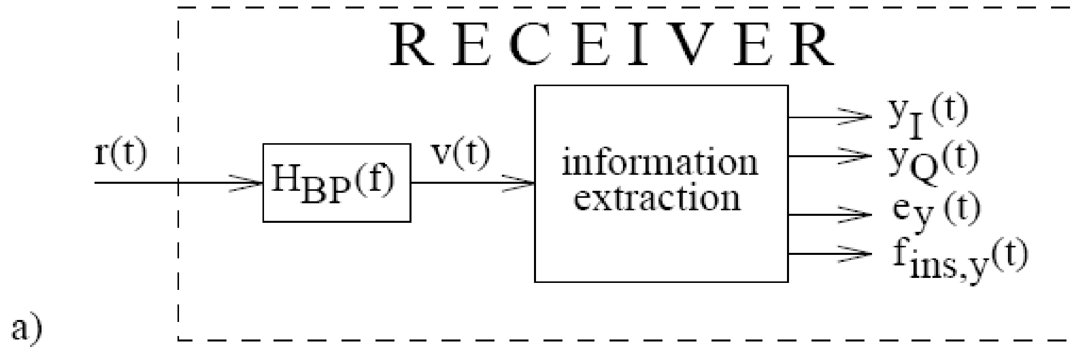
3.6 Receivers for Bandpass Signals



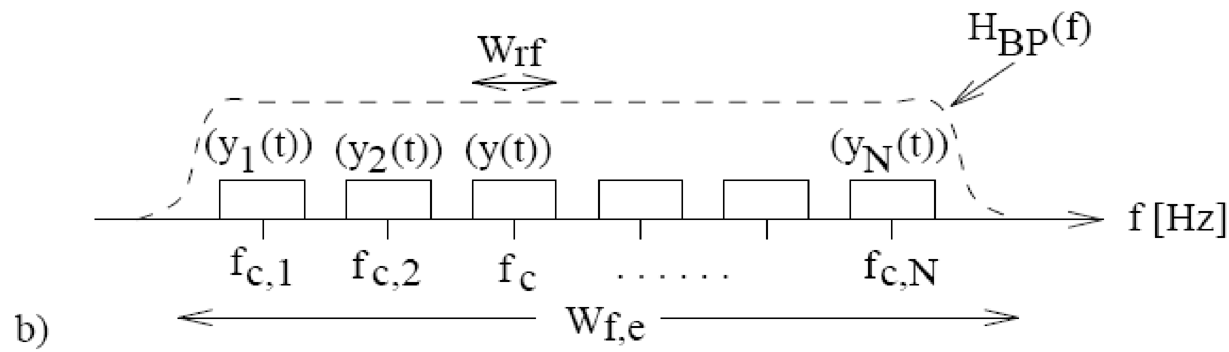


Figure 3.24: a) General blockdiagram of the receiver. b) Illustrating the location of the desired signal $y(t)$ in the frequency domain.

$$\boxed{v(t) = y(t) + y_1(t) + y_2(t) + ... + y_N(t)} \qquad (3.166)$$

$$\boxed{v(t) = y(t) + y_1(t) + y_2(t) + ... + y_N(t)} \qquad (3.166)$$

y(t) is the desired signal and it is located at the carrier frequency fc.

$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t) = e_y(t)\cos(2\pi f_c t + \theta_y(t)) \qquad (3.167)$$

We want to extract y(t) from v(t)!

Homodyne reception:

