Lectures in study period Vt-1 2010: Mondays 13.15 – 15.00 in E:2311.

Problem solving (exercise) class in study period Vt-1 2010: Thursdays 13.15 – 15.00 in E:3139.

2010-01-11: NOTE! Each project group should contact Göran Lindell **early in the first study week**!

2010-01-11: This course continues on Monday 18 January. Lecture on Mondays 13.15 - 15.00 in E:2311, and exercise class on Thursdays 13.15 - 15.00 in E:3139. See the course programme for more details.

2010-01-11: The course programme is updated, and it is now complete.

Week 3 (100118): Study period Vt-1 starts.

Week 5: Lab

Week 7: Deadline for project report (pdf-format, Email) Wednesday 17 February 2010, 15.00.

Week 8+9: Project presentations.

Week 9 (100305): Vt-1 ends.

Week 10: Written examination Tuesday 9 March 2010, 08.00 – 13.00 in Eden 25.

”Attenuation & Rotation” (chapter 3)

Homodyne and Heterodyne receivers (chapter 3)

A basic bit error probability analysis for optical fiber communications (chapter 7)

Mobile communications (chapter 9)

Study period Vt-1 2010 (18 January – 5 March):

Week	**Contents**
3	Lecture (18/1): 8.3 (pages 529-537), 3.4 – 3.5.2 (pages 158 – 174), Problem 5.34. Prob.solv. (21/1): Problems 8.34, 3.16, Example 3.17 on page 166, Example 5.4 on page 343.
4	Lecture (25/1): 3.6 – 3.6.2 (pages 184 - 194), 7.2.2 – 7.3.1 (pages 478 – 486). Prob.solv. (28/1): 5.34 ((5.133) – (5.138)), Example 3.26 on page 191, 3.18, 3.17.
5	**Laboratory lesson this week!** Lecture (1/2): 7.3.2 (pages 486 - 490), 9.1 – 9.1.2 (pages 581 – 589). Prob.solv. (4/2): 7.7, 7.9, 7.10a.
6	Lecture (8/2): 9.1.2 - 9.2 (pages 585 – 596). Prob.solv. (11/2): 9.2, 9.3, 9.4, 9.5.
7	**Deadline for project report this week on Wednesday 17 February, 15.00!** Lecture (15/2): Cancelled. Prob.solv. (18/2): 9.6, 9.7, 9.8.
8	Lecture (22/2): Cancelled. Prob.solv. (25/2): 9.10, 9.9.
9	Lecture (1/3): Summary of the course. Prob.solv. (4/3): Question time. Friday 5 March: Study period Vt-1 ends.
10	**Written examination Tuesday 9 March 2010, 08.00 – 13.00 in Eden 25.**

$$\boxed{D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i] = D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i-1] + D^2_{inc}[i]} \qquad (8.50)$$

