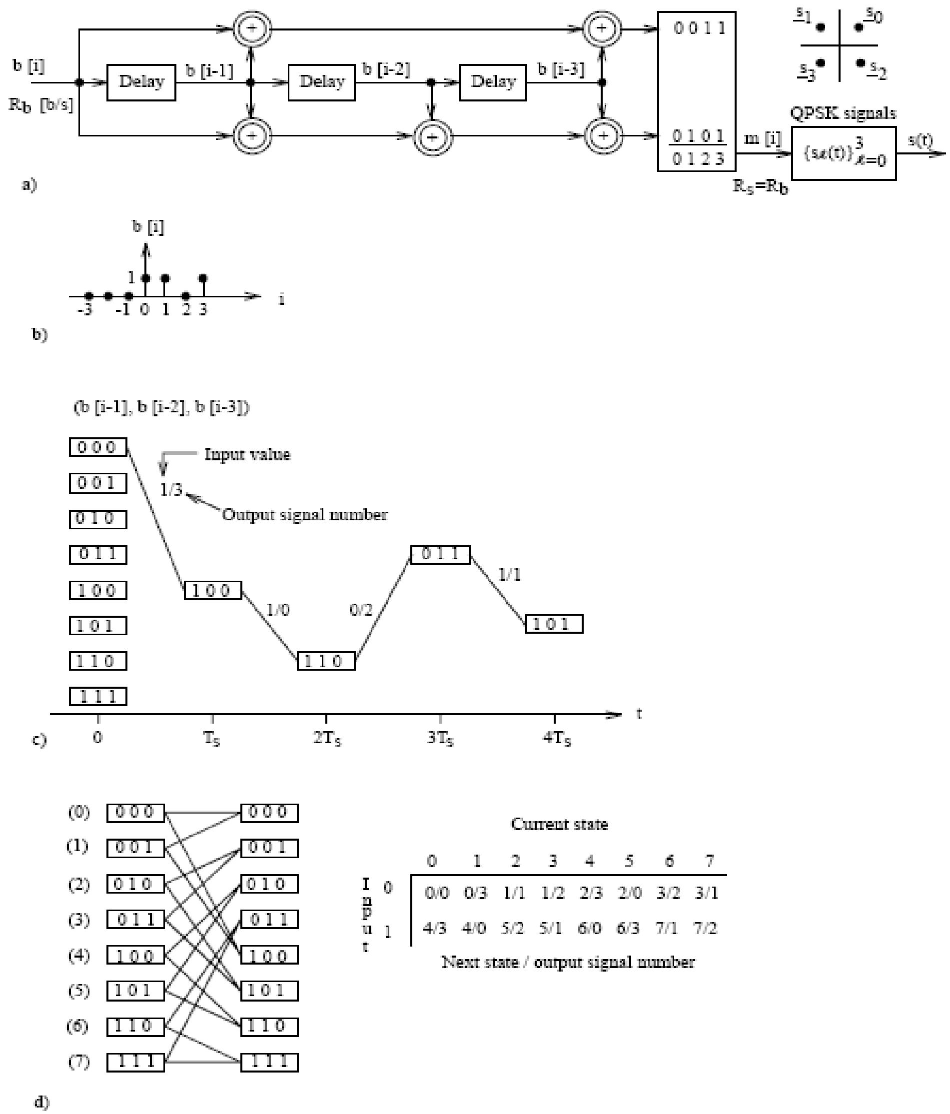


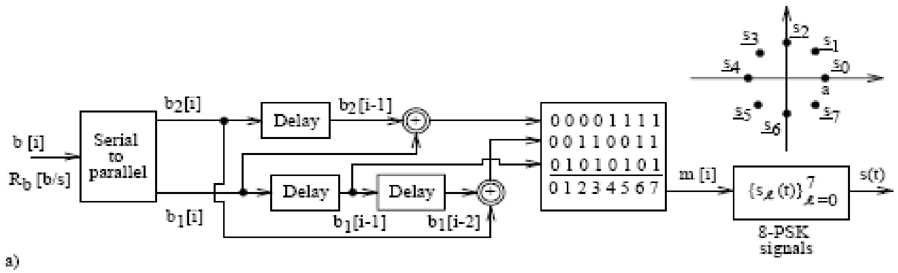
Figure 8.2: a) Rate $r_c = 1/2$ convolutional encoder combined with QPSK; b) Rate $r_c = 2/3$ convolutional encoder combined with 8-PSK, from [63], [64].