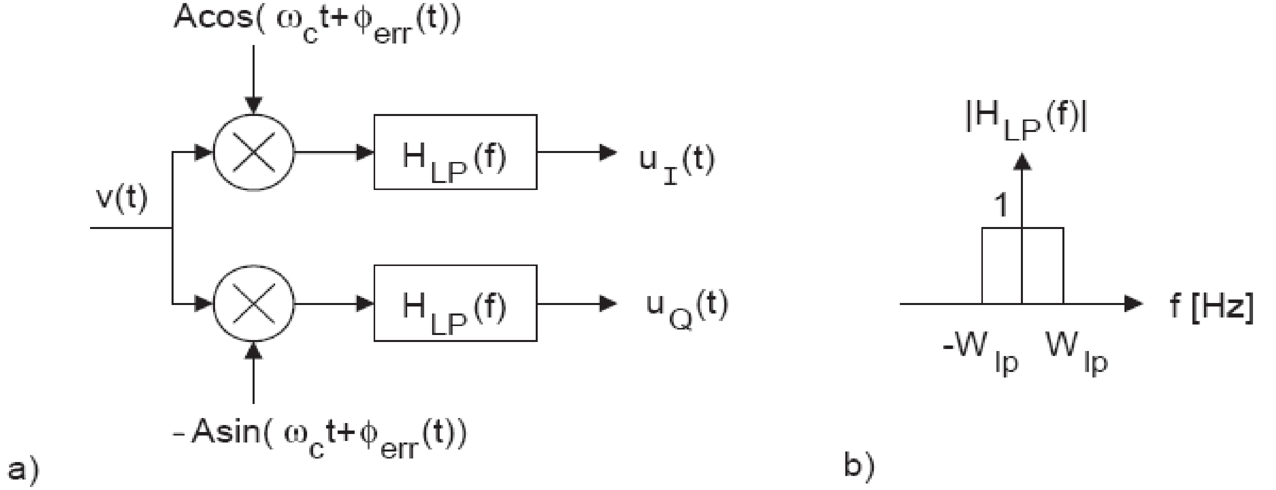


Figure 3.25: a) Homodyne reception. Here a non-ideal phase function $\phi_{err}(t)$ is included. b) Amplitude function of the lowpass filters.

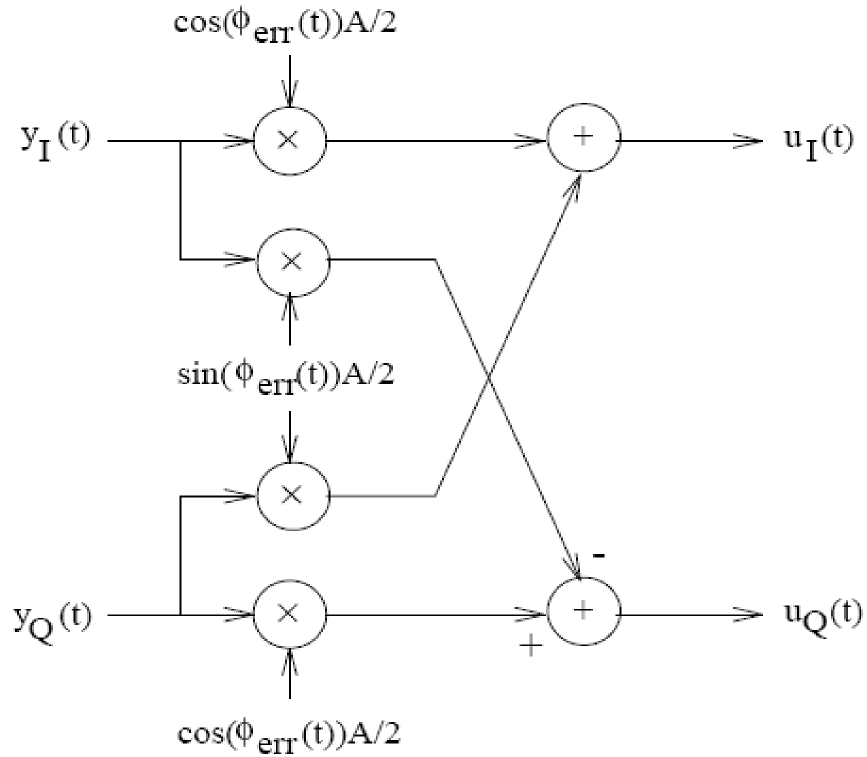


Figure 3.26: Illustrating how the output signals from the homodyne receiver in Figure 3.25a, depend on the desired signals and on the phase function error. See also (3.171)–(3.172).

Channel Filtering, Additive Noise and Homodyne Reception:

