Final exam in

Digital Communications, Advanced Course (ETT055)

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on March 11, 2009, 08–13.

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.

  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.

  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

Good Luck!

Problem 1: Determine for each of the five statements below if it is true or false.
Observe! As usual, motivations to your answers should be given.

a) " The communication system in Figure 8.4a in the compendium has the same bandwidth efficiency as uncoded 8-ary PSK."

b) "Continuous phase modulated (CPM) signals carries information in the envelope if the modulation index $h = 0.85$."

c) "The decision regions for 16-ary PSK are quite sensitive to a random signal attenuation."

d) "Uncoded schemes can be described as schemes having only one state."

e) "The principle of water-filling is very hard to implement for channels that changes fast with time."

(10 points)

Problem 2: Consider the system given in Figure 8.6a in the compendium (rate 2/3 encoder and 8-ary PSK).
A possible sequence of sent signal alternatives is given by: ..., $s_2, s_1, s_2, s_2, x, y, s_3, s_1, s_2, \ldots$.

Determine possible signal alternatives x and y, and also the corresponding path in the trellis.

What could be said about x and y if we instead considered an uncoded 8-ary PSK system?
(10 points)

Problem 3: Assume that the received signal alternatives $\{z_\ell(t)\}_{\ell=0}^{M-1}$ are equally likely, and that they can be described by a two-dimensional signal space. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML symbol receiver is used.

Below, the 4-tuple of information bits carried by each message is given. Furthermore, each message point is also specified in the two-dimensional signal space. The parameter a below is $a > 0$, and the parameter x below is restricted to be in the interval $0 < x < a$.

$z_0(t)$: $(b_1b_2b_3b_4) = (0010)$, $z_{0,1} = -2a - x$, $z_{0,2} = 2a + x$
$z_1(t)$: $(b_1b_2b_3b_4) = (0110)$, $z_{1,1} = -2a + x$, $z_{1,2} = 2a + x$
$z_2(t)$: $(b_1b_2b_3b_4) = (1110)$, $z_{2,1} = 2a - x$, $z_{2,2} = 2a + x$
$z_3(t)$: $(b_1b_2b_3b_4) = (1010)$, $z_{3,1} = 2a + x$, $z_{3,2} = 2a + x$

$z_4(t)$: $(b_1b_2b_3b_4) = (0011)$, $z_{4,1} = -2a - x$, $z_{4,2} = 2a - x$
$z_5(t)$: $(b_1b_2b_3b_4) = (0111)$, $z_{5,1} = -2a + x$, $z_{5,2} = 2a - x$
$z_6(t)$: $(b_1b_2b_3b_4) = (1111)$, $z_{6,1} = 2a - x$, $z_{6,2} = 2a - x$
$z_7(t)$: $(b_1b_2b_3b_4) = (1011)$, $z_{7,1} = 2a + x$, $z_{7,2} = 2a - x$

$z_8(t)$: $(b_1b_2b_3b_4) = (0001)$, $z_{8,1} = -2a - x$, $z_{8,2} = -2a + x$
$z_9(t)$: $(b_1b_2b_3b_4) = (0101)$, $z_{9,1} = -2a + x$, $z_{9,2} = -2a + x$
$z_{10}(t)$: $(b_1b_2b_3b_4) = (1101)$, $z_{10,1} = 2a - x$, $z_{10,2} = -2a + x$
$z_{11}(t)$: $(b_1b_2b_3b_4) = (1001)$, $z_{11,1} = 2a + x$, $z_{11,2} = -2a + x$

$z_{12}(t)$: $(b_1b_2b_3b_4) = (0000)$, $z_{12,1} = -2a - x$, $z_{12,2} = -2a - x$
$z_{13}(t)$: $(b_1b_2b_3b_4) = (0100)$, $z_{13,1} = -2a + x$, $z_{13,2} = -2a - x$
$z_{14}(t)$: $(b_1b_2b_3b_4) = (1100)$, $z_{14,1} = 2a - x$, $z_{14,2} = -2a - x$
$z_{15}(t)$: $(b_1b_2b_3b_4) = (1000)$, $z_{15,1} = 2a + x$, $z_{15,2} = -2a - x$

i) Sketch the decision region for message 5, and for message 11, respectively.

ii) Determine d^2_{min} for the case $x = a/2$.

iii) The received noisy signal point $\mathbf{r}$ has the coordinates r_1 and r_2. Below, two cases are investigated:

Consider a situation when the ML symbol receiver decides that the information bit b_3 is equal to 0. For which values of r_1 and r_2 will this happen?

