

# Kursombud

Kontakta mig i pausen!

- Very fast development of high performing digital communication systems.
- The last 20 years has shown an impressive development, and the next 20 years will be even more dramatic!
- Modern communication systems requires state-of-the-art software and hardware technology and they are among the most advanced technical systems that we have today.

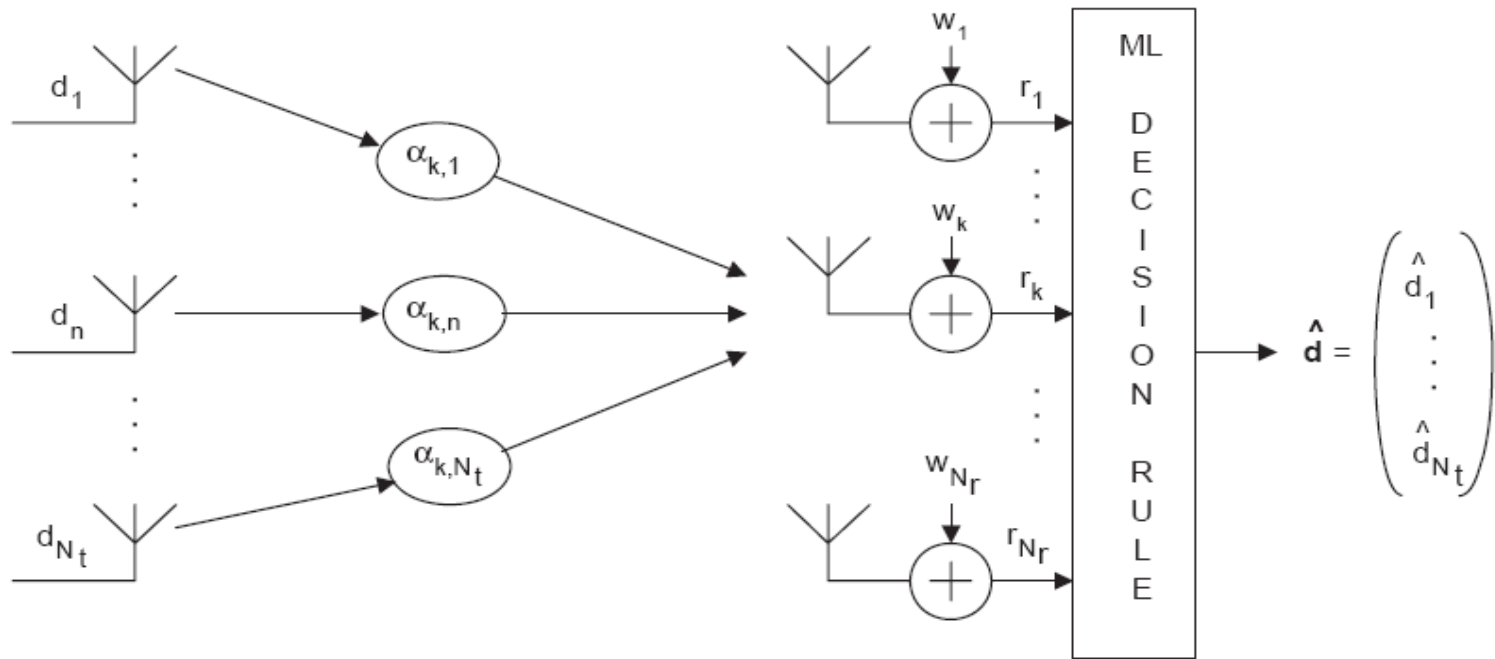
# Communication trends

- **Different users and applications require different bit rates and error protection (imply adaptive systems).**
- **To increase the performance the communication system should be able to adapt to the current quality of the communication link.  
The system should always work close to maximum performance!**
- **MIMO and OFDM are examples of advanced methods.**
- **The demand for higher and higher bitrates drives the development of more advanced and sophisticated communication system solutions.**

- **In this course we will study modern advanced digital communication methods and systems.**
- **Stationary as well as mobile communication system solutions.**
- **This course gives a breadth and a depth so that you can understand today's advanced communication system, and also many future systems.**

The MIMO model is illustrated in the figure below,

### MIMO MODEL



$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\mathbf{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \mathbf{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \mathbf{A}\mathbf{d} + \mathbf{w}$$

# Project work in this course

- 2 students/group.
- A communication application/technical problem/problem area, relevant for the course, is investigated.
- The choice of project is mainly done by the project group.
- Articles and conference papers from IEEE's database "IEEE Xplore"

<http://ieeexplore.ieee.org/Xplore/DynWel.jsp>

is recommended to get additional technical information.

- Written report, oral presentation and be opponent to another group.

# Some examples of applications/systems studied in previous projects:

- Mobile telephony (GSM, EDGE, 3G, 4G,...)
- Internet
- Modem (e.g., ADSL)
- WLAN (Wireless Local Area Network)
- Digital TV
- MIMO, future systems (4G, 5G,...)
- GPS (Global Positioning System)
- Bluetooth
- Home electronics (CD, DVD, remote controls, etc. )

## **A communication link:**

- **Different requirements on error protection.**
- **Different requirement on bit rate (bps) and bandwidth (Hz).**
- **Different qualities of the communication link.**
  
- **What are the technical challenges/problems?**
- **How do we solve these?**



# Course Programme

**Digital Communications, Advanced Course (ETT055), 9 hp, 091026 – 100305**

**First lecture:** Monday 26 October (week 44), 13.15 – 15.00 in E:2311.

**Project** starts in Week 48.

**Laboratory lesson:** LAB (4 hours) starts around 2 February 2010 (Week 5).

**Application** to the laboratory lesson is made on the homepage of this course where you book one available time-slot. Applications can be made one week before the lab starts.