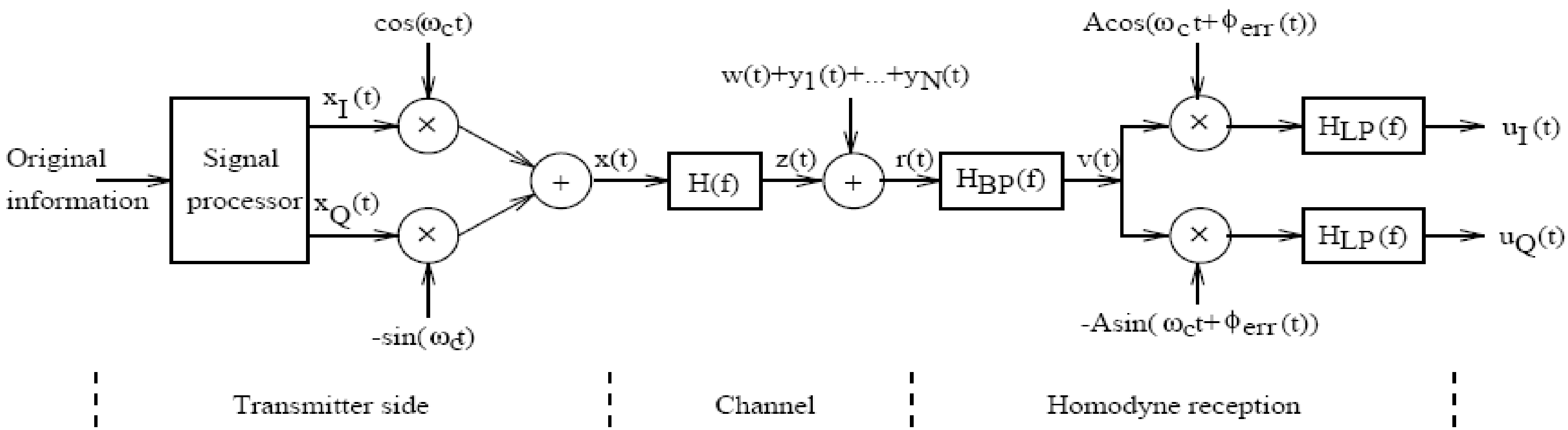


Figure 3.27: Channel filtering, aditive noise and homodyne reception.

3.6.2 Heterodyne Reception

Basically, the carrier frequency fc of the desired signal y(t) is first changed to a lower, so called intermediate frequency.
After that homodyne detection is used.

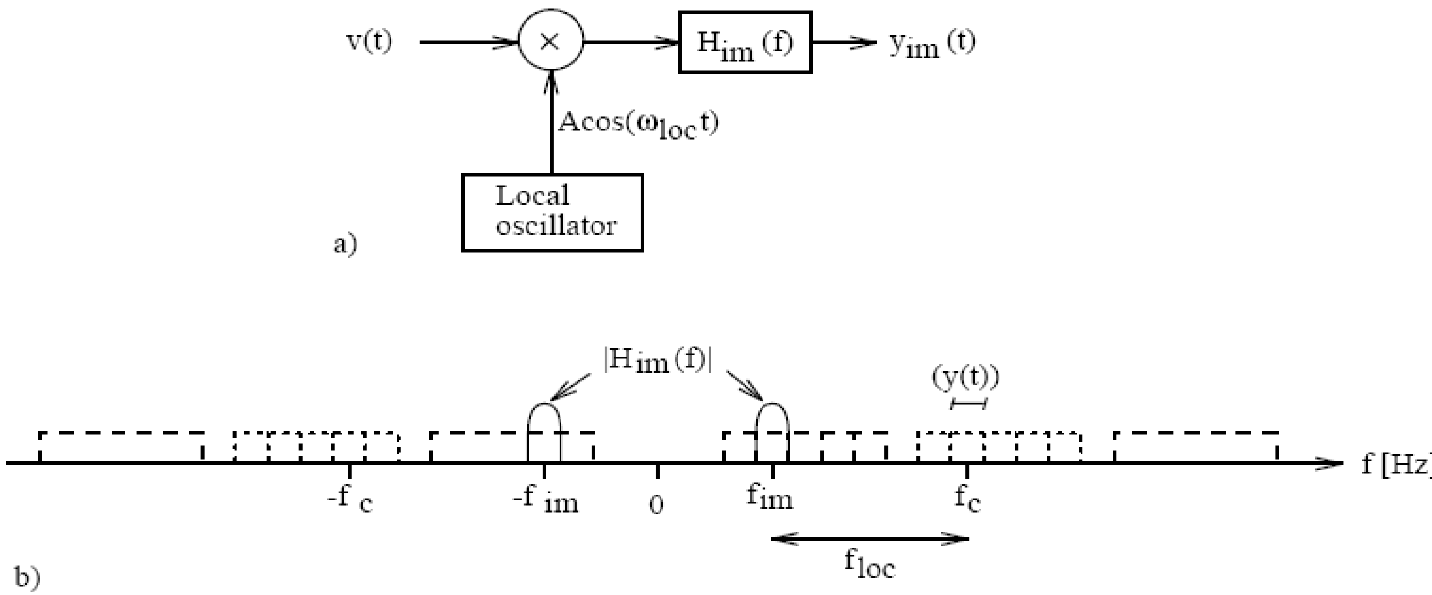


Figure 3.29: a) Mixing down to an intermediate frequency f_{im} [Hz].
b) Illustrating parameters in heterodyne reception.

Additional disturbances may appear after the intermediate filter!!