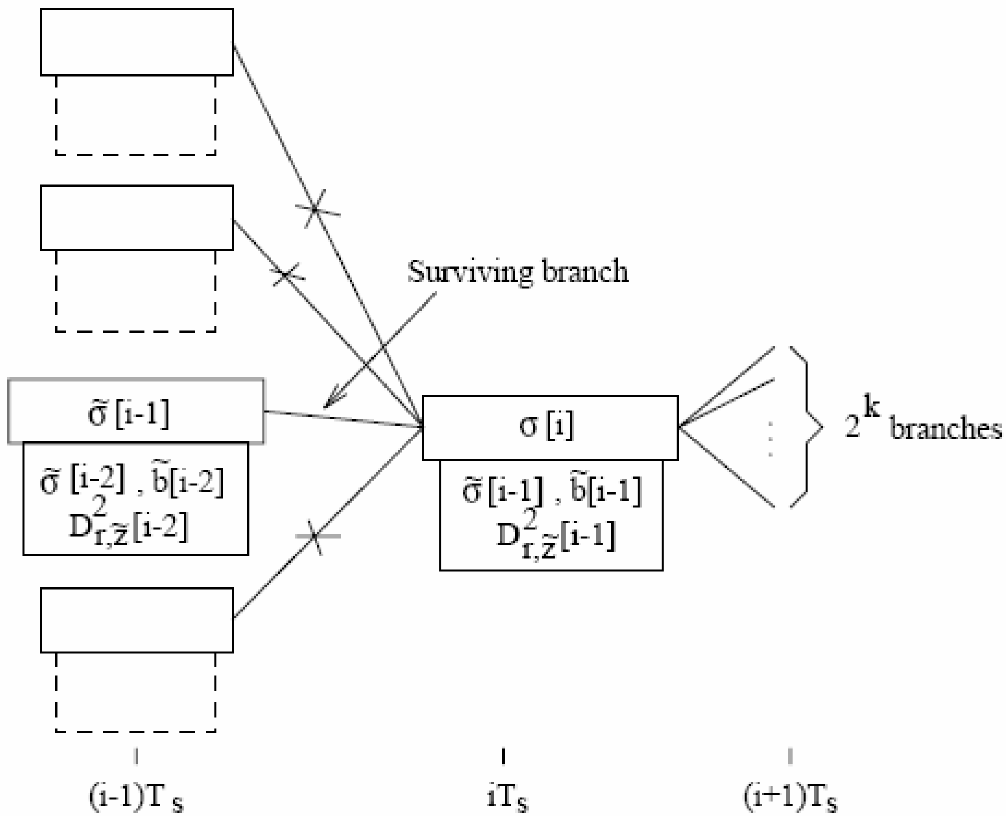


Figure 8.11: Illustrating how branches in the trellis are deleted (x) by the Viterbi algorithm.

8.3.1 The ML Receiver - An Alternative Approach □

In this subsection, an alternative implementation of the coherent ML receiver for trellis coded signals in AWGN is derived. With this implementation the number of matched filters in the receiver is significantly reduced compared with Figure 8.12a, making it particularly interesting in many applications.

$$r(t) = z(t) + N(t) = \sum_{n=-\infty}^{\infty} x_{m[n]}(t - nT_s) + N(t), \quad -\infty \leq t \leq \infty \tag{8.54}$$

$$x_\ell(t) = s_\ell(t) * h(t), \quad \ell = 0, 1, \ldots, \mathcal{M}_{tra} - 1 \tag{8.55}$$

$$\int_{-\infty}^{\infty} (r(t) - z(t))^2 dt \tag{8.56}$$

maximize the expression,

$$\int_{-\infty}^{\infty} \left(r(t)z(t) - \frac{z^2(t)}{2} \right) dt \tag{8.57}$$

$$\int_{-\infty}^{\infty}\left(r(t)z(t)-\frac{z^2(t)}{2}\right)dt=\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} r(t)x_{m[n]}(t-nT_s)dt-$$

$$-\frac{1}{2}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty} x_{m[n]}(t-nT_s)x_{m[\ell]}(t-\ell T_s)dt=$$

$$=\sum_{n=-\infty}^{\infty}\underbrace{\int_{-\infty}^{\infty} r(t)x_{m[n]}(t-nT_s)dt}_{y_{m[n]}[n]}-$$

$$-\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\int_{-\infty}^{\infty} x_{m[n]}(t-nT_s)x_{m[\ell]}(t-\ell T_s)dt=$$

$$=\sum_{n=-\infty}^{\infty} y_{m[n]}[n]-\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\int_{-\infty}^{\infty} x_{m[n]}(\xi)x_{m[\ell]}(\xi+(n-\ell)T_s)d\xi=$$

$$=\sum_{n=-\infty}^{\infty} y_{m[n]}[n]-\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\underbrace{[x_{m[n]}(t)*x_{m[\ell]}(-t)]_{t=(\ell-n)T_s}}_{p_{m[n],m[\ell]}[\ell-n]} \qquad (8.58)$$