Input	Current state: 0	1	2	3	4	5	6	7
0	0/0	0/3	1/1	1/2	2/3	2/0	3/2	3/1
1	4/3	4/0	5/2	5/1	6/0	6/3	7/1	7/2

Next state / output signal number

Figure 8.4: a) A rate 1/2 convolutional encoder combined with QPSK signal alternatives; b) A specific input sequence $b[i]$; c) The corresponding path in the trellis; d) A trellis section, and a table containing all relevant parameters.



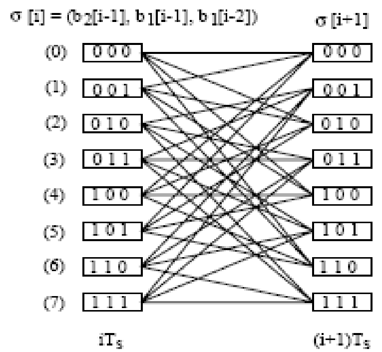
a)

Current state σ [i]

INPUT $\binom{b_2[i]}{b_1[i]}$	(000) 0	(001) 1	(010) 2	(011) 3	(100) 4	(101) 5	(110) 6	(111) 7
$\binom{0}{0}$	0/0	0/2	1/1	1/3	0/4	0/6	1/5	1/7
$\binom{0}{1}$	2/4	2/6	3/5	3/7	2/0	2/2	3/1	3/3
$\binom{1}{0}$	4/2	4/0	5/3	5/1	4/6	4/4	5/7	5/5
$\binom{1}{1}$	6/6	6/4	7/7	7/5	6/2	6/0	7/3	7/1

σ [i+1] / m [i]

b)



c)

Figure 8.6: a) An example of TCM, from [63]–[64]; b) The mappings $F(\cdot,\cdot)$ and $G(\cdot,\cdot)$; c) A trellis section.

<u>Memory (redundancy, dependancy) is introduced among the sent signal alternatives!</u>

This gives us some new properties like, e.g.,:

8.10

Which of the following signal sequences are impossible?

1. $s_3(t), s_2(t-T_b), s_1(t-2T_b), s_1(t-3T_b)$
2. $s_3(t), s_2(t-T_b), s_2(t-2T_b), s_1(t-3T_b)$
3. $s_3(t), s_1(t-T_b), s_0(t-2T_b), s_2(t-3T_b)$
4. $s_3(t), s_1(t-T_b), s_3(t-2T_b), s_1(t-3T_b)$

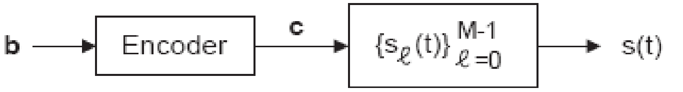
Note: In the uncoded case all signal sequences are possible.

Find the "missing" signal, in the sequence below,

$$s_1(t), s_3(t-T_b),\ ?\ , s_2(t-3T_b), s_3(t-4T_b), s_0(t-5T_b)$$

Note: This is not possible to do in the uncoded case!

2.32 Let us here study adaptive coding and modulation according to the block diagram below.



$$\bar{E}_{sent} = r_c \log_2(M) E_{b,sent} = \frac{k}{n} \log_2(M) E_{b,sent} \tag{8.4}$$

$$R_s = 1/T_s = \frac{1}{r_c} \cdot \frac{1}{\log_2(M)} \cdot R_b = \frac{1}{k/n} \cdot \frac{1}{\log_2(M)} \cdot R_b \tag{8.5}$$

$$W = c \cdot R_s \tag{8.6}$$

Typically, the bandwidth W is fixed and given but:

the rate of the encoder

the number of signal alternatives

and the bit rate can be **ADAPTIVE**, see (8.5)-(8.6)!

We have memory in the sequence of sent signal alternatives!

Some sequences are impossible, see problem!

Only ”good” sequences are sent!