Observe that if any signals are located around the so-called image frequency f_{image}, $f_{image} = |f_{im} - f_{loc}|$, then a disturbing signal may appear at the output of the filter $H_{im}(f)$!

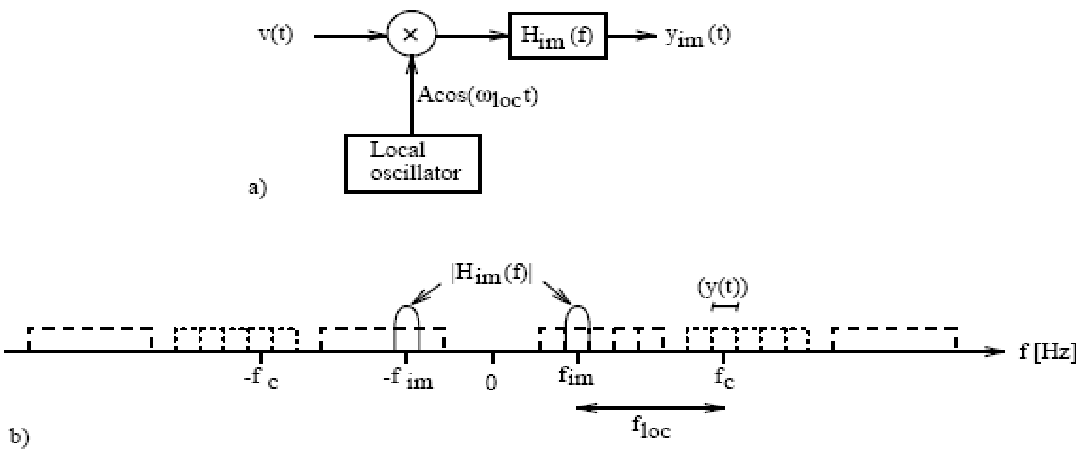
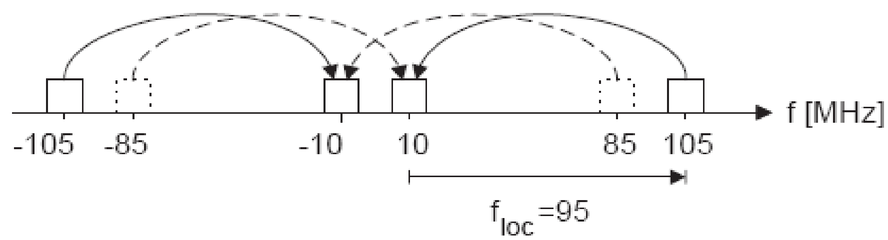


Figure 3.29: a) Mixing down to an intermediate frequency f_{im} [Hz].
b) Illustrating parameters in heterodyne reception.

EXAMPLE 3.26

Assume that $f_{im} = 10$ MHz *in Figure 3.29a. A user wants to tune in a station centered at 105* MHz.
Find the frequency of the local oscillator, and the corrsponding image frequency.

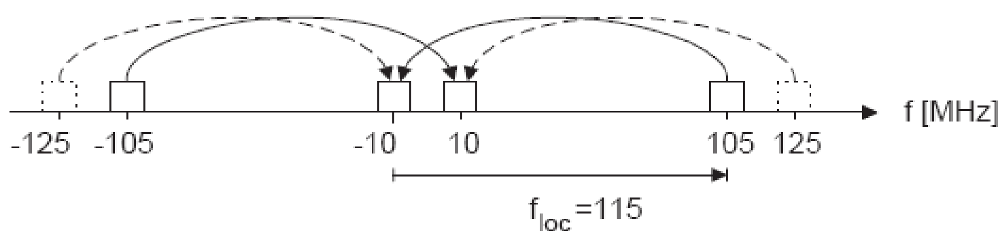
Solution:



From the figure above it is concluded that $f_{loc} = 95$ [MHz] *and* $f_{image} = 85$ [MHz].

Comment:

In principle we may alternatively here choose $f_{loc} = f_c + f_{im} = 105 + 10 = 115$ [MHz].
This is illustrated in the figure below



It is seen that in this case the image frequency equals 125 MHz *which is above the carrier frequency of the desired signal* $f_c = 105$ [MHz].
A consequence of using $f_{loc} = f_c + f_{im}$ *is that the output signal* $y_{im}(t)$ *from the intermediate filter is (ignoring possible signals from the image frequency),*