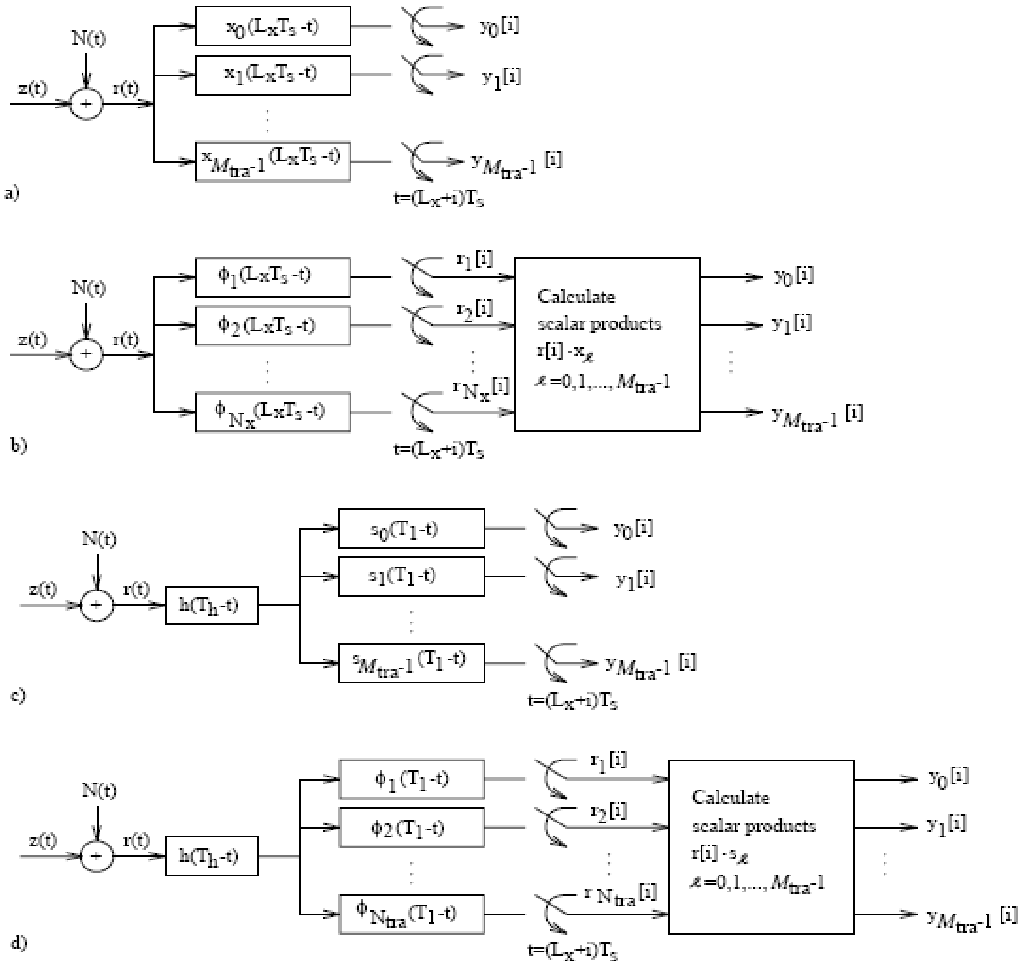


Figure 8.13: The first stage in the ML receiver. a) Filters matched to the signals $x_\ell(t)$; b) Filters matched to the N_x ($N_x \le \mathcal{M}_{tra}$) basis functions of $\{x_\ell(t)\}_{\ell=0}^{\mathcal{M}_{tra}-1}$; c) Filters matched to $h(t)$ and to $s_\ell(t)$; d) Filters matched to $h(t)$ and to the N_{tra} basis functions of $\{s_\ell(t)\}_{\ell=0}^{\mathcal{M}_{tra}-1}$.

$$J_{acc}[i] = \sum_{n=-\infty}^{i} y_{m[n]}[n] - \frac{1}{2} \sum_{n=-\infty}^{i} \sum_{\ell=-\infty}^{i} p_{m[n],m[\ell]}[\ell - n] \qquad (8.68)$$

$$\begin{aligned} J_{acc}[i] &= J_{acc}[i-1] + J_{inc}[i] \\ J_{inc}[i] &= y_{m[i]}[i] - \underbrace{\sum_{n=1}^{L_x-1} p_{m[i-n],m[i]}[n]}_{\text{only if } L_x \geq 2} - \tfrac{1}{2}\, p_{m[i],m[i]}[0] \end{aligned} \qquad (8.71)$$

3.4.1 Low-Rate QAM-Type of Input Signals

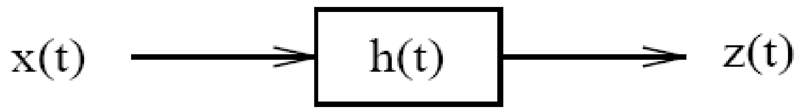


Figure 3.11: Bandpass filtering.