**Messages** will be distributed on the homepage of this course, <http://www.eit.lth.se/kurs/ett055>

### **Written Examination:**

1:st opportunity: Tuesday 9 March 2010, 08.00 – 13.00 in Eden 25.

2:nd opportunity: Saturday 21 August 2010, 08.00 – 13.00, EIT-department.

3:rd opportunity: January 2011, EIT-department.

**Notice:** Application is required for the 2:nd and 3:rd opportunity. Applications are made on the homepage of this course.

**Course Literature:**

- “Introduction to Digital Communications”, compendium August 2006.
- Manual for the laboratory lesson.

The price for the compendium is 300 SEK and it can be bought at the Department of Electrical and Information Technology (EIT). Contact the student reception desk E:3152H. Note that only credit cards are accepted, i.e. no cash!

The manual for the laboratory lesson will be available at the homepage of this course, <http://www.eit.lth.se/kurs/ett055> (it is not available yet).

*You are allowed to use the compendium during the written examination.*

**Teacher:** Göran Lindell, Room E:2360, email: [Goran.Lindell@eit.lth.se](mailto:Goran.Lindell@eit.lth.se)

Lectures in study period Ht-2: Mondays 13.15 – 15.00 in E:2311.

Problem solving (exercise) class in study period Ht-2: Wednesdays 10.15 – 12.00 in E:3139.

### **TIME PLAN**

Week 44: Study period Ht-2 starts.

Week 48: Project info & start-up procedure.

Week 50 (091211): Ht-2 ends.

Week 3 (100118): Study period Vt-1 starts.

Week 5: Lab

Week 7: Deadline for project report (pdf-format, Email) Wednesday 17 February 2010, 15.00.

Week 8+9: Project presentations.

Week 9 (100305): Vt-1 ends.

Week 10: Written examination Tuesday 9 March 2010, 08.00 – 13.00 in Eden 25.

**Preliminary Course Outline for the course Digital Communications, Advanced Course (ETT055)**

**Study period Ht-2 2009 (26 October – 11 December):**

**Week   Contents**

- 44**    Lecture (26/10): Introduction. 5.1 – 5.1.3 (pages 329-347).  
Prob.solv. (28/10): Problems 5.1, 5.11, Example 5.2 on page 334, 5.6i, 5.9, 5.14.
- 45**    Lecture (2/11): 5.1.4 – 5.1.7 (pages 347 – 360), 5.2.1 – 5.3 (pages 360 – 380).  
Prob.solv. (4/11): 5.15a, 5.19, 5.16b, Example 5.4 on page 343, 5.13a, 5.20, 5.18a.
- 46**    Lecture (9/11): 5.4.1 (pages 380 – 392), Figure 5.26 on page 393,  
5.4.4 – 5.4.6 (pages 396 – 405), 8.1 – 8.2 (pages 501 - 512).  
Prob.solv. (11/11): 5.21, 5.23, Example 5.20 on page 373, Example 5.23 on page 384, 4.34i)ii),  
5.33, 5.34, 5.29, 5.30.

- 47 Lecture (16/11): 8.2 – 8.3 (pages 505 – 528).  
Prob.solv. (18/11): 8.1, 8.4, 8.6a,b,c,e, 8.7a,b,c,e, 2.32a,b, 8.8a, Example 8.4 on page 512.
- 48 Lecture (23/11): Project info and start-up procedure, 8.3 (pages 525 – 535).  
Prob.solv. (25/11): 8.25iii), 8.29, 8.10, 8.12, 8.24.
- 49 Lecture (30/11): 8.3 – 8.4 (pages 528 – 537, 548 - 551).  
Prob.solv. (2/12): Example 8.7 on page 516, 8.18a, 8.19a, 8.21, 8.19b.
- 50 Lecture (7/12): Summary of HT2.  
Prob.solv. (9/12): Example 8.16 on page 533, 8.33, 8.34, 8.31, 8.32.  
11/12 2009: Study period Ht-2 ends.

**Study period Vt-1 2010 (18 January – 5 March):**

**To be able to understand more advanced communication system solutions, e.g., systems where coding is used and/or mobile systems, we need to get more knowledge about uncoded systems!**

# Chapter 5

## Receivers in Digital Communication Systems - Part II

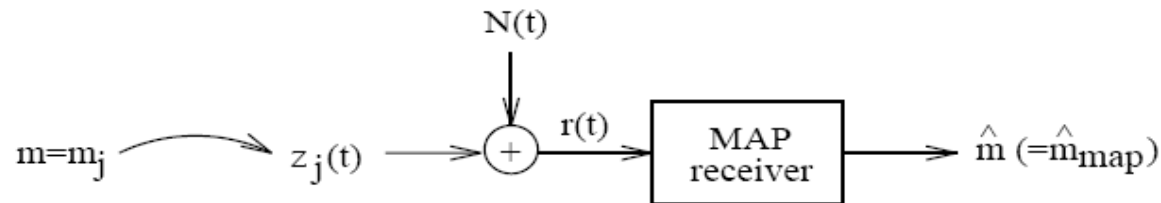


Figure 5.1: Reception of one of  $M$  possible waveforms  $\{z_\ell(t)\}_{\ell=0}^{M-1}$  in AWGN.