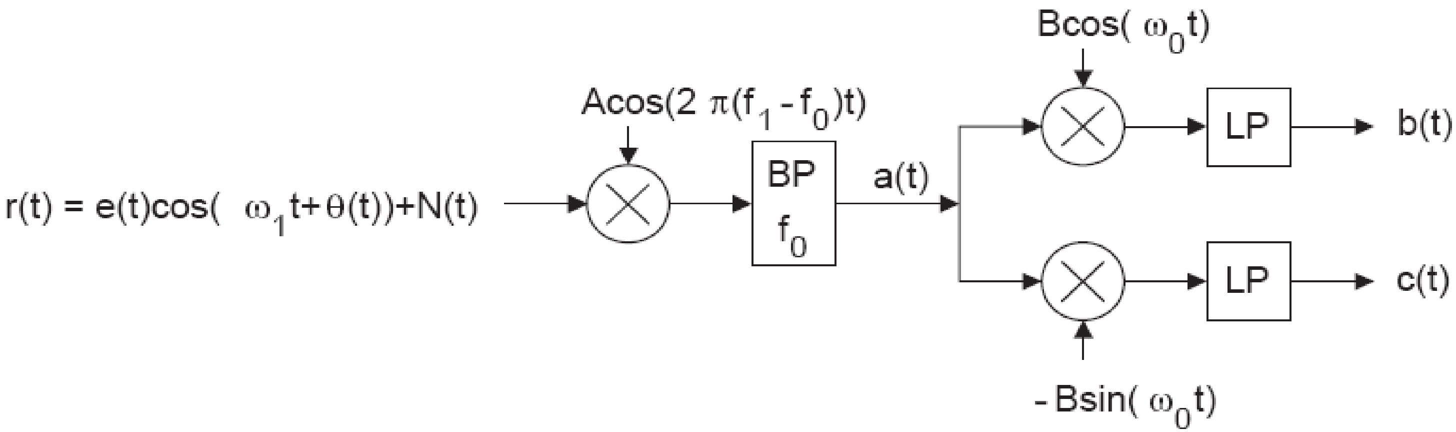
$$y_{im}(t) = [e_y(t)\cos(\omega_c t + \theta_y(t))\cos((\omega_c + \omega_{im})t)] * h_{im}(t) =$$

$$= \left[\frac{e_y(t)}{2}\ \cos(\omega_{im} t - \theta_y(t))\right] * h_{im}(t)$$

So, we obtain a change of sign in the phase. This can be compensated for at a later stage in the receiver. □

EXAMPLE 3.27

The device below is often used in communication systems.



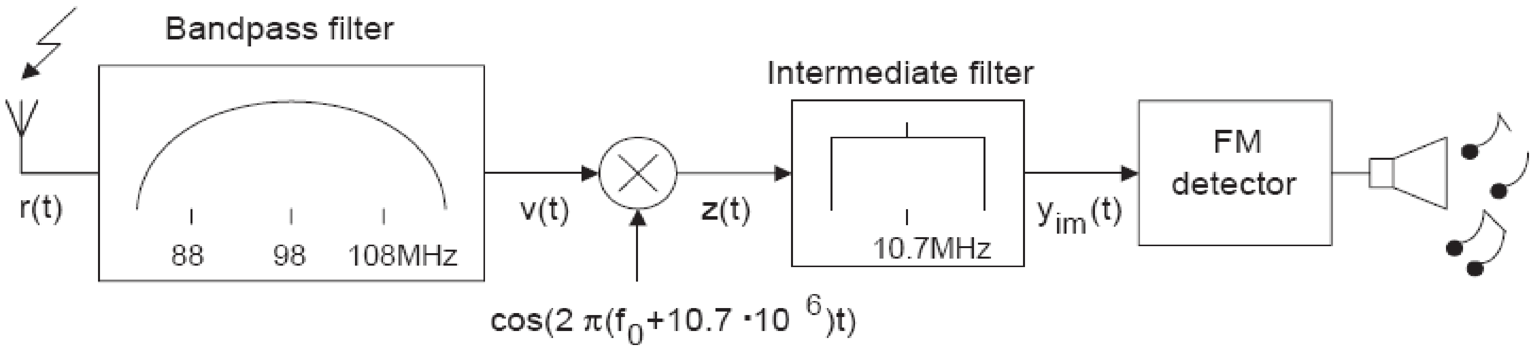
a) *Explain how this device works, and give some examples of advantages and disadvantages with this implementation.*

b) *Give an implementation of an alternative device that realize the same input-output functionality in the noiseless case. Make comparisons with the device in a).*

c) *Calculate* $a(t)$, $b(t)$ *and* $c(t)$ *if* $r(t) = e(t)\cos(\omega_1 t + \theta(t))$ *and if the BP-filter is ideal.*

EXAMPLE 3.28

Let us consider the frequency band 88–108 MHz*, and assume that it contains a number of FM radio stations, each with bandwidth 225* kHz*. Furthermore, assume an FM receiver according to the block-diagram below, where f_0 denotes the carrier frequency of the desired station. The passband of the intermediate filter has width 225* kHz*, and it is centered at 10.7* MHz*.*



Chapter 7
Optical Fiber Communications

Basic bit error probability calculations.