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \qquad (3.103)$$

$$\tilde{x}(t) = x_I(t) + jx_Q(t) \qquad (3.104)$$

This complex signal contains the information!

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \qquad (3.103)$$

$$z(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)Re\{\tilde{x}(t-\tau)e^{j\omega_c(t-\tau)}\}d\tau =$$

$$= Re\left\{e^{j\omega_c t}\int_{-\infty}^{\infty} h(\tau)\tilde{x}(t-\tau)e^{-j\omega_c\tau}d\tau\right\} \qquad (3.105)$$

3 assumptions:

1) The duration of the impulse response $h(t)$ can be considered to be equal to T_h. This means that essentially all the energy in $h(t)$ is assumed to be contained within the time interval $0 \leq t \leq T_h$.

2) The input signal is assumed to be a QAM-type of signal with duration $T = T_s$:

$$x(t) = \begin{cases} 0 & , \quad t < 0 \\ A\cos(\omega_c t) - B\sin(\omega_c t) = \sqrt{A^2+B^2}\cos(\omega_c t + \nu) & , \quad 0 \leq t \leq T_s \\ 0 & , \quad t > T_s \end{cases} \qquad (3.106)$$

3) $T_s > T_h$ ("low" signaling rate).

$$\tilde{x}(t) = \begin{cases} A + jB = \sqrt{A^2 + B^2}\ e^{j\nu} & , \quad 0 \le t \le T_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{3.108}$$

$\boldsymbol{T_h \le t \le T_s}$ **:**

$$z(t) = Re\left\{ e^{j\omega_c t} \int_0^{T_h} h(\tau)\sqrt{A^2 + B^2}\ e^{j\nu} e^{-j\omega_c \tau} d\tau \right\} =$$

$$= Re\{\sqrt{A^2 + B^2}\ e^{j\nu} \cdot H(f_c) e^{j\omega_c t}\} =$$

$$= |H(f_c)|\sqrt{A^2 + B^2} \cos(\omega_c t + \nu + \phi(f_c)) = A_z \cos(\omega_c t) - B_z \sin(\omega_c t) \tag{3.109}$$

Hence, a QAM-signal at the output in this time interval!

However, **attenuation and rotation** compared with the input! Compare with the input x(t) in (3.106)!

$$\begin{aligned} A_z + jB_z &= (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu + \phi(f_c))} = \\ &= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \end{aligned} \tag{3.110}$$

$$A_z + jB_z = (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu+\phi(f_c))} =$$
$$= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \quad (3.110)$$

A COMPACT MODEL WITH A COMPLEX CHANNEL PARAMETER!!

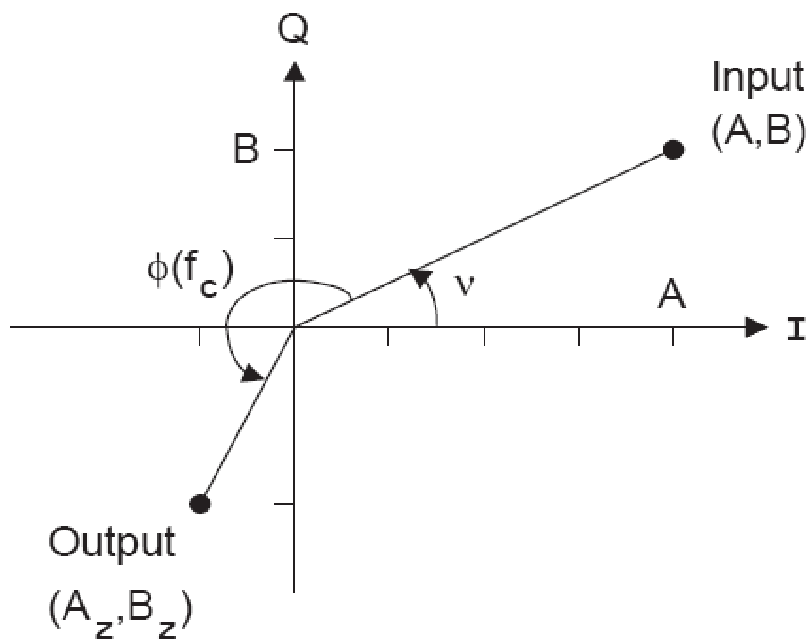


Figure 3.13: Illustrating that the input I-Q amplitudes (A,B) are scaled and rotated by the channel $H(f)$, see (3.109) and (3.110).