The MAP receiver minimizes the symbol error probability!

## Example of a **geometrical description** of M-ary QAM: (QAM – OFDM – MIMO)

### *“From signal waveforms to signal points”*

- The **signal space** concept is general and powerful.
- Increased insight and understanding.
- Improved analysis and implementations.
- We can understand more complicated systems.

$$\boxed{s_\ell(t) = A_\ell g(t) \cos(2\pi f_c t) - B_\ell g(t) \sin(2\pi f_c t)} \quad \ell = 0, 1, \dots, M - 1 \quad (2.87)$$

$$s_\ell(t) = \underbrace{A_\ell \sqrt{E_g/2}}_{s_{\ell,1}} \phi_1(t) + \underbrace{B_\ell \sqrt{E_g/2}}_{s_{\ell,2}} \phi_2(t) \quad (2.99)$$

$$\phi_1(t) = \frac{g(t) \cos(2\pi f_c t)}{\sqrt{E_g/2}} \quad (2.100)$$

$$\phi_2(t) = -\frac{g(t) \sin(2\pi f_c t)}{\sqrt{E_g/2}} \quad (2.101)$$



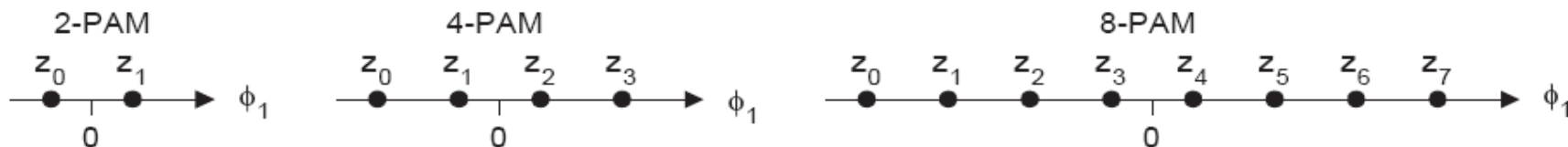
$$z_j(t) = \sum_{\ell=1}^N z_{j,\ell} \phi_{\ell}(t) = z_{j,1} \phi_1(t) + z_{j,2} \phi_2(t) + \dots + z_{j,N} \phi_N(t)$$

(5.1)

$$\int_0^{T_s} \phi_i(t) \phi_j(t) dt = \begin{cases} 1 & , \quad i = j \\ 0 & , \quad i \neq j \end{cases} \quad i, j = 1, 2, \dots, N \quad (5.2)$$

$$z_j(t) \iff \mathbf{z}_j = (z_{j,1}, z_{j,2}, \dots, z_{j,N})^{tr}, \quad j = 0, 1, \dots, M-1 \quad (5.3)$$

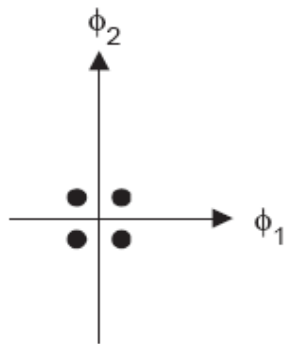
$$s_{\ell}(t) = A_{\ell} g(t) = A_{\ell} \sqrt{E_g} \cdot \underbrace{\frac{g(t)}{\sqrt{E_g}}}_{\phi_1(t)} = \underbrace{A_{\ell} \sqrt{E_g}}_{s_{\ell,1}} \cdot \phi_1(t) = s_{\ell,1} \cdot \phi_1(t) \quad (2.51)$$



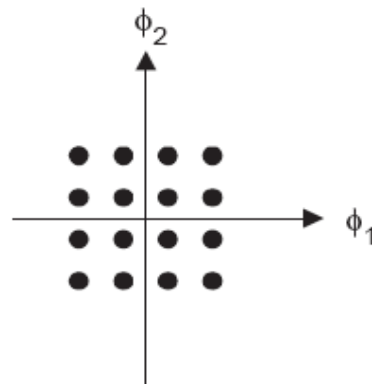
As examples, let us collect some results from subsection 2.4:

$$\begin{aligned}
 \text{M-ary PAM:} \quad & z_j = ((-M + 1 + 2j)\sqrt{E_g}), & N = 1 \\
 \text{M-ary PSK:} \quad & z_j = \left( \cos(\nu_j)\sqrt{\frac{E_g}{2}}, \sin(\nu_j)\sqrt{\frac{E_g}{2}} \right)^{tr}, & N = 2 \\
 \text{M-ary FSK:} \quad & z_j = (0, 0, \dots, \sqrt{E_j}, 0, 0, 0)^{tr}, & N = M \\
 \text{M-ary QAM:} \quad & z_j = \left( A_j\sqrt{\frac{E_g}{2}}, B_j\sqrt{\frac{E_g}{2}} \right)^{tr}, & N = 2
 \end{aligned} \tag{5.4}$$