7.2.2 The Group Delay

Maxwells equations applied to the optical fiber, shows that light propagates along the fiber by one or several modes. The electromagnetic field for a particular mode is periodic in the t variable (time) and in the z variable (the direction of propagation). For a specific mode, the functional dependency is of the form,

$$a_i e^{j(\omega t - \beta_i z)} \tag{7.19}$$

where a_i is a constant, and where β_i is the **propagation constant** for mode number i. In (7.19) complex notation is used for convenience. The period in the t variable is $1/f$, and the period in the z variable is the wavelength $\lambda_i = 2\pi/\beta_i$. By writing (7.19) as

$$a_i e^{j(\omega t - \beta_i z)} = a_i e^{-j\beta_i(z - \frac{\omega t}{\beta_i})} \tag{7.20}$$

it is seen that the wave propagates the distance $\omega t/\beta_i$ in time t. Specifically, in a period $1/f$ it travels the distance λ_i.

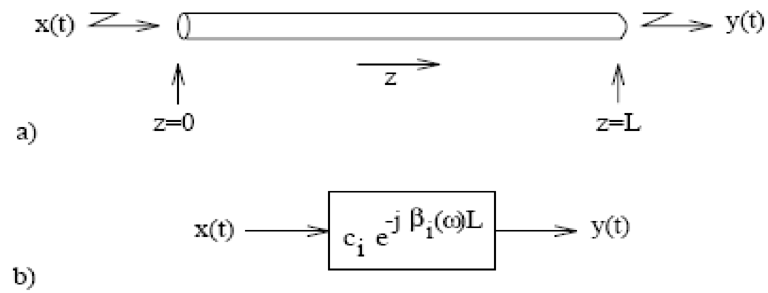


Figure 7.6: a) An optical filter; b) System function for mode number i.

a) optical fiber

$$
\begin{aligned}
x(t) &= a_i e^{j\omega t} \quad &, \quad z = 0 \\
y(t) &= a_i e^{j\omega t} \cdot e^{-j\beta_i L} \quad &, \quad z = L
\end{aligned}
\tag{7.21}
$$

Consequently, a system function for mode number i, $H_i(f)$, can be identified,

$$H_i(f) = c_i e^{-j\beta_i(\omega)L} \tag{7.22}$$

Input:

$$x(t) = \begin{cases} \sqrt{g(t)}\; a_i e^{j(\omega_c t + \nu_i)} & , \quad 0 \le t \le T_b \\ 0 & , \quad \text{otherwise} \end{cases} \qquad (7.23)$$

g(t) here models the signal (optical) power in the pulse.

Output:

$$y(t) = \begin{cases} \sqrt{g(t-\tau_i)}\; a_i c_i e^{j(\omega_c t + \nu_i + \varphi_i)} & , \quad \tau_i \le t \le T_b + \tau_i \\ 0 & , \quad \text{otherwise} \end{cases} \qquad (7.24)$$

$$\begin{aligned} \tau_i &= -\frac{1}{2\pi} \frac{d\phi_i(f)}{df}\bigg|_{f=f_c} = -\frac{1}{2\pi} \frac{d(-\beta_i(\omega)L)}{df}\bigg|_{f=f_c} = \frac{L d\beta_i(\omega)}{d\omega}\bigg|_{\omega=\omega_c} \qquad (7.25) \\ \varphi_i &= \phi_i(f_c) = -\beta_i(\omega_c)L \end{aligned}$$

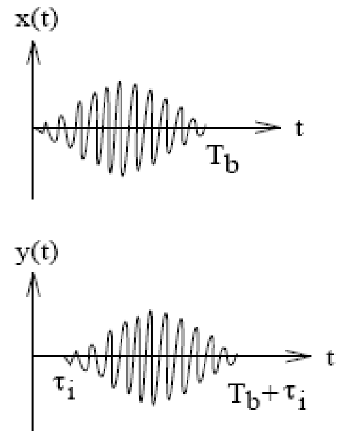


Figure 7.7: Illustrating the effect of the group delay τ_i.

7.3 Reception and Detection

<u>Within a bit interval</u>: A received random number of photons generates a random number of photo-electrons after the photo-detector.