$$
z(t) = \begin{cases} 0 & , \quad t < 0 \\ \text{“non-stationary transient” starting interval} & , \quad 0 \le t \le T_h \\ |H(f_c)|\sqrt{A^2 + B^2} \ \cos(\omega_c t + \nu + \phi(f_c)) & , \quad T_h \le t \le T_s \\ \text{“non-stationary transient” ending interval} & , \quad T_s \le t \le T_s + T_h \\ 0 & , \quad t > T_s + T_h \end{cases}
$$

and within $T_h \le t \le T_s, \ A_z + jB_z = (A + jB)H(f_c)$

(3.111)

An important result here is that the input QAM signal $x(t)$ in (3.106) is changed to a new QAM signal by $|H(f_c)|$ and $\phi(f_c)$ in the interval $T_h \le t \le T_s$, see also Figure 3.13 and (3.110) how the I-Q components are changed. Furthermore, in OFDM applications the signaling rate $1/T_s$ is low such that $T_s \gg T_h$, and many QAM signals with different carrier frequencies are sent in parallel. ***Due to linearity, the result in (3.111) can be applied to each QAM signal in the OFDM signal by replacing f_c with f_n.*** In OFDM applications the receiver uses the time interval $\Delta_h \le t \le T_s$ for detection of the output QAM signals, and the duration of this observation interval is denoted $T_{obs} = T_s - \Delta_h$ (compare with (2.110) on page 51, and $T_h \le \Delta_h$).

So, the n:th QAM signal constellation in a sent OFDM signal is attenuated and rotated by H(fn) which is the value of the channel transfer function H(f) at the carrier frequency fn.

3.4.2 Group and Phase Delay

General narrowband input signal:

Let us approximate $H(f)$, around the carrier frequency f_c, with

$$H(f) \approx |H(f_c)|e^{j(\phi(f_c)+(f-f_c)\phi'(f_c))}, \quad {\scriptstyle f_c - W_{lp} \leq f \leq f_c + W_{lp}} \tag{3.113}$$

according to (3.112). Before doing this let us define the **group delay** $\boldsymbol{\tau_g}$ and the **phase delay** $\boldsymbol{\tau_\phi}$ as

$$\begin{aligned} \tau_g &= -\frac{1}{2\pi}\frac{d\phi(f)}{df}\bigg|_{f=f_c} && (3.117) \\ \tau_\phi &= -\frac{\phi(f_c)}{2\pi f_c} && (3.118) \end{aligned}$$

The final expression for the output signal $z(t)$ is then obtained as,

$$\boxed{\begin{aligned} z(t) &= |H(f_c)|\; x_I(t-\tau_g)\cos(2\pi f_c(t-\tau_\phi)) - \\ &\quad -|H(f_c)|\; x_Q(t-\tau_g)\sin(2\pi f_c(t-\tau_\phi)) = \\ &= |H(f_c)|\; e_x(t-\tau_g)\cos(\omega_c t + \theta_x(t-\tau_g) + \phi(f_c)) = \\ &= Re\{e_x(t-\tau_g)e^{j\theta_x(t-\tau_g)} \cdot H(f_c) \cdot e^{j\omega_c t}\} \end{aligned}} \tag{3.119}$$

where it is assumed that the input signal $x(t)$ is,

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) = \\ &= e_x(t)\cos(2\pi f_c t + \theta_x(t)) \end{aligned} \tag{3.120}$$

$$\boxed{\begin{aligned} z(t) &= |H(f_c)|\ x_I(t-\tau_g)\cos(2\pi f_c(t-\tau_\phi)) - \\ &\quad -|H(f_c)|\ x_Q(t-\tau_g)\sin(2\pi f_c(t-\tau_\phi)) = \\ &= |H(f_c)|\ e_x(t-\tau_g)\cos(\omega_c t + \theta_x(t-\tau_g) + \phi(f_c)) = \\ &= Re\{e_x(t-\tau_g)e^{j\theta_x(t-\tau_g)} \cdot H(f_c) \cdot e^{j\omega_c t}\} \end{aligned}} \tag{3.119}$$