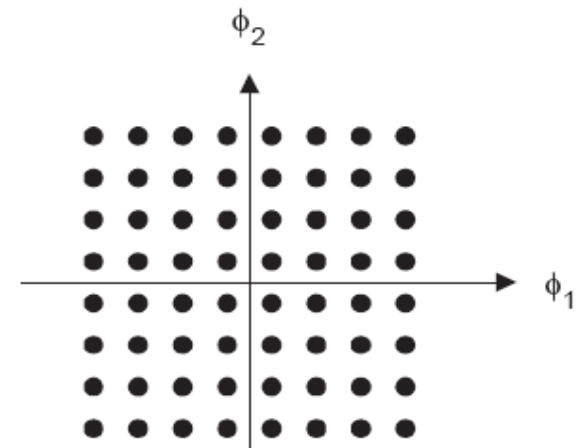
4-QAM



16-QAM



64-QAM



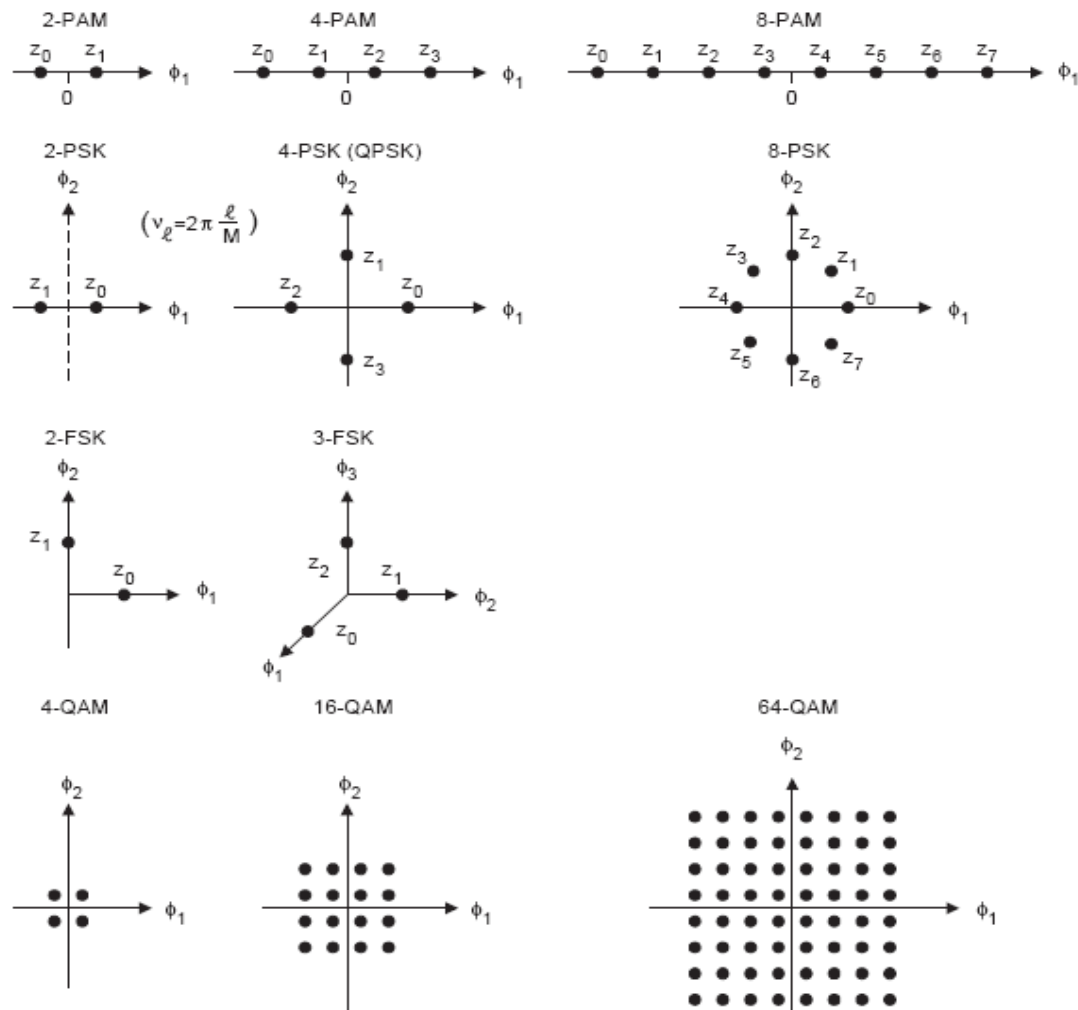


Figure 5.2: Examples of M-ary PAM, M-ary PSK, M-ary FSK and M-ary QAM signal constellations in signal space. See also the corresponding subsections in Chapter 2.

$$\begin{aligned}
 E_j &= \int_0^{T_s} z_j^2(t) dt = \sum_{\ell=1}^N z_{j,\ell}^2 = z_j^{tr} z_j \\
 D_{i,j}^2 &= \int_0^{T_s} (z_i(t) - z_j(t))^2 dt = \sum_{\ell=1}^N (z_{i,\ell} - z_{j,\ell})^2 = \quad , \quad i,j=0,1,\dots,M-1 \\
 &= E_i + E_j - 2z_i^{tr} z_j
 \end{aligned}$$