The Poisson Process:

In (7.27), the arrival times ..., t_{i-1}, t_i, t_{i+1}, are modeled as a **Poisson process** with an intensity $\mathcal{I}(t)$. This means that the number of arrivals $\mathcal{N}_\mathcal{T}$, within a time interval of length $\mathcal{T}$, is a random variable having the properties

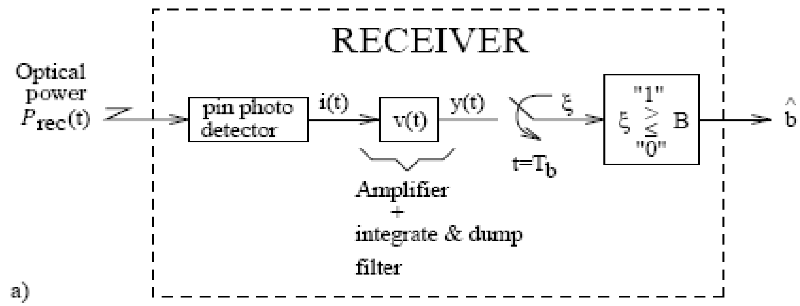
$$Prob\{\mathcal{N}_\mathcal{T} = n\} = \frac{\mu^n e^{-\mu}}{n!}$$

$$\mu = E\{\mathcal{N}_\mathcal{T}\} = \int_{t_0}^{t_0+\mathcal{T}} I(t)dt \qquad (7.29)$$

$$\sigma^2 = E\{(\mathcal{N}_\mathcal{T} - \mu)^2\} = \mu$$

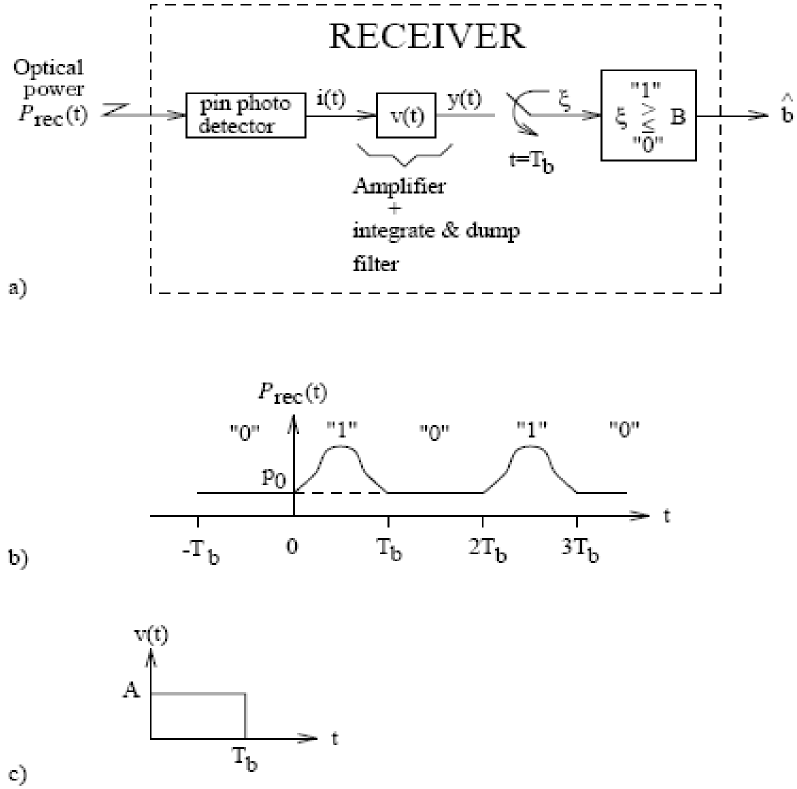
Note that the mean and the variance are identical.

Fig. 7.8a



Compare with Chapter 4!

Fig. 7.8:



"0": po
"1": po+p(t)

Received optical power.

$$\mathcal{P}_{rec}(t) = p_0 + \sum_{i=-\infty}^{\infty} m[i]p(t - iT_b), \quad m[i] \in \{0, 1\}, \quad -\infty \leq t \leq \infty \qquad (7.31)$$

$$\begin{aligned} \xi &= y(T_b) = \int_{-\infty}^{\infty} i(\tau)v(T_b - \tau)d\tau = A\int_0^{T_b} i(\tau)d\tau = \\ &= A\int_0^{T_b} (i_r(t) + i_d(t))dt = Aq\mathcal{N}_{T_b} \end{aligned} \qquad (7.32)$$

q=charge of an electron.
id(t)="dark current".