Consider a situation when the ML symbol receiver decides that the information bit b_4 is equal to 1. For which values of r_1 and r_2 will this happen?

What are your conclusions concerning how to implement the decision making process of the ML symbol receiver?

(10 points)

Problem 4: In this problem binary communication with equally likely antipodal signal alternatives are considered. The average transmitted signal energy per bit is denoted $E_{b,sent}$, and it is equally distributed over L frequency intervals (dimensions) in such a way that the received message points are (after L correlators):

$\mathbf{z}_1 = -\mathbf{z}_0 = (a_1\sqrt{E_{b,sent}/L}, a_2\sqrt{E_{b,sent}/L}, ..., a_L\sqrt{E_{b,sent}/L})$

Above, the channel parameter a_k represents the influence of the k:th frequency interval. It is assumed that all channel parameters are independent random variables.

It is furthermore assumed that the communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and that the ML symbol receiver is used.

Here we consider a communication link where each channel parameter a_k takes the value 0 with probabilty P_0, and the value 1 with probability $(1 - P_0)$.

With such a random channel the bit error probability P_b equals the expected value of $P_b(\mathbf{a})$, i.e. $P_b = E\{P_b(\mathbf{a})\}$, where $\mathbf{a}$ denotes the vector of channel parameters. $P_b(\mathbf{a})$ denotes the bit error probability for a given vector $\mathbf{a}$ of channel parameters.

It is given that for the deterministic special case with $L = 1$ and $a_1 = 1$ the bit error probability equals $P_b(a_1 = 1) = 10^{-6}$.

a) For the system described above an upper bound on P_b can be obtained as:

$P_b = E\{P_b(\mathbf{a})\} \leq 0.5(e^{-11.3/L} + (1 - e^{-11.3/L})P_0)^L$

i) Evaluate numerically the upper bound for the cases $L = 1$, $L = 8$, $L = 16$ if $P_0 = 0.25$.

ii) Use these results and sketch the upper bound versus P_0, $0 \leq P_0 \leq 1$, for the three cases $L = 1$, $L = 8$ and $L = 16$.

iii) What are your conclusions concerning this communication method? Do you recommend it?

b) Calculate $P_b = E\{P_b(\mathbf{a})\}$ exactly for the two cases $L = 1$ and $L = 2$ if $P_0 = 0.25$.

(10 points)

Problem 5:

a) i) What is the goal of the Viterbi algorithm (VA)?

ii) Explain in detail how the VA works, i.e. how it achieves its goal.

iii) Explain advantages and disadvantages with the VA.

b) Consider a MIMO communication system with $N_t = 2$ transmitting antennas and $N_r = 8$ receiving antennas.

Each transmitting antenna uses a conventional QPSK signal constellation (as is illustrated in Subsection 5.1.4), and the two constellations are identical.

The signals from the two antennas are sent during the same time interval.

Here we consider the uncoded case which means that all input bits to the two antennas are independent (and equally likely).

Let us here investigate the situation at the receiver antenna number 5.

The QPSK signal that is sent from transmitting antenna number 1 is attenuated by the channel parameter a when it reaches receiver antenna number 5.

The QPSK signal that is sent from transmitting antenna number 2 is attenuated by the channel parameter b when it reaches receiver antenna number 5.

The receiver at receiving antenna number 5 is equipped with two correlators (which is standard for a QPSK receiver).

We here consider the case when $b = a/4$ and $a > 0$. Hence, no rotation is assumed.

Determine and sketch the location of the message points that appear in the signal space at receiving antenna number 5.

(10 points)