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) = \\ &= e_x(t)\cos(2\pi f_c t + \theta_x(t)) \end{aligned} \tag{3.120}$$

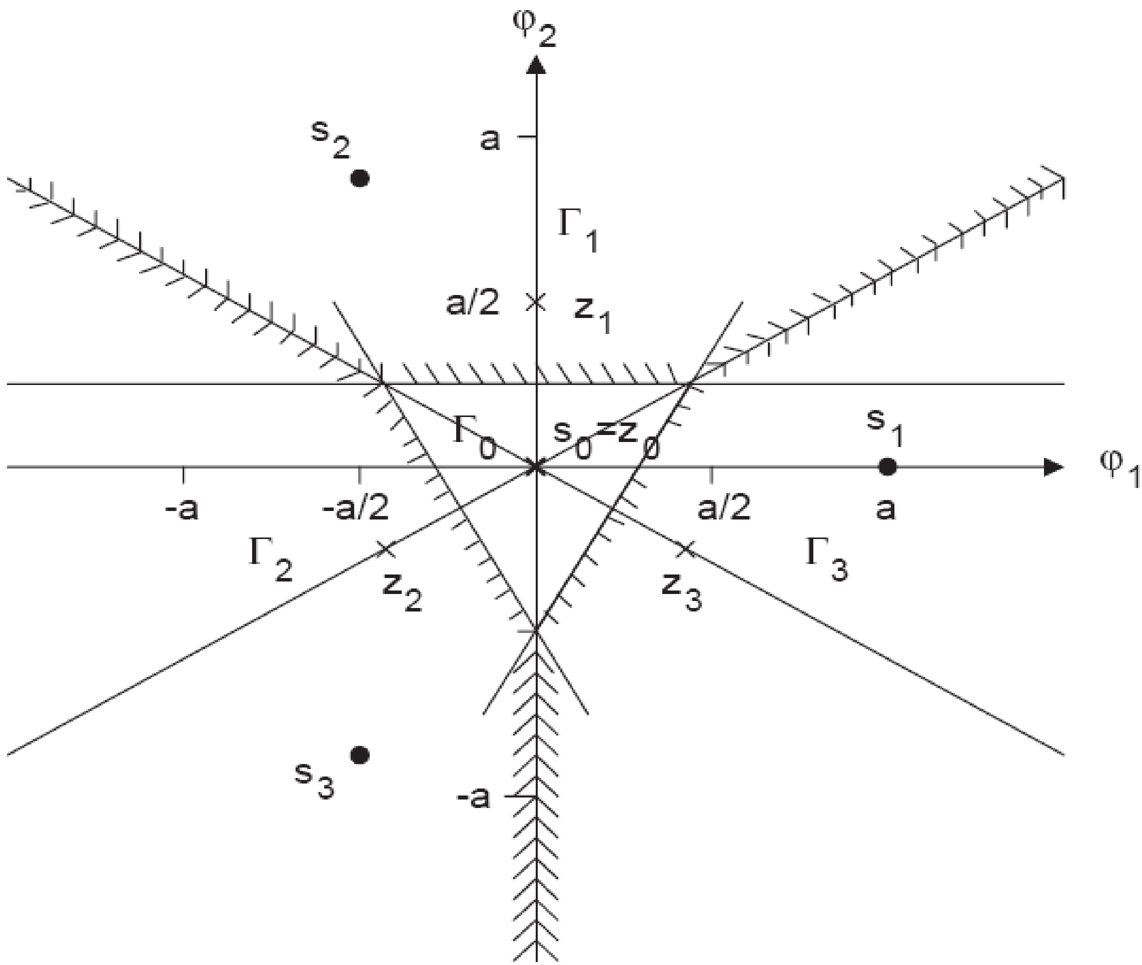
If $z(t)$ is expressed as,

$$z(t) = z_I(t)\cos(2\pi f_c t) - z_Q(t)\sin(2\pi f_c t) \tag{3.124}$$

then the quadrature components of $z(t)$ satisfy,

$$z_I(t) + jz_Q(t) = (x_I(t-\tau_g) + jx_Q(t-\tau_g))H(f_c) \tag{3.125}$$

EXAMPLE 3.17 EXAMPLE 5.4



3.4.3 N-Ray Channel Model

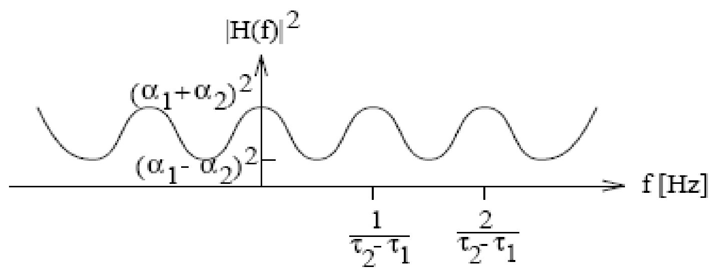
$$z(t) = x(t) * \underbrace{\left(\sum_{i=1}^{N} \alpha_i \delta(t - \tau_i) \right)}_{\text{Impulse response } h(t)} = \sum_{i=1}^{N} \alpha_i x(t - \tau_i) \qquad (3.126)$$

$$H(f) = \mathcal{F}\{h(t)\} = \sum_{i=1}^{N} \alpha_i e^{-j2\pi f \tau_i} \qquad (3.128)$$

So, **H(fc)** is easy to find!

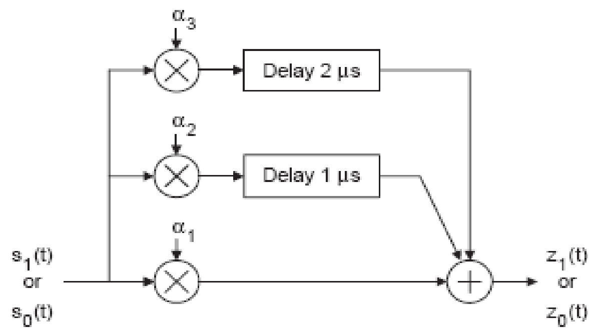
EXAMPLE 3.20

Rough sketch:



It is seen in this figure that the two signal paths add constructively or destructively (fading) depending on the frequency. Furthermore, if $\alpha_1 \approx \alpha_2$ then $|H(f)|$ is very close to zero at certain frequencies (so-called deep fades)!

EXAMPLE 3.19



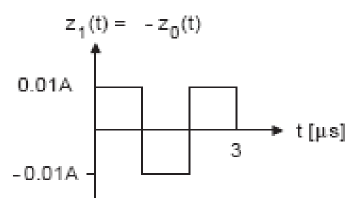
The signal $z_i(t) = s_i(t) * h(t)$ is the output signal corresponding to the input signal $s_i(t)$, $i = 0, 1$. Determine and sketch $z_0(t)$ and $z_1(t)$ if $\alpha_1 = 0.01$, $\alpha_2 = -0.01$, and $\alpha_3 = 0.01$.