Bit error probability:

$$
\begin{aligned}
P_b &= P_0 \underbrace{Prob\{\text{error}|m_0 \text{ sent}\}}_{P_F} + P_1 \underbrace{Prob\{\text{error}|m_1 \text{ sent}\}}_{P_M} \\
&= P_0 Prob\{\xi > B|m_0 \text{ sent}\} + P_1 Prob\{\xi \le B|m_1 \text{ sent}\} = \\
&= P_0 Prob\{\mathcal{N}_{T_b} > (B/Aq)|m_0 \text{ sent}\} + \\
&\quad + P_1 Prob\{\mathcal{N}_{T_b} \le (B/Aq)|m_1 \text{ sent}\} \qquad (7.33)
\end{aligned}
$$

$$
\begin{aligned}
P_F &= Prob\{\mathcal{N}_{T_b} > \alpha|m_0 \text{ sent}\} = \sum_{n=\alpha+1}^{\infty} \frac{\mu_0^n e^{-\mu_0}}{n!} \\
P_M &= Prob\{\mathcal{N}_{T_b} \le \alpha|m_1 \text{ sent}\} = \sum_{n=0}^{\alpha} \frac{\mu_1^n e^{-\mu_1}}{n!} \\
\alpha &= B/Aq \qquad (7.35)
\end{aligned}
$$

Exact expressions!

We need the averages!

$$
\boxed{\begin{aligned}
&Prob\{\mathcal{N}_{\mathcal{T}} = n\} = \frac{\mu^n e^{-\mu}}{n!} \\
&\mu = E\{\mathcal{N}_{\mathcal{T}}\} = \int_{t_0}^{t_0+\mathcal{T}} I(t)dt \\
&\sigma^2 = E\{(\mathcal{N}_{\mathcal{T}} - \mu)^2\} = \mu
\end{aligned}} \qquad (7.29)
$$

$$
\mathcal{I}_e(t) = \eta \cdot \mathcal{M} \cdot \mathcal{I}_{ph}(t) + \mathcal{I}_d = \eta \cdot \mathcal{M} \cdot \frac{\mathcal{P}_{rec}(t)}{hf} + \mathcal{I}_d \text{ [electrons/s]} \qquad (7.8)
$$

Id=id/q
Page 476.

Combining (7.29), (7.8) and (7.31) it is found that

$$
\boxed{\begin{aligned}
&\mu_0 = E\{\mathcal{N}_{T_b} | m_0 \text{ sent}\} = \int_0^{T_b} \left(\frac{\eta}{hf}\, p_0 + \mathcal{I}_d \right) dt = \mathcal{I}_d T_b + \frac{\eta\lambda}{hc}\, p_0 T_b \\
&\mu_1 = E\{\mathcal{N}_{T_b} | m_1 \text{ sent}\} = \mu_0 + \frac{\eta\lambda}{hc} \int_0^{T_b} p(t)dt = \mu_0 + \frac{\eta\lambda}{hc} \cdot \mathcal{E}_p
\end{aligned}} \qquad (7.34)
$$

A very useful approximate expression of the bit error probability:

The key to the Gaussian approximation is to approximate the conditional random variable $\mathcal{N}_{T_b}$ in (7.35), with a Gaussian random variable having the same mean and variance. Doing this, P_F and P_M are approximated by

$$
\begin{aligned}
P_F &= Prob\left\{\frac{\mathcal{N}_{T_b}-\mu_0}{\sqrt{\mu_0}} > \frac{\alpha-\mu_0}{\sqrt{\mu_0}}|m_0 \text{ sent}\right\} \approx Q\left(\frac{\alpha-\mu_0}{\sqrt{\mu_0}}\right) \\
P_M &= Prob\left\{\frac{\mathcal{N}_{T_b}-\mu_1}{\sqrt{\mu_1}} \leq \frac{\alpha-\mu_1}{\sqrt{\mu_1}}|m_1 \text{ sent}\right\} \approx Q\left(\frac{\mu_1-\alpha}{\sqrt{\mu_1}}\right)
\end{aligned}
\qquad (7.37)
$$

A very useful approximation on the bit error probability is obtained by **also** approximating the threshold α in (7.37) by

$$\alpha \approx \sqrt{\mu_0\mu_1} \qquad (7.38)$$

which makes the approximations of P_F and P_M in (7.37) identical. The resulting approximate expression of the bit error probability then becomes

OBS!

$$
\boxed{\begin{aligned} &P_b \approx Q(\varrho) \\ &\varrho = \sqrt{\mu_1} - \sqrt{\mu_0} \end{aligned}} \qquad (7.39)
$$