(5.8)

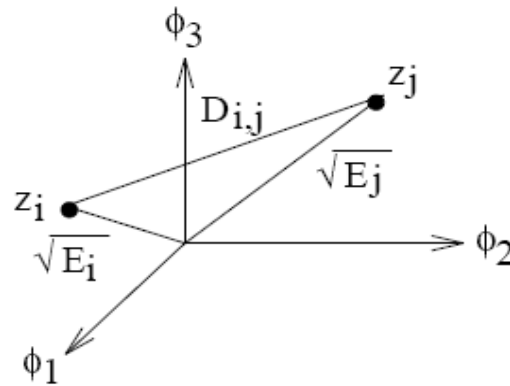
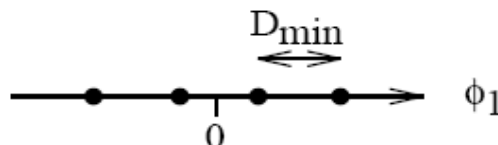


Figure 5.3: Illustrating  $E_\ell$  and  $D_{i,j}$  in signal space.

## EXAMPLE 5.1

The signal space description of 4-ary PAM is given below.



Assume equally likely signal alternatives. Calculate  $d_{\min}^2$ ,  $d_{\min}^2 = D_{\min}^2/2\mathcal{E}_b$ , where  $\mathcal{E}_b$  is (as usual) the average received energy per information bit.

*Solution:*

From the figure we have,  $E_0 = (-3D_{\min}/2)^2$ ,  $E_1 = (-D_{\min}/2)^2$ ,  $E_2 = E_1$ ,  $E_3 = E_0$ .

$$\mathcal{E}_b = \frac{1}{k} \sum_{i=1}^4 P_i E_i = \frac{1}{2} \cdot \frac{1}{4} \left( 2 \cdot \left( \frac{D_{\min}}{2} \right)^2 + 2 \cdot \left( \frac{3D_{\min}}{2} \right)^2 \right) = \frac{5}{8} D_{\min}^2$$

So,

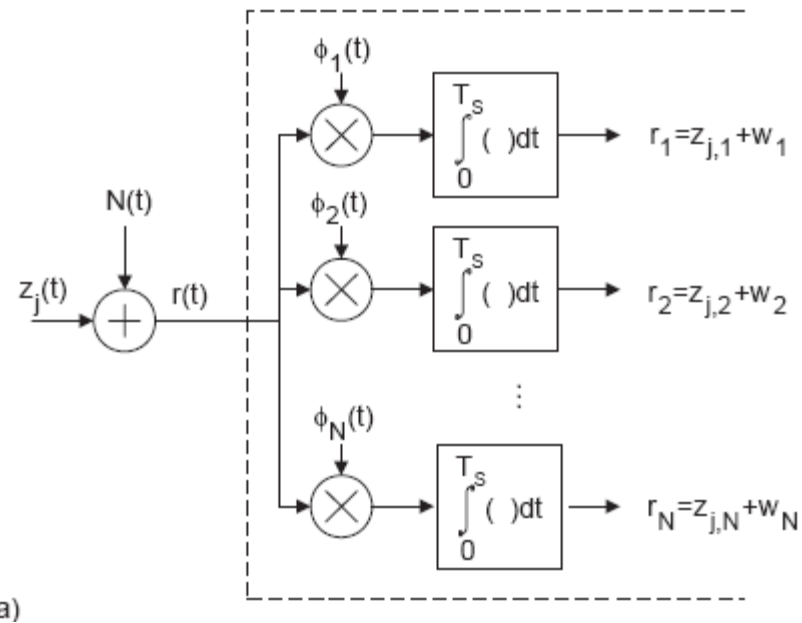
$$d_{\min}^2 = \frac{D_{\min}^2}{2\mathcal{E}_b} = \frac{4}{5}$$

which corresponds to a 3.98 [dB] loss in energy efficiency compared to binary antipodal signaling (or QPSK) for which  $d_{\min}^2 = 2$ .  $\square$

The receiver "looks" in the different dimensions because it is there the information is located!

"look" = scalar product calculation = correlation

$$\int_0^{T_s} z_j(t) \phi_\ell(t) dt = \int_0^{T_s} \sum_{n=1}^N z_{j,n} \phi_n(t) \phi_\ell(t) dt = \sum_{n=1}^N z_{j,n} \int_0^{T_s} \phi_n(t) \phi_\ell(t) dt = z_{j,\ell} \quad (5.12)$$



a)

Figure 5.6: a) The first step in the MAP receiver;

$$\int_0^{T_s} z_j(t) \phi_\ell(t) dt = \int_0^{T_s} \sum_{n=1}^N z_{j,n} \phi_n(t) \phi_\ell(t) dt = \sum_{n=1}^N z_{j,n} \int_0^{T_s} \phi_n(t) \phi_\ell(t) dt = z_{j,\ell} \quad (5.12)$$

After the correlators we obtain a **received noisy signalpoint**  $r$ !

$$\boxed{\begin{aligned} E\{w_\ell\} &= 0 \\ \sigma_\ell^2 &= E\{w_\ell^2\} = N_0/2 \\ E\{w_\ell w_m\} &= 0, \quad \ell \neq m \end{aligned}} \quad \ell = 1, 2, \dots, N \quad (5.22)$$