Your conclusions concerning choice of bit rate to avoid overlapping signal alternatives after the channel?

Solution:

$$z_\ell(t) = \sum_{i=1}^{3} \alpha_i s_\ell(t - \tau_i) = 0.01 s_\ell(t) - 0.01 s_\ell(t - 10^{-6}) + 0.01 s_\ell(t - 2 \cdot 10^{-6}),\ \ \ell = 0, 1$$

yields,



Observe that the signal alternatives are changed significantly by the channel (filtering), and that the duration of both signal alternatives is increased from 1 [μs] before the channel, to 3[μs] after the channel!

If the bit rate is reduced to at most $10^6/3$ bps, *then no overlap of signal alternatives will exist after the channel.* □

3.5.1 Additive Interference

Assume that the information carrying bandpass signal $z(t)$ is corrupted by an additive bandpass signal $w(t)$, resulting in the bandpass signal $y(t)$, see Figure 3.16a,

$$y(t) = z(t) + w(t) \tag{3.129}$$

3.5.2 Multiplicative Interference

Now assume that the information carrying bandpass signal $z(t)$ is corrupted by a multiplicative baseband signal $w(t)$, resulting in the bandpass signal $y(t)$, see Figure 3.17,

$$y(t) = z(t)w(t) = z_I(t)w(t)\cos(2\pi f_c t) - z_Q(t)w(t)\sin(2\pi f_c t) \tag{3.138}$$

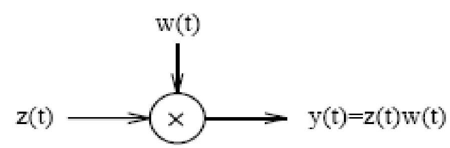


Figure 3.17: Multiplicative (baseband) interference $w(t)$.