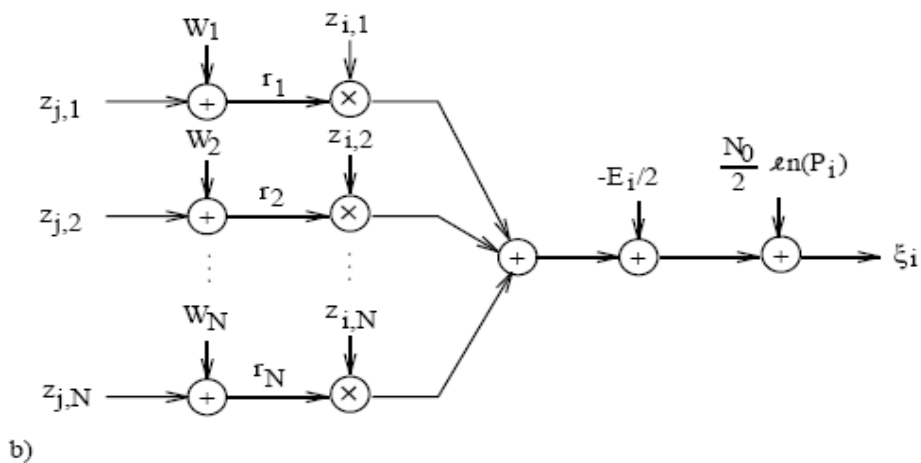
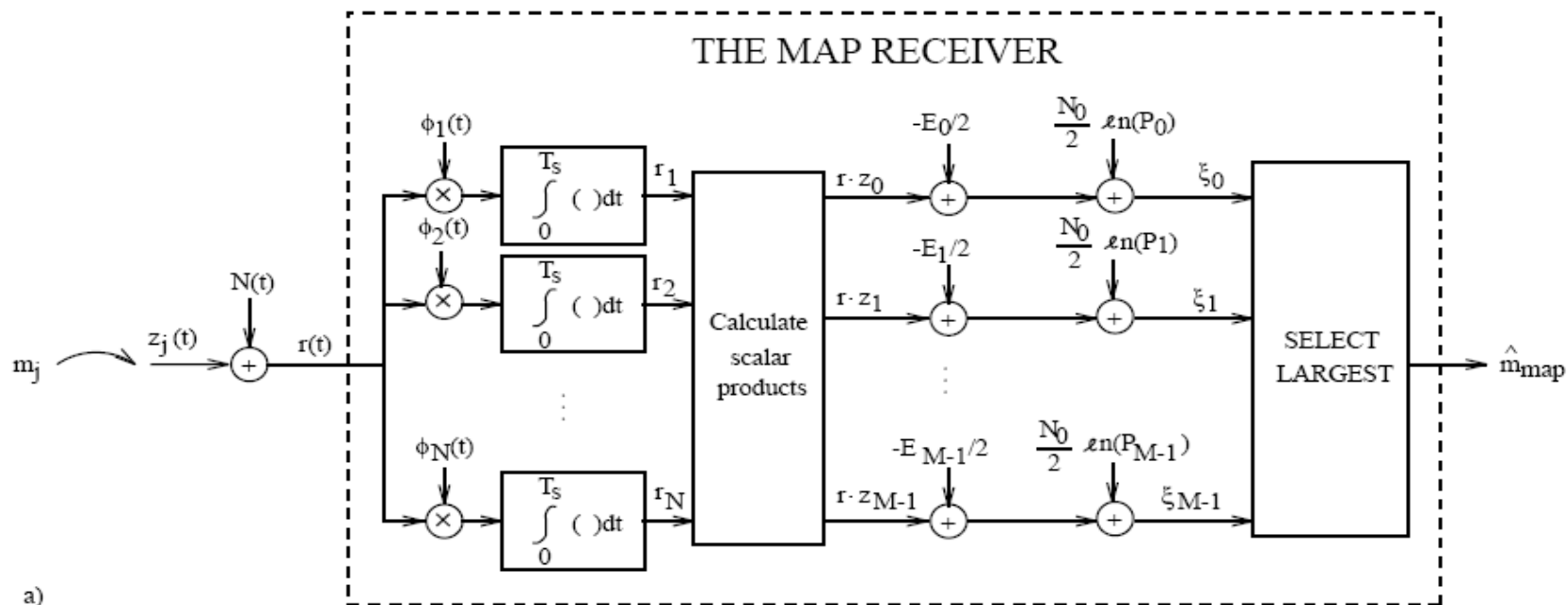


Figure 5.8: a) The MAP receiver; b) A discrete-time model of the decision variable  $\xi_i$ .



$$\boxed{\mathbf{r} = \mathbf{z}_j + \mathbf{w}} \quad (5.18)$$

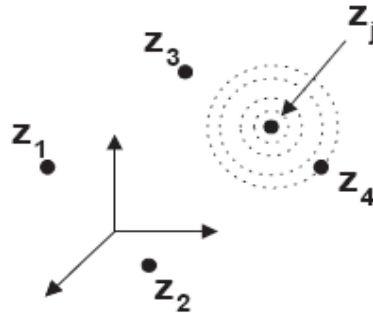


Figure 5.7: Illustrating “the cloud” of noise in  $\mathbf{r}$  if message  $m_j$  is sent.

The distance between the received noisy signal point  $\mathbf{r}$  and the signal point  $\mathbf{z}_j$  is:

$$\boxed{D_{r,j}^2 = (\mathbf{r} - \mathbf{z}_j)^{tr} (\mathbf{r} - \mathbf{z}_j) = \sum_{\ell=1}^N (r_\ell - z_{j,\ell})^2} \quad (5.24)$$

MAP decision rule:

$$\hat{m}(\mathbf{r}) = m_\ell \Leftrightarrow \min_{\{i\}} \{D_{r,i}^2 - N_0 \ln(P_i)\} = D_{r,\ell}^2 - N_0 \ln(P_\ell) \quad (5.25)$$

$$\Downarrow$$

$$\max_{\{i\}} \{\mathbf{r}^{tr} \mathbf{z}_i + c_i\} = \mathbf{r}^{tr} \mathbf{z}_\ell + c_\ell$$

ML decision rule = minimum distance decision rule:

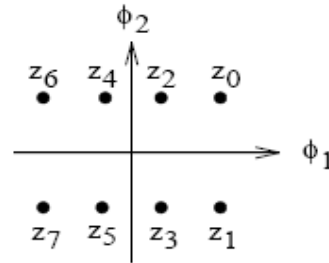
In the MAP decision rule (5.25)–(5.26) we observe that if  $P_i = 1/M$ , then the terms  $N_0 \ln(P_\ell)$  can be ignored, resulting in the decision rule

$$\hat{m}(r) = m_\ell \Leftrightarrow \min_{\{i\}} D_{r,i}^2 = D_{r,\ell}^2 \quad (5.28)$$

Hence, *if  $P_i = 1/M$ , then the ML decision rule is obtained as the minimum Euclidean distance decision rule.* Observe also in (5.25) that

### EXAMPLE 5.3

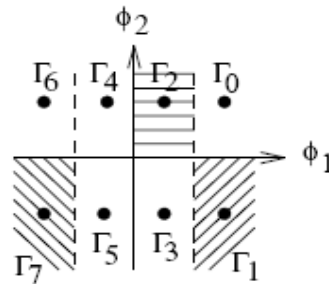
Assume that eight equally likely signal alternatives  $\{z_\ell(t)\}_{\ell=0}^7$  are used, and that  $N = 2$ . The possible noiseless values of the received vector  $\mathbf{r} = (r_1, r_2)^{tr}$  in Figure 5.8a are shown below.



Construct the decision regions used by the MAP-receiver.

#### Solution:

Since the MAP-receiver in this case is identical with the ML receiver, only the Euclidean distances  $D_{r,0}, D_{r,1}, \dots, D_{r,7}$  are used in the decision process. The decision boundaries are therefore drawn exactly in the middle between the different signal points (the ML receiver chooses the signal alternative (message) that is closest to the received noisy signal point  $\mathbf{r}$ ). The result is,



□

### 5.1.3 The Symbol Error Probability for M-ary PAM

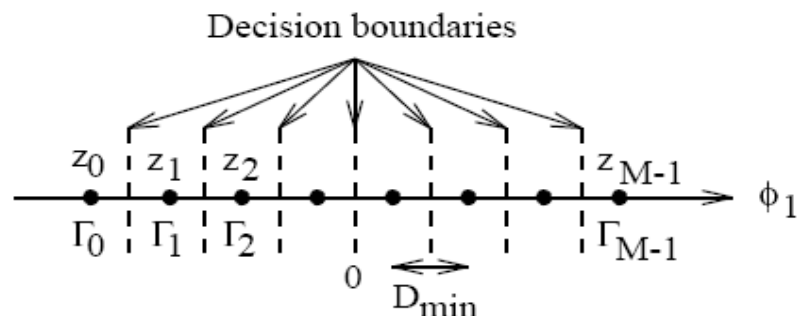


Figure 5.9: The signal space for M-ary PAM with equispaced amplitudes, centered symmetrically around zero (see (5.4)).

$$\begin{aligned}
 \text{Prob}\{\text{error}|m_0 \text{ sent}\} &= \text{Prob}\left\{w_1 > \frac{D_{\min}}{2}\right\} = \\
 &= \text{Prob}\left\{\frac{w_1}{\sqrt{N_0/2}} > \frac{D_{\min}}{\sqrt{2N_0}}\right\} = Q\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right) \quad (5.31)
 \end{aligned}$$

$$\begin{aligned}
\text{Prob}\{\text{error}|m_1 \text{ sent}\} &= \text{Prob}\left\{w_1 < -\frac{D_{\min}}{2} \text{ or } w_1 > \frac{D_{\min}}{2}\right\} = \\
&= \text{Prob}\left\{\frac{w_1}{\sqrt{N_0/2}} < -\frac{D_{\min}}{\sqrt{2N_0}}\right\} + \text{Prob}\left\{\frac{w_1}{\sqrt{N_0/2}} > \frac{D_{\min}}{\sqrt{2N_0}}\right\} = \\
&= 2Q\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right)
\end{aligned} \tag{5.32}$$

$$P_s = \sum_{j=0}^{M-1} P_j \text{Prob}\{\text{error}|m_j \text{ sent}\}$$

$$\boxed{P_s = \frac{2}{M} (M-1)Q\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right)}, \quad \text{M-ary PAM} \tag{5.35}$$

$P_s$  is shown in Figure 5.13 on page 362.