If the interference $w(t)$ is small ($\ll 1$), then the signal $y(t)$ is heavily attenuated, and this is often referred to as **signal fading**.

5.34 Consider a communication system where N_t M-ary QAM signals are sent simultaneously (from N_t antennas). The n:th transmitted M-ary QAM signal is denoted $s_n(t)$,

$$s_n(t) = A(n)g(t)\cos(\omega_c t) - B(n)g(t)\sin(\omega_c t) \tag{5.133}$$

for $n = 1, 2, \ldots, N_t$. Note that the same carrier frequency is used for all N_t transmitted QAM signals!

The receiver is assumed to have N_r receiving antennas. The received signal $r_k(t)$ at the k:th receiving antenna is here modelled as

See (3.109)-(3.110)!

$$r_k(t) = \sum_{n=1}^{N_t} \left(\left[H_{k,n}^{Re} A(n) - H_{k,n}^{Im} B(n)\right] g(t)\cos(\omega_c t) - \right. \\ \left. - \left[H_{k,n}^{Re} B(n) + H_{k,n}^{Im} A(n)\right] g(t)\sin(\omega_c t)\right) + w_k(t) \tag{5.134}$$

for $k = 1, 2, \ldots, N_r$. The variables $H_{k,n}^{Re}$ and $H_{k,n}^{Im}$ models how the n:th transmitted QAM signal is received at the k:th receiving antenna (attenuation and rotation of the I-Q components).

After I and Q demodulation of $r_k(t)$ to baseband, the receiver obtains the noisy signal space coordinates, here collected in r_k as

$$r_k = \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re} A(n) - H_{k,n}^{Im} B(n)\right)}_{\text{received } I \text{ component}} + j \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re} B(n) + H_{k,n}^{Im} A(n)\right)}_{\text{received } Q \text{ component}} \\ + \underbrace{(w_k^{Re} + j w_k^{Im})}_{\text{due to AWGN}} \tag{5.135}$$

Note that complex notation ($j^2 = -1$) is used in (5.135)!

Let us now introduce the complex notations:

See (3.110)!

$$\begin{aligned} d_n &= A(n) + jB(n) \\ \alpha_{k,n} &= H_{k,n}^{Re} + jH_{k,n}^{Im} \\ w_k &= w_k^{Re} + j w_k^{Im} \end{aligned} \tag{5.136}$$

Then (5.135) can be formulated as,

$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k \quad , k = 1, 2, \ldots, N_r \tag{5.137}$$

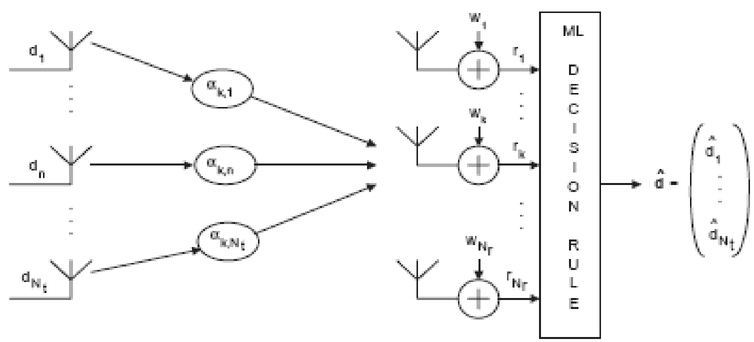
A compact formulation is now obtained as

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w} \tag{5.138}$$

where the $N_r \times N_t$ matrix $\boldsymbol{A}$ contains the channel coefficients $\{\alpha_{k,n}\}$. The relationship in (5.138) is a basic model in so-called multiple-input multiple-output (MIMO) systems.

The MIMO model is illustrated in the figure below,

MIMO MODEL



$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w}$$

64-QAM+Nt=8 (48bits): ML symbol decision rule ....?