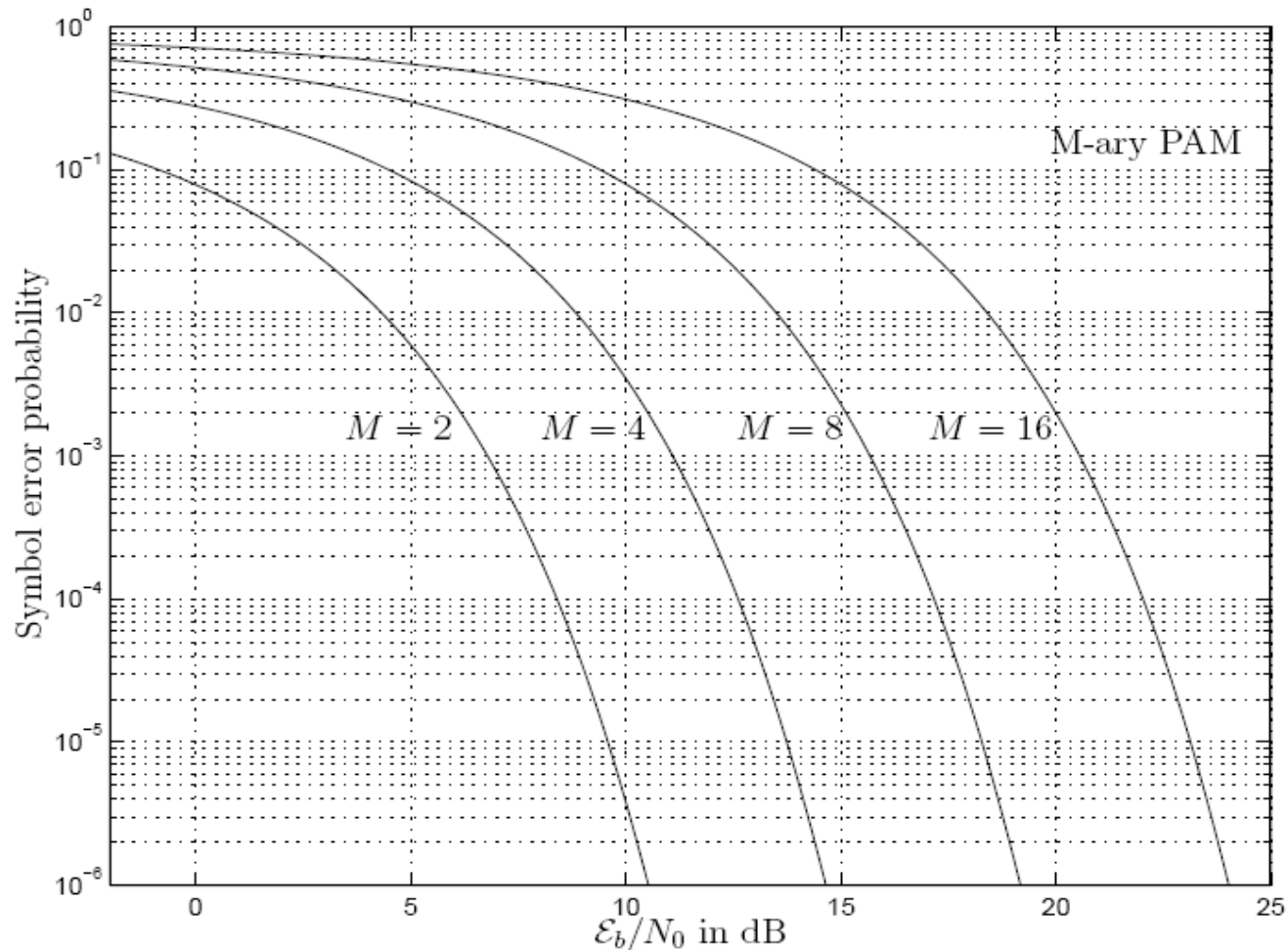
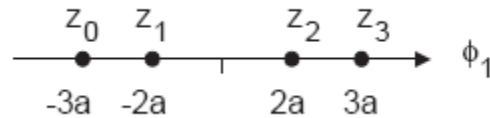


Figure 5.13: The symbol error probability for M-ary PAM,  $M = 2, 4, 8, 16$ , see Table 5.1. The specific assumptions are given in Subsection 2.4.1.1, and in Subsection 5.1.3.



$$5.14 \Pr\{\text{error}|m_0 \text{ sent}\} = \Pr\{w_1 > a/2\} =$$

$$= \Pr\left\{\frac{w_1}{\sqrt{N_0/2}} > \frac{a/2}{\sqrt{N_0/2}}\right\} = Q\left(\sqrt{\frac{a^2}{2N_0}}\right)$$

$$\Pr\{\text{error}|m_1 \text{ sent}\} = \Pr\{w_1 < -a/2 \text{ or } w_1 > 2a\}.$$

So, we obtain

$$P_s = \sum_{j=0}^3 P_j \Pr\{\text{error}|m_j \text{ sent}\} = Q\left(\sqrt{\frac{a^2}{2N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{8a^2}{N_0}}\right)$$